

# Conduct

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Chris Conlon

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Grad IO

# Conduct Testing in Industrial Organization

Foundational Empirical IO Question: How do we observe data on price and quantity and infer which model of firm behavior generated those outcomes?

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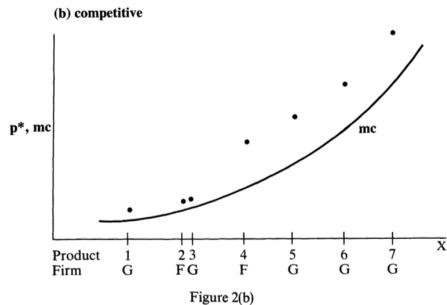
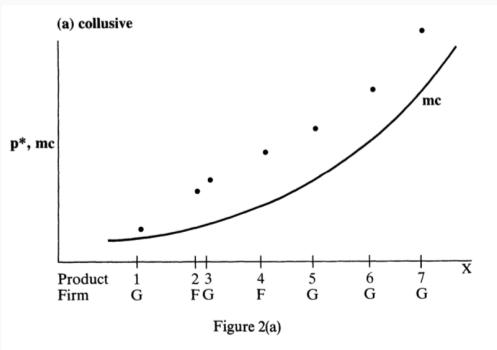
- Early work: Porter (1983), Bresnahan (1982,1987)
- Subsequent work defined the “menu” approach: Nevo (1998, 2001), Villas-Boas (2007)
- Recent revival of “internalization” parameters: Miller and Weinberg (2017), Crawford, Lee, Whinston, and Yurukoglu (2017), Pakes (2017)
- Parallel work by: Duarte, Magnolfi, Sølvssten, Sullivan (2022) which test is best (RV). Magnolfi, Quint, Sullivan, Waldfogel (2022) Should we test or estimate?
- Applications of our test: Starc and Wollman (2022), Scuderi (2022), others?

Is conduct testable? Berry and Haile (2014): yes.

# Conduct Testing in Industrial Organization

- Absent additional restrictions, we cannot generally look at data on  $(P, Q)$  and decide whether or not collusion is taking place.
  - You say we started colluding at date  $t$ , I say we received a correlated shock to  $mc$ .
- We can make progress in two ways: (1) parametric restrictions on marginal costs; (2) exclusion restrictions on supply.
  - Most of the literature focuses on (1) by assuming something like:
$$\ln mc_{jt} = x_{jt}\gamma_1 + w_{jt}\gamma_2 + \omega_{jt}.$$
  - In principle (2) is possible if we have instruments that shift demand for products but not supply. (These are much easier to come up with than “supply shifters”).

## A famous plot (Bresnahan 87)



Bresnahan (1980/1982) recognized this problem: we need “rotations of demand”.

# Conduct Testing in Pictures (Berry Haile 2014)

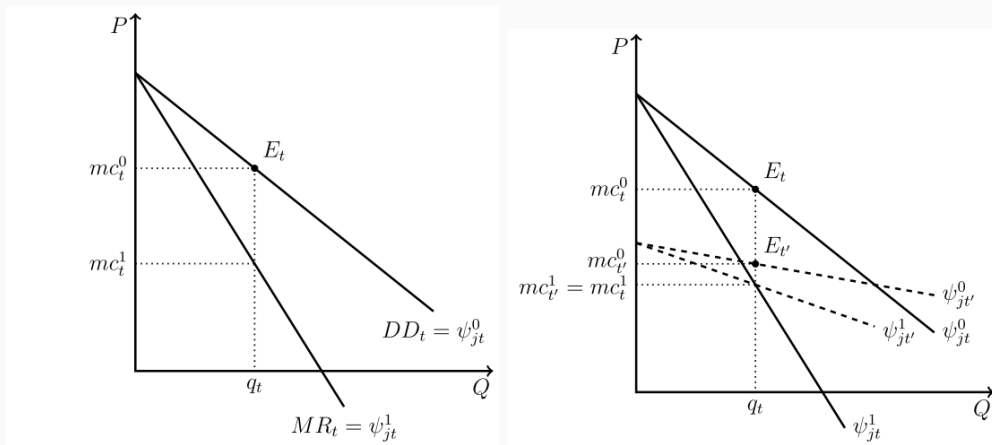


Figure 2(ab) from Berry and Haile (2014), Example 1.

## Setup: Notation and Utility

We begin with a relatively standard BLP-style differentiated products setup.

- Markets  $t$
- Products  $j$
- Data  $\chi_t = \{(x_{jt}, v_{jt}, w_{jt}) \text{ for all } j \in \mathcal{J}_t\}$ .
- Market Shares  $\mathcal{S}_t = [s_{1t}, \dots, s_{Jt}, s_{0t}]$ .
- Prices  $\mathbf{p}_t = [p_{1t}, \dots, p_{Jt}]$ .
- Consumers  $i$  with demographics  $y_{it}$  (income, presence of kids)

## Testing Conduct: Setup

We generalize the  $\mathcal{H}(\kappa)$  and derive multi-product Bertrand FOCs:

$$\begin{aligned}\arg \max_{p \in \mathcal{J}_f} \pi_f(\mathbf{p}) &= \sum_{j \in \mathcal{J}_f} (p_j - c_j) \cdot q_j(\mathbf{p}) + \kappa_{fg} \sum_{j \in \mathcal{J}_g} (p_j - c_j) \cdot q_j(\mathbf{p}) \\ \rightarrow 0 &= q_j(\mathbf{p}) + \sum_{k \in (\mathcal{J}_f, \mathcal{J}_g)} \kappa_{fg} \cdot (p_k - c_k) \frac{\partial q_k}{\partial p_j}(\mathbf{p})\end{aligned}$$

- Instead of 0's and 1's we now have  $\kappa_{fg} \in [0, 1]$  representing how much firm  $f$  cares about the profits of  $g$ .
  - If  $f$  and  $g$  merge (or fully coordinated) then  $\kappa_{fg} = 1$
  - Often in the real world firms cannot reach fully collusive profits and  $\kappa_{fg} \in (0, 1)$ .
  - Evidence that  $\kappa_{fg} > 0$  is not necessarily evidence of malfeasance, just a deviation from static Bertrand pricing



## Testing Conduct: Setup

- Recall the  $\Delta$  matrix which we can write as  $\Delta = \tilde{\Delta} \odot \mathcal{H}(\kappa)$ , where  $\odot$  is the element-wise or Hadamard product of two matrices.
  - $\tilde{\Delta}$  is the matrix of demand derivatives with  $\Delta(j, k) = \frac{\partial q_j}{\partial p_k}$  for all elements.
  - $\mathcal{H}(\kappa) = \kappa_{fg}$  for products owned by  $(f, g)$  where  $\kappa_{ff} = 1$  always.
- Mergers are about changing 0's to 1's in the  $\mathcal{H}(\kappa)$  matrix.
- Matrix form of FOC:  $q(\mathbf{p}) = \Delta(\mathbf{p}, \kappa) \cdot (\mathbf{p} - \mathbf{mc})$
- $\mathbf{mc} = \mathbf{p} - \underbrace{\Delta(\mathbf{p}, \theta_2, \kappa)^{-1} s(\mathbf{p})}_{\eta(\mathbf{p}, \mathbf{s}, \theta_2, \kappa)}$  where  $\eta_{jt}$  is the markup.

# Reasons for Deviations from Static Bertrand

**Biased estimates of own and cross price derivatives:** For anything to work, you have correct estimates of  $\tilde{\Delta}$ . My prior is most papers **underestimate** diversion ratios for close substitutes.

**Vertical Relationships:** Who sets supermarket prices? Just the retailer? Just the manufacturer? Some combination of both? Retailers tend to **soften** downstream price competition.

**Faulty Timing Assumptions:** Bertrand is a simultaneous move pricing game. Lots of alternatives (Stackelberg leader-follower, Edgeworth cycles, etc.).

**Dynamics and Dynamic Pricing:** Forward looking firms or consumers might not set static Nash prices. [e.g. Temporary Sales, Switching Costs, Network Effects, etc.]

**Unmodeled Supergame:** Maybe firms are legally tacitly colluding, higher prices might be about what firms believe will happen in a price war.

# Simultaneous Problem

Assume additivity, and write in terms of structural errors:

$$\begin{aligned}\delta_{jt}(\mathcal{S}_t, \tilde{\theta}_2) + \alpha p_{jt} &= h_d(\mathbf{x}_{jt}, \mathbf{v}_{jt}, \theta_1) + \xi_{jt} \\ f(p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa)) &= h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}, \theta_3) + \omega_{jt}\end{aligned}$$

- To simplify slides we let  $f(x) = x$  (often  $f(x) = \log(x)$ ) but we can put that in  $h_s(\cdot)$ .
- $h(\cdot)$  are often just linear relationships like:  $\theta_1[\mathbf{x}_{jt}, \mathbf{v}_{jt}]$ .
- Endogeneity Problem:  $p_{jt}$  and  $\eta_{jt}$  are functions of  $(\xi, \omega)$ .
- $(\theta_2, \kappa)$  parameters that determine markups

## Approach #1: Demand Side

1. Estimate  $\theta_2$  from demand alone.

$$\delta_{jt}(\mathcal{S}_t, \tilde{\theta}_2) + \alpha p_{jt} = h_d(\mathbf{x}_{jt}, \mathbf{v}_{jt}, \theta_1) + \xi_{jt}$$
$$E[\xi_{jt} | \mathbf{x}_t, \mathbf{v}_t, \mathbf{w}_t] = 0$$

2. Recover marginal costs  $\widehat{\mathbf{mc}} = \mathbf{p} + \boldsymbol{\eta}$

$$\boldsymbol{\eta}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa) \equiv \left( \mathcal{H}(\kappa) \cdot \tilde{\Delta}(\mathbf{p}, \theta_2) \right)^{-1} \mathbf{q}(\mathbf{p})$$

Challenges:

- Given  $[\mathbf{q}, \mathbf{p}, \tilde{\Delta}, \mathcal{H}(\kappa)]$  I can always produce a vector of marginal costs  $\mathbf{mc}$  that rationalizes what we observe. [ie:  $J$  equations  $J$  unknowns].
- Nonparametrically we cannot identify  $\kappa$  without more restrictions (!).

## What do people do?

Maybe some vectors of **mc** look less “reasonable” than others.

- Marginal costs  $\leq 0$  seem problematic. [Might just be that your estimates for demand are too inelastic...]
- or I have a parametric model of MC in mind.

$$f(p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa)) = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}, \theta_3) + \omega_{jt}$$

$$E[\omega_{jt} | \mathbf{x}_t, \mathbf{w}_t, \mathbf{v}_t] = 0$$

- Can test that model with GMM objective of  $mc_{jt}$  on regressors.
- Maybe marginal costs cannot deviate too much within product from period to period. (We can write these as moment restrictions too).

## Approach #2: Simultaneous Supply and Demand

Estimate  $\theta_2$  using both supply and demand. The fit of my supply side will also inform my demand parameters, particularly  $\alpha$  the price coefficient. [BLP 95 used this for additional power with lots of random coefficients and potentially weak instruments].

$$\begin{aligned}\delta_{jt}(\mathcal{S}_t, \tilde{\theta}_2) + \alpha p_{jt} &= h_d(\mathbf{x}_{jt}, \mathbf{v}_{jt}, \theta_1) + \xi_{jt} \\ f(p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa)) &= h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}, \theta_3) + \omega_{jt}\end{aligned}$$

Challenges:

- Should I try to estimate  $\kappa$ ? or just compare objective values at  $\kappa_{fg} \in \{0, 1\}$ ?
- Am I testing conduct? Or am I testing the functional form for my supply model?
- Will a missing IV/restriction change whether or not I believe firms are colluding?

# Estimation vs. Menus

There are two ways to think about conduct:

1. Using moment conditions to estimate  $\hat{\kappa}$  or  $\mathcal{H}(\kappa)$  directly.
  - Often with a small number of parameters (ie:  $\kappa_{fg} = 0$  except for firms I know are in a cartel).
  - Can be challenging to tell similar values of  $\kappa_{fg}$  apart (under-powered).
2. “Menu Approach”
  - Nevo (Economics Letters 1998)
  - Bresnahan (1987)
  - Compare some goodness-of-fit criteria across assumed values of  $\kappa$  (Bertrand vs. Collusion)
3. Rejecting (or failing to reject) a single model.

## Testing a single model of $\kappa$

Put the  $\eta_{jt}$  on the RHS and test whether  $\lambda = 1$ :

$$p_{jt} = h_s(x_{jt}, \mathbf{w}_{jt}, \theta_3) + \lambda \cdot \eta_{jt}(\mathbf{p}, \mathbf{s}, \theta_2, \kappa) + \omega_{jt} \text{ with } E[\omega_{jt} | \mathbf{x}_t, \mathbf{w}_t, \mathbf{z}_t] = 0$$

- We are basically running 2SLS with IV for the endogenous  $\eta_{jt}$
- “Informal” test of Villas Boas (2007):  $\mathbb{E}[\omega_{jt} | \mathbf{x}_{jt}, \mathbf{w}_{jt}, \eta_{jt}] = 0$ .
  - Considers different forms of  $f(\cdot)$ : linear, exponential, logarithmic.
  - Not sure the published paper includes these results (?) WP does?
- Pakes (2017) uses Wollman (2018) data and BLP IV  $\mathbb{E}[\omega_{jt} | x_{jt}, w_{jt}, f(x_{-j})] = 0$ .
- $\lambda \neq 0$  is hard to interpret.



## Table 1: Wollman & Pricing Equilibrium.

Taken from Pakes, 2017, *Journal of Industrial Economics*.

	Price	(S.E.)	Price	(S.E.)
Gross Weight	.36	(0.01)	.36	(.003)
Cab-over	.13	(0.01)	.13	(0.01)
Compact front	-.19	(0.04)	0.21	(0.03)
long cab	-.01	(0.04)	0.03	(0.03)
Wage	.08	(.003)	0.08	(.003)
$\widehat{Markup}$	.92	(0.31)	1.12	(0.22)
Time dummies?	No	n.r.	Yes	n.r.
R <sup>2</sup>	0.86	n.r.	0.94	n.r.

**Note.** There are 1,777 observations; 16 firms over the period 1992-2012. S.E.=Standard error.

These are somewhat reassuring:

- $\lambda \approx 1$  for multiproduct-oligopoly
- Fit is pretty good  $R^2 > 0.8$  and  $R^2 > 0.5$  for within vehicle regressions (not shown).
- As a behavioral model, multiproduct demand estimation seems successful.
- But, do we know that an alternative  $\mathcal{H}(\kappa)$  would have a  $\lambda \neq 1$  or a lower  $R^2$ , and if so how low before we can “reject” the model?

## Goodness of Fit Tests

Another idea (Bonnet and Dubois, Rand 2010) runs the following regression:

$$\log \left( p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \hat{\theta}_2, \kappa) \right) = h_s(x_{jt}, w_{jt}, \theta_3) + \omega_{jt}$$

- Run a regression for each  $\kappa$  and obtain  $Q(\kappa) = \sum_{jt} \hat{\omega}_{jt}^2$
- Employ the **non nested test** of Rivers and Vuong (2002). Why?
- Working out the distribution of  $Q(\kappa_1) - Q(\kappa_2) = T(\kappa_1, \kappa_2)$  is the hard part.
- Also this is OLS (or NLLS) and there are no instruments or **exclusion restrictions** for the supply side. Presumably we could add some and do GMM? (I think this is the “formal” test of Villas Boas (ReStud 2007)).

# Recap

So far three approaches to exploit  $E[\omega_{jt}|x_t, w_t, z_t] = 0$

1. Put the markup on RHS and instrument for it to test  $\lambda = 1$  (Wald)

$$p_{jt} = h_s(x_{jt}, w_{jt}, \theta_3) + \lambda \cdot \eta_{jt}(\mathbf{p}, \mathbf{s}, \hat{\theta}_2, \kappa) + \omega_{jt}$$

2. Put the markup on LHS assuming  $\lambda = 1$  and test goodness of fit of supply equation (Anderson Rubin)

$$p_{jt} - \eta_{jt}(\mathbf{p}, \mathbf{s}, \hat{\theta}_2, \kappa) = h_s(x_{jt}, w_{jt}, \theta_3) + \omega_{jt}$$

3. Estimate supply and demand simultaneously  $[\theta_1, \theta_2, \theta_3]$  and compare goodness of fit for different  $\kappa$ . (Likelihood Ratio)

## Backus Conlon Sinkinson (2022)

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## Basic Setup

We start with marginal revenue and marginal cost (unobserved  $\omega$ , observed  $h(\cdot)$ )

$$\begin{aligned}\psi_{jt}^m &= mc_{jt} \\ p_{jt} - \eta_{jt}^m &= h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}) + \omega_{jt}^m\end{aligned}$$

- Let's be vague/flexible with  $h_s(\cdot)$  for now, but I don't know the production function.
- Assume: Demand and hence  $\eta_{jt}^m$  are **known (given conduct)**.
- Idea  $(\eta^A, \eta^B)$  are monopoly/perfect competition or Cournot/Bertrand.

# The Question

Two competing markups ( $\eta_{jt}^A, \eta_{jt}^B$ ): which fits the data better?  
(both may be misspecified)

$$p_{jt} = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}) + \tau \eta_{jt}^A + (1 - \tau) \eta_{jt}^B + \omega_{jt}$$

Model is defined by a conditional moment restriction  $\mathbb{E}[\omega_{jt} | z_{jt}^s] = 0$

- $H_0 : \tau = 1$  vs  $H_a : \tau = 0$
- This is a **model selection** problem or a **non nested testing** problem.
  - We might want to compare more than two alternatives (too bad).
- Obvious endogeneity problem with  $\eta_{jt}$ !

Compare violations of unconditional moments under  $(\eta_{jt}^A, \eta_{jt}^B)$  and  $A(z_{jt}^s)$ :

$$p_{jt} - \eta_{jt}^A = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}) + \omega_{jt}^A$$

$$p_{jt} - \eta_{jt}^B = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}) + \omega_{jt}^B$$



## Testing Environment

Compare violations of unconditional moments under  $(\eta_{jt}^A, \eta_{jt}^B)$  and  $A(z_{jt}^s)$ :

$$p_{jt} - \eta_{jt}^A = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}) + \omega_{jt}^A$$

$$p_{jt} - \eta_{jt}^B = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}) + \omega_{jt}^B$$

Which gives us

$$g_A = \frac{1}{N} \sum_{jt} \omega_{jt}^A A(z_{jt}^s), \quad g_B = \frac{1}{N} \sum_{jt} \omega_{jt}^B A(z_{jt}^s)$$

$$Q_m = g_m' W_m g_m$$

Now consider a **Rivers Vuong (2002)** type test  $T_{RV} = \sqrt{n} \left( \frac{Q_A - Q_B}{\sigma_{Q_A - Q_B}} \right) \sim N(0, 1)$

$H_0 : Q_A - Q_B = 0$  vs.  $H_A : Q_A > Q_B$  or  $Q_A < Q_B$ .

Getting the SD of the difference is hard  $\rightarrow$  bootstrap

## Comparison to Literature

- Bresnahan (1987): Did LR test to determine collusion vs. competition in 1955 automobile price war
  - No IV, errors were measurement in  $P, Q$ .
- Bonnet and Dubois (2010): RV test
  - But no IV – maximum likelihood with normally distributed  $\omega_{jt}$ 's.
- Villas Boas (2007): Cox test to determine double marginalization or not in yogurt
  - GMM objective, unclear what if any IV are used.
  - Need to “know” the true model.
- Duarte, Magnolfi, Solvsten, Sullivan (2022): RV beats Cox pretty badly in Monte Carlo.

# Main Results: These are $N(0, 1)$

	Others' Cost	Demographics	BLP Inst.	Dmd. Opt. Inst.
Own Profit Max vs.	Panel 1: $A(\mathbf{z}_t) = \mathbf{z}_t$ , linear $h_s(\cdot)$			
Common Ownership	-4.3410	-1.1966	0.5047	-1.2552
Double Marginalization	2.1922	1.0055	-0.0412	7.0897
Double Marginalization + CO	-0.8262	0.6892	0.1428	6.9320
Perfect Competition	3.2995	0.5194	0.7355	3.7223
Monopolist	-2.2264	-1.0528	-0.4525	-0.9202
Own Profit Max vs.	Panel 2: $A(\mathbf{z}_t) = \mathbb{E}[\Delta\eta^{12} \mathbf{z}_t]$ , linear $h_s(\cdot)$ and $g(\cdot)$			
Common Ownership	-2.3044	-0.5105	-0.0384	-1.6133
Double Marginalization	0.8644	0.4421	-0.5311	3.3367
Double Marginalization + CO	-0.9382	-0.2389	-0.3684	-0.0045
Perfect Competition	0.7164	0.6135	-0.1080	-0.3151
Monopolist	-0.8577	-0.4002	-0.3868	-1.2339
Own Profit Max vs.	Panel 3: $A(\mathbf{z}_t) = \mathbb{E}[\Delta\eta^{12} \mathbf{z}_t]$ , random forest $h_s(\cdot)$ and $g(\cdot)$			
Common Ownership	-3.3777	-3.2509	-3.7130	-4.0256
Double Marginalization	-5.9699	-9.9547	-6.5789	-7.8269
Double Marginalization + CO	-5.9264	-6.1550	-6.5231	-7.4760
Perfect Competition	-4.0468	-6.1901	-5.1494	-6.3484
Monopolist	-3.4972	-4.0070	-3.4358	-3.7495

## An Internalization Parameter

Let  $\kappa$  represent the weight a firm places on competitors and  $\tau$  the internalization of those weights.

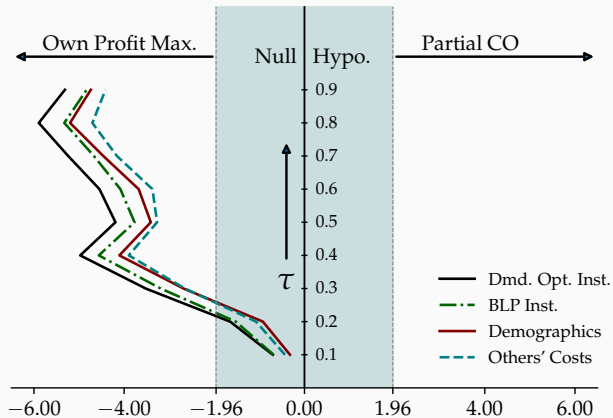
$$\arg \max_{p_j : j \in \mathcal{J}_f} \sum_{j \in \mathcal{J}_f} (p_j - mc_j) \cdot s_j(\mathbf{p}) + \sum_{g \neq f} \tau \kappa_{fg} \sum_{j \in \mathcal{J}_g} (p_k - mc_k) \cdot s_k(\mathbf{p})$$

Now,

- $\tau = 0$  implies own-profit maximization
- $\tau = 1$  implies common ownership pricing
- $\tau$  in between is..? Agency?

We test  $\tau \in (0.1, \dots, 0.9)$  against own-profit maximization.

# Internalization Parameter Results



## Setup: Challenges

The true model for markups (conduct) will satisfy the CMR:  $\mathbb{E}[\omega_{jt}|z_{jt}^s] = 0$

$$p_{jt} - \eta_{jt}^{(m)} = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}; \theta_3) + \omega_{jt}$$

Goal is test two competing markups  $\eta_{jt}^{(A)}, \eta_{jt}^{(B)}$ , but there are challenges:

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$$\mathbb{E}[\omega_{jt} \cdot A(z_{jt}^s)] = 0$$

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- All tests are basically joint tests of the specification for **observed marginal costs** and the **exclusion restriction**.
- Villas Boas (2007) tries log, linear, exponential in  $x\beta$



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3. Choice of  $\eta_{jt}^{(m)}$  will affect our choice of **weighting matrix** and thus the test. (Hall Pelletier (2011))

## A Brief Aside: Chamberlain (1987) in a Slide

What contains as much information as the CMR  $\mathbb{E}[\omega|z_{jt}^s]$  and moments of the form  $\mathbb{E}[\omega_{jt} \cdot A(z_{jt}^s)]$ .

- For linear models  $A(z_{jt}^s) = z_{jt}^s$  is generally without loss.
- For nonlinear models, Chamberlain (1987) shows that the efficient estimator uses

$$A(z_{jt}^s) = \mathbb{E} \left[ \frac{\partial \omega_{jt}}{\partial \theta} | z_{jt}^s \right]$$

- That is not too helpful (its a function of the unknown  $\theta$ ).
- Much of the follow-up work has been about feasible approximations to this “optimal instrument” (e.g., Newey 1990)

For us a similar concern arises, but it is about **power** to distinguish conduct models rather than **efficiency** of estimation.

## Our Idea: Motivation #1 (Optimal IV)

The model is given by

$$p_{jt} = h_s(x_{jt}, w_{jt}, \theta_3) + \tau \cdot \eta_{jt}^A + (1 - \tau) \cdot \eta_{jt}^B + \omega_{jt}^m$$

where  $H_0 : \tau = 1$  and  $H_a : \tau = 0$

- The optimal IV in the Chamberlain (1987) sense is given by  $\mathbb{E} \left[ \frac{\partial \omega_{jt}}{\partial \tau} | z_t \right] = \mathbb{E} \left[ \eta_{jt}^A - \eta_{jt}^B | z_t \right]$ .
- In words: The IV need to predict the **difference in markups** (beyond observed  $h_s(x_{jt}, w_{jt}, \theta_3)$ ).

## Our Idea: Motivation #2 (Misspecification)

Index the **true** model by 0. Then,

$$p_{jt} - \eta_{jt}^0 = h_s(x_{jt}, w_{jt}) + \omega_{jt}^0.$$

To motivate a useful test, we ask what happens when we estimate supply with the **wrong** conduct model (1):

## Our Idea: Motivation #2 (Misspecification)

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To motivate a useful test, we ask what happens when we estimate supply with the **wrong** conduct model (1):

$$p_{jt} - \eta_{jt}^1 = h_s(x_{jt}, w_{jt}) + \underbrace{\eta_{jt}^0 - \eta_{jt}^1}_{\equiv \Delta \eta_{jt}^{0,1}} + \underbrace{\omega_{jt}^0}_{\omega_{jt}^1}.$$

- Misspecifying conduct introduces an omitted variable: the difference in markups.
- Our test is premised on detection of this omitted variable.

## Our Innovation: How does this help?

The model is given by

$$p_{jt} - \eta_{jt}^m = h_s(\cdot) + \omega_{jt}^m, \text{ and } \mathbb{E}[\omega_{jt}^{(m)} \cdot A(z_t)] = 0.$$

We suggest  $A(z_t) = \mathbb{E}[\eta_{jt}^1 - \eta_{jt}^2 | z_t]$ ; several advantages:

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$$p_{jt} - \eta_{jt}^m = h_s(\cdot) + \omega_{jt}^m, \text{ and } \mathbb{E}[\omega_{jt}^{(m)} \cdot A(z_t)] = 0.$$

We suggest  $A(z_t) = \mathbb{E}[\eta_{jt}^1 - \eta_{jt}^2 | z_t]$ ; several advantages:

- Reduces potentially many moments ( $\mathbb{E}[\omega_{jt}' z_t] = 0$ ) to a single, scalar moment. No need for a weighting matrix, or associated problems.
- Testing is reduced to two prediction exercises:  $\mathbb{E}[\eta_{jt}^1 - \eta_{jt}^2 | z_t]$  and  $\widehat{\omega}_{jt}^{(m)}$ .
- Show in the paper that this leads to the most powerful test (maximizes distance between two GMM objective functions conditional on weight matrix).
- Downside: Our choice of instrument is **model specific**! UMP is not going to happen.

## Possible Exclusion Restrictions

We are looking for variables which affect demand but not supply:

$$\sigma_j^{-1}(\mathcal{S}_t, \mathbf{p}_t, \mathbf{y}_t, \mathbf{x}_t, \mathbf{v}_t, \tilde{\theta}_2) = h_d(\mathbf{x}_{jt}, \mathbf{v}_{jt}; \theta_1) - \alpha p_{jt} + \lambda \log(\text{ad}_{jt}) + \xi_{jt}$$

$$p_{jt} - \eta_{jt}(\mathcal{S}_t, \mathbf{p}_t; \theta_2, \mathcal{H}_t(\kappa)) = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}; \theta_3) + \omega_{jt}$$

Things we use:

- Obvious choice:  $\mathbf{v}_{jt}$  (things like product recalls are relatively weak)
- Demographics (enter nonlinearly):  $\mathbf{y}_t$  (chain-level income works well)
- Characteristics of other goods:  $f(\mathbf{x}_{-j,t})$  (BLP instruments).
- Characteristics of other goods:  $\mathbf{w}_{-j,t}$  (commodity price of oats for Rice Krispies)



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Things we don't use:

- Unobserved demand shocks  $\xi_{jt}$  (see MacKay Miller 2020 for  $\text{Cov}(\xi_j, \omega_j) = 0$ ).
- Observable  $\kappa$  conduct shifters (financial mergers/events, see Miller Weinberg (2018))

# Algorithm

(1) Split the sample by markets  $t$  into 70% *test* and 30% *train*.

(2) On the *training sample*:

(a) Approximate the optimal instruments  $a(z_{jt}^s) = \mathbb{E}[\Delta\eta_{jt}^{(1,2)} \mid z_{jt}^s]$  as the fitted values from:

$$\Delta\eta_{jt}^{1,2} = g(z_{jt}^s) + \zeta_{jt}.$$

(b) Estimate the marginal cost function, under models 1 and 2 to obtain residuals  $\hat{\omega}_{jt}^1$  and  $\hat{\omega}_{jt}^2$ :

$$p_{jt} - \eta_{jt}^m = h_s(\mathbf{x}_{jt}, \mathbf{w}_{jt}; \theta_3) + \omega_{jt}^m.$$

(3) On the *test sample*:

(a) For each candidate model, compute the value of the scalar moment:<sup>1</sup>

$$Q(\eta^m) = \left( \sum_{j,t} \hat{\omega}_{jt}^m \cdot \hat{g}(\mathbf{z}_t) \right)^2.$$

(b) Repeat the previous step on bootstrapped samples and estimate  $\hat{\sigma}/\sqrt{n}$  the standard error of the difference  $\hat{Q}(\eta^1) - \hat{Q}(\eta^2)$ .

(c) Compute the test statistic

$$T = \frac{\sqrt{n}(Q(\eta^1) - Q(\eta^2))}{\hat{\sigma}} \sim \mathcal{N}(0, 1).$$

*Note: Steps 2(a) and 2(b) can be done in any order via non-parametric regression.*

# Limitations

Not everything is testable:

- If  $\Delta\eta_{jt}$  cannot be explained by  $z_{jt}^s$  beyond contents of  $(x_j, w_j)$  we have nothing
- Flexible demand models are required to generate cross sectional variation in markups
- Beware of “accidental” exclusion restrictions.

## Alternate Justifications

1. Model Misspecification: if one of the two models is correct,  $\eta_{jt}^1 - \eta_{jt}^2$  exactly corresponds to the misspecification error when using the other. [» detail](#).
2. Difference in Test Statistics: We can also show it maximizes the difference in GMM objectives ( $Q_1 - Q_2$ ). (Which is almost power for fixed  $\sigma$ ). Also we need that either  $h_s(\cdot)$  doesn't depend on  $\eta^{(m)}$  or that it is linear so we can residualize on  $(\mathbf{x}, \mathbf{w})$ .