Tutorial 1

Sonntag, 16. Oktober 2022 10:14

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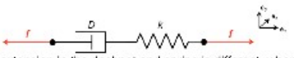
Introduction to Biomechanics VU 317.043

Tutorial 1

11.10.2022

1 Mechanical elements - Maxwell body

Consider the following lumped parameter model of the Maxwell body:



Hint: The extension in the dashpot and spring is different, whereas their force is the same.

Assuming the extension to instantaneously rise to a magnitude x_0 (relaxation experiment):

- derive an expression for the force $f_1(t)$ of the body
- sketch the progression of $f_1(t)$ qualitatively

Now assume the force to instantaneously rise to a magnitude f_0, where it stays constant (creep experiment):

- derive an expression for the extension $x_1(t)$ of the body
- sketch the progression of $x_1(t)$ qualitatively

To check your results: In your sketches of $f_1(t)$ and $x_1(t)$, how does the system behave for $t=0$? How does the system behave for $t \to \infty$? Is this behaviour what you would intuitively expect to happen?

1)

$$\boxed{\begin{aligned} f_D(t) &= D\,\dot{x}_D(t) \Rightarrow \dot{x}_D(t) = \frac{f_D(t)}{D} \\ f_k(t) &= k\,x_k(t) \Rightarrow x_k(t) = \frac{f_k(t)}{k} \end{aligned}}$$

$f_1 = f_D = f_k$

$x_k + x_D = x_0$

$\dot{x}_k + \dot{x}_D = 0$

$$\Rightarrow \frac{1}{k}\dot{f}_k(t) + \frac{1}{D}f_D(t) = 0$$

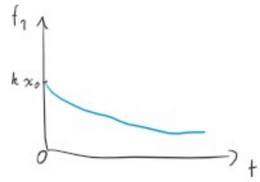
$$\frac{1}{k}\dot{f}_1(t) + \frac{1}{D}f_1(t) = 0$$

$$\dot{f}_1(t) = -\frac{k}{D}f_1(t) \quad \| \quad f_1(t) = Ce^{\lambda t}$$

$$\lambda C e^{\lambda t} = -\frac{k}{D}Ce^{\lambda t}$$

$$\lambda = -\frac{k}{D} \Rightarrow f_1(t) = Ce^{-\frac{k}{D}t}$$

$$f_1(0) = C = k x_0 \Rightarrow f_1(t) = k x_0 e^{-\frac{k}{D}t}$$

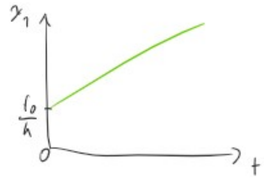


$$f_0 = f_k = f_D$$

$$x_k + x_D = x_1$$

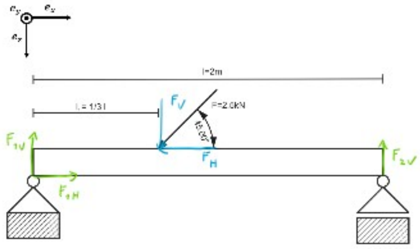
$$\Rightarrow x_1(t) = \frac{f_k(t)}{k} + \int \frac{f_D(t)}{D}\,dt = \frac{f_0}{k} + \frac{f_0 t}{D} + C$$

$$x_1(0) = C = 0 \Rightarrow x_1(t) = f_0\left(\frac{1}{k} + \frac{t}{D}\right)$$



2 Statics- Beams

Determine the required reaction forces and moments of the weightless beam. Sketch the internal forces $n(x)$ (normal force) and $v(x)$ (shear force), and the moment curve $M(x)$.



To check your results: There should be no "jumps" in the moment curve. What should the moment at the ends of the beam be?

3 Dynamics

3.1 Jump

A person with a mass of 60kg is performing a jump from rest in a crouched position. The duration of the take-off phase is τ = 180 ms and the vertical ground reaction force f_1 [N] can be described by:

$$f_1(t) = 2400\,\sin\left(\frac{\pi t}{\tau}\right) + 600\left(1 - \frac{t}{\tau}\right)$$

Calculate the peak height h, that the center of mass (COM) of the person raises above its position at the end of the take-off phase.

To check your results: h≈0.68 m

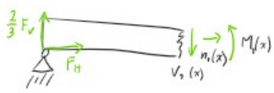
2)

$$H: \quad F_{1H} = F_H = F\cos(45°) = \frac{1}{\sqrt{2}}F$$

$$V: \quad F_V = F_{1V} + F_{2V} = F\sin(45°) = \frac{1}{\sqrt{2}}F$$

$$M: \quad F_V \frac{l}{3} = F_{2V}\,l \Rightarrow F_{2V} = \frac{1}{3\sqrt{2}}F \Rightarrow F_{1V} = \left(\frac{1}{\sqrt{2}} - \frac{1}{3\sqrt{2}}\right)F = \frac{2}{3\sqrt{2}}F = \frac{\sqrt{2}}{3}F = 2F_{2V}$$

$$\boxed{0 < x < \frac{l}{3}}$$



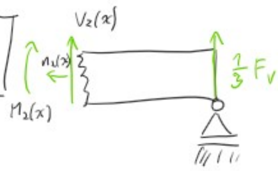
$$n_1(x) = -F_H = -\frac{1}{\sqrt{2}}F$$

$$V_1(x) = \frac{2}{3}F_V = \frac{\sqrt{2}}{3}F$$

$$M_1(x) = \frac{2}{3}F_V \cdot x = \frac{\sqrt{2}}{3}F x \Rightarrow M_1\left(\frac{l}{3}\right) = \frac{\sqrt{2}}{9}lF$$

$$\frac{dM_1(x)}{dx} = \frac{\sqrt{2}}{3}F$$

$$\boxed{\frac{l}{3} < x\, l}$$



$$n_2(x) = 0$$

$$V_2(x) = -\frac{1}{3}F_V = -\frac{1}{3\sqrt{2}}F$$

$$M_2(x) = \frac{1}{3}F_V(l - x) = \frac{1}{3\sqrt{2}}F(l - x) \Rightarrow M_2\left(\frac{l}{3}\right) = \frac{1}{3\sqrt{2}}F\left(l - \frac{l}{3}\right) = \frac{\sqrt{2}}{9}lF = M_1\left(\frac{l}{3}\right) \checkmark$$

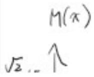
$$\frac{dM_2}{dx} = -\frac{1}{3\sqrt{2}}F$$

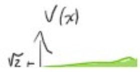
3.2 Inertia

A slender, circular rod with cross-section $A = \frac{d^2\pi}{4}$, length l and density ρ is located with one end at the origin and oriented parallel to axis e_z.

a) For a rotation around axis e_1, with the rotation centre located at the centre of mass calculate the moment of inertia.

b) Calculate the moment of inertia around axis e_y with the rotation centre located at the centre of mass and at the origin. Further, calculate the radius of gyration, with the assumption that d is much smaller than l.

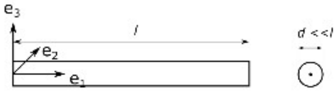




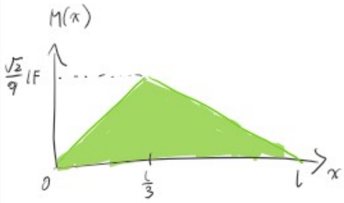
a) For a rotation around axis e_1, with the rotation centre located at the centre of mass calculate the moment of inertia.

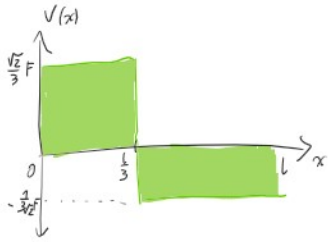
b) Calculate the moment of inertia around axis e_3 with the rotation centre located at the centre of mass and at the origin. Further, calculate the radius of gyration, with the assumption that d is much smaller than l.

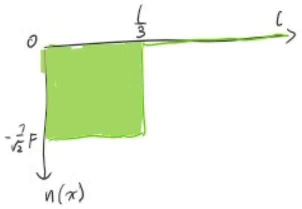
Hint: Use the parallel axis theorem to calculate the moment of inertia with respect to a new axis. $I = I_{cm} + md^2$, where I_{cm} is the moment of inertia at the centre of mass. m is the mass and d is the perpendicular distance to the new axis.



To check your results: Look up the moments of inertia, that you are supposed to calculate here. For your solution in b) you will need the parallel axis theorem and the perpendicular axis theorem.







3.1)

$$f_3(t) = ma(t) = 2400 \sin\left(\frac{\pi t}{\tau}\right) + 600\left(1 - \frac{t}{\tau}\right) \quad /:m$$

$$\int a(t)\,dt = \int 40 \sin\left(\frac{\pi t}{\tau}\right) + 10\left(1 - \frac{t}{\tau}\right) \overbrace{- g}^{\text{gravitational force}}\,dt$$

$$v(t) = -40 \cos\left(\frac{\pi t}{\tau}\right)\frac{\tau}{\pi} + 10\left(t - \frac{t^2}{2\tau}\right) - gt + C$$

$$v(0) = 0 \Rightarrow -40\frac{\tau}{\pi} + C = 0 \Rightarrow C = 40\frac{\tau}{\pi}$$

$$v(\tau) = -40\cos(\pi)\frac{\tau}{\pi} + 10\tau - \frac{5\tau^2}{\tau} + 40\frac{\tau}{\pi} - g\tau$$

$$= 10\tau - 5\tau + 80\frac{\tau}{\pi} - g\tau$$

$$\frac{mv^2(\tau)}{2} = mgh$$

$$\frac{m}{2}\tau^2\left(5 + \frac{80}{\pi} - g\right)^2 = mgh$$

$$h = \frac{\tau^2}{2g}\left(5 + \frac{80}{\pi} - g\right)^2 \approx 0{,}68\,m$$

3.2 a)

$$I_{e_1} = \rho\int_V r^2\,dV = \rho\int_{\varphi=0}^{2\pi}\int_{z=0}^{l}\int_{r=0}^{\frac{d}{2}} r^2\, r\,dr\,dz\,d\varphi =$$

$$= \rho\int_{\varphi=0}^{2\pi}\int_{z=0}^{l}\frac{\left(\frac{d}{2}\right)^4}{4}\,dz\,d\varphi = \rho\, 2\pi l\,\frac{d^4}{4\cdot 2^4} = \frac{\pi}{32}\rho l d^4$$

b)

$$I^{(c)}_{e_2} = I^{(c)}_{e_3} = \frac{1}{2}\left(I^{(c)}_{e_2} + I^{(c)}_{e_3}\right)$$

$$\left.\begin{array}{l} I^{(c)}_{e_2} = \rho\int_V e_1^2 + e_3^2\,dV \\ I^{(c)}_{e_3} = \rho\int_V e_2^2 + e_3^2\,dV \end{array}\right\} I^{(c)}_{e_3} = \underbrace{\frac{\rho}{2}\int_V e_1^2 + e_2^2}_{I_{e_1}/2} + e_3^2 + e_3^2\,dV$$

$$I^{(c)}_{e_3} = \frac{\pi}{64}\rho l d^4 + \rho\int_V e_3^2\,dV$$

$$\int_V e_3^2\,dV = \int_{\varphi=0}^{2\pi}\int_{z=0}^{l}\int_{r=0}^{\frac{d}{2}} z^2 r\,dr\,dz\,d\varphi = \int_{\varphi=0}^{2\pi}\int_{z=-\frac{l}{2}}^{\frac{l}{2}}\frac{z^2}{2}\left(\frac{d}{2}\right)^2 dz\,d\varphi =$$

$$= \int_{\varphi=0}^{2\pi}\frac{1}{3}\left(\left(\frac{l}{2}\right)^3 + \left(\frac{l}{2}\right)^3\right)\frac{d^2}{8}\,d\varphi = \frac{2\pi}{3}\; 2\frac{l^3}{8}\;\frac{d^2}{8} = \frac{\pi}{48}l^3 d^2$$

$$\Rightarrow I^{(c)}_{e_3} = \frac{\pi}{64}\rho l d^4 + \frac{\pi}{48}\rho l^3 d^2 = \frac{\pi}{16}\rho l d^2\left(\frac{d^2}{4} + \frac{l^2}{3}\right) = \frac{m}{4}\left(\frac{d^2}{4} + \frac{l^2}{3}\right)$$

$$\boxed{I = I_{cm} + md^2}$$

$$I_{e_3} = I^{(c)}_{e_3} + m\left(\frac{l}{2}\right)^2 = I^{(c)}_{e_3} + \frac{d^2\pi l}{4}\rho\left(\frac{l}{2}\right)^2 =$$

$$= \frac{\pi}{16}\rho l d^2\left(\frac{d^2}{4} + \frac{l^2}{3}\right) + \frac{\pi}{16}\rho l^3 d^2 = \frac{\pi}{16}\rho l d^2\left(\frac{d^2}{4} + \frac{4l^2}{3}\right) = \frac{m}{4}\left(\frac{d^2}{4} + \frac{4l^2}{3}\right)$$

$$I^{(c)}_{e_3} = m r^{(c)^2}_{e_3} = \frac{m}{4}\left(\frac{d^2}{4} + \frac{l^2}{3}\right) \Rightarrow \sqrt{\frac{1}{4}\left(\frac{d^2}{4} + \frac{l^2}{3}\right)} \approx \frac{1}{2}\sqrt{\frac{l^2}{3}} = \frac{l}{2\sqrt{3}}$$

$$I_{e_3} = m r_{e_3}^2 = \frac{m}{4}\left(\frac{d^2}{4} + \frac{4l^2}{3}\right) \Rightarrow \sqrt{\frac{1}{4}\left(\frac{d^2}{4} + \frac{4l^2}{3}\right)} \approx \frac{1}{2}\sqrt{\frac{4l^2}{3}} = \frac{l}{\sqrt{3}}$$