# CS 5110/6110 Program 4: Vickrey Clark Grove Auctions `

You want to sell weekly placements of ads on your website. You have three slots, call them S1, S2, and S3, where you will display the ads. From past experience, you know that the ad in S1 will get 500 clicks per week, S2 will get 300 clicks per week, and S3 will get 100 clicks per week. (For simplicity, we'll assume you don't want to run the same ad in two different spots, that the number of clicks per week is constant and independent of the ad, and that you're running the same ad in each spot for the entire week.)  
  
Suppose you have found ten advertisers, A1, A2, ... A10, who want to advertise on your site. You decide to use VCG to determine whose ads are placed in each slot and what rate to charge them.  
  
You ask each advertiser to make a bid of how much they are willing to pay per click. We will assume that they are willing to pay for any of the slots. The bids you get back are:  
  
A1: $.50  
A2: $.40  
A3: $.30  
A4: $.20  
A5: $.10

A6: $.09

A7: $.09

A8: $.08

A9: $.08

A10: $.05

For convenience, let's call Pn the price that advertiser An is willing to pay". For example, P2 = .40. We'll also call C1 = 500, C2 = 300, and C3 = 100, the number of clicks for each ad slot (S1,S2, S3).

Part 1

Repeat for each of the three bid distributions shown in the starter code.

* Generate 10 bid prices (for A1..A10) from the given distribution
* Under VCG, the rule for assigning the winners is what you would expect: you give the better slots to the better bids. Thus, you would assign slots S1, S2, and S3 to advertisers A1, A2, and A3 respectively. Other ads would not appear at all. Let R1, R2, R3 be the rate charged for each slot (per click).
* Compute the social welfare of the allocation which is where Wi is the price the agent who got slot i was willing to pay. In our case, Wi=Pi because the bids are ordered.  
    
  The rules for determining what each winner pays are more complicated. Think of each winner as "displacing" the bidders below it. For example, if A3 hadn't bid, A4 would have taken the slot S3 instead of getting nothing. Thus A3’s bid represents a loss of C3 clicks to A4, the displacement deprives A4 of clicks worth C3 \* P4 = 100 \* $.20 = $20. A3 pays $20, the "cost" of its displacement for the C3 clicks  
    
  Likewise, if A2 hadn't bid, A4 would have taken S3 as before, but additionally A3 would have taken S2. Since being in S2 would result in (C2 - C3) = 200 more clicks than slot S3, A3 is deprived of clicks worth 200 \* $.3 = $60. We have already calculated the cost of depriving A4 of S3 to be $20 above, so the amount A2 pays is $60 + $20 = $80 for the C2 clicks.  
    
  Algebraically, you start to see a pattern:  
    
  A3 pays P4 \* (C3 - 0)   
  = .2 \* (100 - 0)   
  = .2 \* 100  
  = 20 (R3 = 20/C3 = .2)  
    
  A2 pays P3 \* (C2 - C3) + P4 \* (C3 - 0)   
  = .3 \* (300 - 100) + .2 \* (100 - 0)   
  = .3 \* 200 + .2 \* 100  
  = 60 + 20  
  = 80 (R2=80/C2 = .2667)  
    
  A1 pays P2 \* (C1 - C2) + P3 \* (C2 - C3) + P4 \* (C3 - 0)   
  = .4 \* (500 - 300) + .3 \* (300 - 100) + .2 \* (100 - 0)  
  = .4 \* 200 + .3 \* 200 + .2 \* 100  
  = 80 + 60 + 20  
  = 160 (R1=160/C1 = .32)  
    
  Other ads don't run, so they pay nothing.

So to compute social welfare:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | Ci | Wi | Ri | Ci(Wi-Ri) |
|  | 500 | 0.5 | 0.32 | 90 |
|  | 300 | 0.4 | 0.2667 | 39.99 |
|  | 100 | 0.3 | 0.2 | 10 |
|  |  |  |  |  |
|  |  |  | TOTAL | 139.99 |

Consider controlling the following parameters:

1. Number of bidders and the value per click for each bidder.
2. Closeness of bids (or pattern of bids). For example, (.5,.4,.3, .2, .1) or (.5, .45, .2, .1, .1)
3. Number of clicks expected for each slot (in decreasing order)
4. Number of advertising slots

Come up with an interesting parameter combination.

In the video you turn in, demonstrate what you learned.

Part 2

Suppose the bidders decide they don’t trust the mechanism and would rather pay a first price bid. The bidders can’t bid their true evaluation, as their social welfare would be zero (if they pay exactly what it is worth).They will need to bid strategically, knowing the number of bidders and the type of bidder distribution (without knowing specific bid values). No bidder will bid more than their valuation. Explain the bidding strategy used for each distribution. Show the actual valuation, the amount bid, and the social welfare of each case.

Hint:

I found it easier to think of it this way. Using Vickrey Clark Groves, the idea is that a player should pay the difference of what the system would make “Without them” – “what the system makes from the others pay the player does bid”

For player 1, when he doesn’t play, the system makes:

C1\*P2 + C2\*P3 + C3\*P4 (as everybody shifts up to the better slot)

When Player 1 does play, the system makes:

C1\*P1 + C2\*P2 + C3\*P3

The amounts shown in red are what the system makes from the others when player 1 bids.

Vickrey Clarke Grove says to subtract them:

C1\*P2 + C2\*P3 + C3\*P4 - C2\*P2 + C3\*P3 = (C1-C2)\*P2 + (C2-C3)\*P3 + C3\*P4

which is exactly what we got before.