Tutorial 2

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22-23_Int...

Introduction to Biomechanics VU 317.043

Tutorial 2

25.10.2022

1 Knee Joint

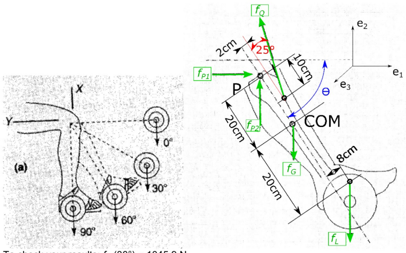
Consider a simple strengthening exercise for the quadriceps, as illustrated in the left figure below. The weight is lifted by the lower leg at different angles. Use the free body diagram in the right figure to answer the following points:

i) Derive an expression for the muscle tension f_Q of the quadriceps required for static equilibrium as a function of the flexion angle Θ

ii) Determine the reaction forces in joint *P*

iii) For f_L = 150 N, f_G = 30 N and Θ = 0°, 30°, 60°, 90° calculate the required muscle tension.

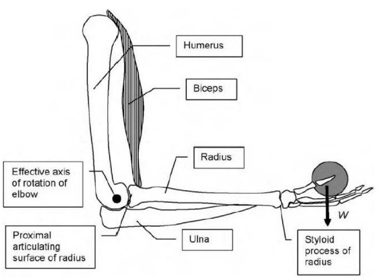
iv) Sketch $f_Q(\Theta)$ (with the inserted force values) from Θ = 0° to 90°



To check your results: f_Q (30°) = 1045.9 N

2 Elbow Joint

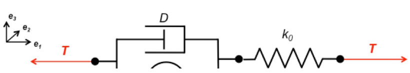
Consider the following simplified diagram of an arm holding a weight. Calculate the force exerted by the bices on the radius in order to hold the arm plus a weight of 1.5 kg in place.



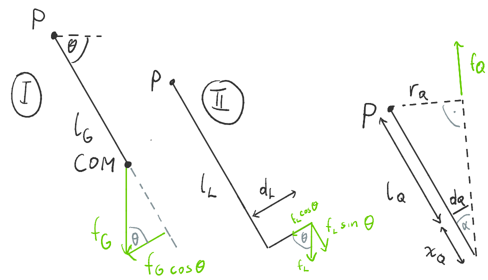
Begin by drawing an appropriate free body diagram. Estimate all the lengths and weights needed for your calculation. For this you can either research literature or measure on your own arm. In any case you need to be able to justify our assumptions.

3 Muscle lumped parameter model

Consider the three-element model of a muscle:



1i)



$x_Q = \frac{d_Q}{\tan\alpha}$

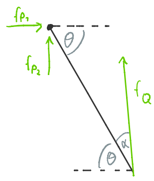
$r_Q = \sin\alpha\,(l_Q + x) = \sin\alpha\left(l_Q + \frac{d_Q}{\tan\alpha}\right)$

$= l_Q \sin\alpha + d_Q \cos\alpha$

$$M:\ \underbrace{l_G f_G \cos\theta}_{\text{I}} + \underbrace{d_L f_L \sin\theta + l_L f_L \cos\theta}_{\text{II}} - \underbrace{r_Q f_Q}_{\text{III}} = 0$$

$$f_Q = \frac{l_G f_G \cos\theta + f_L (d_L \sin\theta + l_L \cos\theta)}{l_Q \sin\alpha + d_Q \cos\alpha}$$

ii)



$f_{P_1} = f_Q \cos(\theta + \alpha)$

$f_{P_2} = f_G + f_L - f_Q \sin(\theta + \alpha)$

iii)

$f_L = 150N$

$l_L = 0{,}4m$

$d_L = 0{,}08m$

$f_G = 30N$

$l_G = 0{,}2m$

$\left.\begin{matrix} l_Q = 0{,}1m \\ d_Q = 0{,}02m \\ \alpha = 25° \end{matrix}\right\} r_Q = 0{,}06m$

$\Rightarrow$

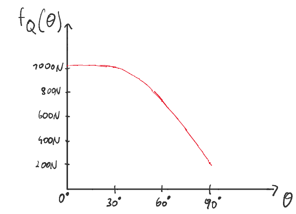
$f_Q(\theta = 0°) = 1092{,}93N$

$f_Q(\theta = 30°) = 1045{,}87N$

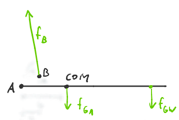
$f_Q(\theta = 60°) = 718{,}56N$

$f_Q(\theta = 90°) = 198{,}72N$

iv)



2)



$b = 0{,}2m$

$c = 0{,}03m$

$d = 0{,}38m$

$a = 0{,}01m$

$\alpha = 85°$

$f_{GW} = 14{,}715\ N$

$f_{GA} = 15N$

$\Rightarrow f_B = 279{,}01\ N$

$M:\ a f_B \cos\alpha + c f_B \sin\alpha - b f_{GA} - d f_{GW} = 0$

$f_B (a\cos\alpha + c\sin\alpha) = b f_{GA} + d f_{GW}$

$f_B = \frac{b f_{GA} + d f_{GW}}{a\cos\alpha + c\sin\alpha} =$

3)

$f_D = D \dot{x}_D \qquad f_k = k x_k$

$\dot{x}_D = \frac{f_D}{D} \qquad x_k = \frac{f_k}{k}$

$x_D + x_k = x$

$\dot{x}_D + \dot{x}_k = 0 \Rightarrow \dot{x}_D = -\dot{x}_k$

$T(t) = f_k = f_D + T_0$

$k x_k = D \dot{x}_D + T_0$

$k x_k = -D \dot{x}_k + T_0$

$\dot{x}_k = \frac{T_0}{D} - \frac{k}{D} x_k$

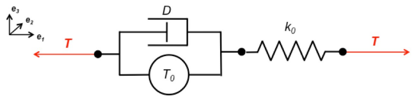
homogeniouse Solution:

$\dot{x}_k^{(H)} = -\frac{k}{D} x_k^{(H)} \Rightarrow x_k^{(H)} = C(t) e^{-\frac{k}{D}t}$

$\dot{x}_k^{(H)} = \dot{C}(t) e^{-\frac{k}{D}t} - C(t) \frac{k}{D} e^{-\frac{k}{D}t}$

$\Rightarrow \dot{C}(t) e^{-\frac{k}{D}t} - C(t) \frac{k}{D} e^{-\frac{k}{D}t} = \frac{T_0}{D} - \frac{k}{D} C(t) e^{-\frac{k}{D}t}$

Consider the three-element model of a muscle.



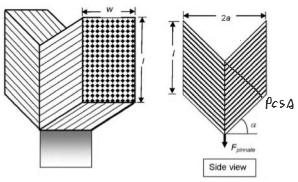
The contractile element produces a constant tension T_0 during activation – otherwise it produces no tension.

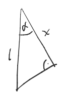
- For an isometric contraction derive an expression for $T(t)$, assuming that the contractile element is activated from $t = 0$ until $t = C$.
- Sketch $T(t)$

4 Pennate Muscle

Prove that the force generated (per unit depth) of a pennate muscle arrangement (as per the sketch below) is:

$$F_{pennate} = 2wf_{fiber/area} l \cos(a)\sin(a)$$





$x = \cos\alpha \cdot l\, w$

$$\Rightarrow \dot{c}(t) e^{-\frac{k}{D}t} - c(t)\frac{k}{D} e^{-\frac{k}{D}t} = \frac{T_0}{D} - \frac{k}{D} c(t) e^{-\frac{k}{D}t}$$

$$\dot{c}(t) e^{-\frac{k}{D}t} = \frac{T_0}{D}$$

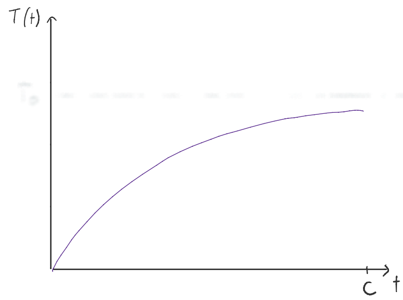
$$\dot{c}(t) = \frac{T_0}{D} e^{\frac{k}{D}t} \Rightarrow c(t) = \frac{T_0}{D}\frac{D}{k} e^{\frac{k}{D}t} + c$$

$$\Rightarrow x_k = \left(\frac{T_0}{k} e^{\frac{k}{D}t} + c\right) e^{-\frac{k}{D}t} = \frac{T_0}{k} + c e^{-\frac{k}{D}t}$$

$$x_k(0) = \frac{T_0}{k} + c = 0 = c = -\frac{T_0}{k}$$

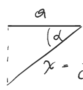
$$x_k = \frac{T_0}{k}\left(1 - e^{-\frac{k}{D}t}\right)$$

$$T(t) = f_k = k x_k = T_0\left(1 - e^{-\frac{k}{D}t}\right)$$



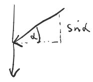
4)

$$PCSA = \frac{\text{muscle volume}}{\text{fibre length}} = \frac{a w l}{\frac{a}{\cos\alpha}} = w l \cos\alpha$$



$x = \frac{a}{\cos\alpha}$

$$F_{muscle} = f_{fibre/area} \cdot PCSA = w l f_{fibre/area} \cos\alpha$$



$$F_{Pinnate} = 2 F_{muscle} \sin\alpha = 2 w f_{fibre/area} l \cos\alpha \sin\alpha$$