

Introduction to Biomechanics VU 317.043

Tutorial 3

8.11.2022

Leg – joint reaction forces and moments

Given figure 1 below calculate the joint reaction forces at the knee and the hip at the 50% stance phase of the gait cycle.

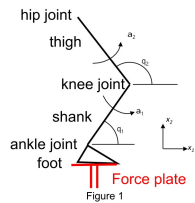


Figure 1

Use the following data for your calculations:

Anthropometric data

Limb segment	Segment length	Segment mass*	Centre of mass**	Radius of gyration Iy #
Thigh	32 cm	0.1 M	0.433	0.323
Shank	43 cm	0.0465 M	0.433	0.302

* Expressed as a fraction of the total body mass, M, where M = 70 kg

** Expressed as a fraction of the total segment length, measured from the proximal end of the segment

Expressed as a fraction of the segment length and for rotation around the center of mass

Thigh and shank kinematic data

% of stance phase of gait cycle	Shank angle Θ_1 [°]	Thigh angle Θ_2 [°]	Shank acceleration α_1 [rad/s ²]	Thigh acceleration α_2 [rad/s ²]
0	108.4	109.4	-32.87	0.13
10	100.3	108.4	-2.78	3.58
20	91.9	106.9	18.47	-28.36
30	86.5	101.2	7.87	-4.84
40	82.5	94.3	-0.22	7.81
50	78.7	88.7	-0.22	0.01
60	74.9	83.1	-2.99	-1.71
70	70.2	77.5	-13.88	11.95
80	63.2	73.8	-13.69	16.49
90	53.6	73.4	-8.16	27.13
100	42.9	78.1	25.25	30.36

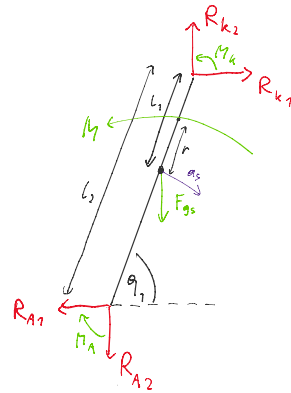
Ankle reaction forces and muscle moments for the foot segment

% of stance phase of gait cycle	Reaction force, x1 direction [N]	Reaction force, x2 direction [N]	Muscle moment, [Nm]
0	-60.1	-101.7	-1.7
10	109.1	-572.1	6.9
20	135.1	-728.3	-0.5
30	98.3	-628.0	-6.8
40	27.7	-456.0	-18.7
50	21.6	-453.5	-30.3
60	-2.6	-534.2	-45.4
70	-49.9	-670.2	-71.2
80	-107.9	-744.2	-89.7
90	-130.5	-555.8	-68.6
100	-32.9	-125.7	-12.3

Shank and thigh acceleration data

% of stance phase of gait cycle	Shank acceleration x1 direction [ms ⁻²]	Shank acceleration x2 direction [ms ⁻²]	Thigh acceleration x1 direction [ms ⁻²]	Thigh acceleration x2 direction [ms ⁻²]
0	-5.87	2.26	-0.31	4.29
10	-4.01	2.75	-4.05	2.63
20	-6.62	-1.74	-4.85	-2.32
30	-3.89	0.40	-4.13	-1.01
40	0.82	-0.53	-0.65	-2.12
50	1.14	0.12	1.16	1.28

Shank:



$$x: R_{k1} - R_{A1} = m_s \cdot a_{s1}$$

$$R_{k1} = m_s \cdot a_{s1} + R_{A1} = \underline{25,54 \text{ N}}$$

$$y: R_{k2} - R_{A2} - F_{gs} = m_s \cdot a_{s2}$$

$$R_{k2} = m_s \cdot a_{s2} + R_{A2} + m_s \cdot g = \underline{-427,67}$$

$$M: I_s = m_s r^2$$

$$M = I_s \cdot \alpha_1 = m_s r^2 \alpha_1 = -0,012 \text{ Nm}$$

$$M_1 = R_{A2} (l_2 - l_1) \cos \theta_1 - R_{A1} (l_2 - l_1) \sin \theta_1 = -26,83 \text{ Nm}$$

$$M_2 = R_{k2} l_1 \cos \theta_1 - R_{k1} l_1 \sin \theta_1 = -20,76 \text{ Nm}$$

$$M = M_k - M_A + M_1 + M_2$$

$$M_k = m_s r^2 \alpha_1 + M_A - R_{A2} (l_2 - l_1) \cos \theta_1 + R_{A1} (l_2 - l_1) \sin \theta_1 - R_{k2} l_1 \cos \theta_1 + R_{k1} l_1 \sin \theta_1 = \underline{76,68 \text{ Nm}}$$

$$m = 70 \text{ kg}$$

$$m_s = 0,0465 \text{ m}$$

$$a_{s1} = 7,27 \text{ m/s}^2$$

$$a_{s2} = -0,95 \text{ m/s}^2$$

$$R_{A1} = 27,6 \text{ N}$$

$$R_{A2} = -453,5 \text{ N}$$

$$r = 0,302 l_2$$

$$l_2 = 0,43 \text{ m}$$

$$\alpha_1 = -0,22 \text{ rad/s}^2$$

$$l_1 = 0,433 l_2$$

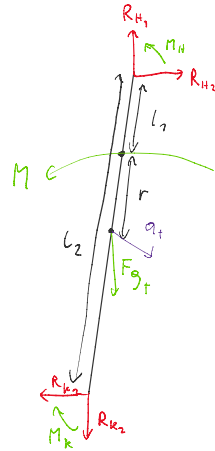
$$\theta_1 = 78,7^\circ$$

$$M_A = -30,3$$

% of stance phase of gait cycle	Shank acceleration x1 direction [ms ⁻²]	Shank acceleration x2 direction [ms ⁻²]	Thigh acceleration x1 direction [ms ⁻²]	Thigh acceleration x2 direction [ms ⁻²]
0	-5.87	2.26	-0.31	4.29
10	-4.01	2.75	-4.05	2.63
20	-6.62	-1.74	-4.85	-2.32
30	-3.89	0.40	-4.13	-1.01
40	0.82	-0.53	-0.65	-2.12
50	1.21	-0.95	1.15	-1.48
60	0.45	1.32	1.16	0.67
70	3.58	1.43	3.58	0.20
80	9.13	0.09	7.98	-1.12
90	14.31	0.82	9.09	-0.68
100	8.47	1.66	-2.10	3.27

Hint Reaction forces and muscle moments measured via a force plate or calculated for a certain segment need to be inverted to be valid for the next segment.

Thigh:



$$x: R_{H1} - R_{K1} = m_t \cdot a_{t1}$$

$$R_{H1} = m_t \cdot a_{t1} + R_{K1} = \underline{33,59 \text{ N}}$$

$$y: R_{H2} - R_{K2} - F_{g_t} = m_t \cdot a_{t2}$$

$$R_{H2} = m_t \cdot a_{t2} + R_{K2} + m_t \cdot g = \underline{-366,22 \text{ N}}$$

$$M: I_t = m_t \cdot r^2$$

$$M = I_t \cdot \alpha_2 = m_t \cdot r^2 \cdot \alpha_2 = 0,00075 \text{ Nm}$$

$$M_1 = R_{K2} (l_2 - l_1) \cos \theta_2 - R_{K1} (l_2 - l_1) \sin \theta_2 = -6,38 \text{ Nm}$$

$$M_2 = R_{H2} l_1 \cos \theta_2 - R_{H1} l_1 \sin \theta_2 = -5,8 \text{ Nm}$$

$$M = M_H - M_K + M_1 + M_2$$

$$M_H = m_t \cdot r^2 \cdot \alpha_2 + M_K - R_{K2} (l_2 - l_1) \cos \theta_2 + R_{K1} (l_2 - l_1) \sin \theta_2 - R_{H2} l_1 \cos \theta_2 + R_{H1} l_1 \sin \theta_2 = \underline{28,86 \text{ Nm}}$$

$$m_t = 0,1 \text{ m}$$

$$a_{t1} = 1,15 \text{ m/s}^2$$

$$a_{t2} = -1,16$$

$$R_{K1} = 25,54 \text{ N}$$

$$R_{K2} = -424,67 \text{ N}$$

$$r = 0,323 \text{ l}_2$$

$$l_2 = 0,32 \text{ m}$$

$$\alpha_2 = 0,01 \text{ rad/s}^2$$

$$l_1 = 0,433 \text{ l}_2$$

$$\theta_2 = 88,7$$