Homework 5 - Robin Steiner (11778873)

Sonntag, 18. Dezember 2022 12:17

1 i) $y \uparrow \curvearrowleft$, $\rightarrow x$

For two feet

$$F_{floor} = mg\left(\frac{h}{\Delta h}+1\right)$$

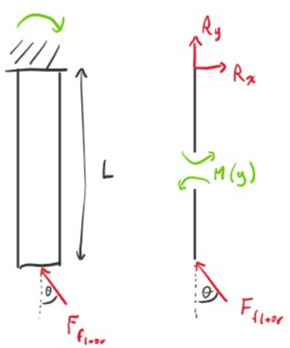
$$A = \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$$

$$\sigma_1 = \frac{F_{floor}}{2A} = \underline{78,3\ MPa}$$

$m = 120\,kg$
$h = 7\,m$
$\Delta h = 0,15\,m$
$E = 17\,GPa$
$D_o = 0,025\,m$
$D_i = 0,013\,m$

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{2mg\left(\frac{h}{\Delta h}+1\right)}{\pi\left(D_o^2 - D_i^2\right)E} = 4,608 \cdot 10^{-3} = \underline{0,4608\ \%} < 1,6\ \% \rightarrow \text{no fracture}$$

ii)



$$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$

$$M(y) = \frac{F_{floor}}{2}\sin\theta\,(y-L)$$

$$\sigma_2(y) = \frac{M(y)}{I}x + \frac{F_{floor}}{2A}\cos\theta$$

max y: $y = 0$

$$\Rightarrow M(0) = -\frac{F_{floor}}{2}\sin\theta\, L$$

max x: $x \in \left[-\frac{D_o}{2}, \frac{D_o}{2}\right]$

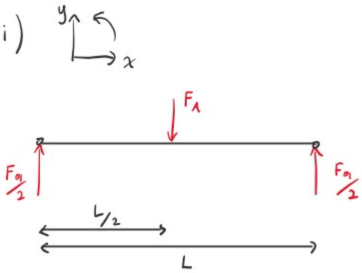
$$\sigma_{2max} = \frac{F_{floor}}{2A}\cos\theta \pm \frac{F_{floor}}{2I}\sin\theta\, L\left(\frac{D_o}{2}\right)$$

$$= \frac{F_{floor}}{2}\left(\frac{\cos\theta}{A} \pm \frac{\sin\theta\, L D_o}{2I}\right) = \begin{cases} \underline{4\ GPa} \\ -3,9\ GPa \end{cases}$$

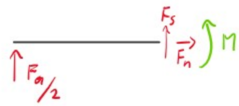
$L = 0,4\,m$
$\theta = 30°$

$$\varepsilon_{2max} = \frac{F_{floor}}{2E}\left(\frac{\cos\theta}{A} \pm \frac{\sin\theta\, L D_o}{2I}\right) = \begin{cases} 0,2361 = \underline{23,61}\ \% > 1,6\ \% \rightarrow \text{fracture} \\ -0,2281 \end{cases}$$

2 i)



$0 < x < \frac{L}{2}$



$\frac{L}{2} < x < L$

$\sum M = 0$

$$\left.\begin{aligned} M_1(x) &= \frac{F_a}{2} x \\ M_2(x) &= \frac{F_a}{2}(L - x) \end{aligned}\right\} \quad \text{max}: \quad M\left(\frac{L}{2}\right) = \frac{L F_a}{4}$$

ii)

$$\sigma_{max} = \frac{M_{max}\, y}{I} = \frac{M_{max}\, d_o}{2I} = \left| \begin{aligned} M_{max} &= \frac{L F_a}{4} \\ I &= \frac{\pi}{64}(d_o^4 - d_i^4) \end{aligned} \right| = \frac{8 L F_a d_o}{\pi (d_o^4 - d_i^4)}$$

offset from neutral axis

$$\sigma_{max} = \varepsilon_{max} E$$

$$\Rightarrow \varepsilon_{max} E = \frac{8 L F_a d_o}{\pi (d_o^4 - d_i^4)}$$

$$F_a = \frac{\varepsilon_{max} E \pi (d_o^4 - d_i^4)}{8 L d_o} = \underline{\underline{15{,}85\ N}}$$

$L = 0{,}05\,m$

$d_o = 0{,}004\,m$

$d_i = 0{,}0038\,m$

$E = 8{,}5\,GPa$

$\varepsilon_{max} = 0{,}02$

iii)

$I_x = \frac{\pi}{4} a b^3$ — bending happens along the x-axis

$$I = I_o - I_i = \frac{\pi}{4} \frac{d_1}{2} \left(\frac{d_2}{2}\right)^3 - \frac{\pi}{4}\left(\frac{d_1}{2} - w\right)\left(\frac{d_2}{2} - w\right)^3 = \frac{\pi}{4}\left(\frac{d_1 d_2^3}{16} - \left(\frac{d_1}{2} - w\right)\left(\frac{d_2}{2} - w\right)^3\right)$$

$$\Rightarrow \sigma_{max} = \frac{L F_a d_2}{2\pi \left(\frac{d_1 d_2^3}{16} - \left(\frac{d_1}{2} - w\right)\left(\frac{d_2}{2} - w\right)^3\right)}$$

offset from neutral axis

$$\Rightarrow F_a = \frac{2 \varepsilon_{max} E \pi \left(\frac{d_1 d_2^3}{16} - \left(\frac{d_1}{2} - w\right)\left(\frac{d_2}{2} - w\right)^3\right)}{L d_2} = \underline{\underline{25{,}33\,N}}$$

$d_1 = 0{,}0048\,m$

$d_2 = 0{,}0032\,m$

$w = 0{,}0002\,m$