Robin Rognerud

Ex 6 $\quad A \cup B = (A-B) \cup (B-A) \cup (A \cap B)$

$$= (x \in A \wedge x \notin B) \cup (x \in B \wedge x \notin A) \cup (x \in A \wedge x \in B)$$

$$= (x \in A) \cup (x \in B) \cup (x \in A \wedge x \in B)$$

$$= (A \cup B) \cup A \cap B$$

$$= \underline{\underline{A \cup B \quad \text{from absorption law.}}}$$

Ex 7 $\quad A-(B \cap C) = (A-B) \cup (A-C)$

$$\cancel{(x \in A) - (x \in B \wedge x \in C)} = (x \in A \wedge x \notin B) \cup (x \in A \wedge x \notin C)$$

$$= (x \in A) \wedge (x \notin B \wedge x \notin C)$$

$$\underline{\underline{= A - (B \cap C)}}$$

Ex 8 i) $\quad U \Delta V := (U-V) \cup (V-U)$

Check if $U \Delta V = V \Delta U$

$$U \Delta V = (U-V) \cup (V-U)$$

$$= (V-U) \cup (U-V) \quad \text{commutativity law}$$

$$\underline{\underline{= V \Delta U}}$$

8 ii) $U\Delta(V\Delta W) = (U\Delta V)\Delta W$

- Right: $(U\Delta V)\Delta W$

$= (((U\cap\bar V)\cup(\bar U\cap V))\cap\bar W)\cup(\overline{((U\cup V)\cap(\bar U\cup\bar V))}\cap W)$ def. of symmetric diff.

$= (U\cap\bar V\cap\bar W)\cup(\bar U\cap V\cap\bar W)\cup(((\overline{U\cup V})\cup(\overline{\bar U\cup\bar V}))\cap W)$ distributive and de Morgan.

$= (U\cap\bar V\cap\bar W)\cup(\bar U\cap V\cap\bar W)\cup(((\bar U\cap\bar V)\cup(U\cap V))\cap W)$ de Morgan and double negation

$= (U\cap\bar V\cap\bar W)\cup(\bar U\cap V\cap\bar W)\cup(\bar U\cap\bar V\cap W)\cup(U\cap V\cap W)$ distributive

- Left: $U\Delta(V\Delta W)$

$= (U\cap\overline{((V\cup W)\cap(\bar V\cup\bar W))})\cup(\bar U\cap((V\cap\bar W)\cup(\bar V\cap W)))$ def of symmetric diff.

$= (U\cap((\overline{V\cup W})\cup(\overline{\bar V\cup\bar W})))\cup(\bar U\cap V\cap\bar W)\cup(\bar U\cap\bar V\cap W)$ distributive and de Morgan

$= (U\cap((\bar V\cap\bar W)\cup(V\cap W)))\cup(\bar U\cap V\cap\bar W)\cup(\bar U\cap\bar V\cap W)$ de Morgan.

$= (U\cap\bar V\cap\bar W)\cup(U\cap V\cap W)\cup(\bar U\cap V\cap\bar W)\cup(\bar U\cap\bar V\cap W)$ distributivity

- Due to ~~distr~~ commutativity we can see left and right is equal and therefore the symmetric difference is associative.

Ex 9 For

$$x \in X \wedge x \in Y = X \cap Y$$

Therefore the largest set containing both x and Y, is X intersected with Y