Øving 8

1

$R^{-1} = \{(2,0), (5,0), (9,0), (9,1), (12,1), (15,1), (2,2)\}$

$S^{-1} = \{(0,2), (6,2), (6,5), (8,9), (1,12), (7,12), (4,15)\}$

$(S \circ R)^{-1}$

$S \circ R = \{\cancel{(0,0)}, \cancel{(0,6)}, \cancel{(0,8)}, \cancel{(1,1)}, (1,4), \cancel{(2,6)}\}$

$\cancel{S \circ R = \{(0,0), (0,6), (0,8), (1,8), (1,7), (1,4), (2,6)\}}$

$S \circ R = \{(0,0), (0,6), (0,6), (0,8), (1,8), (1,1), (1,7), (1,4), (2,0), (2,6)\}$

$(S \circ R)^{-1} = \{(0,0), (6,0), (6,0), (8,0), (8,1), (1,1), (7,1), (4,1), (0,2), (6,2)\}$

2 $f: \mathbb{Q} \to \mathbb{Q}, \; x \mapsto f(x) = \frac{x}{3}$

$g: \mathbb{R} \to \mathbb{R}, \quad x \mapsto b(x) = x^3$

$h: \mathbb{R} \times \mathbb{R} \to \mathbb{R} \; (x,y) \mapsto c(x,y) = xy$

1) $g \circ f = \left(\frac{x}{3}\right)^3 = \frac{x^3}{9}$ domain $= \mathbb{Q}$ Range $= \mathbb{Q}$

2) $g \circ h = (xy)^3 = x^3 y^3$ domain $= \mathbb{R}$ Range $= \langle -\infty, \infty \rangle$ $\mathbb{R}$

3) $\left(\frac{x}{3}\right)^3 = \frac{x^3}{3}$, it cannot be true.

4) $h \circ f = \frac{x}{3} \cdot y = \frac{xy}{3}$ $\quad f \circ g = \frac{x^3}{3}$

3.

1) ~~a~~ Range of a $= \mathbb{R}$

Range of $b = \mathbb{R}_+$

Range of $c = \mathbb{R}$

2) a and b are injective, c is not because if x is 0, y can be all different numbers but still give out 0.

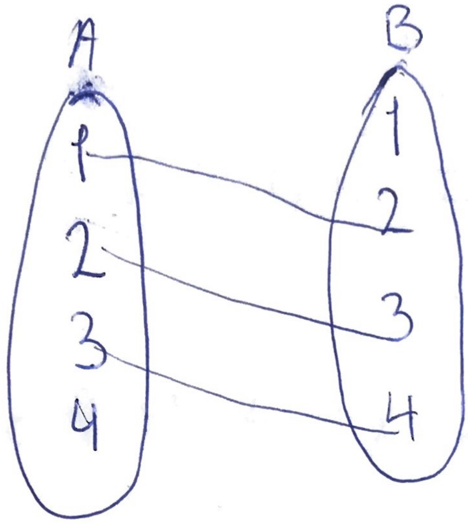
3) a and c is surjective, b is not while it only gives out positive integers

4) a is the only function that is both injective and Surjective, therefore it is bijective.

5

a) Range = $[2, \infty\rangle$

b)

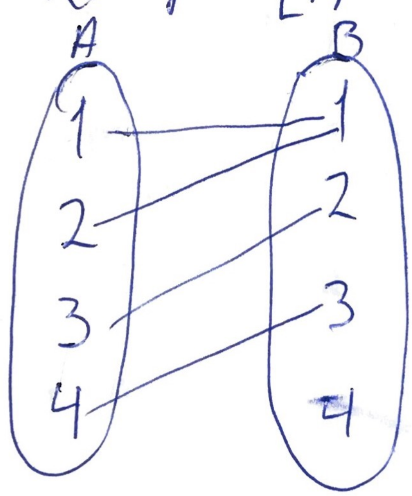


1 is not mapped, so it is not an onto function.

c) As we see over, $a \in A$ will never map the same $b \in B$. Therefore the function is one-to-one.

d) Range = $[1, \infty\rangle$

e)



Every $b \in B$ is mapped, therefore g is an onto function.

f) Both $a_1, a_2 \in A$ maps $b_1 \in B$, therefore g is not one-to-one.

5) g)

$g \circ f = \max\{1, (x+1)-1\} = \max\{1, x\}$

$g \circ f = \{1, 2, 3, 4, 5 \ldots, Z_+\} = \underline{\underline{Z_+}}$

h) $(f \circ g)(x)$ for $x = 2, 3, 4, 7, 12, 25$

$f \circ g = \max\{1, x-1\} + 1$

$x=2: \max\{1, 2-1\} + 1 = 2$

$x=3: \max\{1, 3-1\} + 1 = 3$

$x=4: \text{mas} \{1, 4-1\} + 1 = 4$

$x=7: \text{maa}\{1, 7-1\} + 1 = 7$

$x=12: \max\{1, 12-1\} + 1 = 12$

$x=25: \max\{1, 25-1\} + 1 = 25$

Therefore

$\underline{\underline{(f \circ g)(x) = x \text{ for } x > 1}}$

i) no

8 $n^3 - n = 3t$

$1^3 - 1 = 0 \qquad \frac{0}{3} = 0 \qquad$ ok!

Inductive step: $n = k$

$$k^3 - k = 3t$$

$n = (k+1)$:

$$(k+1)^3 - (k+1) = 3t$$

$(k^2 + 2k + 1)(k+1) - (k+1) =$

$k^3 + 2k^2 + k + k^2 + 2k + \cancel{1} - k \cancel{- 1} =$

$k^3 - k + 3k^2 + 3k =$

$3t + 3k^2 + 3k =$

$\underline{\underline{3(t + k^2 + k)}}$

4

$f(a,b)$

$f(a,a) = a^2 + a^2 = a^2 + a^2$ Reflexive. ✓

$f(a,b) = a^2 + b^2 = b^2 + a^2$ Symmetric ✓

$f(a,b) = f(b,a)$ — " —

$f(a,c)$ der $f(a,b) \to f(b,c)$

$f(a,b) = a^2 + b^2$ $f(b,c) = b^2 + c^2$ $\to$ $f(a,c) = a^2 + c^2$

transative ✓

C is equivalent.

6 $f_{A \cup B} = f_A(x) + f_B(x) - (f_A f_B)(x)$

$S = \{(0,0), (0,1), (1,0), (1,1)\}$

$f_{0 \cup 0} = 0 + 0 - 0 \cdot 0 = 0$

$f_{0 \cup 1} = 0 + 1 - 0 \cdot 1 = 1$

$f_{1 \cup 0} = 1 + 0 - 1 \cdot 0 = 1$

$f_{1 \cup 1} = 1 + 1 - 1 \cdot 1 = 1$

$\forall x \in \{0,1\} = \forall x \in U$

7

$$5 \cdot F_{n+1} = L_{n+4} - L_n$$

Base: $n=1$

$$5 \cdot F_2 = L_5 - L_1$$

$$5 = 7 - 2$$

$$5 = 5$$

assume $n=k$

$$5F_{k+1} = L_{k+4} - L_k$$

$n = k+1$

$$5F_{k+2} = L_{k+5} - L_{k+1}$$

$$5F_{k+2} = F_{k+1} + F_k$$

$$5F_{k+2} = L_{k+4} - L_k + 5F_k$$

$$\cancel{5F_{k} = L_{k+3} - L_{k-1} + L_{k+2}}$$

~~Lk+~~

$$L_{k+4} - L_k + L_{k+3} - L_{k-1}$$

$$\Downarrow$$

$$L_{k+5} - L_{k+1}$$

9

6 a) $wy \leq xy$

$w \leq \cancel{x}$ $\Rightarrow$ $wy \leq xy$

$0 \cdot 1 \leq 0 \cdot 1$

1: $0 \leq 0$

$0 \cdot 0 \leq 0 \cdot 0$

2: $1 \leq 1$

$1 \cdot 1 \leq 1 \cdot 1$

3: $0 \leq 1$

$1 \cdot 0 \leq 1 \cdot 0$

$0 \cdot 1 \leq 1 \cdot 1$

$0 \cdot 0 \leq 1 \cdot 0$

$\Downarrow$

~~6 b) $w+y \leq x+z$~~

$$\left.\begin{array}{l} 0 \leq 0 \\ 0 \leq 0 \\ 1 \leq 1 \\ 0 \leq 0 \\ 0 \leq 1 \\ 0 \leq 0 \end{array}\right\} \text{True}$$

9 6b) $w + y \le x + z$

$w \le x$	$y \le z$
$0 \le 0$	$0 \le 0$
$1 \le 1$	$1 \le 1$
$0 \le 1$	$0 \le 1$

$$\Downarrow$$

$$\begin{aligned} 0+0 &\le 0+0 \\ 0+1 &\le 0+1 \\ 0+0 &\le 0+1 \\ 1+0 &\le 1+0 \\ 1+1 &\le 1+1 \\ 1+0 &\le 1+1 \\ 0+0 &\le 1+0 \\ 0+1 &\le 1+1 \\ 0+0 &\le 1+1 \end{aligned} \quad = \quad \left.\begin{aligned} 0 &\le 0 \\ 1 &\le 1 \\ 0 &\le 1 \\ 1 &\le 1 \\ 1 &\le 1 \\ 1 &\le 1 \\ 0 &\le 1 \\ 1 &\le 1 \\ 0 &\le 1 \end{aligned}\right\} \text{True}$$

10. $f: A \to A$ $\quad R \times A$: (x, y) s.a. $f(x) = f(y)$

R: if $f(x) = f(y)$ then (x, y) also can be (x, x)
therefore it is reflexive.

S: $f(x) = f(y) \Leftrightarrow f(y) = f(x)$

$(x, y) \Leftrightarrow (y, x)$, therefore it is symmetric

T: if $f(x) = f(y)$ is true, the $f(y) = f(z)$ can be true
Hence, $f(x) = f(z)$, and therefore it is transative

R is an equivalence relation on A.