Computational Biomaterials and Biomechanics

Intravoxel analysis of clinical CT scan

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Contents

1 Introduction

This report discusses the application of computational techniques to analyze a clinical CT scan of a vertebrae. The basis is a dataset which assigns frequency counts to grey values within the range of 0 to 256. The report will discuss several steps necessary to gain insight on the longitudinal elastic modulus in the bone include plotting a histograms of grey values, generating a probability density function, identifying characteristic grey values, assessing vascular porosity, and creating a histogram for voxel-specific longitudinal elastic modulus in bony regions. All results and plots where generated using `MATLAB`.

2 Solution to the Tasks

2.1 Histogram of the Grey Values

First we plot the histogram of the CT dataset of the vertebrae. The result can be seen in Figure 1

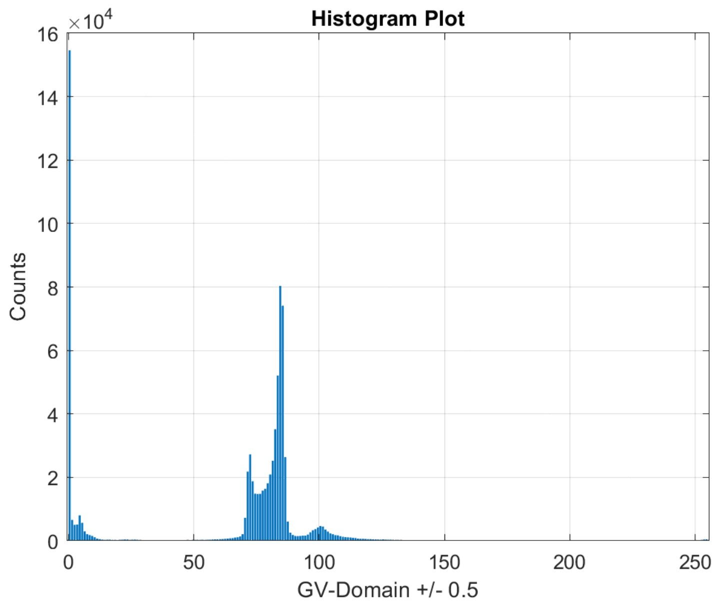

Figure 1: Histogram of the grey values

As we can see, besides the complex between the grey values from 50 to 150, which represent the vertebrae, there is also a big amount of grey values close to zero. This indicates that a large part of the scan must have been air.

2.2 Probability Density Function

The next step is to normalize the data by the total counts in the dataset, this gives us the probability density function, which can be seen in Figure 2.

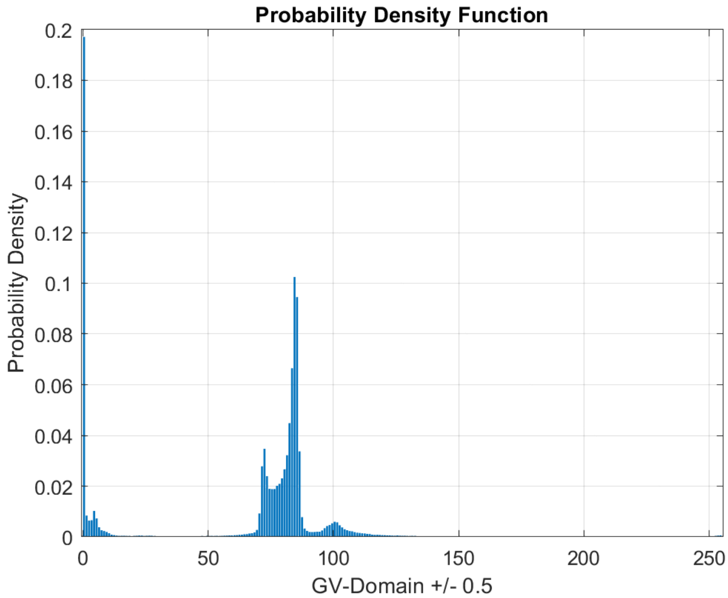

Figure 2: Probability Density Function of the grey values

2.3 Identifying Characteristic Grey Values

From this probability density function we can identify four characteristic grey values which represent the vertebrae:

1. $GV_{fat} = (72.5, 0.0347)$
2. $GV_{soft} = (84.5, 0.1025)$
3. $GV_{thr} = (91.5, 0.0019)$
4. $GV_{max} = (166.5, 0)$

GV_{fat} is identified, by finding the first maxima of the relevant section of the probability density function. It is associated with fat tissue. Likewise, GV_{soft} represents the second maxima and is associated with soft tissue. GV_{thr} is the dip (minima) after GV_{soft} and marks the start of the region of the probability density function which represents the bone. Finally, GV_{max} represents the highest grey value that can be attributed to the vertebrae. It is the first grey value after GV_{thr} which exhibits a probability of zero.

All the values where obtained using code and are plotted in Figure 3

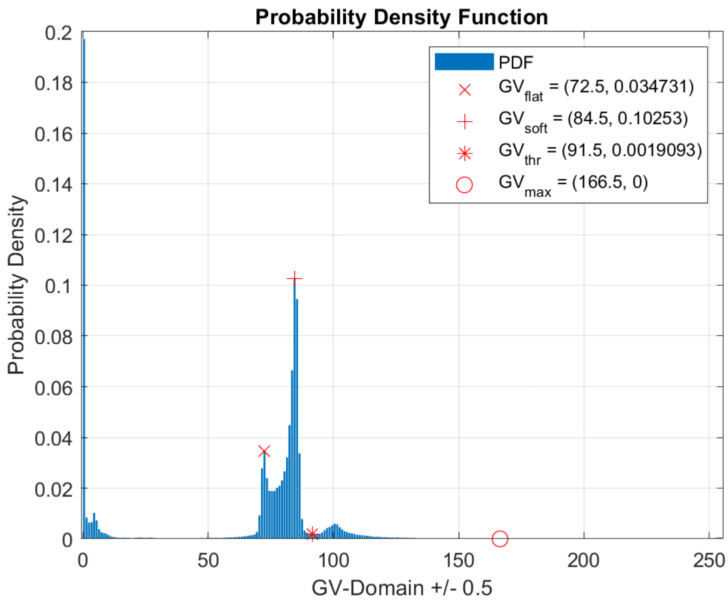

Figure 3: Probability Density Function with specific grey values describing the vertebrae

2.4 Vascular Porosity

Next we consider GV_{soft} as being relevant for the vascular porosity. We calculate the probability density function for the vascular porosity using the averaging rule:

$$GV = \sum_{i=1}^{N_C} GV_i \times f_i$$

combining this with voxel partitioning rules for the cortical shell we obtain the following system of equations:

$$\frac{l_1}{l_{voxel}} \times GV_{ev} + \left(1 + \frac{l_1}{l_{voxel}}\right) \times GV_{soft} = GV_{max}$$
$$\frac{l_2}{l_{voxel}} \times GV_{ev} + \left(1 + \frac{l_2}{l_{voxel}}\right) \times GV_{soft} = GV_{max-1}$$
$$l_1 + l_2 = l_{cort},$$

with GV_{ev} being the grey value associated with the extravascular tissue. [1]

Solving this system we get:

$$GV_{ev} = \frac{l_{voxel}}{l_{cort}} \times (GV_{max} + GV_{max-1} - 2GV_{soft}) + GV_{soft}$$

GV_{max-1} is defined as the densest neighbor of GV_{max-1}. Its value is taken from literature as $GV_{max-1} = 156$ Similarly we take the values for the width of the cortical shell $l_{cort} = 0.23\ mm$ and the width of the voxel $l_{voxel} = 0.324mm$ from literature [1] Calculating this for our previously found GV_{max}, and GV_{soft} we get $GV_{ev} = 300.53$

Finally, we are interested in the vascular porosity ϕ_{vas} of the bony region, which can be obtained with:

$$\phi_{vas} = \frac{\mu_{macro} - \mu_{ev}}{\mu_{H2O} - \mu_{ev}}$$

We can assume a linear relation between the grey value and the attenuation coefficient and therefore use the previously found grey value of the extravascular tissue directly. Additionally, we can replace μ_{H2O} with GV_{soft}, since we can assume similar grey values for them. [1] Since we are only interested in the vascular porosity of the bone, we will use the grey values in the region between GV_{thr} and GV_{max}, which we assume to be the bony part. Doing all this we get:

$$\phi_{vas} = \frac{GV_{bone} - GV_{ev}}{GV_{soft} - GV_{ev}}$$

To get the probability density function for the vascular porosity we multiply the result with 100. The final plot can be seen in Figure 4.

2.5 Longitudinal Elastic Modulus

Lastly we want to find a histogram for the voxel-specific longitudinal elastic modulus in the bony region of the vertebrae. For this we utilize a function

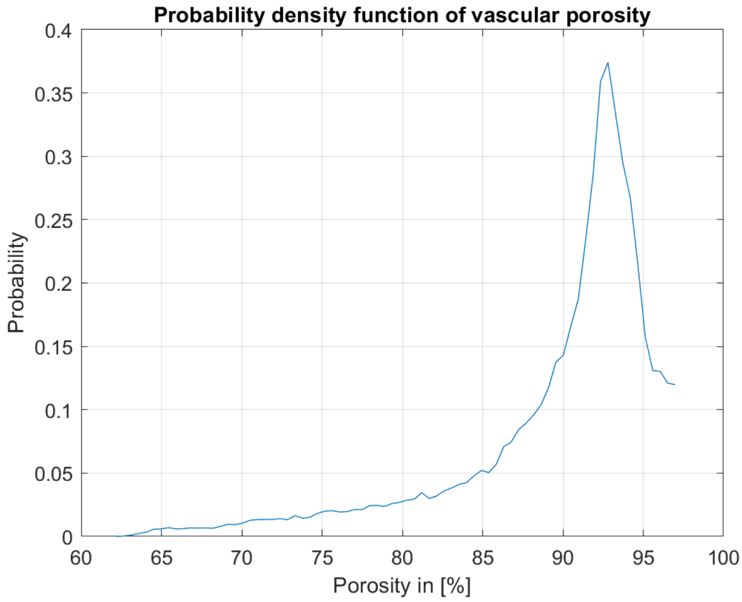

Figure 4: Probability Density Function with specific grey values describing the vertebrae

which calculates the stiffness matrices for a given porosity value. The result was inverted and the longitudinal Young's modulus was extracted from the matrix.

$$E_{macro,3} = \frac{1}{C^{-1}_{macro,3333}}$$

Doing this for all the previously obtained vascular porosities, we finally get the plot shown in Figure 5

3 Conclusion

The grey values obtained in the Section 2.3 are very similar to what was found in [1]:

Source	GV_{fat}	GV_{soft}	GV_{thr}	GV_{max}
Our Results	72.5	84.5	91.5	166.5
Blanchard et al.	72	84	93	164

This is expected considering the same anatomical structure (vertebrae) wa analyzed.

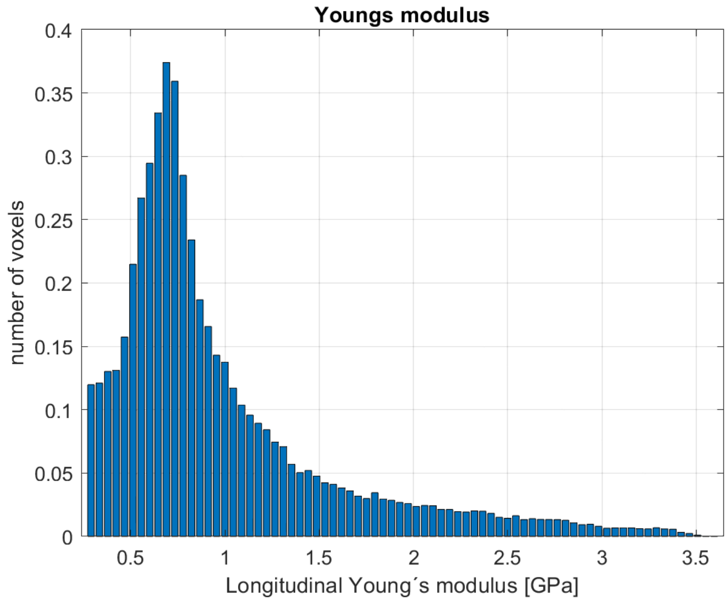

Figure 5: Probability Density Function with specific grey values describing the vertebrae

In Section 2.4, we found the grey value of the extravascular tissue to be $GV_{ev} = 300.53$. This again it very close to the result from Blanchard et al. [1], where GV_{ev} was found to be 298. This grey value was then used to further calculate the vascular porosity of the bony region of our sample. The calculated porosity spans between 62% and 97%. This is because the calculations were performed on a region limited to the grey values representing the bone, so between GV_{thr} and GV_{max}.

The Young's modulus plot was generated using the provided function `hom_exvas_to_macro`. The input porosity values were those calculated in Section 2.4 for the bony region. Therefore, the Young's modulus values plotted alongside their corresponding voxel counts pertain solely to the bony region within the CT scan. The observed modulus values range from approximately 0.2 GPa to 3.5 GPa.

4 References

[1] Romane Blanchard, Claire Morin, Andrea Malandrino, Alain Vella, Zdenka Sant, and Christian Hellmich. Patient-specific fracture risk assessment of vertebrae: a multiscale approach coupling x-ray physics and continuum micromechanics. *International journal for numerical methods in biomedical engineering*, 2016.