Homework 2 - Robin Steiner (11778873)

Mittwoch, 19. Oktober 2022 14:47

1)

$$x_0 = x_k = x_1$$
$$f_\circ = f_0 + f_k$$
$$f_0(t) = D\,\dot{x}_0(t) = D\,\dot{x}_1(t)$$
$$f_k(t) = k\,x_k(t) = k\,x_1(t)$$

$$f_\circ = D\,\dot{x}_1(t) + k\,x_1(t)$$

homogeneous solution:

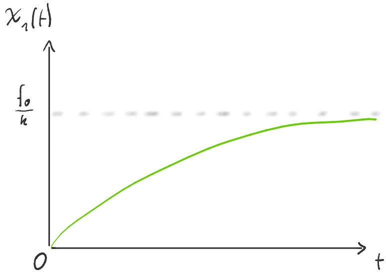
$$\left[\dot{x}_H(t) + \frac{k}{D}x_H(t) = 0\right]$$
$$\Rightarrow x_H(t) = c\,e^{-\frac{k}{D}t}$$
$$\Rightarrow x_1(t) = c(t)\,e^{-\frac{k}{D}t}$$
$$\dot{x}_1(t) = \dot{c}(t)\,e^{-\frac{k}{D}t} - \frac{k}{D}c(t)\,e^{-\frac{k}{D}t}$$

$$\dot{c}(t)\,e^{-\frac{k}{D}t} - \cancel{\frac{k}{D}c(t)\,e^{-\frac{k}{D}t}} = \frac{f_\circ}{D} - \cancel{\frac{k}{D}c(t)\,e^{-\frac{k}{D}t}}$$

$$\dot{c}(t) = \frac{f_\circ}{D}e^{\frac{k}{D}t} \Rightarrow c(t) = \frac{f_\circ}{k}e^{\frac{k}{D}t} + c' \Rightarrow x_1(t) = \left(\frac{f_\circ}{k}e^{\frac{k}{D}t} + c'\right)e^{-\frac{k}{D}t}$$

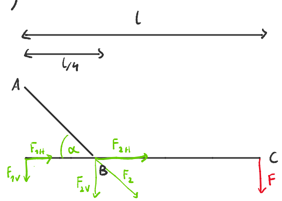
$$x_1(0) = 0 = \frac{f_\circ}{k} + c' \Rightarrow c' = -\frac{f_\circ}{k}$$

$$\Rightarrow \boxed{x_1(t) = \frac{f_\circ}{k}\left(1 - e^{-\frac{k}{D}t}\right)}$$



$$\lim_{t\to\infty} x_1(t) = \frac{f_\circ}{k}$$

2)



$$F_{2H} = \cos(\alpha)\,F_2 \qquad F_{2V} = \sin(\alpha)\,F_2$$

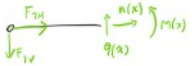
$$M: \quad \sin(\alpha)\,F_2\,\frac{l}{4} = -F\,l \Rightarrow F_2 = -\frac{4F}{\sin(\alpha)} = -1{,}7\,kN$$

$$\Rightarrow F_{2H} = -\frac{4F}{\tan(\alpha)} = -1{,}2\,kN$$

$$\Rightarrow F_{2V} = -4F = -1{,}2\,kN$$

$H: F_{1H} = -F_{2H} \quad \Rightarrow F_{1H} = \frac{4F}{\tan(\alpha)} = 1{,}2\,kN$

$V: F_{1V} + F_{2V} = -F \quad \Rightarrow F_{1V} = -F + 4F = 3F = 0{,}9\,kN$

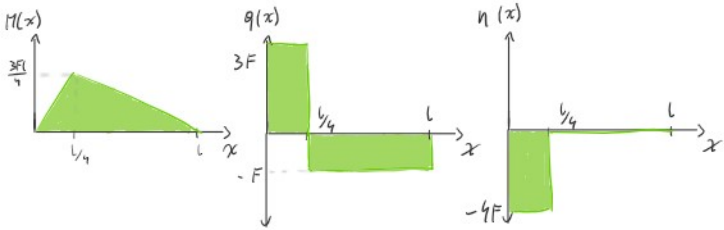


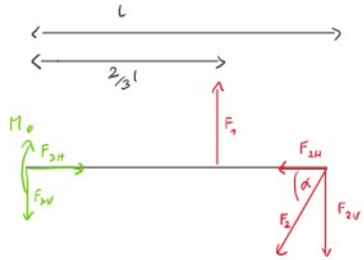
$M(x) = F_{1V}\,x = 3F\,x$ $\qquad M(x) = (l-x)\,F$

$M(\frac{l}{4}) = \frac{3Fl}{4} = 0{,}675\,kNm$ $\qquad M(\frac{l}{4}) = \frac{3Fl}{4} = 0{,}675\,kNm$ ✓

$q(x) = F_{1V} = 3F = 0{,}9\,kN$ $\qquad q(x) = -F = -0{,}3\,kN$

$n(x) = -F_{1H} = -4F = -1{,}2\,kN$ $\qquad n(x) = 0$



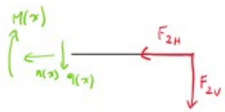


$F_{2H} = \cos(\alpha)\,F_2 \qquad F_{2V} = \sin(\alpha)\,F_2$

$H: F_{3H} = F_{2H} = \cos(\alpha)\,F_2 = 50N$

$V: F_{3V} = F_1 - F_{2V} = F_1 - \sin(\alpha)\,F_2 = -26{,}6N$

$M: M_0 = \frac{2}{3}l\,F_1 - l\,F_{2V} = l\left(\frac{2}{3}F_1 - \sin(\alpha)F_2\right) = -69{,}9Nm$



$n(x) = -F_{3H} = -50N$ $\qquad n(x) = -F_{2H} = -50N$

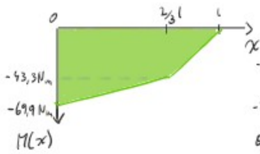
$q(x) = F_{3V} = -26{,}6N$ $\qquad q(x) = -F_{2V} = -86{,}6N$

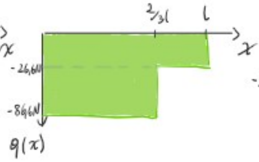
$M(x) = -F_{3V}\,x + M_0$ $\qquad M(x) = -(l-x)\,F_{2V} = -(l-x)\sin(\alpha)F_2$

$= -(F_1 - \sin(\alpha)F_2)\,x + \left(\frac{2}{3}F_1 - \sin(\alpha)F_2\right)l$ $\qquad \Rightarrow M(\frac{2}{3}l) = -\frac{1}{3}l\sin(\alpha)F_2 = -43{,}3Nm$

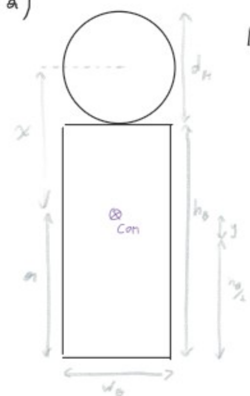
$= \sin(\alpha)F_2\,(x-l) - F_1\left(\frac{2}{3}l - x\right)$ $\qquad \frac{dM(x)}{dx} = \sin(\alpha)\,F_2 > 0$

$\Rightarrow M(\frac{2}{3}l) = -\frac{1}{3}l\sin(\alpha)F_2 = -43{,}3\,Nm$ $\qquad \frac{dM(x)}{dx} = \sin(\alpha)F_2 - F_1 > 0$





3 a)



Body : Cuboid - Inertia

$$I_{CM}^{(c)} = \frac{1}{12} m_B \left(h_B^2 + w_B^2\right)$$

$$I_B = I_{CM}^{(c)} + m_B y^2 = \frac{1}{12} m_B \left(h_B^2 + w_B^2\right) + m_B \left(a - \frac{h_B}{2}\right)^2$$

$$= 15,25 \text{ kg m}^2$$

Head : Sphere - Inertia

$$I_{CM}^{(s)} = \frac{2}{5} m_H r^2 = \frac{1}{10} m_H d_H^2$$

$$\Rightarrow I_H = I_{CM}^{(s)} + m_H x^2 = \frac{1}{10} m_H d_H^2 + m_H \left(h_B - a + \frac{d_H}{2}\right)^2$$

$$= 5,565 \text{ kgm}^2$$

$$\Rightarrow I = I_B + I_H = \frac{m_B}{12}\left(h_B^2 + w_B^2\right) + m_B \left(a - \frac{h_B}{2}\right)^2 + \frac{m_H d_H^2}{10} + m_H \left(h_B - a + \frac{d_H}{2}\right)^2$$

$$\underline{= 20,815 \text{ kgm}^2}$$

b)

$$\vec{V}_{12} = \vec{V}_2 - \vec{V}_1 = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 3,7 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 1,3 \end{pmatrix}$$

$$\Rightarrow |\vec{V}_{12}| = \sqrt{(-3)^2 + 1,3^2} = 3,27 \text{ m/s}$$

$$w = \frac{V}{r} = \frac{|\vec{V}_{12}|}{a} = \underline{4,09 \text{ rad/s}}$$

$$L = w I = 85,13 \text{ kgm}^2\text{/s}$$

c)

Conservation of angular momentum:

$$\boxed{L_1 = L_2} \Rightarrow I_1 w_1 = I_2 w_2$$

$$I_2 = I_{(sphere)} = \frac{2}{5} m r^2 = \frac{1}{10}(m_H + m_B) d_s^2 = 5{,}184 \text{ kg m}^2$$

$$\Rightarrow w_2 = \frac{I_1 w_1}{I_2} = 16{,}42 \text{ rad/s}$$

d)

$$\frac{m {v_1^{(2)}}^2}{2} = mgh$$

$$h = \frac{{v_1^{(2)}}^2}{2g} = \frac{3{,}7^2}{2 \cdot 9{,}81} = 0{,}7 \text{ m}$$