Homework 1 - Robin Steiner (11778873)

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1)

$$\boxed{\begin{aligned} [v] &= \frac{T}{L} \\ [\rho] &= \frac{M}{L^3} \\ [\mu] &= \frac{M}{L \cdot T} \\ [d] &= L \\ [R_e] &= 1 \end{aligned}}$$

$$R_e = v^x \rho^y \mu^z d$$

$$[R_e] = [v]^x [\rho]^y [\mu]^z [d]$$

$$1 = \left(\frac{T}{L}\right)^x \left(\frac{M}{L^3}\right)^y \left(\frac{M}{L \cdot T}\right)^z L$$

$$1 = \frac{T^x}{L^x} \frac{M^y}{L^{3y}} \frac{M^z}{L^z T^z} L = \frac{T^{x-z} M^{y+z}}{L^{x+3y+z-1}}$$

$$\Rightarrow \begin{aligned} x - z &= 0 \\ y + z &= 0 \\ x + 3y + z - 1 &= 0 \end{aligned} \quad \Rightarrow \begin{aligned} x &= z \\ y &= -z = -x \\ x + 3x + x &= 1 \end{aligned} \quad \Rightarrow \begin{aligned} x &= -1 \\ y &= 1 \\ z &= -1 \end{aligned} \quad \Rightarrow \underline{\underline{R_e = \frac{\rho d}{v \mu}}}$$

2 a)

$$V_{min}: \quad F_g = F_l \qquad (S = L B)$$

$$m \cdot g = \frac{1}{2} C_L L B \rho_{air} V_{min}^2$$

$$V_{min} = \sqrt{\frac{2 m g}{C_L L B \rho_{air}}} \Rightarrow V_{min} \propto \sqrt{\frac{1}{L}} = L^{-\frac{1}{2}}$$

2b)

$$P_s = F_{D(s)} V_s \qquad (A = LB)$$

$$c\, m^{0,73} = \frac{1}{2} C_D LB \rho_{air} V_s^3$$

$$V_s = \sqrt[3]{\frac{2 c m^{0,73}}{C_D LB \rho_{air}}} \Rightarrow V_s \propto \sqrt[3]{\frac{1}{L}} = L^{-\frac{1}{3}}$$

2c)

For a bird to be able to fly, its metabolically sustainable flight velocity (V_s) needs to be bigger then the speed needed for lift off (V_{min})

$$\Rightarrow V_s > V_{min}$$

$$L^{-\frac{1}{3}} > L^{-\frac{1}{2}}$$

$$L > 1$$

$\Rightarrow$ for $L > 1$ this is always the case, so there is no limit when looking at the size (wingspan) of the bird

However when looking at the weight:

$$V_S > V_{min}$$

$$m^{\frac{0,73}{3}} > m^{\frac{1}{2}}$$

$$m < 1$$

$\Rightarrow$ since V_{min} grows faster then V_S there is an upper bound in weight