Homework 6 - Robin Steiner (11778873)

Sonntag, 15. Jänner 2023 08:34

1a) $5\,l = 0{,}005\,m^3$

$2\,\%: 0{,}005 \cdot 0{,}02 = 0{,}0001\,m^3$

Zylinder Volume: $V = \pi r^2 L = \frac{\pi d^2 L}{4}$

$\Rightarrow L = \frac{4V}{\pi d^2} \approx \underline{2{,}598 \cdot 10^6\,m}$

b)

$N = \frac{L}{l} = \frac{2{,}598 \cdot 10^6}{1{,}2 \cdot 10^{-3}} \approx \underline{2{,}165 \cdot 10^9 \text{ capillaries}}$

c)

$Q = \frac{\pi R^4}{8\mu_{eff}} \frac{\Delta p}{L}$

$\Delta p = \frac{8\mu_{eff} L Q}{\pi R^4} \approx \underline{2672{,}69\,Pa}$

$\approx \underline{20{,}047\,mmHg}$

$Q = \frac{4{,}5\,l/min}{2 \cdot 10^9} = \frac{0{,}0045/60\ m^3/s}{2 \cdot 10^9} = 3{,}75 \cdot 10^{-14}\,m^3/s$

$L = 1{,}2 \cdot 10^{-3}\,m$

$R = 3{,}5 \cdot 10^{-6}\,m$

$\mu_{eff} = 0{,}0035\,Pa \cdot s$

$\Delta P_{tot} = 85\,mm\,Hg$

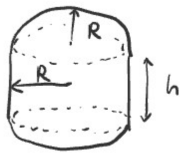
$\frac{\Delta p}{\Delta P_{tot}} \approx \underline{23{,}59\,\%}$

2 a)

$$V_{sphere} = \frac{4\pi R^3}{3} \Rightarrow R = \sqrt[3]{\frac{3V}{4\pi}} \approx \underline{2{,}86\,\mu m}$$

b)

minimum pore radius is there for R



$$V_{tot} = \frac{4\pi R^3}{3} + \pi R^2 h$$

$$A_{tot} = 4\pi R^2 + 2\pi R h$$

$$h = \frac{A_{tot} - 4\pi R^2}{2\pi R}$$

$$V_{tot} = \frac{4\pi R^3}{3} + \pi R^2 \left(\frac{A_{tot} - 4\pi R^2}{2\pi R}\right)$$

$$V_{tot} = \frac{4\pi}{3} R^3 + \frac{A_{tot}}{2} R - 2\pi R^3$$

$$-\frac{2\pi}{3} R^3 + \frac{A_{tot}}{2} R - V_{tot} = 0$$

$$R \approx \begin{cases} -6{,}21\,\mu m \\ \underline{1{,}65\,\mu m} \\ 4{,}55\,\mu m \end{cases} \Leftarrow \boxed{R^3 - \frac{3A_{tot}}{4\pi} R + \frac{3V_{tot}}{2\pi} = 0}$$

c) As shown in a) and b) non-spherical red blood cells can pass through smaller pores even if the Volume of the blood cells are the same. This ensures better blood flow and therefore oxygen transport.

3a)

$$\tau = \tau_y + \mu \dot{\gamma} \quad \Big| \quad \dot{\gamma} = \frac{dv}{dr}$$

$\tau(r) = \frac{r}{2} \frac{dp}{dx}$ ... steady flow

$$\Rightarrow \frac{r}{2} \frac{dp}{dx} = \tau_y + \mu \frac{dv}{dr}$$

$$\mu \, dv = \left(\frac{r}{2} \frac{dp}{dx} - \tau_y \right) dr$$

$$v = \frac{1}{\mu} \int \left(\frac{r}{2} \frac{dp}{dx} - \tau_y \right) dr$$

$$v = \frac{1}{\mu} \left(\frac{r^2}{4} \frac{dp}{dx} - \tau_y r + c \right)$$

$$v(R) = 0 = \frac{1}{\mu} \left(\frac{R^2}{4} \frac{dp}{dx} - \tau_y R + c \right)$$

$$\Rightarrow c = \tau_y R - \frac{R^2}{4} \frac{dp}{dx}$$

$$\Rightarrow \boxed{v(r) = \frac{1}{\mu} \left(\frac{1}{4} \frac{dp}{dx} (r^2 - R^2) - \tau_y (r - R) \right)}$$

b)

$$\tau_y = \frac{R_c}{2}\frac{dp}{dx} \quad \Big| \quad \hat{r} = \frac{r}{R} \quad ; \quad \hat{R}_c = \frac{R_c}{R}$$

$$\frac{dp}{dx} = \frac{2\tau_y}{R_c}$$

$$v(\hat{r}) = \frac{R\tau_y}{\mu}\left(\frac{1}{2\hat{R}_c}(\hat{r}^2 - 1) - \hat{r} + 1\right)$$

$$v(\hat{R}_c) = \frac{R\tau_y}{\mu}\left(\frac{1}{2\hat{R}_c}(\hat{R}_c^2 - 1) - \hat{R}_c + 1\right) = \frac{R\tau_y}{\mu}\left(1 - \frac{\hat{R}_c}{2} - \frac{1}{2\hat{R}_c}\right)$$

$$Q = \int_0^{R_c} v(\hat{R}_c)\, 2\pi r\, dr + \int_{R_c}^{R} v(r)\, 2\pi r\, dr = \quad \Big| \quad dr = R\, d\hat{r}$$

$$= 2\pi R^2 \left(\int_0^{\hat{R}_c} v(\hat{R}_c)\,\hat{r}\, d\hat{r} + \int_{\hat{R}_c}^{1} v(\hat{r})\,\hat{r}\, d\hat{r}\right) =$$

$$= \frac{2\pi R^3 \tau_y}{\mu}\left(\int_0^{\hat{R}_c}\left(1 - \frac{\hat{R}_c}{2} - \frac{1}{2\hat{R}_c}\right)\hat{r}\, d\hat{r} + \int_{\hat{R}_c}^{1}\left(\frac{1}{2\hat{R}_c}(\hat{r}^2 - 1) - \hat{r} + 1\right)\hat{r}\, d\hat{r}\right) =$$

$$= \frac{2\pi R^3 \tau_y}{\mu}\left(\frac{\hat{R}_c^2}{2}\left(1 - \frac{\hat{R}_c}{2} - \frac{1}{2\hat{R}_c}\right) + \frac{1}{2R_c}\left(\frac{\hat{r}^4}{4} - \frac{\hat{r}^2}{2}\right) - \frac{\hat{r}^3}{3} + \frac{\hat{r}^2}{2}\Big|_{\hat{R}_c}^{1}\right) =$$

$$= \frac{2\pi R^3 \tau_y}{\mu}\left(\cancel{\frac{\hat{R}_c^2}{2}} - \frac{\hat{R}_c^3}{4} - \cancel{\frac{\hat{R}_c}{4}} + \left(-\frac{1}{8\hat{R}_c} + \frac{1}{6} - \left(\frac{\hat{R}_c^3}{8} - \cancel{\frac{\hat{R}_c}{4}} - \frac{\hat{R}_c^3}{3} + \cancel{\frac{\hat{R}_c^2}{2}}\right)\right)\right)$$

$$= \frac{2\pi R^3 \tau_y}{\mu}\left(-\frac{1}{8\hat{R}_c} + \frac{1}{6} - \frac{\hat{R}_c^3}{24}\right)$$

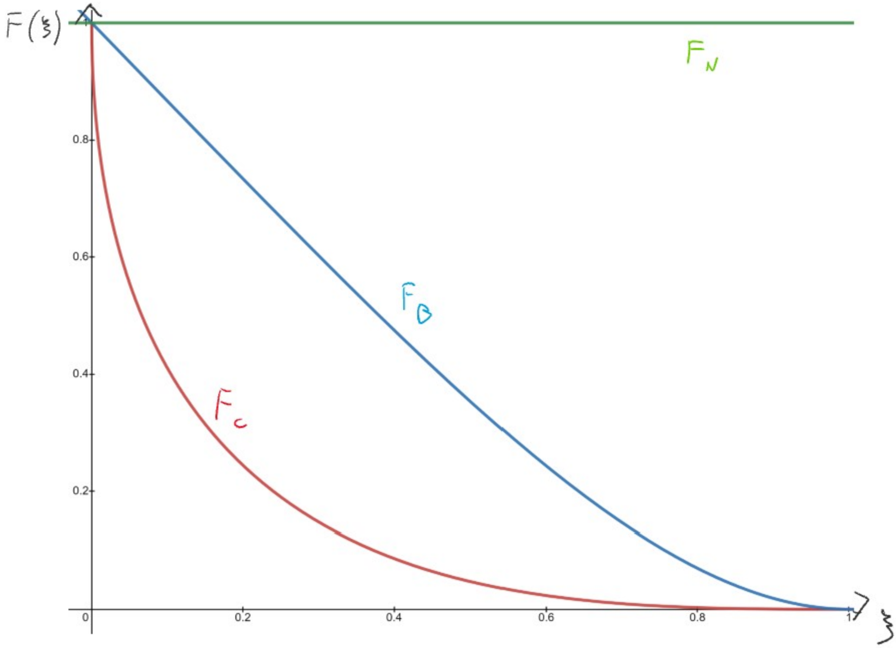
$$= -\frac{2\pi R^3 \tau_y}{8\mu \hat{R}_c}\left(\frac{1}{3}\hat{R}_c^4 - \frac{4}{3}\hat{R}_c + 1\right) \quad \Big| \quad \tau_y = \frac{R\hat{R}_c}{2}\frac{dp}{dx}$$

$$\boxed{Q = -\frac{\pi R^4}{8\mu}\frac{dp}{dx}F(\hat{R}_c); \quad F(\hat{R}_c) = \frac{1}{3}\hat{R}_c^4 - \frac{4}{3}\hat{R}_c + 1}$$

c) $F_B(\xi) = \frac{1}{3}\xi^4 - \frac{4}{3}\xi + 1$ ... Bingham

$F_c(\xi) = 1 - \frac{16}{7}\sqrt{\xi} + \frac{4}{3}\xi - \frac{1}{21}\xi^4$ .... Casson

$F_N(\xi) = 1$ ... Newtonian



It can be seen from the plot, that the flow rate of a Bingham-Fluid lies between the flowrate of a Newtonian-Fluid and a Casson-Fluid. (assuming all other variables to be equal) For small ξ ($\xi \to 0$) they all tend towards 1 ($F(\xi) \to 1$) and for $\xi \to 1$ the Bingham and Casson flow rate both approach zero ($F(\xi) \to 0$).

This difference is due to the difference in the shear stress