# Game Physics Notes 01

CSCI 321

WWU

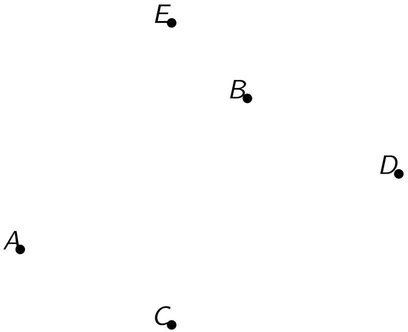
April 7, 2016

# numpy

A nice python library for dealing with mathematical vectors and matrices.

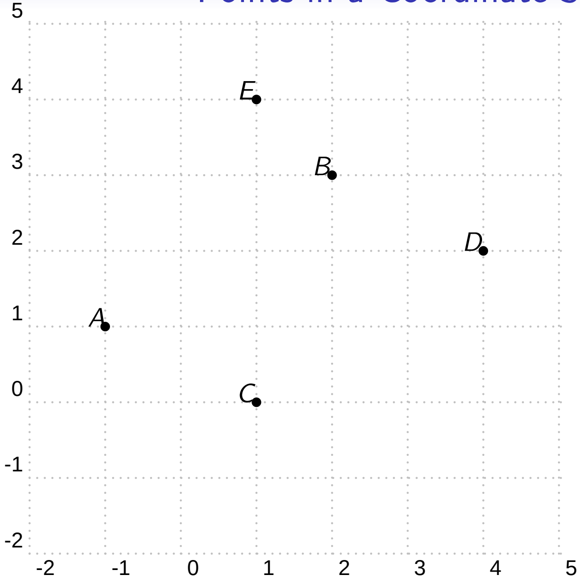
```
>>> import numpy
>>> x = (1,2,3)
>>> y = (3,2,1)
>>> 3*x
(1, 2, 3, 1, 2, 3, 1, 2, 3)
>>> x + y
(1, 2, 3, 3, 2, 1)
>>> xvec = numpy.array(x)
>>> yvec = numpy.array(y)
>>> 3*xvec
array([3, 6, 9])
>>> xvec + yvec
array([4, 4, 4])
>>> numpy.dot(xvec, yvec)
10
```

# Points in Space



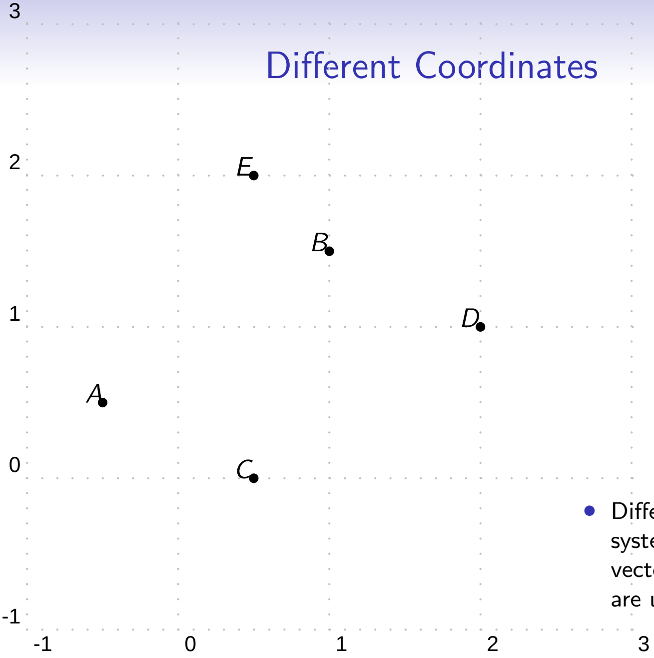

- Points exist in space without a coordinate system.
- But with only labels it's difficult to compute with them.

# Points in a Coordinate System



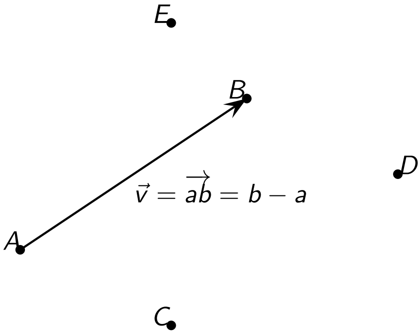

- A coordinate system gives positions to points.
- Relates *points* to *tuples of numbers*, or *mathematical vectors*.
- However, points are *not* vectors!

# Different Coordinates



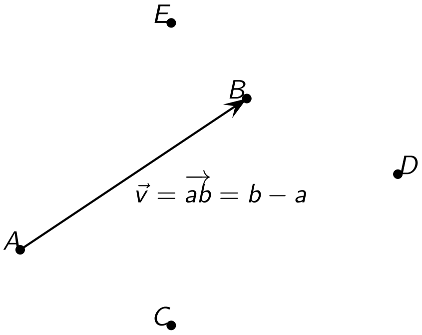

- Different coordinate systems give different vectors, but the *points* are unchanged.

# Physical vectors are differences between points.



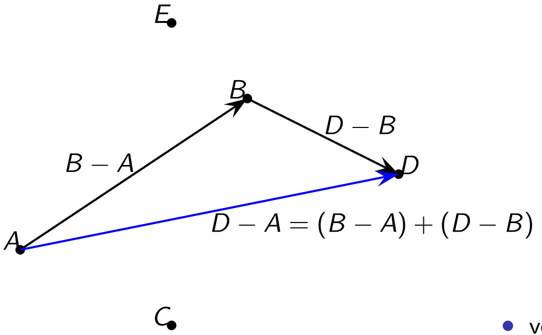

- Physical vectors are *not* mathematical vectors.
- But given a coordinate system, you can represent the points as mathematical vectors, and then subtract.
- But these mathematical vectors are not the same thing!
- Different coordinate systems will give you different mathematical vectors for the *same* physical vector.

# Points and vectors are not the same thing!



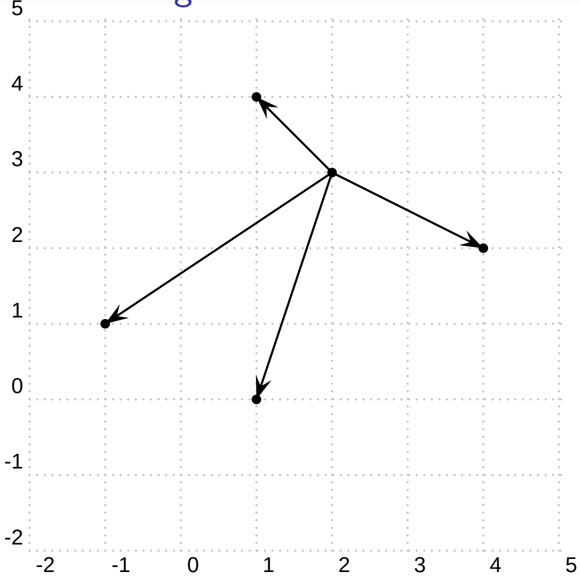

- A point is a position in space.
- A vector has a magnitude and a direction.
- The vector from $a$ to $b$ is the point difference: $\vec{v} = \overrightarrow{ab} = b - a$
- You can add two vectors, but you *cannot* add two points!
- You can add points and vectors: $b = a + \vec{v} = a + (b - a)$

# Vector Addition



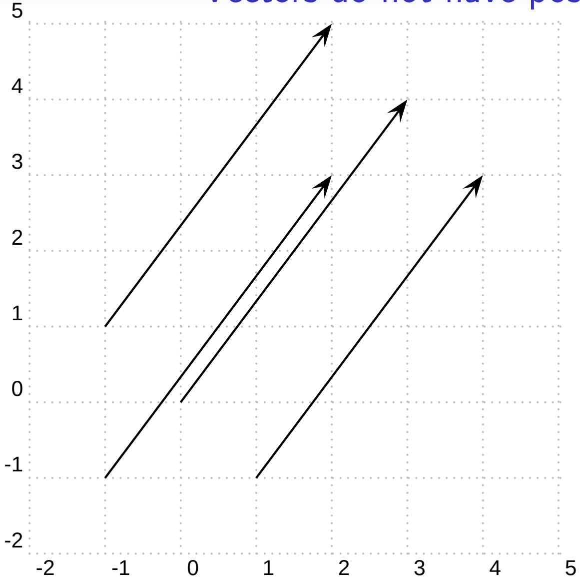

- vector + vector = vector
- point + vector = point
- point - point = vector
- point + point = *nonsense*

# Coordinates give mathematical vectors to physical vectors.



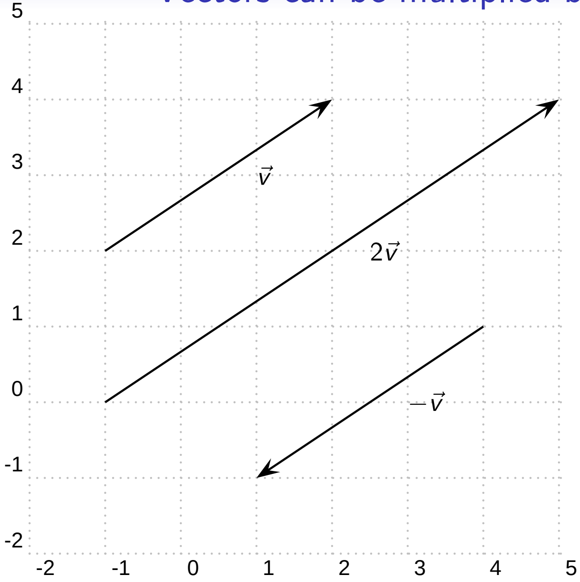

- Subtract the components.
- $(1, 4) - (2, 3) = (-1, 1)$
- $(-1, 1) - (2, 3) = (-1, -2)$
- $(1, 0) - (2, 3) = (-1, -3)$
- $(4, 2) - (2, 3) = (2, -1)$
- Note: we subtract two *points* to get a *vector*.

# Vectors do not have positions



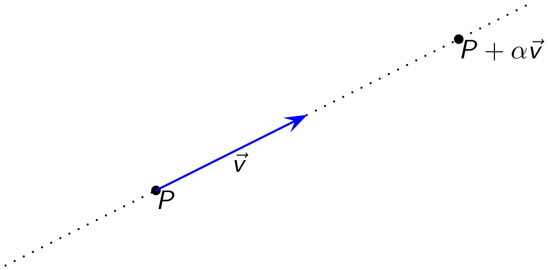

- Each of these vectors is the *same* vector.

# Vectors can be multiplied by scalars



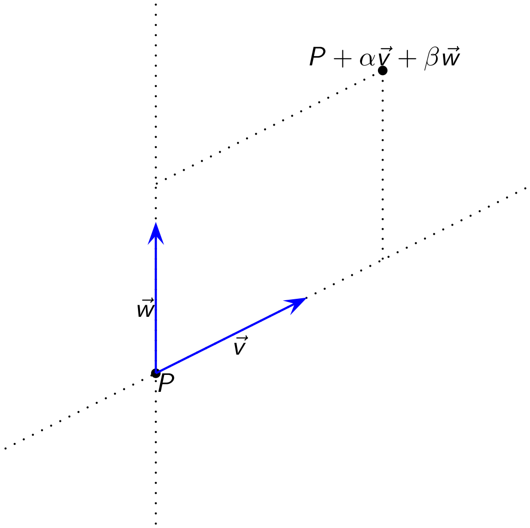

- Multiplication is repeated addition.

# Lines



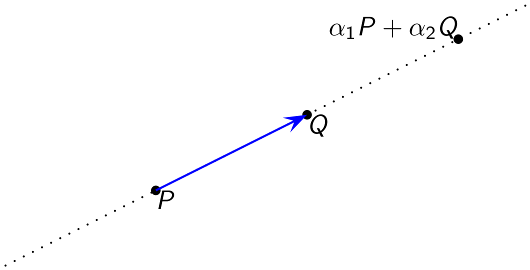

- The line through $P$ in the direction $v$ is the set of all points $P + \alpha v$ for some $\alpha \in \mathbb{R}$

# Planes (in 3 dimensions)



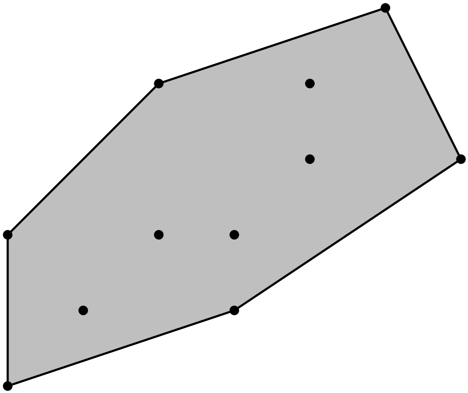

- The plane through $P$ spanned by $v$ and $w$ is the set of all points $P + \alpha v + \beta w$ for some $\alpha, \beta \in \mathbb{R}$

# Affine sums



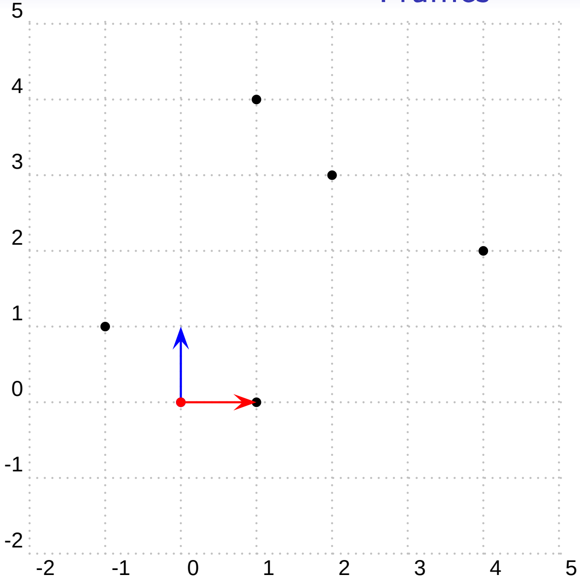

- $P + \alpha(Q - P)$
  $= (1 - \alpha)P + \alpha Q$
  $= \alpha_1 P + \alpha_2 Q$
- $\alpha_1 + \alpha_2 = 1$
- Think of each point as the vector from some arbitrary point:
  $P \equiv P - O$
  $Q \equiv Q - O$
- If $0 \le \alpha_i$ then the point is between $P$ and $Q$.

# Convex hull



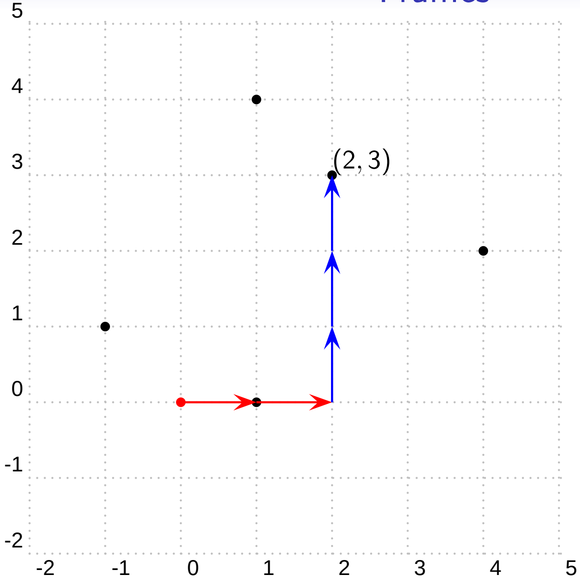
- $P = \alpha_1 P_1 + \alpha_2 P_2 + \ldots + \alpha_n P_n$
- $\alpha_1 + \alpha_2 + \ldots + \alpha_n = 1$
- $0 \leq \alpha_i$

# Frames



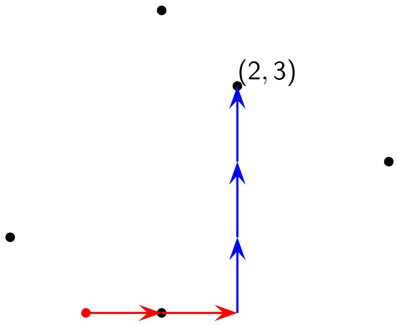

- A coordinate system can be thought of as a single point, the *origin*, and a set of *basis vectors*.
- Such a set is called a *frame*.

# Frames



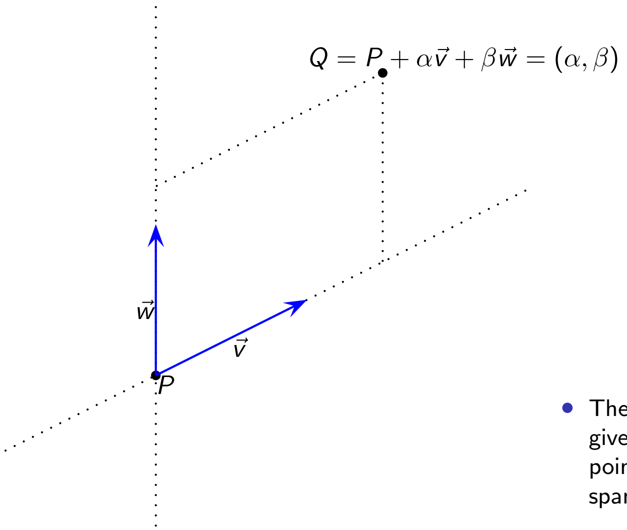

- The coordinates of a point are how many copies of the basis vectors you have to add to the origin.

# Frames



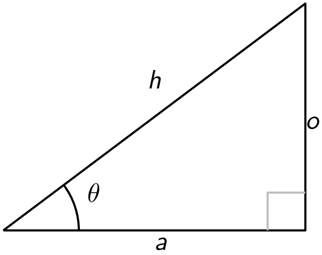

- Note that a frame gives sense to coordinates without anything other than points and vectors.
- A coordinate system is nothing more than an origin and a set of basis vectors, a *frame*.
- An *orthonormal* frame is one in which all the vectors are of unit length and perpendicular to each other.

# Frames do not have to be orthonormal



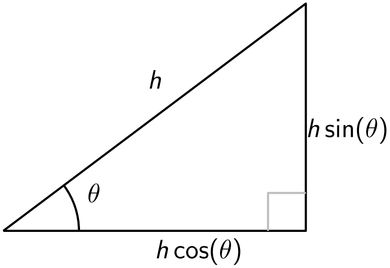

- The frame $F = (P, \vec{v}, \vec{w})$ gives coordinates to any point in the plane it spans.

# Trigonometry



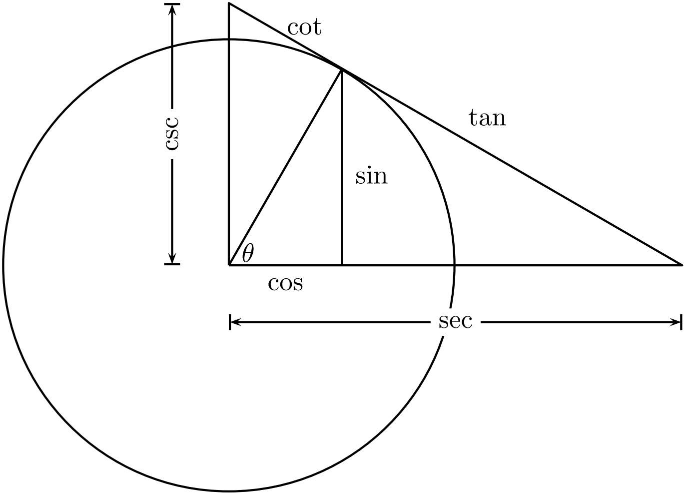

- $\sin(\theta) = o/h$
- $\cos(\theta) = a/h$
- $\tan(\theta) = o/a$

# Trigonometry

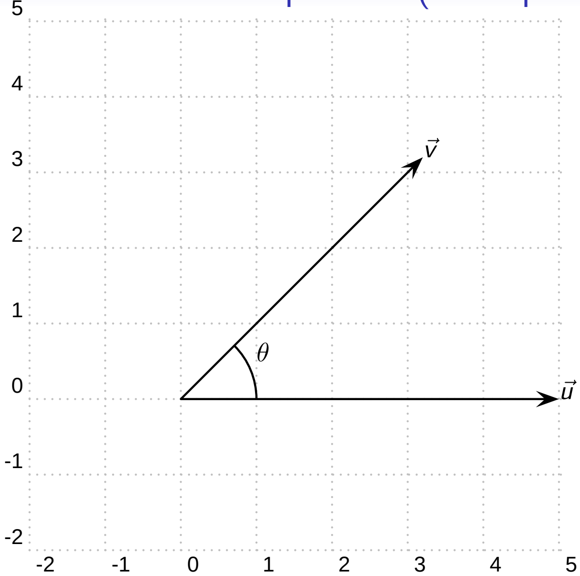



- $\tan(\theta) = \sin(\theta)/\cos(\theta)$

# Trigonometry

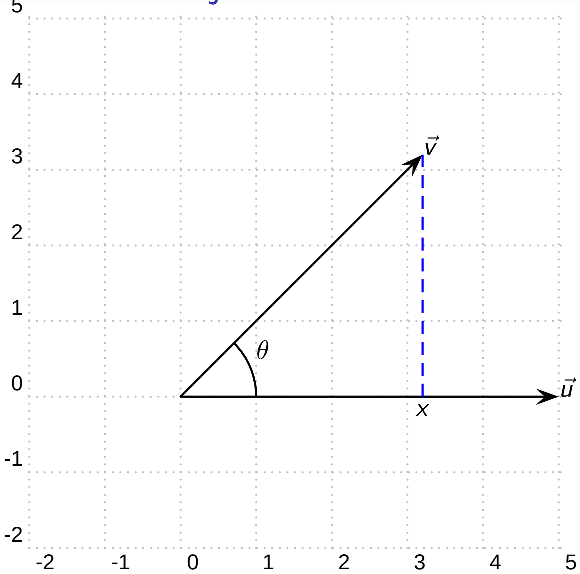


# Dot product (Inner product)



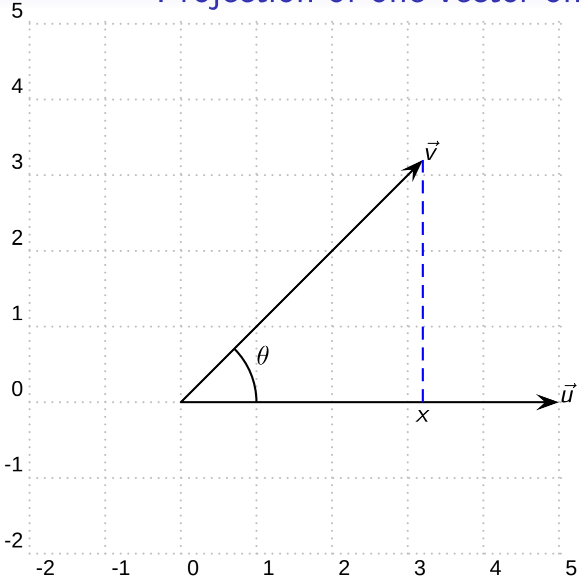

- $u \cdot v = \cos(\theta)|u||v|$
- $|u| = \sqrt{u \cdot u}$

# Projection of one vector on another



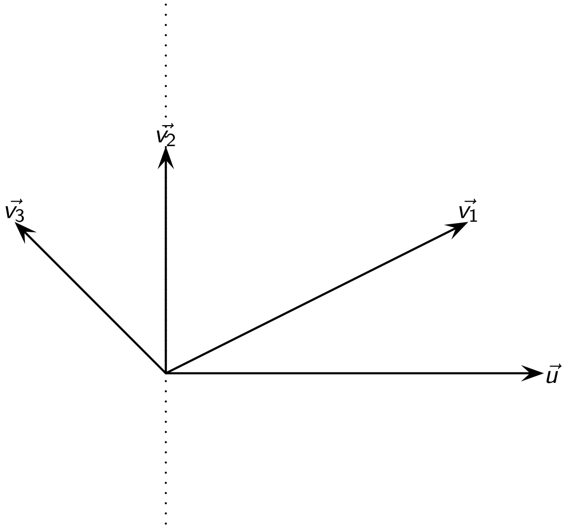

- What is $x$?

# Projection of one vector on another



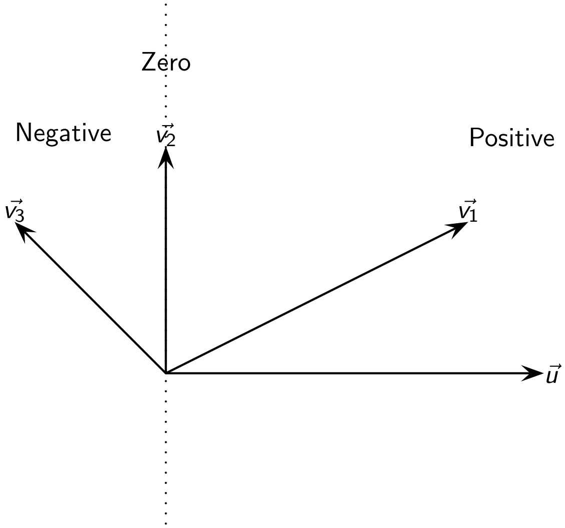

- $x = \cos(\theta)|v|$
- $x = u \cdot v/|u|$

# Same direction, opposite direction




- What is the sign of $u \cdot v_i$?

# Same direction, opposite direction



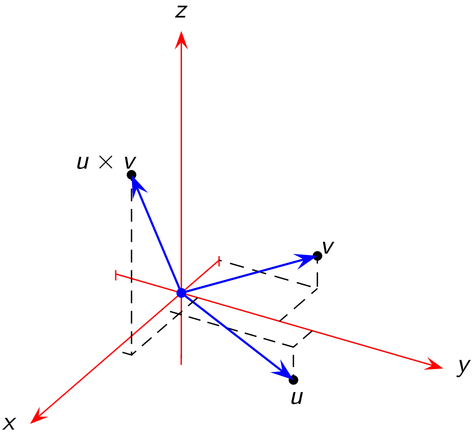

- Sign of $u \cdot v_i$

# *AMAZING* theorem about the dot product.

- In any coordinate system whatsoever:

$$\begin{aligned}
u \cdot v &= (u_x, u_y, u_z) \cdot (v_x, v_y, v_z) \\
&= u_x v_x + u_y v_y + u_z v_z \\
&= [u_x \ u_y \ u_z] \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \\
&= u^T v
\end{aligned}$$

# Cross product (vector product)




- A vector at right angles to $u$ and $v$.
- $u \times v =$ $(u_2 v_3 - u_3 v_2,$ $u_3 v_1 - u_1 v_3,$ $u_1 v_2 - u_2 v_1)$
- Mnemonic:

$$u \times v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

- $|u \times v| = |u||v| \sin(\theta)$