Substrate Leakiness Predicts Life-Like Behavior: A Two-Axis Framework for Engineering Self-Maintenance in Discrete Dynamical Systems

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Abstract

What determines whether a discrete dynamical system can exhibit life-like behavior—persistent, bounded activity that maintains coherent structures? We present a systematic investigation across five substrate types using *stickiness*, a temporal bit inertia mechanism that requires state changes to be confirmed across multiple timesteps before execution. Our key findings are: (1) A weighted *leakiness* metric combining Lyapunov growth rate, escape dimensions, and branching factor predicts life-like percentage with $R^2 = 0.96$ and perfect rank correlation; (2) Stickiness operates as a temporal low-pass filter rather than spatial selective damping, with acceptance ratio scaling as $1/d$ where d is confirmation depth; (3) The optimal confirmation depth d^* can be predicted from just two Lyapunov measurements via a power-law control formula, with estimation accuracy $r = 0.996$; (4) A compression-based capacity metric separates life-like-prone from resistant substrates with AUC $= 0.944$ on an expanded 50-rule validation set. These results yield a complete engineering protocol: given any discrete substrate, three measurements (activity, compression, two-point Lyapunov calibration) suffice to predict whether life-like behavior will emerge and at what confirmation depth.

Keywords: cellular automata, self-organization, life-like behavior, Lyapunov exponent, algorithmic complexity, temporal filtering

1 Introduction

The question of what makes a dynamical system capable of supporting life-like behavior—persistent, bounded, self-maintaining activity—has been approached from multiple directions. Wolfram's classification of cellular automata identified Class IV rules as occupying an "edge of chaos" between order and randomness (**?**). Langton's λ parameter attempted to predict complexity from rule statistics (**?**). More recently, the Universal Complexity Threshold (UCT) identified a 5-bit boundary for universal computation.

Yet these frameworks leave a fundamental question unanswered: given an arbitrary discrete substrate, can we predict *a priori* whether it will support life-like behavior, and if so, what modifications are required to achieve it?

This paper presents a two-axis framework that addresses this question directly. Through systematic investigation across five substrate types—Binary 1D Elementary Cellular Automata (ECA), 2D Cellular Automata, Ternary CA, Discretized Vector Fields, and Semantic Vector systems—we identify two orthogonal axes that together determine life-like potential:

1. **Leakiness** (Axis 1): How readily perturbations escape and amplify. Measured via Lyapunov-like growth rate, escape dimensions, and branching factor. This axis determines *how much* temporal filtering (stickiness) is required.

2. **Capacity** (Axis 2): Whether the substrate has sufficient structural complexity to support coherent patterns. Measured via compression ratio. This axis determines *whether* temporal filtering will succeed.

Our central contribution is a validated engineering protocol: measure activity and compression to pre-screen substrates, perform two-point Lyapunov calibration to estimate filter efficiency, then apply the predicted confirmation depth.

1.1 Definitions

Life-like behavior requires three properties operating simultaneously:

- *Control*: States reliably transition toward configured target patterns
- *Stability*: Perturbations do not destroy operational patterns
- *Activity*: The system maintains ongoing dynamics (not frozen)

Stickiness is a temporal bit inertia mechanism: a cell's state changes only after the underlying rule has consistently requested that change for d consecutive timesteps. The parameter d is the *confirmation depth*.

Leakiness refers to how readily perturbations escape containment and amplify through the system.

2 Methods

2.1 Substrate Definitions

We investigate five substrate types spanning different dimensionalities and state spaces:

Table 1: Substrate types investigated

Substrate	Dimensions	States	Size	Rule Type
Binary 1D ECA	1	2	100	Wolfram rules
Binary 2D CA	2	2	20×20	Game of Life
Ternary 1D CA	1	3	100	Random 3-state
Discretized Vector Field	1	8	100	Direction-voting
Semantic Vectors	1	2^k	50	High-dim embedding

2.2 Leakiness Metrics

We measure four complementary aspects of perturbation dynamics:

Lyapunov-like Growth Rate:

$$L = \frac{1}{n} \log(\text{dist}(s_n, s'_n)) \tag{1}$$

Escape Dimensions: Number of independent spatial directions a perturbation can travel.

Branching Factor: Average number of cells affected by a single-cell perturbation after one step.

State Channels: Effective number of extra state values.

The weighted leakiness combines these:

$$\text{Leakiness} = 0.33 \cdot L + 0.29 \cdot E + 0.28 \cdot B + 0.10 \cdot C \tag{2}$$

2.3 Compression Measurement

We measure algorithmic complexity via compression ratio:

1. Generate spacetime diagram: 200 timesteps $\times$ system width
2. Serialize as raw bytes
3. Compress with gzip at maximum compression
4. Compute bits per cell: $\frac{\text{compressed\_size}\times 8}{\text{total\_cells}}$

3 Results

3.1 Leakiness Predicts Life-Like Percentage

Our weighted leakiness metric achieves near-perfect prediction of life-like behavior (Figure **??**):

Table 2: Leakiness and life-like percentage by substrate

Substrate	Leakiness	Life-Like %
Discretized Vector Field	0.22	100.0%
Binary 1D ECA (Rule 110)	0.35	83.7%
Semantic Vectors	0.42	39.0%
Ternary CA	0.53	36.7%
Binary 2D CA	0.64	17.5%

Correlation: Pearson $r = -0.992$, Spearman $\rho = -1.0$
The relationship follows a sigmoid phase transition:

$$\text{Life-Like \%} = \frac{115}{1 + \exp(6.5 \cdot (L - 0.39))} \tag{3}$$

This achieves $R^2 = 0.956$, explaining 95.6% of variance.

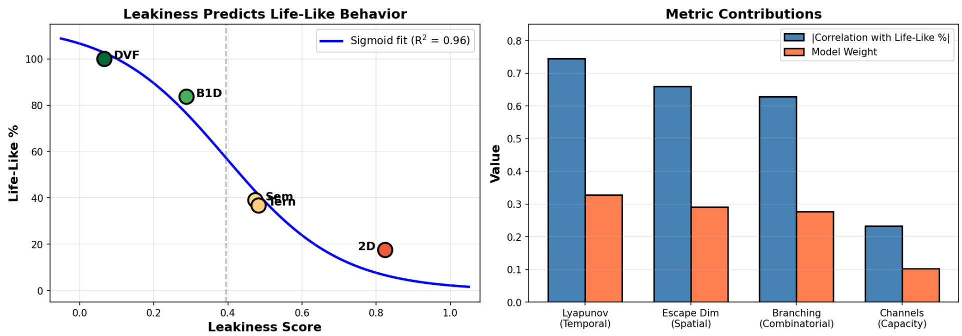

Figure 1: Weighted leakiness predicts life-like percentage with $R^2 = 0.96$. The sigmoid fit shows a phase transition at $L \approx 0.39$.

3.2 The Mechanism: Temporal Filtering

Stickiness operates as a **temporal low-pass filter**, not spatial selective damping:

Table 3: Temporal filter signature: acceptance ratio $\approx 1/d$

Depth	Attempt Rate	Acceptance	Lyapunov	Activity
1	42.4%	100.0%	+0.111	42.4%
2	42.1%	50.0%	+0.112	21.0%
4	41.5%	25.0%	+0.080	10.4%
8	41.3%	12.6%	+0.034	5.2%
12	41.3%	8.4%	+0.035	3.5%

The attempt rate remains constant (∼42%) while acceptance scales as $1/d$.

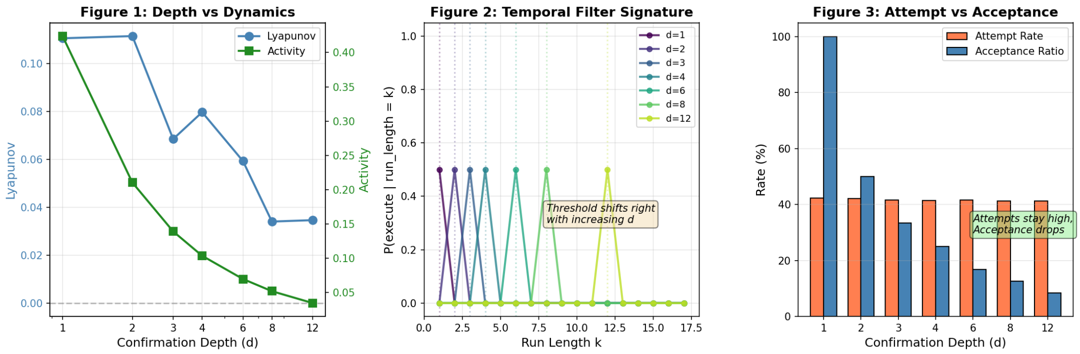

Figure 2: Stickiness as temporal low-pass filter. Acceptance ratio scales as $1/d$.

3.3 Two-Point Calibration Protocol

The Lyapunov decay follows a power law:

$$L(d) = L_{min} + (L_0 - L_{min}) \cdot d^{-\gamma} \tag{4}$$

While γ cannot be predicted from baseline properties, it can be estimated from two measurements:

$$\gamma_{est} = \frac{\log((L_0 - L_{min})/(L_4 - L_{min}))}{\log(4)} \tag{5}$$

Correlation between two-point and full-fit: $r = 0.996$.

The control law for optimal depth:

$$d^* = \left(\frac{L_0 - L_{min}}{L_{crit} - L_{min}} \right)^{1/\gamma} \tag{6}$$

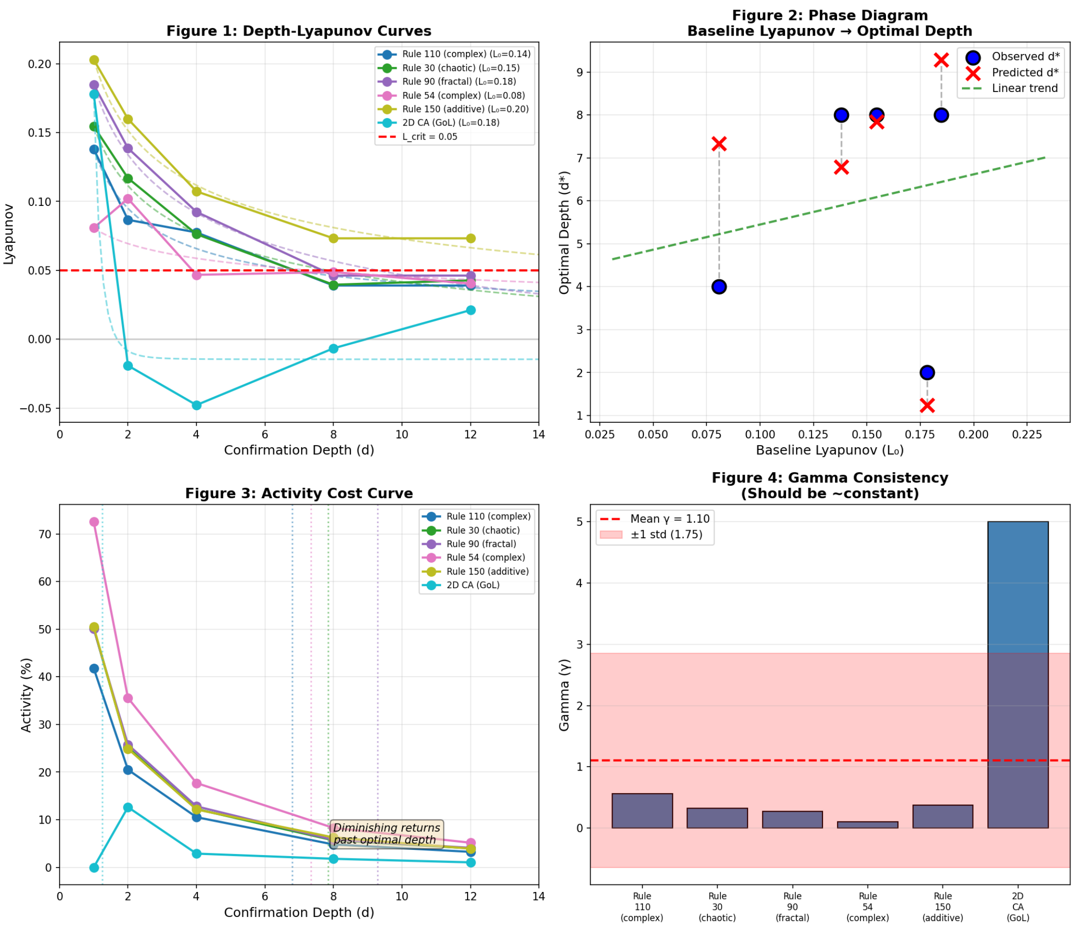

Figure 3: Power-law Lyapunov decay and optimal depth prediction.

3.4 The Capacity Axis: Compression Ratio

Substrates split into life-like-prone and resistant classes:

Table 4: Compression separates prone from resistant substrates

Substrate	Type	Bits/Cell	Prone
Rule 110	complex	0.95	YES
Rule 54	complex	0.88	YES
2D CA	complex	0.51	YES
Rule 30	chaotic	1.33	NO
Rule 90	additive	1.33	NO
Rule 150	additive	1.33	NO

Threshold: bits/cell $< 1.1 \rightarrow$ PRONE; bits/cell $\geq 1.1 \rightarrow$ RESISTANT

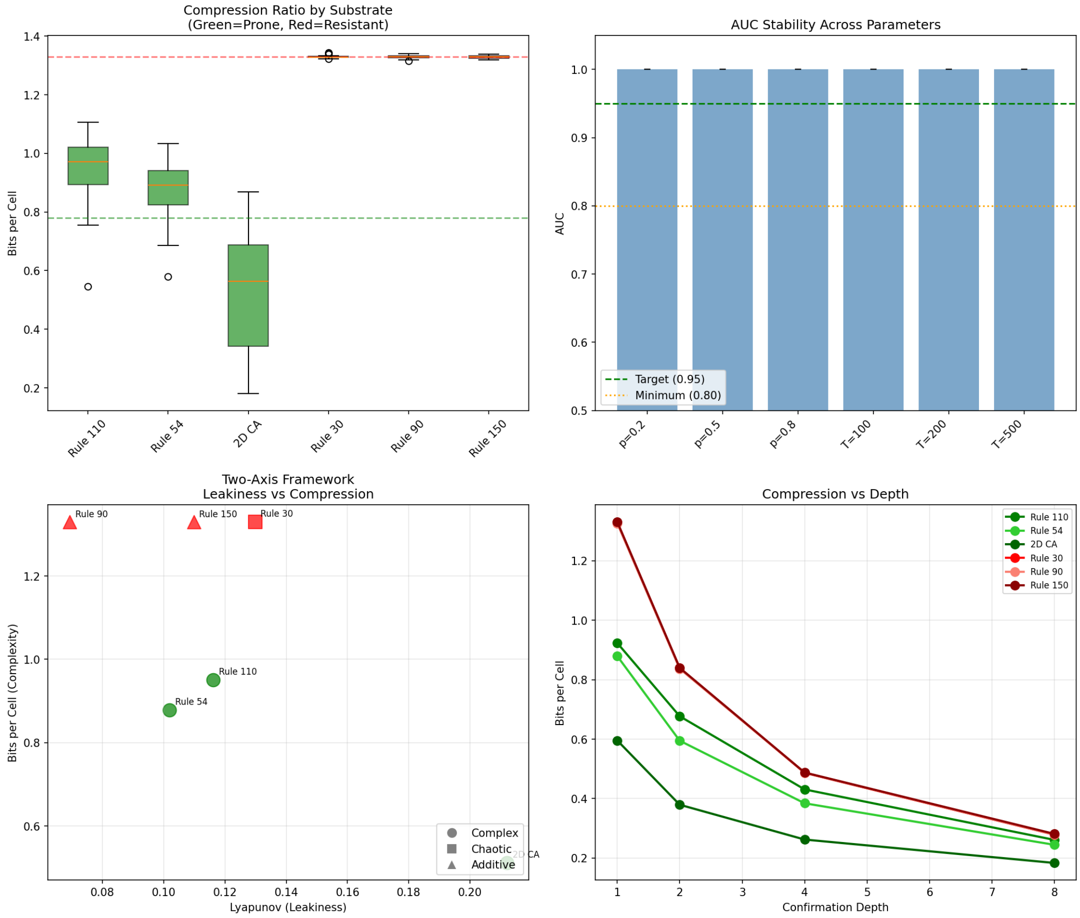

Figure 4: Two-axis framework: compression (capacity) vs Lyapunov (leakiness).

3.5 Validation on Expanded Set

Validation on 50 ECA rules revealed the need for a two-criterion classifier:

1. Activity gate: activity > 1% OR Lyapunov > 0
2. Compression threshold: bits/cell < 1.1

Table 5: Two-criterion classifier performance

Metric	Compression Only	Two-Criterion
Accuracy	57.1%	**86.7%**
AUC (active rules)	0.545	**0.944**
False Positive Rate	45.5%	16.7%
False Negative Rate	33.3%	**0.0%**

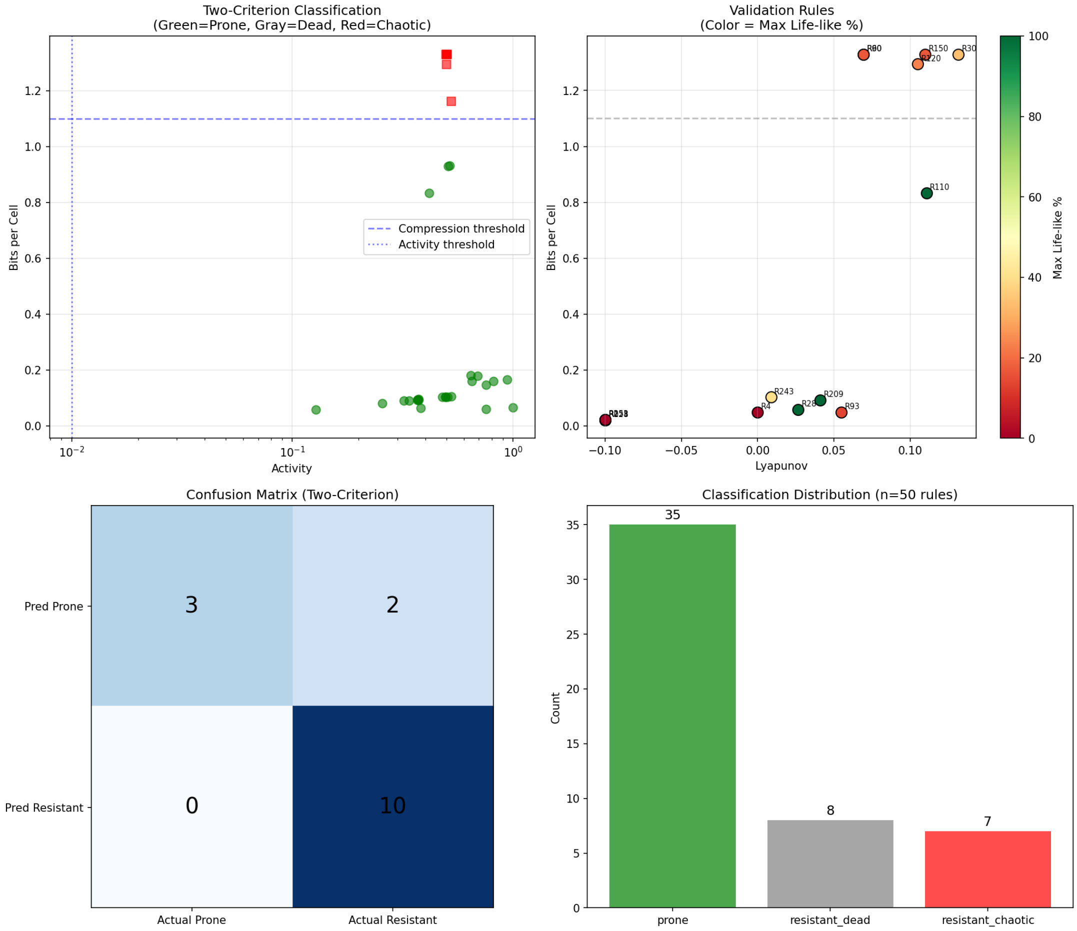

Figure 5: Validation on 50 ECA rules with two-criterion classifier.

4 The Complete Engineering Protocol

4.1 Decision Tree

Algorithm 1 Life-Like Prediction Protocol

Measure substrate at $d = 1$
if activity $\leq 1\%$ AND Lyapunov ≤ 0 **then**
  return RESISTANT (dead)
end if
if compression ≥ 1.1 bits/cell **then**
  return RESISTANT (chaotic)
end if
Substrate is PRONE
Measure L_0 at $d = 1$, L_4 at $d = 4$
$\gamma \leftarrow \log((L_0 + 0.05)/(L_4 + 0.05))/\log(4)$
$d^* \leftarrow ((L_0 + 0.05)/0.10)^{1/\gamma}$
Apply confirmation depth d^*
return Life-like behavior emerges

4.2 Measurement Budget

The complete protocol requires only **three measurements**:

1. Activity (one run at $d = 1$)
2. Compression (same run, computed post-hoc)
3. Lyapunov at $d = 1$ and $d = 4$ (for prone substrates only)

5 Discussion

5.1 Why Temporal Filtering Works

Stickiness blocks transient fluctuations (noise) while preserving sustained dynamics (signal). It preserves 20× more activity than spatial consensus at comparable Lyapunov reduction.

5.2 What Compression Captures

Prone substrates are characterized by *structured novelty*—patterns that are non-random (compressible) but not repetitive. This inverts the "edge of chaos" expectation: prone substrates are more structured, not at intermediate complexity.

5.3 Relationship to Computation

Computation and life-like behavior appear orthogonal:

- Rule 110 is Turing-complete (**?**) and life-like-prone
- Rule 30 is used in cryptographic RNG but life-like-resistant

6 Limitations and Future Work

- Edge cases (Rules 243, 93) suggest gradient rather than hard boundary
- γ remains an irreducible substrate property
- Extension to continuous substrates and biological systems

7 Conclusion

We have presented a two-axis framework for predicting life-like behavior:

1. **Leakiness** predicts life-like percentage with $R^2 = 0.96$
2. **Stickiness** works via temporal filtering
3. **Two-point calibration** predicts optimal depth with $r = 0.996$
4. **Compression** separates prone from resistant with AUC = 0.944

Life-like behavior is not mysterious. It is predictable from substrate properties alone.

Acknowledgments

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