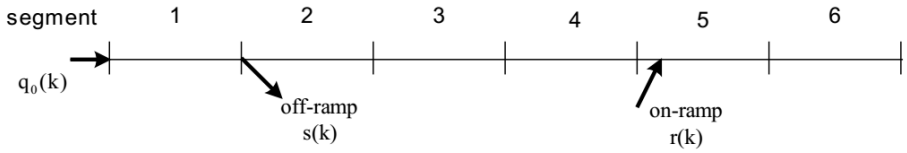
We consider a two-lane freeway stretch of 3 km in length. For modelling purposes the stretch is subdivided into 6 segments of $\Delta = 500\,\text{m}$ each. There is an on-ramp located at the beginning of segment No. 5 and an off-ramp located at the beginning of section No. 2 (see sketch). The fundamental diagram of the stretch is given by

$$Q(\rho) = \rho \cdot v_f \left(1 - \frac{\rho}{\rho_{max}}\right)$$

where v_f is the free speed and ρ_{max} the maximum density. The mainstream demand $q_0(k)$, the on-ramp demand r(k) and the constant exit rate β are known. The exit rate determines the off-ramp volume via

$$s(k) = \beta \cdot q_1(k).$$



1. What is the critical density ρ_{cr} and capacity q_{cap} for the fundamental diagram ?
2. Write the equations describing the dynamic evolution of the traffic flow in the freeway stretch according to the space/time-discretized LW model under the assumption of free-flow traffic conditions downstream of the stretch. Which are the state variables **x** and external (input) variables **d** of the model?
3. Write a computer program to simulate the traffic conditions over a horizon of K = 180 time steps using the following values:

   Initial condition: $\rho_i(0) = 20$ veh/km, i = 1,…, 6

   v_f = 100 km/h, ρ_{max} = 160 veh/km, β = 0.1

$$q_0(k) = \begin{cases} 4000 \ \text{veh/h} & 0 \le k \le 100 \\ 2000 \ \text{veh/h} & 100 < k \le 180 \end{cases}$$

   r(k) = 800 veh/h ∀k

   T = 10 s = 0.00278 h.

   Provide 3-dimensional time-space plots for the mean speed and density.

4. Besides the above 3-dimensional plots, provide also the following 2-densional plots: the density and flow profiles (over time) for each segment, and make some comprehensive analyses of the evolution of traffic flow dynamics across segments based on these plots.