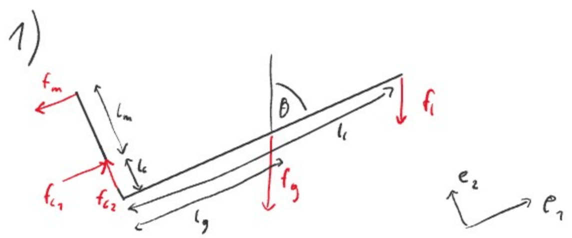
Homework 3 - Robin Steiner (11778873)

Sonntag, 6. November 2022 08:21

1)



i)

$$\sum M = \sum F \times r = 0$$

$$f_m \cdot l_m - \cos\theta f_g l_c - \sin\theta f_g l_g - \cos\theta f_l l_c - \sin\theta\, l_l f_l = 0$$

$$f_m = \frac{\cos\theta\left(l_c(f_g + f_l)\right) + \sin\theta\left(f_g l_g + f_l l_l\right)}{l_m}$$

$$f_m = \frac{4\cos\theta(f_g + f_l) + 5\sin\theta(7 f_g + 9 f_l)}{3}$$

ii)

$$f_{c_1} - f_m - \cos\theta f_g - \cos\theta f_l \qquad f_{c_2} - \sin\theta f_g - \sin\theta f_l$$

$$f_{c_1} = f_m + \cos\theta f_g + \cos\theta f_l \qquad f_{c_2} = \sin\theta f_g + \sin\theta f_l$$

iii)

$$\left.\begin{array}{l} f_l = 200N \\ f_g = 480N \\ \theta = 60° \end{array}\right\} \quad f_m(60°) = \frac{4\cos(60°)\cdot 680 + 5\sin\theta(7\cdot 480 + 9\cdot 200)}{3} =$$

$$\frac{2720}{3}\cos(60°) + 8600\sin(60°) = 7{,}901\ kN$$

iv)

$f_m(0°) = 0{,}907\ kN$

$f_m(10°) = 2{,}386\ kN$

$f_m(20°) = 3{,}793\ kN$

$f_m(30°) = 5{,}085\ kN$

$f_m(40°) = 6{,}223\ kN$

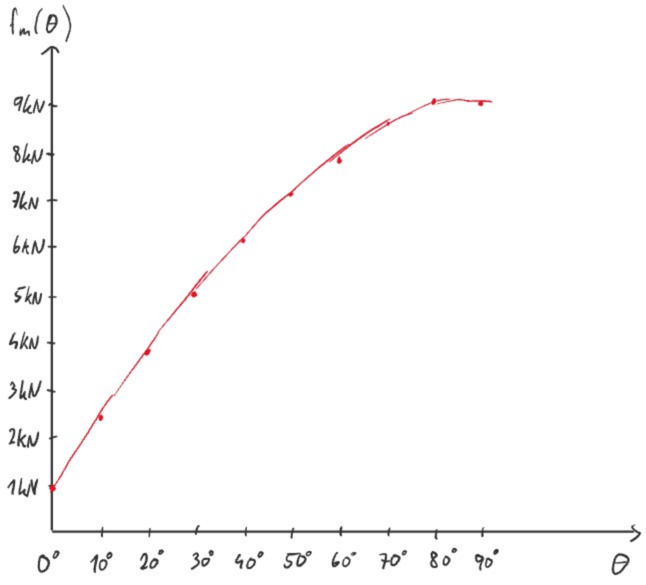
$f_m(50°) = 7{,}171\ kN$

$f_m(60°) = 7{,}901\ kN$

$f_m(70°) = 8{,}391\ kN$

$f_m(80°) = 8{,}627\ kN$

$f_m(90°) = 8{,}6\ kN$



v)

$$f_{c_2} = \sin\theta f_g + \sin\theta f_l = \sin\theta(f_g + f_l)$$

$$f_g + f_l = \frac{f_{c_2}}{\sin\theta} = \frac{1150}{\sin(30°)} = 2300N$$

2)

①

$$F = 0 = T_0 + F_\eta + F_k \qquad F_\eta = \eta_0 \dot{x}^{(1)}(t)$$

$$F_k = k_0\left(x^{(1)}(t) - x_0\right)$$

$$\Rightarrow \dot{x}^{(1)}(t) = \frac{x_0 k_0 - T_0}{\eta_0} - \frac{k_0}{\eta_0} x(t)$$

homogeneous solution:

$$x_H^{(1)}(t) = \lambda e^{-\frac{k_0}{\eta_0} t}$$

particulate solution:

$$x_p^{(1)}(t) = \lambda \Rightarrow \dot{x}^{(1)}(t) = 0$$

$$\Rightarrow x_p^{(1)}(t) = x_0 - \frac{T_0}{k_0}$$

$$\Rightarrow x^{(1)}(t) = x_H^{(1)}(t) + x_p^{(1)}(t) = \lambda e^{-\frac{k_0}{\eta_0} t} + x_0 - \frac{T_0}{k_0}$$

$$x^{(1)}(0) = \lambda + x_0 - \frac{T_0}{k_0} = x_0 \Rightarrow \lambda = \frac{T_0}{k_0}$$

$$\Rightarrow x^{(1)}(t) = \frac{T_0}{k_0}\left(e^{-\frac{k_0}{\eta_0} t} - 1\right) + x_0$$

②

$$x^{(1)}(c) = \frac{T_0}{k_0}\left(e^{-\frac{k_0}{\eta_0} c} - 1\right) + x_0 = x_1$$

$$F = 0 = F_\eta + F_k \qquad F_\eta = \eta_0 \dot{x}^{(2)}(t)$$

$$F_k = k_0\left(x^{(2)}(t) - x_0\right)$$

$$\Rightarrow \dot{x}^{(2)}(t) = \frac{x_0 k_0}{\eta_0} - \frac{k_0}{\eta_0} x^{(2)}(t)$$

homogeneous solution:

$$x_H^{(2)}(t) = \lambda e^{-\frac{k_0}{\eta_0} t}$$

particulate solution:

$$x_p^{(2)}(t) = \lambda \Rightarrow \dot{x}^{(2)}(t) = 0$$

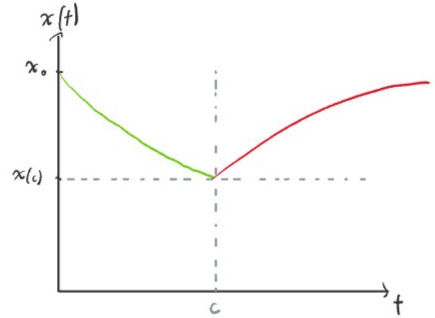
$$\Rightarrow x_p^{(2)}(t) = x_0$$

$$\Rightarrow x^{(2)}(t) = x_H^{(2)}(t) + x_p^{(2)}(t) = \lambda e^{-\frac{k_0}{\eta_0} t} + x_0$$

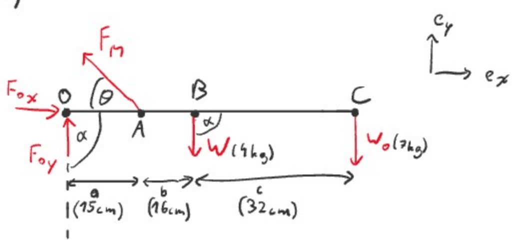
$$x^{(2)}(c) = \lambda e^{-\frac{k_0}{\eta_0} c} + \cancel{x_0} = x_1 = \frac{T_0}{k_0}\left(e^{-\frac{k_0}{\eta_0} c} - 1\right) + \cancel{x_0}$$

$$\Rightarrow \lambda = \frac{T_0}{k_0}\left(1 - e^{\frac{k_0}{\eta_0} c}\right)$$

$$\Rightarrow x^{(2)}(t) = \frac{T_0}{k_0}\left(1 - e^{\frac{k_0}{\eta_0} c}\right) e^{-\frac{k_0}{\eta_0} t} + x_0$$



3i)



$$\sum M = \sum F \times r = 0$$

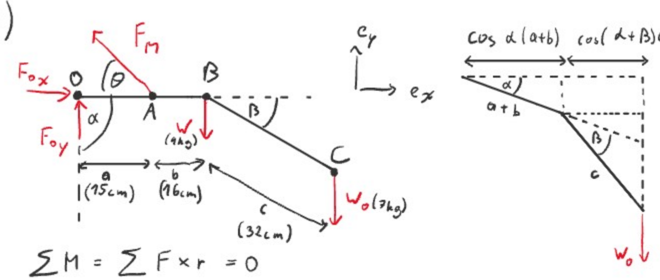
$$F_M \sin\theta\, a - W \sin\alpha\,(a+b) - W_0 \sin\alpha\,(a+b+c) = 0$$

$$F_M = \frac{\sin\alpha}{a \sin\theta}\left(W(a+b) + W_0(a+b+c)\right)$$

$$F_M = \sin\alpha \,\frac{1}{15 \cdot \sin(20°)}\left(4 \cdot 9{,}81\,(15+16) + 7 \cdot 9{,}81\,(15+16+32)\right)$$

$$= 1080{,}37 \sin\alpha$$

ii)



$$\sum M = \sum F \times r = 0$$

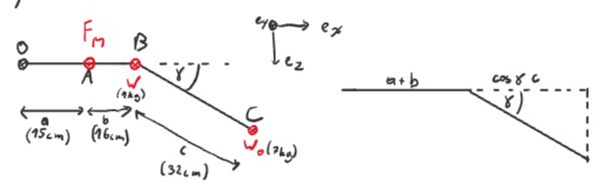
$$F_M \sin\theta\, a - W \sin\alpha\,(a+b) - W_0\left(\sin\alpha\,(a+b) + \cos(\alpha+\beta)\,c\right) = 0$$

$$F_M = \frac{W \sin\alpha\,(a+b)}{\sin\theta\, a} + \frac{W_0\left(\cos\alpha\,(a+b) + \cos(\alpha+\beta)\,c\right)}{\sin\theta\, a}$$

$$F_M = 237{,}11 \sin\alpha + 414{,}94 \cos\alpha + 428{,}33 \cos(\alpha+\beta)$$

$$= 571{,}93\ N$$

iii)



$$\sum M = \sum F \times r = 0 \quad (z\text{-axis})$$

$$F_M \sin\theta\, a - W \sin\alpha\,(a+b) + W_0\,(a+b+\cos\gamma\, c)$$

$$F_M = \frac{\sin\alpha}{a \sin\theta}\left(W(a+b) + W_0(a+b+\cos\gamma\, c)\right)$$

$$F_M = \frac{\sin\alpha}{15 \cdot \sin(20°)}\left(4 \cdot 9{,}81 \cdot 31 + 7 \cdot 9{,}81\,(31 + \cos(45°)\,32)\right)$$

$$F_M = 954{,}91 \sin\alpha\ N$$

$$F_M(\alpha = 90°) = 954{,}91\ N$$