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Exponential Stability Analysis and Controller Design for LTI Positive System with Controller Failure

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Abstract— **This paper studies exponential stability and fault tolerant control for linear time-invariant (LTI) positive system with controller failure. Through dwell time (DT) and mode-dependent average dwell time (MDADT) approach, sufficient conditions of globally uniformly exponential stability (GUES) are proposed. We treat the mode changing between controller work and failure as a switch. The main idea is using switching behavior to compensate the increment of Lyapunov function (LF) produced by controller failure, introducing the multiple linear copositive Lyapunov function (MLCLF) method to obtain less conservative results. By solving the linear programming problem, the switched positive system can be GUES under the constraint of DT and MDADT. On this basis, a fault tolerant controller is designed. Numerical examples are provided to show the effectiveness of the results.**

Index Terms— **Positive linear system; Mode-dependent average dwell time; Dwell time; Controller failure; Multiple linear copositive Lyapunov function.**

I. INTRODUCTION

Positive systems are the systems whose state variable are always required to be positive. Since a great number of applications in many areas, such as economy, biology and communication [1], it has received extensive attention from scholars in recent years. Stability is one of the most significant indexes to evaluate positive system performance. In all of the approaches of stability analysis, linear copositive Lyapunov function (LCLF) method is one of the most effective tools on positive linear system that have been widely used [2]. After that, multiple linear copositive Lyapunov function method (MLCLF) is put forward [3]. Via this method, the conservatism can be greatly reduced.

In practical engineering, controller failure phenomenon is inevitable and often occurs, mainly due to some environmental factors [4] or human factors. When controller failure happens, the controller signal cannot reach the actuator efficiently, making the system turn to an unstable open-loop system. It may cause the entire system to become unstable. So how to maintain and analyze the stability of system when there exist unstable parts of it? This bring us to the switching system approach. We can model the system with controller failure as a switching system. Regarding the closed-loop portion of system as one subsystem, and the open-loop failure portion as another. Switching behavior occurs when controller failure beginning or ending. Dwell time switching belongs to constrained switching, which is one of the switching rules. This concept is derived by [5] and let the time interval between two consecutive switching time no larger or smaller than a given constant. In 1999, average dwell time (ADT) approach was proposed by [6]. Comparing with DT, this method is able to allow some successive switching intervals less than or more than a given constant. Besides, based on ADT approach, MDADT was formulated by [7] to release average dwell time restriction, which can get less conservative results.

So far in stability analysis, there have been a large number of conclusions on the study of controller failure. [8] analyzed stability of LTI system. By confining unavailable rate of controller and ADT, exponential stability condition is obtained; [9] set minimum DT for controller working interval and maximum DT for controller failure interval, namely, it made the controller available time long enough to compensate the unavailable part to stabilize the system. Similarly, based on the minimum and maximum dwell time to maintain stability of system, [10] constructs a LF which is always decreasing throughout the system running time. Furthermore, [11] [12] and [13] separately discussed the case of L_2 gain, dynamical output feedback and time-varying delay. Nevertheless, in all of the above studies, none of which has investigated controller failure problem on positive system. For some purpose on control system such as economics, people artificially suspend the function of controller when necessary. It is this kind of system model that attracts us to investigate the topic. Inspired by the theory of switching systems stability analysis in [14], using MLCLF, the stability analysis and the controller design of LTI positive system are derived.

This paper investigates exponential stability of LTI positive system with controller failure under minimum DT and MDADT switching. MLCLF is applied to obtain less conservative results. State-feedback controller with time-varying controller gain is also designed to stabilize the system.

The organization of this article is as follows. In section 2, controller failure problem and system model are described with some definitions and lemmas. In section 3, stability analysis on LTI positive system is derived by using MLCLF with DT and MDADT approach. In section 4, instead of the constant controller gain that we have known in section 3, time-varying controller gain is obtained by solving LP problem. In section 5, two examples are provided to show the effectiveness of our results. Finally, the conclusion is given in section 6.

Notations: R represents the real numbers. R^n and R^n_+ stands for the set of n-dimensional vectors and n-dimensional

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positive vectors of real entries. N denotes positive integer. $\|\bullet\|$ denotes the Euclidean norm. A^T denotes the transport of A. $A \geq 0$ (≤ 0) denotes all elements of matrix are nonnegative (nonpositive).

II. PROBLEM DESCRIPTION

Consider the linear time-invariant (LTI) system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in R_+^n$ is the state vector, $u(t) \in \mathrm{R}^m$ is the control input, A, B are known constant matrices with the appropriate dimensions. The control input is formulated as

$$u(t) = Kx(t) \tag{2}$$

where the state feedback gain K is the constant matrix which has been known.

Because the controller fails intermittently, it makes the system transform between two modes during the entire operation. Thus, we come up with a switching system approach, treating these two phases model as two subsystems, the switching occurs when mode changes. Let t_k, $k \in N$ be the system switching instant. Without loss of generality, we consider the controller works well from the beginning. Then we separate the operation of LTI system into two parts: During the time interval $\Gamma_{1,k} = [t_{2k}, t_{2k+1}),\ k \in N$ the controller works; during the time interval $\Gamma_{2,k} = [t_{2k+1}, t_{2k+2}),\ k \in N$ the controller fails. When the controller failure happens, the control input changing to zero. Thus, system (1) could be depicted by

$$\dot{x}(t) = A_{\sigma(t)}x(t) \tag{3}$$

where $\sigma(t) \in \{1, 2\}$ represents the switching signal, which is right-continuous piecewise constant function. When $\sigma(t)$=1, the controller works; when $\sigma(t)$= 2, the control input changes to zero. A_1, A_2 are given as

$$A_1 = A + BK,\ A_2 = A$$

For the sake of analyzing the stability of system (3) and designing the controller, the following definitions are necessary to introduce.

Definition 1 [15] System (1) is said to be positive, if any initial condition $x(0) \geq 0$, implies that the corresponding trajectory $x(t) \geq 0$ for any $t \geq 0$.

Lemma 1 [16] System (1) is positive if and only if A is Metzler matrix, of which the off-diagonal entries are nonnegative, and $B \geq 0$.

Lemma 2 [17] Let A be Metzler. Then the following condition are equivalent

(i)A is Hurwitz.

(ii) There exists some vector $v \in \mathrm{R}_+^n$ with $Av > 0$.

Definition 2 [18] For any $t_2 \geq t_1 \geq 0$, let $N_{\sigma_i}(t_2, t_1)$ denote the switching numbers that the ith subsystem is activated, and $T_i(t_2, t_1)$ denote the total running time of the ith subsystem during the interval (t_1, t_2). N_{0_i} denotes the chatter bounds. If there exist $\tau_{a_i} > 0$ such that

$$N_{\sigma_i}(t_2, t_1) \leq N_{0_i} + \frac{t_2 - t_1}{\tau_{a_i}}$$

Then we say that $\tau_{a_i} > 0$ the slow MDADT of switching signal $\sigma(t)$.

Definition 3 [18] For any $t_2 \geq t_1 \geq 0$, let $N_{\sigma_i}(t_2, t_1)$ denote the switching numbers that the ith subsystem is activated, and $T_i(t_2, t_1)$ denote the total running time of the ith subsystem during the interval (t_1, t_2). N_{0_i} denotes the chatter bounds. If there exist $\tau_{a_i} > 0$ such that

$$N_{\sigma_i}(t_2, t_1) \geq N_{0_i} + \frac{t_2 - t_1}{\tau_{a_i}}$$

Then we say that $\tau_{a_i} > 0$ the fast MDADT of switching signal $\sigma(t)$.

Definition 4 [19] The equilibrium $x^* = 0$ of system (3) is globally uniformly exponentially stable (GUES) under switching signal if the solution of system (3) satisfies

$$\|x(t)\| \leq ke^{-\gamma(t-t_0)} \|x(t_0)\|, \forall t \geq t_0$$

for constants $k > 0, \gamma > 0$ with any initial conditions $x(t_0) \geq 0$.

We establish time-scheduled copositive Lyapunov function as follows

$$V(t) = x^T(t)v_{\sigma(t)}(t) \tag{4}$$

where $\sigma(t)$ is switching signal as described above. The form of $v_{\sigma(t)}(t)$ are given below.

Define the time interval between two consecutive switches is not less than a forced dwell time which is specified in advance. Specifically, ψ_s is defined to be minimum dwell time of $\Gamma_{1,k}$. ψ_u is defined to be minimum dwell time of $\Gamma_{2,k}$.

For $\Gamma_{1,k}$, divide ψ_s into H_1 segments averagely, and let $\theta_{1,h} = h\psi_s/H_1, h = 0, 1, ..., H_1 - 1$, then

$$v_1(t) = \begin{cases} (1-\alpha_1)v_{1,h} + \alpha_1 v_{1,h+1}, \\ \qquad t \in [t_{2k} + \theta_{1,h}, t_{2k} + \theta_{1,h+1}) \\ v_{1,H_1},\ t \in [t_{2k} + \psi_s, t_{2k+1}) \end{cases} \tag{5}$$

where $\alpha_1 = \frac{t-t_{2k}-\theta_{1,h}}{\psi_s/H_1}$, $v_{1,h}, v_{1,H_1} \in R_+^n$.

For $\Gamma_{2,k}$, divide ψ_u into H_2 segments averagely, and let $\theta_{2,h} = h\psi_u/H_2, h = 0, 1, ..., H_2 - 1$, then

$$v_2(t) = \begin{cases} (1-\alpha_2)v_{2,h} + \alpha_2 v_{2,h+1}, \\ \qquad t \in [t_{2k+1} + \theta_{2,h}, t_{2k+1} + \theta_{2,h+1}) \\ v_{2,H_2}, t \in [t_{2k+1} + \psi_u, t_{2k+2}) \end{cases} \tag{6}$$

where $\alpha_2 = \frac{t-t_{2k+1}-\theta_{2,h}}{\psi_u/H_2}$, $v_{2,h}, v_{2,H_2} \in R_+^n$.

III. STABILITY ANALYSIS

In this section, we assume that system matrix A is Metzler, namely, the open-loop system is positive. There exist controller gain K can stabilize the system which is a given constant vector, and the system with controller is also positive. Based on the Lyapunov function we propose, the theorem is presented as follows.

Theorem 1 Consider the positive system (3), for given scalars $0 < \mu_1 < 1, \mu_2 > 1, \lambda_1 > 0, \lambda_2 > 0, H_1, H_2 \in N\backslash\{0\}$, if there exist a set of vectors $v_{1,h} \in R_+^n, h = 0, 1, ..., H_1 - 1, v_{2,h} \in R_+^n, h = 0, 1, ..., H_2 - 1$ such that

$$A_1^T v_{1,h} + \frac{v_{1,h+1} - v_{1,h}}{\Delta_1} + \lambda_1 v_{1,h} < 0 \quad (7)$$

$$A_1^T v_{1,h+1} + \frac{v_{1,h+1} - v_{1,h}}{\Delta_1} + \lambda_1 v_{1,h+1} < 0 \quad (8)$$

$$A_1^T v_{1,H_1} + \lambda_1 v_{1,H_1} < 0 \quad (9)$$

$$A_2^T v_{2,h} + \frac{v_{2,h+1} - v_{2,h}}{\Delta_2} - \lambda_2 v_{2,h} < 0 \quad (10)$$

$$A_2^T v_{2,h+1} + \frac{v_{2,h+1} - v_{2,h}}{\Delta_2} - \lambda_2 v_{2,h+1} < 0 \quad (11)$$

$$A_2^T v_{2,H_2} - \lambda_2 v_{2,H_2} < 0 \quad (12)$$

$$v_{1,0} - \mu_1 v_{2,H_2} \leq 0 \quad (13)$$

$$v_{2,0} - \mu_2 v_{1,H_1} \leq 0 \quad (14)$$

where $\Delta_1 = \psi_s/H_1,\ \Delta_2 = \psi_u/H_2$. Then the system (3) is GUES with MDADT

$$\begin{cases} \tau_s \geq \tau_s^* = \frac{\ln \mu_2}{2\lambda_1} \\ \tau_u \leq \tau_u^* = -\frac{\ln \mu_1}{2\lambda_2} \end{cases}. \quad (15)$$

Proof When $t \in \Gamma_{1,k}$, the controller works. Consider the time interval $t \in [t_{2k} + \theta_{1,h}, t_{2k} + \theta_{1,h+1})$, by (7) and (8), we have

$$\begin{aligned} &\dot{V}(t) + \lambda_1 V(t) \\ &= \dot{x}^T(t)v_1(t) + x^T(t)\dot{v}_1(t) + \lambda_1 x^T(t)v_1(t) \\ &= x^T(t)A_1^T v_1(t) + \lambda_1 x^T(t)v_1(t) + x^T(t)\tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1} \\ &= [x^T(t)A_1^T + \lambda_1 x^T(t)][(1-\alpha_1)v_{1,h} + \alpha_1 v_{1,h+1}] \\ &\quad + x^T(t)\tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1} \\ &= x^T(t)\Big\{(1-\alpha_1)[A_1^T v_{1,h} + \lambda_1 v_{1,h} + \tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1}] \\ &\quad + \alpha_1[A_1^T v_{1,h+1} + \lambda_1 v_{1,h+1} + \tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1}]\Big\} < 0. \end{aligned} \quad (16)$$

Integrating both sides of the inequality, we have

$$V(t) < e^{-\lambda_1(t - t_{2k} - \theta_{1,h})} V(t_{2k} + \theta_{1,h}). \quad (17)$$

By (5) and (6), we know Lyapunov function is continuous in successive switching intervals. Then we get $V(t_{2k} + \theta_{1,h+1}) \leq e^{-\lambda_1(\theta_{1,h+1} - \theta_{1,h})} V(t_{2k} + \theta_{1,h})$. Also because $V((t_{2k} + \psi_s)^-) = V(t_{2k} + \psi_s)$, we have

$$V(t_{2k} + \psi_s) \leq e^{-\lambda_1 \psi_s} V(t_{2k}). \quad (18)$$

When $t \in [t_{2k} + \psi_s, t_{2k+1})$, by (9), we have

$$\begin{aligned} &\dot{V}(t) + \lambda_1 V(t) \\ &= \dot{x}^T(t)v_{1,H_1} + \lambda_1 x^T(t)v_{1,H_1} \\ &= x^T(t)[A_1^T v_{1,H_1} + \lambda_1 v_{1,H_1}] < 0. \end{aligned} \quad (19)$$

That means

$$V(t) < e^{-\lambda_1(t - t_{2k} - \psi_s)} V(t_{2k} + \psi_s). \quad (20)$$

When $t = t_{2k+1}^-$, we have

$$V(t_{2k+1}^-) < e^{-\lambda_1(t_{2k+1} - t_{2k} - \psi_s)} V(t_{2k} + \psi_s). \quad (21)$$

With (18) and (21), we obtain

$$V(t_{2k+1}^-) < e^{-\lambda_1(t_{2k+1} - t_{2k})} V(t_{2k}). \quad (22)$$

When $t \in \Gamma_{2,k}$, the controller failure happens. Consider the time interval $t \in [t_{2k+1} + \theta_{2,h}, t_{2k+1} + \theta_{2,h+1})$, by (10) and (11), we have

$$\begin{aligned} &\dot{V}(t) - \lambda_2 V(t) \\ &= \dot{x}^T(t)v_2(t) + x^T(t)\dot{v}_2(t) - \lambda_2 x^T(t)v_2(t) \\ &= x^T(t)A_2^T v_2(t) - \lambda_2 x^T(t)v_2(t) + x^T(t)\tfrac{v_{2,h+1}-v_{2,h}}{\Delta_2} \\ &= [x^T(t)A_2^T - \lambda_2 x^T(t)][(1-\alpha_2)v_{2,h} - \alpha_2 v_{2,h+1}] \\ &\quad + x^T(t)\tfrac{v_{2,h+1}-v_{2,h}}{\Delta_2} \\ &= x^T(t)\Big\{(1-\alpha_2)[A_2^T v_{2,h} + \lambda_2 v_{2,h} + \tfrac{v_{2,h+1}-v_{2,h}}{\Delta_2}] \\ &\quad + \alpha_2[A_2^T v_{2,h+1} + \lambda_2 v_{2,h+1} + \tfrac{v_{2,h+1}-v_{2,h}}{\Delta_2}]\Big\} < 0. \end{aligned} \quad (23)$$

Be similar to $t \in \Gamma_{1,k}$, we have

$$V(t_{2k+1} + \psi_u) \leq e^{\lambda_2 \psi_u} V(t_{2k+1}). \quad (24)$$

When $t \in [t_{2k+1} + \psi_u, t_{2k+2})$, we have

$$V(t_{2k+2}^-) < e^{\lambda_2(t_{2k+2} - t_{2k+1} - \psi_u)} V(t_{2k+1} + \psi_u). \quad (25)$$

By (24) and (25), we obtain

$$V(t_{2k+2}^-) < e^{\lambda_2(t_{2k+2} - t_{2k+1})} V(t_{2k+1}). \quad (26)$$

Hence, it concludes that

$$V(t) \leq \begin{cases} e^{-\lambda_1(t - t_{2k})} v(t_{2k}),\ t \in \Gamma_{1,k} \\ e^{\lambda_2(t - t_{2k+1})} v(t_{2k+1}),\ t \in \Gamma_{2,k} \end{cases} \quad (27)$$

By (13) and (14), we have

$$V(t_{2k}) \leq \mu_1 V(t_{2k}^-), V(t_{2k+1}) \leq \mu_2 V(t_{2k+1}^-). \quad (28)$$

For initial time t_0, there exist two possibilities of the instant time t.

1) When $t \in [t_{2k+1}, t_{2k+2})$, by Definitions 2 and 3, we can conclude that

$$\begin{aligned} &V(t) \\ &\leq \mu_2 \exp(\lambda_2(t - t_{2k+1}))V(t_{2k+1}^-) \\ &\leq \mu_1\mu_2 \exp(\lambda_2(t - t_{2k+1}))\exp(-\lambda_1(t_{2k+1} - t_{2k})) \\ &\quad \times V(t_{2k}^-)\cdots \\ &\leq \mu_1^k \mu_2^{k+1} \exp\{\lambda_2[(t - t_{2k+1}) + (t_{2k} - t_{2k-1}) + ... \\ &\quad + (t_2 - t_1)]\} \times \exp\{-\lambda_1[(t_{2k+1} - t_{2k}) + ... \\ &\quad + (t_1 - t_0)]\}V(t_0) \\ &= \mu_1^{\frac{N_\sigma - 1}{2}} \mu_2^{\frac{N_\sigma + 1}{2}} \exp(\lambda_2 t_u)\exp(-\lambda_1(t - t_0 - t_u))V(t_0) \\ &\leq \mu_1^{\frac{1}{2}(N_0 + \frac{t_u}{\tau_u}) - \frac{1}{2}} \exp(\lambda_2 t_u) \mu_2^{\frac{1}{2}(N_0 + \frac{t - t_0 - t_u}{\tau_s}) + \frac{1}{2}} \\ &\quad \times \exp(-\lambda_1(t - t_0 - t_u))V(t_0). \end{aligned} \quad (29)$$

Under MDADT constraint (15), it yields

$$V(t) \leq k_1 \exp[-\gamma(t-t_0)]V(t_0), \tag{30}$$

where $k_1 = \exp\left[\frac{N_0-1}{2}\ln\mu_1 + \frac{N_0+1}{2}\ln\mu_2\right] > 0$,
$-\gamma = \max\left\{\frac{\ln\mu_1}{2\tau_u} + \lambda_2, \frac{\ln\mu_2}{2\tau_s} - \lambda_1\right\} < 0$.
2) When $t \in [t_{2k}, t_{2k+1})$, by Definitions 2 and 3, we have

$$\begin{aligned} & V(t) \\ & \leq \mu_1^k\mu_2^k \exp(\lambda_2 t_u)\exp(-\lambda_1(t-t_0-t_u))V(t_0) \\ & = \exp\left[\tfrac{N_0}{2}\ln\mu_1\right]\exp\left[\left(\tfrac{\ln\mu_1}{2\tau_u}+\lambda_2\right)t_u\right] \\ & \quad \times \exp\left[\tfrac{N_0}{2}\ln\mu_2\right]\exp[\left(\tfrac{\ln\mu_2}{2\tau_s}-\lambda_1\right) \\ & \quad \times (t-t_0-t_u)]V(t_0), \end{aligned} \tag{31}$$

Under MDADT constraint (15), we have

$$V(t) \leq k_2 \exp\left[-\gamma(t-t_0)\right]V(t_0), \tag{32}$$

where $k_2 = \exp\left[\frac{N_0}{2}\ln\mu_1\mu_2\right] > 0$,
$-\gamma = \max\left\{\frac{\ln\mu_1}{2\tau_u} + \lambda_2, \frac{\ln\mu_2}{2\tau_s} - \lambda_1\right\} < 0$.

Combining (29) and (30) into the following form: For any time $t \in [t_{2k}, t_{2k+2})$, we have

$$V(t) \leq ke^{-\gamma(t-t_0)}V(t_0), \tag{33}$$

where $k = \max\{k_1, k_2\}$. Therefore, GUES of system (3) can be easily obtained by Definition 4.

IV. CONTROLLER DESIGN

For system (1), we design the state feedback control law as

$$u(t) = \begin{cases} K_{1,h}x(t),\ t \in [t_{2k}+\theta_{1,h}, t_{2k}+\theta_{1,h+1}) \\ K_{1,H_1}x(t),\ t \in [t_{2k}+\psi_s, t_{2k+1}) \end{cases} \quad h = 0,1,...,H_1-1 \tag{34}$$

where $K_{1,h}$ and K_{1,H_1} are controller gains to be determined. Then the system is

$$\dot{x}(t) = \begin{cases} [A+BK(t)]x(t),\ t \in \Gamma_{1,k} \\ Ax(t),\ t \in \Gamma_{2,k} \end{cases} \tag{35}$$

where A is Metzler matrix. Based on Lyapunov function (5) and (6), the following theorem with controller design is presented.

Theorem 2 Consider system (35), given scalars $0 < \mu_1 < 1, \mu_2 > 1, \lambda_1 > 0, \lambda_2 > 0, \bar{v}_{1,h} > 0,\ \rho \in R_+$. $H_1, H_2 \in N\backslash\{0\}$. If there exist series of vectors $v_{1,h} \in R_+^n, h = 0,1,...,H_1-1, v_{2,h} \in R_+^n, h = 0,1,...,H_2-1$, such that

$$\bar{v}_{1,h}^T B^T v_{1,h} A + B\bar{v}_{1,h} g_{1,h}^T + \rho I_n \geq 0 \tag{36}$$

$$A^T v_{1,h} + g_{1,h} + \frac{v_{1,h+1}-v_{1,h}}{\Delta_1} + \lambda_1 v_{1,h} < 0 \tag{37}$$

$$A^T v_{1,h+1} + g_{1,h+1} + \frac{v_{1,h+1}-v_{1,h}}{\Delta_1} + \lambda_1 v_{1,h+1} < 0 \tag{38}$$

$$A^T v_{1,H_1} + g_{1,H_1} + \lambda_1 v_{1,H_1} < 0 \tag{39}$$

where $\Delta_1 = \psi_s/H_1, \Delta_2 = \psi_u/H_2$. With the constraints of (10)-(14), system (35) is positive system and GUES with controller (41) and MDADT

$$\begin{cases} \tau_s \geq \tau_s^* = \frac{\ln\mu_2}{2\lambda_1} \\ \tau_u \leq \tau_u^* = -\frac{\ln\mu_1}{2\lambda_2} \end{cases}. \tag{40}$$

The controller gain is

$$K_{1,h} = \frac{1}{\bar{v}_{1,h}^T B^T v_{1,h}}\bar{v}_{1,h} g_{1,h}^T,\ \ h = 0,1,...,H_1 \tag{41}$$

Proof We know that $\bar{v}_{1,h} > 0, B \geq 0, v_{1,h} > 0$, then

$$\bar{v}_{1,h}^T B^T v_{1,h} \geq 0$$

could be easily obtained. By inequality (36) in Theorem 2, we can get

$$A + \frac{1}{\bar{v}_{1,h}^T B^T v_{1,h}} B\bar{v}_{1,h} g_{1,h}^T + \frac{\rho}{\bar{v}_{1,h}^T B^T v_{1,h}} I_n \geq 0. \tag{42}$$

Then we can conclude that $A + BK_{1,h}$ is Metzler matrix. According to Lemma 2, system (35) is a positive system. When $t \in [t_{2k}+\theta_{1,h}, t_{2k}+\theta_{1,h+1})$, by (37) and (38), we have

$$\begin{aligned} & \dot{V}(t) + \lambda_1 V(t) \\ & = \dot{x}^T(t)v_1(t) + x^T(t)\dot{v}_1(t) + \lambda_1 x^T(t)v_1(t) \\ & = x^T(t)(A+BK(t))^T v_1(t) \\ & \quad + \lambda_1 x^T(t)v_1(t) + x^T(t) \times \tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1} \\ & = [x^T(t)(A+BK(t))^T + \lambda_1 x^T(t)] \\ & \quad \times [(1-\alpha_1)v_{1,h} + \alpha_1 v_{1,h+1}] + x^T(t)\tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1} \\ & = x^T(t)\{(1-\alpha_1)[A^T v_{1,h} + K_{1,h}^T B^T v_{1,h} + \lambda_1 v_{1,h} \\ & \quad + \tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1}] + \alpha_1[A^T v_{1,h+1} + K_{1,h+1}^T B^T v_{1,h+1} \\ & \quad + \lambda_1 v_{1,h+1} + \tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1}]\} \\ & = x^T(t)\left\{(1-\alpha_1)[A^T v_{1,h} + g_{1,h} + \lambda_1 v_{1,h} + \tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1}]\right. \\ & \quad \left. + \alpha_1[A^T v_{1,h+1} + g_{1,h+1} + \lambda_1 v_{1,h+1} + \tfrac{v_{1,h+1}-v_{1,h}}{\Delta_1}]\right\} \\ & < 0. \end{aligned} \tag{43}$$

When $t \in [t_{2k}+\psi_s, t_{2k+1})$, by (39), we have

$$\begin{aligned} & \dot{V}(t) + \lambda_1 V(t) \\ & = \dot{x}^T(t)v_{1,H_1} + \lambda_1 x^T(t)v_{1,H_1} \\ & = x^T(t)[A^T v_{1,H_1} + K_{1,H_1}^T B^T v_{1,H_1} + \lambda_1 v_{1,H_1}] \\ & = x^T(t)[A^T v_{1,H_1} + g_{1,H_1} + \lambda_1 v_{1,H_1}] < 0. \end{aligned} \tag{44}$$

When the controller is unavailable, the situation is the same as Theorem 1. So we omit the proof here.

V. SIMULATION EXAMPLE

In this section, two illustrative examples are represented to show the effectiveness of our results.

Example 1 Consider the positive system (3) with the following parameters:

$$A = \begin{bmatrix} -1.05 & 1.1 \\ 1.2 & -1.1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, K = [-0.1, -0.6],$$
$$\tilde{A} = A + BK = \begin{bmatrix} -1.15 & 0.5 \\ 1 & -2.3 \end{bmatrix},$$

where $A, \tilde{A}$ are both Metzler matrices. The eigenvalues of A are $0.0742, -2.2242$, which means the open-loop system is unstable. The eigenvalues of $\tilde{A}$ are $-0.8136, -2.6364$, which means the closed-loop system is stable. Let $\lambda_1 = 0.5,\ \lambda_2 = 0.25,\ \mu_1 = 0.75,\ \mu_2 = 2,\ \psi_s = 0.5,\ \psi_u = 0.25,\ H_1 = 1,\ H_2 = 1$. After solving the linear programming problem, we obtain

$$v_{1,0} = \begin{bmatrix} 1.9106 \\ 1.0219 \end{bmatrix}, v_{1,1} = \begin{bmatrix} 1.8605 \\ 1.1716 \end{bmatrix},$$
$$v_{2,0} = \begin{bmatrix} 2.6516 \\ 2.2889 \end{bmatrix}, v_{2,1} = \begin{bmatrix} 2.6326 \\ 2.2243 \end{bmatrix}.$$

According to Theorem 1, it can be easily calculated that $\tau_s^* = 0.6931, \tau_u^* = 0.5754$. We construct a switching sequence, where switching times $N_\sigma = 17$. Let the chatter bound $N_0 = 3$, then we have $\tau_s \leq \frac{t-t_0}{N_\sigma - N_0} = \frac{10}{17-3} \approx 0.7142$. Hence, we can make τ_s equal to 0.7, which satisfies the constraint of MDADT. Let $x_0 = [25, 15]^T$ as the initial state, the state trajectories of positive system (3) are presented by figure 1. It shows that the positive system in Example 1 can be stabilized by the switching signal we construct.

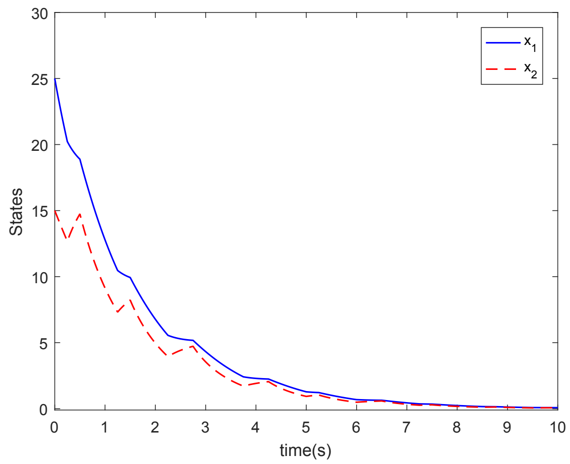

Fig. 1. State trajectories of example 1

Example 2. Consider the positive system (35) with the following parameters:

$$A = \begin{bmatrix} -1.05 & 1.1 \\ 1.2 & -1.1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

Let $\lambda_1 = 0.5,\ \lambda_2 = 0.25,\ \mu_1 = 0.75,\ \mu_2 = 2,\ \psi_s = 0.5,\ \psi_u = 0.25,\ H_1 = 1,\ H_2 = 1$. Solving the linear programming problem, we obtain

$$v_{1,0} = \begin{bmatrix} 16.1998 \\ 16.4514 \end{bmatrix}, v_{1,1} = \begin{bmatrix} 18.6498 \\ 18.2558 \end{bmatrix},$$
$$v_{2,0} = \begin{bmatrix} 29.7634 \\ 28.1072 \end{bmatrix}, v_{2,1} = \begin{bmatrix} 27.6434 \\ 26.1460 \end{bmatrix},$$
$$g_{1,0} = \begin{bmatrix} -23.9534 \\ -18.5746 \end{bmatrix}, g_{1,1} = \begin{bmatrix} -29.1920 \\ -27.1038 \end{bmatrix},$$

substituting the solution into (41), it arrives

$$K_{1,0} = [-0.4878, -0.3783],\ K_{1,1} = [-0.5292, -0.4914].$$

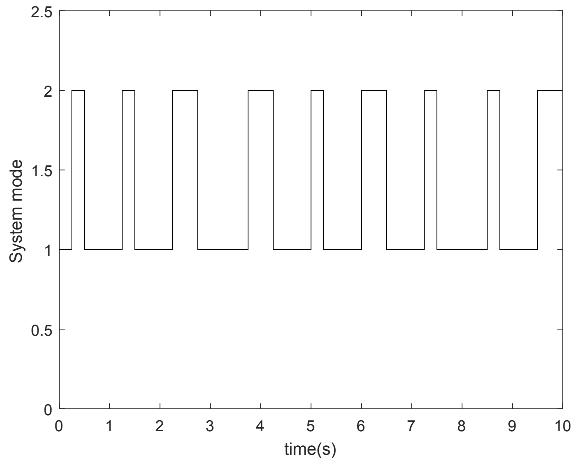

Fig. 2. Switching signal of example 1

According to Theorem 2, it can be calculated that $\tau_s^* = 0.6931, \tau_u^* = 0.5754$. Be similar to Example 1, we construct another switching sequence, where switching times $N_\sigma = 16$. Let the chatter bound $N_0 = 3$, then we have $\tau_s \leq \frac{t-t_0}{N_\sigma - N_0} = \frac{10}{17-3} \approx 0.7692$. Hence, we can make τ_s equal to 0.7, which satisfies the constraint of MDADT. Under the initial condition $x_0 = [25, 15]^T$, the state trajectories of the positive system (35) are depicted by the figure 3. It is shown that the positive system in Example 2 can be stabilized by the switching signal that we construct.

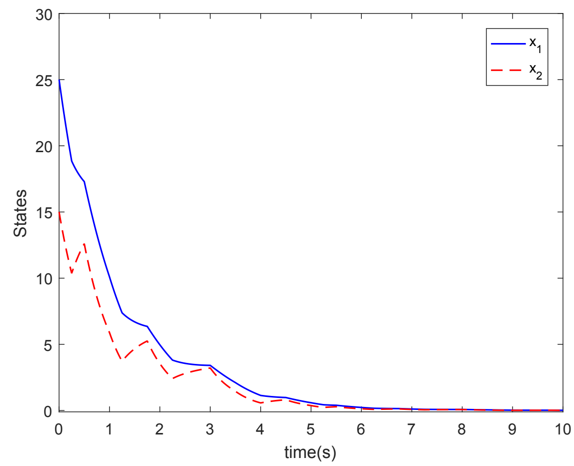

Fig. 3. State trajectories of example 2

VI. CONCLUSIONS

This paper has been concerned with exponential stability for LTI positive system on controller failure problem. By the constraints of DT and MDADT, GUES of the system has been obtained. MLCLFs are proposed to get less conservative results. Numerical examples have been given to show the effectiveness of our results. Moreover, discussing partial failure of controller on positive system will be our future work.

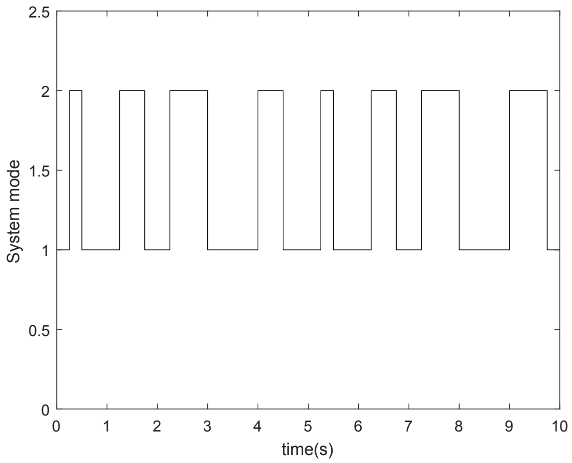

Fig. 4. Switching signal of example 2

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