Dynamic Demand I: Overview

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Grad IO

Dynamic Demand

- Earlier this term we looked at *static models of product differentiation* such as BLP (1995).
- We have also looked at single agent models of dynamic behavior such as Rust (1987).
- What if we could put those two together? Why?

Dynamic Demand

What about the following questions?

- Secondary Markets: Good or bad for sellers? overall welfare?
  - I may pay more for a new car today because I can sell it tomorrow.
  - I may pay less for a new car today because of competition with used cars.
- Durability: Should products be built to last longer or shorter?
  - I pay more for an appliance (dishwasher, car, microwave, IPhone, Computer, etc.) if it lasts longer.
  - But, I will re-purchase less frequently (Maytag Repairman)

Dynamic Demand

What about the following questions?

- Adoption or Scrappage Subsidies
  - Cash-For-Clunkers paid a rebate to replace your old car (Explorer/Caravan) with a new fuel-efficient one (Corolla)
  - Was this about being green or about bailing out auto industry?
- Temporary Sales: Why do some products have them?
  - Pure price discrimination (attracting low-value buyers?)
  - Attracting switchers/ state dependence?
  - Intertemporal price discrimination with storage?

Dynamic Demand

Thus far we have implicitly assumed you buy a product and you receive all utility from consumption immediately. We could think about each period receiving a flow payment f_{ijt}. However, most of the time we could just write the NPV of future discounted flow payoffs as a lump sum:

$$v_{ijt} = \sum_{t=0}^{\infty} \beta^t \cdot f_{ijt}$$

and compare lump sum / NPV payoffs: v_{ijt} vs p_{jt} for different goods.

- For things like yogurt –we probably don't need to model when you choose to consume the yogurt separately from purchase.
- Maybe the good depreciates over time $f_{ijt} \geqslant f_{ij,t+1}$ (fine).
- Maybe it breaks with some probability ρ_t (in which case I could use "expected NPV").
- But...

Rentals?

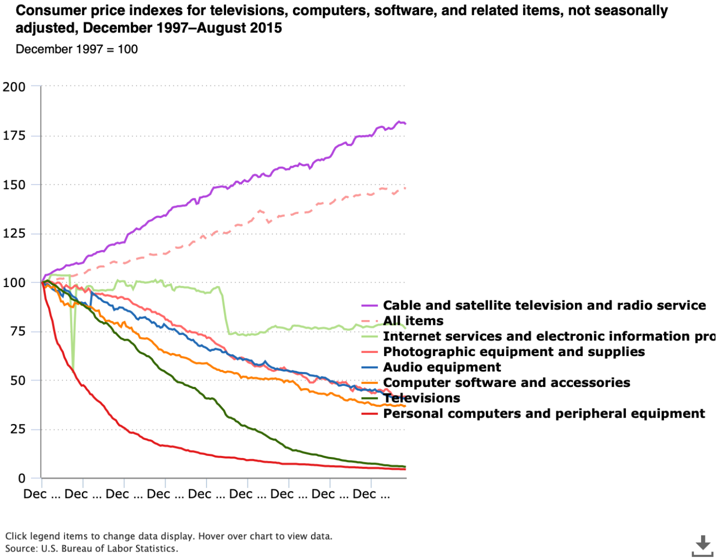
Another way to think about dynamics is to think about the rental rate:

- A house or car or other durable has a per-period price

$$\Delta p_{j,t} = p_{j,t+1} - p_{j,t}$$

- You buy it and pay $p_{j,t}$ and sell it to the market at $p_{j,t+1}$ each period.
- Each period you "rent" the product to yourself at $\Delta p_{j,t}$.
- This only makes sense if the secondary market is frictionless (or we have to include a "switching" term)
  - Gavazza, Lizzeri, Roketskiy (AER 2014) do this for cars.
  - Kalouptsidi (AER 2014) does this to pin down the value function.

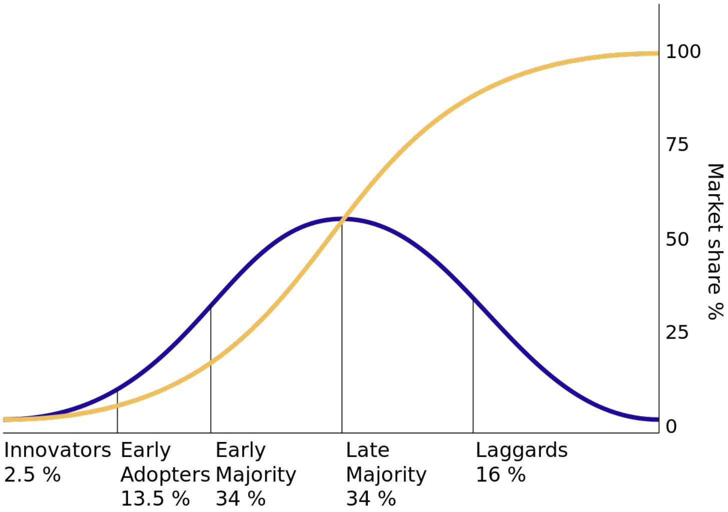
CPI: High Tech Durables



High Tech Durables

- Today a 55" 4K LCD TV is \$239. In 2006, you could buy a 32" 720P TV for $> \$10,000$.
- In December 2011 TV prices fell 17% on an annual basis and other A/V equipment fell 11%, and computer equipment fell 14%.
- From August 2005 to August 2015 prices declined by 87.2%.
- We might also find that over time consumers buy better cameras or larger TV's
- The BLS tries to do *chaining* and *quality adjustments* but in high-tech products this can be very difficult.
- This has a potentially large impact on price indices (a small bias in the CPI can be billions of dollars in SSA/Medicare payments).

Adoption Curve

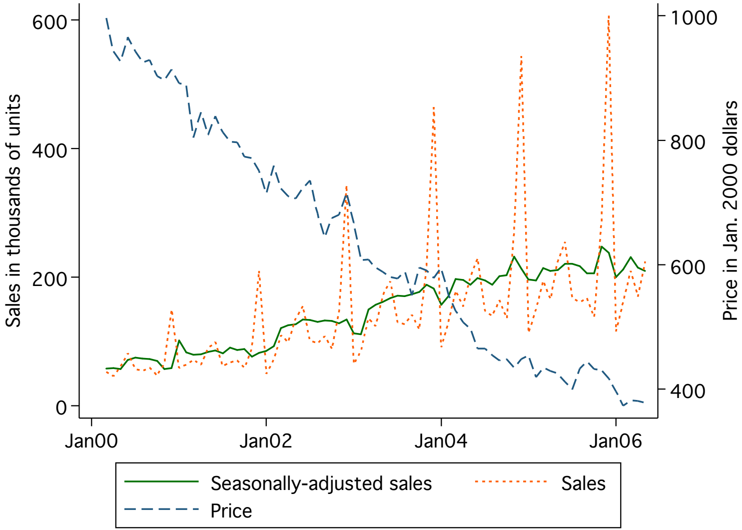


Dynamic Demand (Gowrisankaran Rysman)

Figure 1: Average non-indicator characteristics over time

Dynamic Demand (Gowrisankaran Rysman)

Figure 3: Prices and sales for camcorders



High-Tech Durables

Imagine if we regressed P on Q (with the usual static IV):

- In early periods P falls and Q rises.
- In later periods P falls and Q also falls (top of the S-curve).
- Depending on the time period we might find that demand slopes upwards (lower prices lead to lower sales)

Dynamic Demand: Infeasible Static Approach

Think about this model:

$$u_{ijt} = \alpha_i x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$
$$u_{i0t} = \overline{u}_{i0t} + \varepsilon_{i0t}$$

- The real problem is that $\overline{u}_{i0t} = 0$ for all (i, t) is a bad assumption.
- If we knew $\overline{u}_{i0t}$, we could plug it in, estimate a static model and be fine.
- $\overline{u}_{i0t}$ includes several things:
  - How good is my current TV/Camera/Car today? f_{i0t}. (Initial conditions may differ)
  - What will happen tomorrow / what do I anticipate (new IPhone debut, price cuts for Black Friday, etc.)
- Idea: use the realization of $t + 1$ to inform outside option today (Rust!)

Ad-Hoc approach

- Just proxy with a time trend or sieve (Lou Prentice Ying 2012), (Eizenberg 2011) etc. That is $u_{i0t} = \gamma_{0i} + \gamma_{1i}t + \gamma_{2i}t^2 + \dots$
- We can get the elasticity correct.
- Not structural! Not helpful if we want to do counterfactuals! Can't get the elasticities under different conditions.
- Is u_{i0t} about current durable value? or Equilibrium beliefs about the future? (both!)
- Do we have i specific coefficients (we should!)

Dynamic Demand: Stripped Down Version

Let's start with some very strong assumptions to get our intuition clear:

1. There are $t = 1, 2$ periods.
2. Consumers can purchase at most one unit of an (infinitely) durable good
3. After purchasing the durable good they leave the market forever.

Dynamic Demand: Naive Static Approach

$$u_{ijt} = \alpha_i x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$
$$u_{i0t} = \varepsilon_{i0t}$$

- Suppose we estimate demand treating each period $t = 1, 2$ as a separate market.
- ie: close our eyes and do BLP.
- What do we get wrong?
  - How does $\frac{\partial q_{j,t}}{\partial p_{k,s}}$ look? How should it look?
- Three problems:
  - period $t = 1$ and $t = 2$ are substitutes
  - distribution of $f(\alpha_{it})$ is likely different in $t = 1, 2$.
  - Is $E[u_{i0t}]$ the same for all (i, t)?

Dynamic Demand: Complete Information

Suppose consumers have full information about all shocks $(x_{j1}, x_{j2}, \xi_{j1}, \xi_{j2}, \varepsilon_{ij1}, \varepsilon_{ij2})$ in both periods.

$$u_{ijt} = \alpha_i x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$

What would we do?

- Why not estimate static demand among $\bigcup_{t=1,2} \mathcal{J}_t$ alternatives?
- Remaining issues:
  - Buying in period 1 allows me an additional period of consumption
  - Need to discount period 2 utility $\beta \times (\alpha_i x_{jt} + \xi_{jt} + \varepsilon_{ijt})$ or $\beta \times (\alpha_i x_{jt} + \xi_{jt}) + \varepsilon_{ijt}$
  - A single outside good of not purchasing in either period $u_{i0} = \varepsilon_{i0}$
- How bad is this model? Compared to the last one?

Dynamic Demand: Relaxing some assumptions

Problematic assumption was probably full information. Suppose instead that only $(\varepsilon_{ij2}, \varepsilon_{i0})$ are unobserved in $t = 1$.

$$v_{i0,t=1} \equiv \mathbb{E}_{\varepsilon}[\max_j u_{ij2} | \Omega_{t=1}] = \log\left(\sum_j \exp[\alpha_i x_{j2} + \xi_{j2}]\right) + \eta$$

- $\Omega_{t=1} = \{x_{j1}, x_{j2}, \xi_{j1}, \xi_{j2}, \varepsilon_{ij1}\}$ (everything known at $t = 1$)
- η is Euler's constant (per usual).
- What about outside good?
  - Either just another good in $t = 2$ with $\alpha_i x_{j2} + \xi_{j2} = 0$
  - Or we consider $v_{i0,t=2} = \mathbb{E}[\max\{\varepsilon_{i0}, \max_j u_{ij2}\} | \Omega_{t=1}]$

Dynamic Demand: What about Beliefs?

- This assumed that x_{j2} and ξ_{j2} were observed in $\Omega_{t=1}$.
- Maybe we want to make some component of x_{j2} unobserved (such as price or ξ).

$$\mathbb{E}_t \left[\log \left(\sum_j \exp[\alpha_i p_{j2} + \xi_{j2}] \right) | \Omega_t \right] = \int \log \left(\sum_j \exp[\alpha_i p_{j2} + \xi_{j2}] \right) g(\mathbf{p_2} | \Omega_t)$$

Integrate out over the unknown (distribution depends on information Ω_t)

Dynamic Demand: Rational Expectations

$$\mathbb{E}_t\left[\log\left(\sum_j \exp[\alpha_i x_{j2} + \xi_{j2}]\right)\big|\Omega_t\right] = \log\left(\sum_j \exp[\alpha_i x_{j2} + \xi_{j2}]\right) + \eta + \zeta_{it}$$

Rational expectations implies that expectational error ζ_{it} is orthogonal to everything known at Ω_t

- Can't use anything in Ω_t to predict ζ_{it} so $\mathbb{E}_t[\zeta_{it} \times A(\Omega_t)] = 0$

Dynamic Demand: Relaxing some assumptions

Now what?

$$
\begin{aligned}
u_{ij,t=1} &= \alpha_i x_{j1} + \xi_{j1} + \varepsilon_{ij1} \\
u_{i0,t=1} &= \overline{u}_{i0,t=1} + \beta v_{i0,t=1} + \zeta_{i,t=1} + \varepsilon_{i01} \\
u_{ij,t=2} &= \alpha_i x_{j2} + \xi_{j2} + \varepsilon_{ij2} \\
u_{i0,t=2} &= \overline{u}_{i0,t=2} + \underbrace{\beta v_{i0,t=2} + \zeta_{i,t=2}}_{=0} + \varepsilon_{i02}
\end{aligned}
$$

- If $(\overline{u}_{i0,t}, \beta v_{i0,t=1})$ are known we can estimate static demand with each t as a separate "market".
- I pulled an extra ε_{i01} out of my hat.
- What is the other dynamic linkage?

Dynamic Demand: What about cream skimming?

Also need to account for the fact that $f(\alpha_{i,t=1})$ and $f(\alpha_{i,t=2})$ are not the same:

- If goods are perfectly durable, and consumers permanently exit the market...
- Assume that $f(\alpha_{i,t=1}) = w_{i,t=1}$ is a discrete distribution of "types"
  - Note: "type" does not include ε.
  - then $w_{i,t=2} = w_{i,t=1} \cdot s_{i0,t=1}$

Dynamic Demand: Can we go further?

Replacement / Upgrades:

- Suppose we allow the people who purchase in $t = 1$ to remain in the market.
- Now part of your "type" α_i includes your existing stock of the durable good $\overline{u}_{i0t}$
  - Think about as a RC on the constant β_{i0}
- A purchase increases the value of outside good (previously to ∞)
- The transitions become more complicated $w_{t=2} = f(w_{t=1}, s_{ij,t=1})$.
  - You still throw the old durable into the trash when you are done.
    - Could also allow for a scrap value.
  - Need to be a bit careful about NPV of expected stream of payments vs. "flow utility" now.

Dynamic Demand: Storable Goods

- Same idea:

$$u_{ijt} = \alpha_i x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$
$$u_{i0t} = \overline{u}_{i0t} + \mathbb{E}[\max_j u_{ij,t+1} | \Omega_t] + \varepsilon_{i0t}$$

- My type $\overline{u}_{i0t}$: how much laundry detergent I have left.
- My beliefs $\mathbb{E}[\max_j u_{ij,t+1} | \Omega_t]$: is my preferred brand likely to be on discount in the near future?
- Bias from $u_{i0t} = \varepsilon_{i0t}$. Sales are high during discounts and low following discounts. We think this implies demand is too elastic relative to a permanent price change.
- Durables $Corr(u_{i0t}, p_{jt}) < 0$ (time trend).
  Storables: $Corr(u_{i0t}, p_{jt}) > 0$ (sale in adjacent period).