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Stochastic stability of positive Markov jump linear systems with fixed dwell time[☆]

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ABSTRACT

This paper studies the stochastic stability of positive Markov jump linear systems with a fixed dwell time. By constructing an auxiliary system that originated from the initial system with state jumps, sufficient and necessary conditions of stochastic stability for positive Markov jump linear systems are obtained with both exactly known and partially known transition rates. The main idea in the latter case is applying a convex combination to convert bilinear programming into linear programming problems. On this basis, multiple piecewise linear co-positive Lyapunov functions are provided to achieve less conservative results. Then state feedback controller is designed to stabilize the positive Markov jump linear systems by solving linear programming problems. Numerical examples are presented to illustrate the viability of our conclusions.

1. Introduction

Positive system is the system whose state variable is always required to be nonnegative. It has received extensive attention from scholars in recent years due to quite a few applications in many fields [1–3]. As is well-known, stability is one of the most requisite indexes to evaluate positive system performance. Different from the Lyapunov function (LF) applied to analyze the general system, co-positive Lyapunov function (CLF) [4] is considered as a preferable tool employed for stability analysis of positive systems, since it drastically reduces the computation complexity. Up to now, a lot of literature studying positive system utilize it to analyze positive system stability. Afterward, some innovative methods were proposed, such as multiple co-positive Lyapunov functions (MCLF) [5], the approach that reduces the conservatism comparing with CLF.

On the other hand, the stochastic switching system [6–8] is a prevalent system model in multiple areas, like communication networks [9], economics, and power systems. Markov jump linear system (MJLS) is a particular stochastic system that consists of finite subsystems manipulated by the Markov chain. That is, the Markov process grips switching signals in such systems. When these signals alter, the system will switch to other modes to operate. Heretofore, a great number of essays on MJLS [10–13] have been published. It is noted that transition rates (TRs) in MJLS do not change over time. While if TRs are time-varying, such system is regarded as semi-Markov jump linear system (S-MJLS) [14–18]. What is worth mentioning, systems in these articles are idealized mathematic models from a theoretical standpoint. In fact, switching too many times in a finite time period would destabilize the entire system in practice. Moreover, as

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one class of constrained switching, the existence of dwell time (DT) [19] in stable subsystems makes the stability of the system be guaranteed. Both of these factors necessitate the consideration of fixed DT. However, it is not hard to realize the conventional method handling stochastic stability (SS) of MJLS is not suitable to evaluate system performance when fixed DT is included in each subsystem. For all such dilemmas, the article [20], which studied SS of a continuous-time switched system with both fixed and random DT, has grappled with this difficulty successfully. Sufficient and necessary conditions of SS for MJLS were obtained in the article [20]. Then based on the theory above, the thesis [21] in 2018 achieved H_∞ control by introducing multiple piecewise linear Lyapunov function (MPLLF) [22] offered by Allerhand and Shaked. Furthermore, positive Markov jump linear system (PMJLS) belongs to a special MJLS, where states of all subsystems are confined to the first orthant. Lots of conclusions have been made on PMJLS [23–25], especially in recent years, huge amounts of these were flooded into people's view. For instance, [26] attained necessary and sufficient conditions of SS for PMJLS with time-delay; [27] shed light on the relationships among Mean stability, 1-moment stability, and Almost-sure stability for PMJLS; The book [28] published in 2015 brought exhaustive elaboration on several traits of PMJLS as well as some motivating applications. Yet most of the above discussions are the results under the premise of completely known TRs. While, as a contingency, the case of partially known TRs is ubiquitous in the application. This makes the linear matrix inequation (LMI) cannot be solved by the original approach because of the missing terms. Also, it could be more practical by resolving this problem on the theoretical level. In the majority of results hitherto, the most prevailing approach to deal with such a situation was put forward in 2010 [29], in which sufficient and necessary conditions of SS for MJLS with partially known TRs were attained. The most remarkable point in [29] is the internal relation among elements in the TR matrix was exploited properly, which made the solution of LMI become no longer encumbered by unknown terms.

In spite of this, few aforementioned papers, however, have addressed PMJLS with partially known TRs when fixed DT was taken into account. Many positive systems can be modeled as PMJLS, and contain both given and random DT. Notwithstanding, when considering SS of PMJLS with determined DT, positive constraints are entailed as an additional requirement. It is this limitation of positivity condition results in theories analyzing systems with unrestricted state trajectories cannot be applicable for positive systems. (Even if it could be used to analyze positive system performance, computational complexity will be much higher than that treating positive systems.) As a result, we are propelled to search for some new methods to figure out the issue. In this article, unique features of positive system are employed to fit DT into PMJLS, which constructs one of our main contributions. Additionally, the results of [20] failed to tackle with state feedback controller since exponential terms interfere with the solution of LMI. While in this paper, we are able to overcome this conundrum by introducing multiple piecewise linear co-positive Lyapunov function (MPLCLF) approaches to solve linear programming (LP), then state feedback controllers are going to be designed smoothly.

This paper investigates SS of PMJLS with a fixed DT. In this article, an auxiliary positive system with state jumps is established, which guarantees the continuity of Markov process. Thus, the equivalence of SS for the auxiliary positive system and the original positive system is able to be verified. Then SS of the system mentioned above with partially known TRs is studied, which enables us to release the restriction of completely known TRs. The application of convex combination makes bilinear programming (BP) problems brought by unknown terms easier to resolve. Besides, MPLCLF approaches are applied to make previous results less conservative. Finally, the state feedback controller design is also discussed later in the article.

Notations: R represents the real numbers. R^n and R^n_+ stands for the set of n-dimensional vectors and n-dimensional positive vectors of real entries. $R^{n\times m}$ is the space of $n \times m$ matrices with real entries. N denotes positive integer. $\|\cdot\|$ denotes the Euclidean norm. A^T denotes the transport of A. $A \geq 0$ (≤ 0) denotes all elements of matrix are nonnegative (nonpositive). a_{ij} stands for the element of matrix A, $i, j = 0, 1, 2, \ldots, n$. $\bar{\sigma}(\cdot)$ is the maximum singular values of square matrices. $E\{\cdot\}$ denotes the mathematical expectation.

2. Problem description

Consider continuous PMJLS

$$\dot{x}(t) = A_{\sigma(t)}x(t) + E_{\sigma(t)}u(t) \tag{1}$$

where $x(t) \in R^n_+$ is state vector, $u(t) \in \mathrm{R}^m$ is control input, stochastic process $\{\sigma(t), t \geq 0\}$ is taking values in set $I \triangleq \{1, \ldots, N\}$. In PMJLS, $\{\sigma(t), t \geq 0\}$ is continuous-time, discrete-state homogeneous Markov process, the transition probabilities are

$$\Pr\{\sigma(t+\Delta t) = j | \sigma(t) = i\} = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t), j \neq i \\ 1 + \lambda_{ii}\Delta t + o(\Delta t), j = i \end{cases} \tag{2}$$

where $\Delta t > 0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t) = 0$, $\lambda_{ij} \geq 0$ $(i, j \in I, j \neq i)$, and $\lambda_{ii} = -\sum_{j=1,j\neq i}^{N} \lambda_{ij}$. When the ith subsystem is activated, $\sigma(t_k) = i \in I$, where t_k $(k = 0, 1, 2, \ldots)$ is the switching instant, then the system matrices can be denoted by $A_i \in R^{n\times n}$, $E_i \in R^{n\times m}$. TR matrix is presented as

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & & \lambda_{2N} \\ \vdots & & \ddots & \\ \lambda_{N1} & \lambda_{N2} & \cdots & \lambda_{NN} \end{bmatrix}.$$

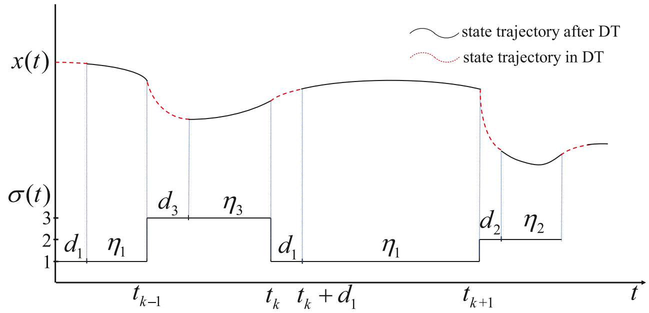

Fig. 1. State trajectories and switching signal of the original system.

In most of this article, TRs are assumed to be partially known, i.e. some elements in Λ are unknown. In order to discern from known elements in proofs afterward, the unknown elements are labeled. For example, if the element of the 3rd row and the 4th column of Λ is unknown, it will be expressed as $\hat{\lambda}_{34}$. For convenience, define

$$I_K^{(i)} = \left\{j : \lambda_{ij} \text{ is known}\right\}, I_{UK}^{(i)} = \left\{j : \lambda_{ij} \text{ is unknown}\right\}, \tag{3}$$

where $I_K^{(i)} \neq \emptyset$. Assume the number of elements in $I_K^{(i)}$ is m_i, then m_i may take the value of $1, 2, \ldots, N-2$. It should be noticed that the sum of elements in every row is zero. So, there is no such situation that only one element is unknown in the given row, i.e. $m_i < N-1$. Denote

$$\lambda_k^{(i)} = \sum_{j\in I_k^{(i)}, j\neq i} \lambda_{ij}. \tag{4}$$

Besides, a fixed DT is considered. Compared to PMJLS only with random DT, the running time of each subsystem in our system obeys DT d_i, during which the entire system is not permitted to switch. In other words, Markov jump does not occur in this time period. When the fixed DT ends, the Markov process begins until the next switched behavior occurs. Thus, two time periods constitute the time interval between two consecutive switching. If t_{k+1} is the next switching instant, then $[t_k, t_{k+1}) = [t_k, t_k + d_i) \cup [t_k + d_i, t_{k+1})$. Let η_i be the second time period and equal to $t_{k+1} - t_k - d_i$, during which the transition probabilities are identical to (2). One possible state trajectory of such system with 3 subsystems is shown in Fig. 1.

Definition 1 ([30]). System (1) ($u(t) \equiv 0$) is said to be positive if, for any initial condition $x(0) \geq 0$, the state trajectory $x(t) \geq 0$ for any $t \geq 0$.

Lemma 1 ([31]). *System (1) is positive if and only if A_i is Metzler matrix, of which the off-diagonal entries are nonnegative, and $E_i \geq 0$.*

Definition 2 ([32]). System (1) ($u(t) \equiv 0$) is said to be SS for every initial condition $x_0 \in R_+^n$ and $r_0 \in I$, when the following holds:

$$E\left\{\int_0^\infty \|x(t)\|_1 dt | x_0, r_0\right\} < \infty.$$

3. Stability analysis

In the first part of the section, all elements in the TR matrix are presumed to be known. Lemma 2 is stated as follows by serving as a prerequisite for the establishment of Lemma 3.

Lemma 2. *Given a Metzler matrix $A \in R^{n\times n}$, and arbitrary vector $x(t) \in R_+^n$, then there exists a constant c_0, satisfying $\left\|e^{At}x(t)\right\|_1 \geq e^{c_0 t}\|x(t)\|_1$ for any $t \in [0, \infty)$. When $c \in (-\infty, c_0]$, we have $\left\|e^{At}x(t)\right\|_1 \geq e^{ct}\|x(t)\|_1$, $t \in [0, \infty)$.*

Proof. Let $c_0 = \min_{i=1,\ldots,n}\{a_{ii}\}$, then $A - c_0 I > 0$. Setting the function $g(t) = e^{(A-c_0 I)t} > 0$, then

$$\frac{dg(t)}{dt} = (A - c_0 I)e^{(A-c_0 I)t}x(t) > 0.$$

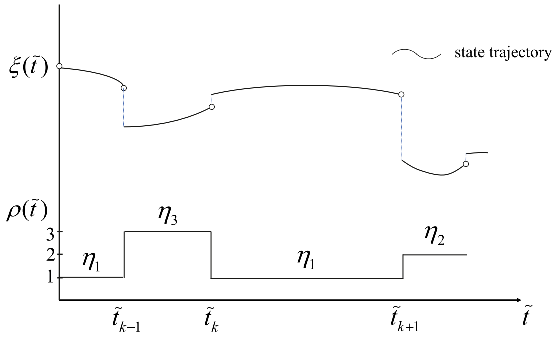

Fig. 2. State trajectories and switching signal of the auxiliary system.

Since $A - c_0 I > 0$, we have $\frac{dg(t)}{dt} > 0$, the function $g(t)$ is proved to be monotonically increasing. Then $g(t) \geq g(0)$, where $t \in [0, \infty)$, i.e. $e^{(A-c_0 I)t}x(t) \geq x(t)$, thus it can be concluded that $\left\| e^{(A-c_0 I)t}x(t) \right\|_1 \geq \|x(t)\|_1 \Rightarrow e^{-c_0 t}\left\| e^{At}x(t) \right\|_1 \geq \|x(t)\|_1 \Rightarrow \left\| e^{At}x(t) \right\|_1 \geq e^{c_0 t}\|x(t)\|_1$. $\square$

Remark 1. Distinguish from general systems whose state vectors are not limited by positivity, positive systems have their own one-of-a-kind features. When the stability of positive systems is evaluated, co-positive Lyapunov function is usually employed instead of quadratic Lyapunov function. Thus, one norm of state vectors rather than two norms is utilized in the calculation, and its derivation is not identical to general systems. In [20], two norms are used to prove monotonicity of function $g(t)$. Whereas for positive systems in this paper, we also acquire distinctive results by setting c_0 as the minimum value of a_{ii}, $i = 1, 2, \ldots, N$.

Lemma 3. *System (1) ($u(t) \equiv 0$) is said to be SS if and only if there exist vectors $v_i \in R^n_+$, $i \in N$, satisfying*

$$A_i^T v_i + \lambda_{ii} v_i + \sum_{j=1, j\neq i}^{N} \lambda_{ij} e^{A_j^T d_j} v_j < 0 \tag{5}$$

for all $i \in N$ with the fixed DT d_i.

Proof. Inspired by [20], an auxiliary positive system with state jumps is transformed from the original system established above. The state equation of it is shown below

$$\begin{cases} \dot{\xi}(\tilde{t}) = A_{\rho(\tilde{t})}\xi(\tilde{t}), \ \tilde{t}_k < \tilde{t} < \tilde{t}_{k+1} \\ \xi(\tilde{t}_k) = e^{A_{\rho(\tilde{t})} d_{\rho(\tilde{t})}}\xi(\tilde{t}_k^-), \ \tilde{t} = \tilde{t}_k \end{cases} \tag{6}$$

where $\xi(\tilde{t})$ is system state, $\tilde{t}_k$ is switching instant. $\{\rho(\tilde{t}), \tilde{t} \geq 0\}$ is stochastic process derived from finite set $I \triangleq \{1, \ldots, N\}$. The transition probabilities are

$$\Pr\left\{\rho(\tilde{t} + \Delta t) = j | \rho(\tilde{t}) = i\right\} = \begin{cases} \lambda_{ij}\Delta t + o(\Delta t), j \neq i \\ 1 + \lambda_{ii}\Delta t + o(\Delta t), j = i \end{cases} \tag{7}$$

The definition of λ_{ij} is the same as (2). At the switching instant $\tilde{t}_k$, the system state directly shifts to a certain point, which the original system state should become at the terminus of DT, i.e. $\xi(\tilde{t}_k) = x(t_k + d_{\sigma_k})$, where $\rho_k = \sigma_k$. η_i is the time period whose value is the same as defined in the original system, namely, $\eta_i = t_{k+1} - t_k - d_i = \tilde{t}_{k+1} - \tilde{t}_k$. For the sake of comparison, Fig. 2 is the state trajectory of the auxiliary system corresponding to the system in Fig. 1.

Then it can be proved that SS of these two positive systems is equivalent.

($\Rightarrow$) Assume system (1) is SS, then for system (6), we have

$$E\left\{\int_0^{\infty} \left\|\xi(\tilde{t})\right\|_1 d\tilde{t} | \xi_0, \rho_0\right\} \leq E\left\{\int_0^{\infty} \|x(t)\|_1 dt | x_0, \sigma_0\right\} < \infty.$$

($\Leftarrow$) Conversely, assume system (6) is SS. Relying on Lemma 2 in this paper and Lemma 3 in [20], i.e. $E\left\{\int_b^{b+X} e^{at}dt\right\} = \frac{e^{\kappa b}}{\kappa - a}$, where X is exponentially distributed random variable with parameter κ, $a, b \in R$, $a < \kappa$, there exists $c_i < \kappa_i$, such that

$$\begin{aligned}
& E\left\{\int_{t_k+d_i}^{t_{k+1}} \|x(t)\|_1 dt | x(t_k)\right\} \\
&= E\left\{\int_{t_k+d_i}^{t_k+d_i+\eta_i} \left\|e^{A_i(t-t_k)}x(t_k)\right\|_1 dt\right\} \\
&= E\left\{\int_{d_i}^{d_i+\eta_i} \left\|e^{A_i\tau}x(t_k)\right\|_1 d\tau\right\} \\
&\geq E\left\{\int_{d_i}^{d_i+\eta_i} e^{c_i\tau}\|x(t_k)\|_1 d\tau\right\} \\
&= \frac{e^{\kappa_i d_i}}{\kappa_i - c_i} \cdot \|x(t_k)\|_1 \\
&\geq \alpha_{\min}\|x(t_k)\|_1,
\end{aligned}$$

holds for any $x(t_k) \neq 0$ and $i \in N$, where $\alpha_{\min} = \min_{i\in N}\{e^{\kappa_i d_i}/(\kappa_i - c_i)\} > 0$. So, we have

$$E\left\{\int_{t_k}^{t_k+d_i} \|x(t)\|_1 dt | x(t_k)\right\} \leq (\alpha_{\max} d_{\max}/\alpha_{\min}) E\left\{\int_{t_k+d_i}^{t_{k+1}} \|x(t)\|_1 dt\right\},$$

where $\alpha_{\max} = \max_{i\in N}\left\{\max_{\tau\in[0,d_i]}\bar{\sigma}(e^{A_i\tau})\right\}$, $d_{\max} = \max_{i\in N}\{d_i\}$. Thus

$$E\left\{\int_{t_k}^{t_{k+1}} \|x(t)\|_1 dt\right\} \leq (\alpha_{\max} d_{\max}/\alpha_{\min} + 1) E\left\{\int_{\tilde{t}_k}^{\tilde{t}_{k+1}} \left\|\xi(\tilde{t})\right\|_1 d\tilde{t}\right\}.$$

Then it is obtained

$$E\left\{\int_0^{\infty} \|x(t)\|_1 dt | x_0, \sigma_0\right\} \leq (\alpha_{\max} d_{\max}/\alpha_{\min} + 1) E\left\{\int_0^{\infty} \left\|\xi(\tilde{t})\right\|_1 d\tilde{t} | \xi_0, \rho_0\right\} < \infty.$$

Thus, system (1) is proved to be SS. Therefore, the equivalence of SS for system (1) and (6) is eventually verified.

Then the sufficiency and necessity of SS for system (1) with fixed DT can be proved.

($\Leftarrow$) Consider LF

$$V(\xi(\tilde{t}), \rho(\tilde{t})) = \xi^T(\tilde{t})v(\rho(\tilde{t})) \tag{8}$$

where $v(\rho(\tilde{t})) = v_i > 0$. We have

$$\begin{aligned}
& E\left\{V(\xi(\tilde{t} + \Delta t), \rho(\tilde{t} + \Delta t)) | \xi(\tilde{t}), \rho(\tilde{t}) = i\right\} \\
&= \xi^T(\tilde{t})\left[\left(\sum_{j=1, j\neq i}^{N} \lambda_{ij} e^{A_j^T d_j} v_j + \lambda_{ii} v_i + A_i^T v_i\right)\Delta t + v_i\right] + o(\Delta t).
\end{aligned}$$

Hence,

$$\begin{aligned}
& \Delta V(\xi(\tilde{t}), \rho(\tilde{t}) = i) \\
&= -\xi^T(\tilde{t})\left[-\left(\sum_{j=1, j\neq i}^{N} \lambda_{ij} e^{A_j^T d_j} v_j + \lambda_{ii} v_i + A_i^T v_i\right)\right] \\
&\leq -\mu_i \left\|\xi(\tilde{t})\right\|_1,
\end{aligned}$$

where μ_i stands for the minimal element of $\sum_{j=1,j\neq i}^{N} \lambda_{ij} e^{A_j^T d_j} v_j + \lambda_{ii} v_i + A_i^T v_i$. It is obtained

$$\lim_{\tilde{t}\to\infty} E\left\{V(\xi(\tilde{t}), \rho(\tilde{t}) = i) | \xi_0, \rho_0\right\} = 0.$$

Thus, we have

$$E\left\{\int_0^{\infty} \left\|\xi(\tilde{t})\right\|_1 dt | \xi_0, \rho_0\right\} \leq -\frac{1}{\mu_i}\left[\lim_{\tilde{t}\to\infty} E\left\{V(\xi(\tilde{t}), \rho(\tilde{t}) = i) | \xi_0, \rho_0\right\} - V(\xi_0, \rho_0)\right] < \infty.$$

Therefore, SS of system (6) is proved. In this case, SS of system (1) is verified by virtue of the conclusion attained by Lemma 3.

($\Rightarrow$) Assume system (1) is known to be SS. System (6) is also SS from the previous conclusion. According to [20], it can be proved that there exist vectors $v_i \in R_+^n$, such that (5) holds for all $i \in N$. So, the necessity of SS for system (1) is proved. $\square$

Based on the lemma above, SS of system (1) with partially known TRs is able to be discussed.

Theorem 1. *The positive system (1) ($u(t) \equiv 0$) with partially known TRs is SS if and only if there exist vectors $v_i \in R_+^n$, $i \in I$, such that*

$$A_i^T v_i + \lambda_{ii} v_i + \nu_k^{(i)} - (\lambda_{ii} + \lambda_k^{(i)}) e^{A_j^T d_j} v_j < 0, \text{ for } i \in I_K^{(i)} \tag{9}$$

$$A_i^T v_i - \lambda_k^{(i)} v_i + \nu_k^{(i)} < 0, \text{ for } i \in I_{UK}^{(i)} \tag{10}$$

$$A_i^T v_i + \nu_k^{(i)} + \lambda_d^{(i)} v_i - \lambda_d^{(i)} e^{A_j^T d_j} v_j - \lambda_k^{(i)} e^{A_j^T d_j} v_j < 0, \text{ for } i \in I_{UK}^{(i)} \tag{11}$$

where $\nu_k^{(i)} = \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} e^{A_j^T d_j} v_j$, $\lambda_d^{(i)}$ is the lower bound for the diagonal element λ_{ii}.

Proof. It is known that $\lambda_{ii} + \lambda_k^{(i)} < 0$. Rewriting in Eq. (5) as

$$\begin{aligned}
\Theta_i &= A_i^T v_i + \lambda_{ii} v_i + \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} e^{A_j^T d_j} v_j + \sum_{j \in I_{uk}^{(i)}, j \neq i} \hat{\lambda}_{ij} e^{A_j^T d_j} v_j \\
&= A_i^T v_i + \lambda_{ii} v_i + \nu_k^{(i)} + (-\lambda_{ii} - \lambda_k^{(i)}) \sum_{j \in I_{uk}^{(i)}, j \neq i} \frac{\hat{\lambda}_{ij}}{(-\lambda_{ii} - \lambda_k^{(i)})} e^{A_j^T d_j} v_j \\
&= \sum_{j \in I_{uk}^{(i)}, j \neq i} \frac{\hat{\lambda}_{ij}}{(-\lambda_{ii} - \lambda_k^{(i)})} \left[A_i^T v_i + \lambda_{ii} v_i + \nu_k^{(i)} - (\lambda_{ii} + \lambda_k^{(i)}) e^{A_j^T d_j} v_j \right] < 0.
\end{aligned}$$

(1) When $i \in I_K^{(i)}$, $\Theta_i < 0$ is equivalent to

$$A_i^T v_i + \lambda_{ii} v_i + \nu_k^{(i)} - (\lambda_{ii} + \lambda_k^{(i)}) e^{A_j^T d_j} v_j < 0.$$

Then the equivalent condition of SS of system (1) is obtained.
(2) When $i \in I_{UK}^{(i)}$, $\Theta_i < 0$ is equivalent to

$$A_i^T v_i + \hat{\lambda}_{ii} v_i + \nu_k^{(i)} - (\hat{\lambda}_{ii} + \lambda_k^{(i)}) e^{A_j^T d_j} v_j < 0. \tag{12}$$

Since $\lambda_d^{(i)} \le \hat{\lambda}_{ii} < -\lambda_k^{(i)}$, for $\forall \varepsilon > 0$, $\hat{\lambda}_{ii} \in [\lambda_d^{(i)}, -\lambda_k^{(i)} + \varepsilon]$. Then $\hat{\lambda}_{ii}$ could be represented as a convex combination

$$\hat{\lambda}_{ii} = -\alpha \lambda_k^{(i)} + \alpha \varepsilon + (1 - \alpha) \lambda_d^{(i)},$$

where $\alpha \in [0, 1]$. So, (12) is equivalent to

$$A_i^T v_i - \lambda_k^{(i)} v_i + \nu_k^{(i)} + \varepsilon (v_i - e^{A_j^T d_j} v_j) < 0, \tag{13}$$

and

$$A_i^T v_i + \nu_k^{(i)} + \lambda_d^{(i)} v_i - \lambda_d^{(i)} e^{A_j^T d_j} v_j - \lambda_k^{(i)} e^{A_j^T d_j} v_j < 0.$$

When $\varepsilon \to 0^+$, (13) is equivalent to

$$A_i^T v_i - \lambda_k^{(i)} v_i + \nu_k^{(i)} < 0.$$

Thus, system (1) is SS if and only if inequality (9)–(11) holds. □

Remark 2. Despite the fact that specific value of $\hat{\lambda}_{ii}$ cannot be obtained directly as an unknown term, being the only negative element in each row, it can still be confined to a limited range due to the restriction that the transition probabilities are not allowed to exceed 1. Under such situation, $\lambda_d^{(i)}$ is defined *a priori* as the lower bound of $\hat{\lambda}_{ii}$.

Some flaws exist nevertheless. Not only the path of LF cannot be tracked by the approach stated above, but the controller is not able to be designed as well. This reminds us of the method put forward by [22], in which a piecewise linearly LF is arranged to be decreasing between two consecutive switching instants. Then MPLCLF is devised to deal with the positive system with DT. In this article, we divide DT into several segments evenly. That is, the time interval d_i is divided into H_i equal time periods. Let $\theta_{i,h} = h d_i / H_i$, $h = 0, 1, \ldots, H_i - 1$, then in each segment of d_i, LF can be represented by

$$v_i(t) = (1 - \alpha_i) v_{i,h} + \alpha_i v_{i,h+1}, \tag{14}$$

where $\alpha_i = (t - t_k - \theta_{i,h})/(d_i/H_i)$, $v_{i,h}$, $v_{i,H_i} \in R_+^n$. Then Theorem 2 could be derived by substituting time-varying LF into Theorem 1.

Theorem 2. *For given scalars $d_i \in R_+$, $H_i \in N\backslash\{0\}$, system (1) ($u(t) \equiv 0$) with partially known TRs is SS if there exist vectors $v_{i,h} \in R_+^n$, $i \in I$, $h = 1, 2, \ldots, H_i - 1$, such that*

$$A_i^T v_{i,h} + (v_{i,h+1} - v_{i,h})/\Delta_i < 0, \tag{15}$$

$$A_i^T v_{i,h+1} + (v_{i,h+1} - v_{i,h})/\Delta_i < 0, \tag{16}$$

$$A_i^T v_{i,H_i} + \lambda_{ii} v_{i,H_i} + v_k^{(i)} - (\lambda_{ii} + \lambda_k^{(i)}) v_{j,0} < 0, \text{ for } i \in I_K^{(i)} \tag{17}$$

$$A_i^T v_{i,H_i} - \lambda_k^{(i)} v_{i,H_i} + v_k^{(i)} < 0, \text{ for } i \in I_{UK}^{(i)} \tag{18}$$

$$A_i^T v_{i,H_i} + \lambda_d^{(i)} (v_{i,H_i} - v_{j,0}) + v_k^{(i)} - \lambda_k^{(i)} v_{j,0} < 0, \text{ for } i \in I_{UK}^{(i)} \tag{19}$$

where $v_k^{(i)} = \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} v_{j,0}$, $\Delta_i = d_i/H_i$, $\lambda_d^{(i)}$ is the lower bound for the diagonal element λ_{ii}.

Proof. Since $v_{i,h}$, $v_{i,h+1}$, and v_{i,H_i} exist, then according to (14), $v_i(t)$ also exists. Enlightened by the method of [21], given continuous vector $u_i(t) \in R_+^n$, where $t \in [0, d_i]$, define

$$v_i(t) = e^{A_i^T (d_i - t)} v_i(d_i) + \varphi_i(t),$$

where $\varphi_i(t) = \int_t^{d_i} e^{A_i^T (s-t)} u_i(s) ds$. It can be calculated that

$$A_i^T v_i(t) + \dot{v}_i(t) = -u_i(t),$$

Thus, we have

$$A_i^T v_i(t) + \dot{v}_i(t) < 0. \tag{20}$$

In this case, if (15)–(19) hold, the following inequalities

$$A_i^T v_i(d_i) + \lambda_{ii} v_i(d_i) + \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} v_j(0) - (\lambda_{ii} + \lambda_k^{(i)}) v_j(0) < 0, \text{ for } i \in I_K^{(i)} \tag{21}$$

$$A_i^T v_i(d_i) - \lambda_k^{(i)} v_i(d_i) + \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} v_j(0) < 0, \text{ for } i \in I_{UK}^{(i)} \tag{22}$$

$$A_i^T v_i(d_i) + \lambda_d^{(i)} (v_i(d_i) - v_j(0)) + \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} v_j(0) - \lambda_k^{(i)} v_j(0) < 0, \text{ for } i \in I_{UK}^{(i)} \tag{23}$$

hold. Multiply (20) by $e^{A_i^T t}$ to the left, it is obtained

$$e^{A_i^T t} A_i^T v_i(t) + e^{A_i^T t} \dot{v}_i(t) < 0. \tag{24}$$

Integrating the variable t between $[0, d_i]$ on both sides of (24), we have

$$e^{A_i^T d_i} v_i(d_i) - v_i(0) < 0. \tag{25}$$

Notice that $\lambda_{ii} + \lambda_k^{(i)} < 0$, $\lambda_d^{(i)} + \lambda_k^{(i)} < 0$. Substitute (25) into (21), (22) and (23) respectively, then we have

$$A_i^T v_i(d_i) + \lambda_{ii} v_i(d_i) + \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} e^{A_j^T d_j} v_j(d_j) - (\lambda_{ii} + \lambda_k^{(i)}) e^{A_j^T d_j} v_j(d_j) < 0, \text{ for } i \in I_K^{(i)}$$

$$A_i^T v_i(d_i) - \lambda_k^{(i)} v_i(d_i) + \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} e^{A_j^T d_j} v_j(d_j) < 0, \text{ for } i \in I_{UK}^{(i)}$$

$$A_i^T v_i(d_i) + \lambda_d^{(i)} (v_i(d_i) - e^{A_j^T d_j} v_j(d_j)) + \sum_{j \in I_k^{(i)}, j \neq i} \lambda_{ij} e^{A_j^T d_j} v_j(d_j) - \lambda_k^{(i)} e^{A_j^T d_j} v_j(d_j) < 0, \text{ for } i \in I_{UK}^{(i)}$$

Let $v_i = v_i(d_i)$. then (9)–(11) hold. From the conclusion of Theorem 1, SS of system (1) is attained, which completes the proof. □

Remark 3. It is known that multiple linear co-positive Lyapunov function (MLCLF) allows every subsystem to hold its linear co-positive Lyapunov function (LCLF), which is not the same as common LCLF since all of the subsystems share only one LCLF. But it is still required to be invariable during the period when a certain subsystem is activated. While for MPLCLF, LF is not a constant value and traceable between two switching instants. That is where MPLCLF is superior to MLCLF.

Remark 4. Time-varying Lyapunov function method is also adopted by some other works like [33–36]. [35] studies the stability as well as the stabilization of positive impulsive linear systems. The piecewise approach is used to relax infinite-dimensional LMI conditions into finite-dimensional LMI conditions. [36] attained a new adaptive law comparing to previous works of adaptive control for the switched system.

If TRs are completely known, BP problems would not occur. No convex combination is needed to convert BP into LP problems. As a result, sufficient conditions of SS for our system could be expressed directly as LP inequalities. The corollary is as follows.

Corollary 1. *For given scalars $d_i \in R_+, H_i \in N\backslash\{0\}$, system (1) ($u(t) \equiv 0$) is SS if there exist vectors $v_{i,h} \in R^n_+, i \in I, h = 1, 2, \ldots, H_i - 1$, such that*

$$A_i^T v_{i,h} + (v_{i,h+1} - v_{i,h})/\Delta_i < 0, \tag{26}$$

$$A_i^T v_{i,h+1} + (v_{i,h+1} - v_{i,h})/\Delta_i < 0, \tag{27}$$

$$A_i^T v_{i,H_i} + \lambda_{ii} v_{i,H_i} + \sum_{j=1, j\neq i}^{N} \left\{\lambda_{ij} v_{j,0}\right\} < 0, \tag{28}$$

where $v_k^{(i)} = \sum_{j\in I_k^{(i)}, j\neq i} \lambda_{ij} v_{j,0}$, $\Delta_i = d_i/H_i$.

4. Controller design

In this section, time-varying controllers are proposed to stabilize system (1). The state feedback control law is regulated as

$$u(t) = \begin{cases} K_{i,h}x(t), & t \in [t_k + \theta_{i,h}, t_k + \theta_{i,h+1}) \\ K_{i,H_i}x(t), & t \in [t_k + d_i, t_{k+1}) \end{cases} \tag{29}$$

where $h = 0, 1, \ldots, H_i - 1$, $K_{i,h}$ and K_{i,H_i} are controller gains to be determined, $\theta_{i,h} = hd_i/H_i$ as defined above. Then the system can be expressed as

$$\dot{x}(t) = (A_i + E_i K_{i,h}(t))x(t), \; t \in [t_k, t_{k+1}), h = 0, 1, 2, \ldots, H_i \tag{30}$$

Based on MPLCLF (14), the following theorem is presented.

Theorem 3. *Consider system (1), given scalars $H_i \in N\backslash\{0\}$, $d_i \in R_+$, vectors $\bar{v}_{i,h} \in R^m_+$. If there exist series of vectors $v_{i,h} \in R^n_+$, $g_{i,h} \in R^n$, $i \in I$, $h = 0, 1, \ldots, H_i - 1$, scalar $\rho \in R_+$, such that*

$$\bar{v}_{i,h}^T E_i^T v_{i,h} A_i + E_i \bar{v}_{i,h} g_{i,h}^T + \rho I_n \geq 0 \tag{31}$$

$$A_i^T v_{i,h} + g_{i,h} + (v_{i,h+1} - v_{i,h})/\Delta_i < 0 \tag{32}$$

$$A_i^T v_{i,h+1} + g_{i,h+1} + (v_{i,h+1} - v_{i,h})/\Delta_i < 0 \tag{33}$$

$$A_i^T v_{i,H_i} + g_{i,H_i} + \lambda_{ii} v_{i,H_i} + v_k^{(i)} - (\lambda_{ii} + \lambda_k^{(i)}) v_{j,0} < 0, \; for\ i \in I_K^{(i)} \tag{34}$$

$$A_i^T v_{i,H_i} + g_{i,H_i} - \lambda_k^{(i)} v_{i,H_i} + v_k^{(i)} < 0, \; for\ i \in I_{UK}^{(i)} \tag{35}$$

$$A_i^T v_{i,H_i} + g_{i,H_i} + \lambda_d^{(i)} v_{i,H_i} + v_k^{(i)} - (\lambda_d^{(i)} + \lambda_k^{(i)}) v_{j,0} < 0, \; for\ i \in I_{UK}^{(i)} \tag{36}$$

where $v_k^{(i)} = \sum_{j\in I_k^{(i)}, j\neq i} \lambda_{ij} v_{j,0}$, $\Delta_i = d_i/H_i$, $\lambda_d^{(i)}$ is the lower bound for the diagonal element λ_{ii}. Then system (1) is a positive system and SS with controller

$$K_{i,h} = \frac{1}{\bar{v}_{i,h}^T E_i^T v_{i,h}} \bar{v}_{i,h} g_{i,h}^T, \; h = 0, 1, \ldots, H_i \tag{37}$$

Proof. It is known that $\bar{v}_{i,h} > 0, E_i \geq 0, v_{i,h} > 0$, then

$$\bar{v}_{i,h}^T E_i^T v_{i,h} \geq 0$$

could be easily obtained. According to in Eq. (31), we have

$$A_i + \frac{1}{\bar{v}_{i,h}^T E_i^T v_{i,h}} E_i \bar{v}_{i,h} g_{i,h}^T + \frac{\rho}{\bar{v}_{i,h}^T E_i^T v_{i,h}} I_n \geq 0.$$

Then it can be concluded that $A_i + E_i K_{i,h}$ is Metzler matrix. Due to Lemma 1, system (1) is positive system.

According to Theorem 2, (15)–(19) are sufficient conditions of SS for system (1) when $u(t) \equiv 0$. While in Theorem 3, the control input is included. Let $\tilde{A}_i = A_i + E_i K_{i,h}$. Since $\tilde{A}_i$ could be proved to be Metzler matrix, the same as A_i. So, conditions of (32)–(36) ensure that (15)–(19) hold. As a result, system (1) with state feedback controller is verified to be SS, which completes the proof. □

Remark 5. When the fixed DT is equal to zero ($d_i = 0$), only random DT is left. Then our system turns to normal PMJLS, which has been studied by [3,23,28,32], etc. Basically, the situation of $d_i = 0$ could be regarded as a special case of our system model.

Remark 6. It has to be conceded that the sojourn time between two successive jumps in the system above is subject to the exponential distribution. It may not necessarily be related to time in semi-Markov jump linear system (S-MJLS), and some studies of stability analysis, as well as control synthesis for S-MJLS, have been published in recent years. For instance, [14] studies σ-error mean square stability with bounded sojourn times. [15–18] discuss hidden S-MJLS where the hidden semi-Markov chain is defined as a two-layer stochastic process, of which the application scope is further expanded compared with S-MJLS. Moreover, the study on semi-positive Markov jump linear system (S-PMJLS) [37–41] is at the research frontier and also challenging. Thus, analyzing S-PMJLS performance with the approach above could generalize our results.

Remark 7. Note that each parameter of the system established above is pre-known. In fact, system matrices A_i, B_i could be partly known or limited to a certain range in many practical cases. Also, the system with parameter uncertainties has been considered in other system models [42–44]. Since DT is considered in our system, the condition obtained in Lemma 3 cannot be utilized to the uncertain case owing to the exponential term. However, when the time-varying LF is employed, the method of dealing with uncertainty in the above studies can be applied to our system model.

5. Simulation example

In this section, the efficiency of our results is proved by two numerical examples presented.

Example 1. Consider system (1) ($u(t) \equiv 0$) with the following parameters:

$$A_1 = \begin{bmatrix} -0.45 & 0.8 \\ 0.5 & -1.2 \end{bmatrix}, A_2 = \begin{bmatrix} -0.75 & 0.04 \\ 0.03 & 0.05 \end{bmatrix}, A_3 = \begin{bmatrix} -0.2 & 0.1 \\ 1 & -1 \end{bmatrix},$$

where A_1, A_2 and A_3 are Metzler matrices. TR matrix is

$$\Lambda = \begin{bmatrix} -1.3 & \hat{\lambda}_{12} & \hat{\lambda}_{13} \\ 0.7 & -1.2 & 0.5 \\ \hat{\lambda}_{31} & 0.3 & \hat{\lambda}_{33} \end{bmatrix},$$

where $\hat{\lambda}_{12}, \hat{\lambda}_{13}, \hat{\lambda}_{31}$ and $\hat{\lambda}_{33}$ represent the unknown terms in Λ. Let $d_1 = 5, d_2 = 0.5, d_3 = 3, H_1 = H_2 = H_3 = 1$. After solving LP problems, it is obtained

$$v_{1,0} = \begin{bmatrix} 2.4099 \\ 1.7376 \end{bmatrix}, v_{1,1} = \begin{bmatrix} 3.3628 \\ 2.3889 \end{bmatrix},$$
$$v_{2,0} = \begin{bmatrix} 1.5348 \\ 1.6151 \end{bmatrix}, v_{2,1} = \begin{bmatrix} 1.9229 \\ 1.4390 \end{bmatrix},$$
$$v_{3,0} = \begin{bmatrix} 3.5525 \\ 0.5701 \end{bmatrix}, v_{3,1} = \begin{bmatrix} 2.6748 \\ 0.8069 \end{bmatrix}.$$

Set $x_0 = [25, 15]^T$ as the initial state, the state trajectories, and the switching signal of system (1) are presented by Fig. 3. It indicates that the positive system in Example 1 can be stabilized, albeit with unknown TRs.

Example 2. Consider positive system (1) with the following parameters:

$$A_1 = \begin{bmatrix} -0.45 & 0.8 \\ 0.5 & -1.2 \end{bmatrix}, A_2 = \begin{bmatrix} -0.75 & 0.04 \\ 0.03 & 0.05 \end{bmatrix}, A_3 = \begin{bmatrix} -0.2 & 0.1 \\ 1 & -1 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 0.10 \\ 0.22 \end{bmatrix}, E_2 = \begin{bmatrix} 0.12 \\ 0.35 \end{bmatrix}, E_3 = \begin{bmatrix} 0.20 \\ 0.24 \end{bmatrix},$$

where A_1, A_2 and A_3 are Metzler matrices. TR matrix is the same as in Example 1. After solving LP problems, it is obtained

$$v_{1,0} = \begin{bmatrix} 123.7832 \\ 101.7790 \end{bmatrix}, v_{1,1} = \begin{bmatrix} 61.2200 \\ 54.0368 \end{bmatrix},$$

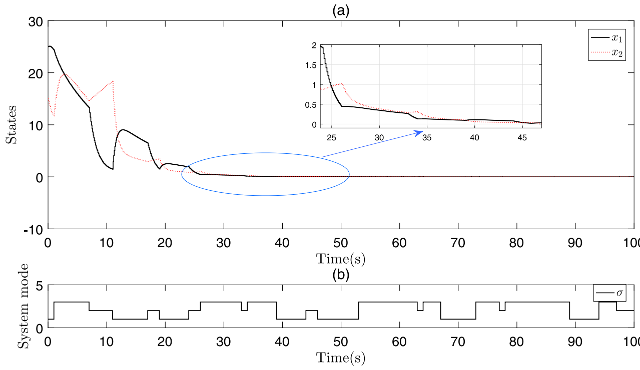

Fig. 3. State trajectories and switching signal of Example 1.

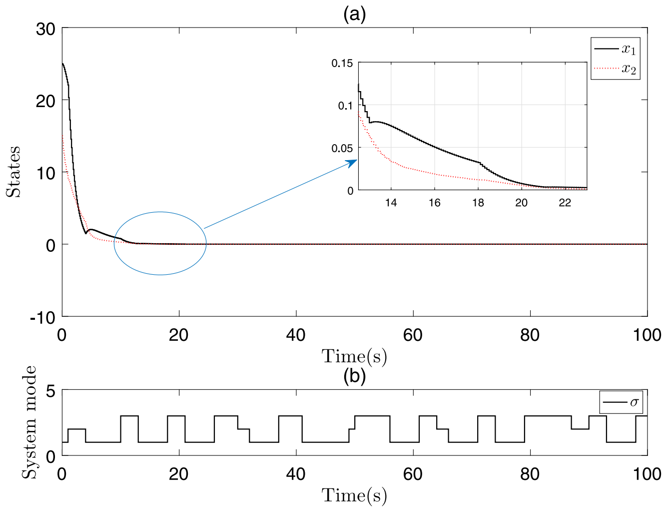

Fig. 4. State trajectories and switching signal of Example 2.

$$
\begin{aligned}
v_{2,0} &= \begin{bmatrix} 64.1943 \\ 54.9715 \end{bmatrix}, v_{2,1} = \begin{bmatrix} 66.6271 \\ 53.0535 \end{bmatrix}, \\
v_{3,0} &= \begin{bmatrix} 61.0029 \\ 38.3094 \end{bmatrix}, v_{3,1} = \begin{bmatrix} 51.5920 \\ 52.4471 \end{bmatrix}, \\
g_{1,0} &= \begin{bmatrix} -16.3460 \\ -15.3703 \end{bmatrix}, g_{1,1} = \begin{bmatrix} -50.4159 \\ -29.0678 \end{bmatrix}, \\
g_{2,0} &= \begin{bmatrix} 2.6891 \\ -38.1361 \end{bmatrix}, g_{2,1} = \begin{bmatrix} -29.9819 \\ -73.5685 \end{bmatrix}, \\
g_{3,0} &= \begin{bmatrix} -52.5332 \\ -16.3711 \end{bmatrix}, g_{3,1} = \begin{bmatrix} -106.0087 \\ -12.1607 \end{bmatrix}.
\end{aligned}
$$

substituting the solution into (37), it arrives

$$
\begin{aligned}
K_{1,0} &= \begin{bmatrix} -0.4701 & -0.4421 \end{bmatrix}, K_{2,0} = \begin{bmatrix} 0.0998 & -1.4154 \end{bmatrix}, K_{3,0} = \begin{bmatrix} -2.4554 & -0.7652 \end{bmatrix}, \\
K_{1,1} &= \begin{bmatrix} -2.7993 & 1.6140 \end{bmatrix}, K_{2,1} = \begin{bmatrix} -1.1287 & -2.7695 \end{bmatrix}, K_{3,1} = \begin{bmatrix} -4.6280 & -0.5309 \end{bmatrix}.
\end{aligned}
$$

Set $x_0 = [25, 15]^T$ as the initial state. Apparently, by learning the description of state traces in Fig. 4, it can be easily proved that positive system (1) with the time-varying controller tends to be stable ultimately.

6. Conclusions

This article has attained sufficient and necessary conditions of SS for PMJLS with the fixed DT. By converting the original positive system to an auxiliary positive system with state jumps, the problem of discontinuous Markov chain due to the fixed DT has been resolved. Then partially known TRs have been considered, and the utilization of MPLCLF has enabled the state feedback controller to stabilize the system. Numerical examples have exemplified the feasibility of our results. Moreover, discussing the stability and stabilization problem of the semi-positive Markov jump system with parameter uncertainties will be our future work.

CRediT authorship contribution statement

Jie Lian: Methodology, Writing - review & editing, Supervision. **Renke Wang:** Writing- original draft, Conceptualization, Software.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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