

# Homogenous Products

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# Introduction

One of the earliest exercises in econometrics is the estimation of supply and demand for a homogenous product

- ▶ According to Stock and Trebbi (2003) IV regression first appeared in a book by Phillip G. Wright in 1928 entitled *The Tariff on Animal and Vegetable Oils* [neatly tucked away in Appendix B: Supply and Demand for Butter and Flaxseed.]
- ▶ Lots of similar studies of simultaneity of supply + demand for similar agricultural products or commodities.

## Working (1927)

Supply and Demand For Coffee, everything is linear

$$Q_t^d = \alpha_0 + \alpha_1 P_t + U_t$$

$$Q_t^s = \beta_0 + \beta_1 P_t + V_t$$

$$Q_t^d = Q_t^s$$

Solving for  $P_t, Q_t$ :

$$P_t = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{V_t - U_t}{\alpha_1 - \beta_1}$$
$$Q_t = \frac{\alpha_1 \beta_0 - \alpha_0 \beta_1}{\alpha_1 - \beta_1} + \frac{\alpha_1 V_t - \beta_1 U_t}{\alpha_1 - \beta_1}$$

Price is a function of both error terms, and we can't use a clever substitution to cancel things out.

To make things really obvious:

$$\begin{aligned}Cov(P_t, U_t) &= -\frac{Var(U_t)}{\alpha_1 - \beta_1} \\Cov(P_t, V_t) &= \frac{Var(V_t)}{\alpha_1 - \beta_1}\end{aligned}$$

- ▶ When demand slopes down ( $\alpha_1 < 0$ ) and supply slopes up ( $\beta_1 > 0$ ) then price is positively correlated with demand shifter  $U_t$  and negatively correlated with supply shifter  $V_t$ .

## Working (1927)

$$Cov(P_t, Q_t) = \alpha_1 Var P_t + Cov(P_t, U_t)$$

$$Cov(P_t, Q_t) = \beta_1 Var P_t + Cov(P_t, V_t)$$

- ▶ Bias in OLS estimate (Demand)  $Bias(\alpha_1) = \frac{Cov(P_t, U_t)}{Var P_t}$ .
- ▶ Bias in OLS estimate (Supply)  $Bias(\beta_1) = \frac{Cov(P_t, V_t)}{Var P_t}$ .
- ▶ We can actually write both this way when  $Cov(U_t, V_t) = 0$ :

$$OLS \text{ Estimate} = \frac{\alpha_1 Var(V_t) + \beta_1 Var(U_t)}{Var(V_t) + Var(U_t)}$$

- ▶ More variation in supply  $V_t \rightarrow$  better estimate of demand.
- ▶ More variation in demand  $U_t \rightarrow$  better estimate of supply.
- ▶ Led Working to say the **statistical demand function** (OLS) is not informative about the economic demand function (or supply function).

- ▶ For most of you, this was probably a review.
- ▶ We know what the solution is going to be to the simultaneity problem.
- ▶ We need an **excluded instrument** that shifts one curve without affecting the other.
- ▶ We can use this to form a 2SLS estimate.
- ▶ Instead let's look at something a little different...

# Simultaneity and Identification

Angrist, Imbens, and Graddy (ReStud 2000).

- ▶ Demand for Whiting (fish) at Fulton Fish Market
- ▶ Do not place functional form restrictions on demand (log-log, log-linear, linear, etc.).
- ▶ “What does linear IV regression of  $Q$  on  $P$  identify, even if the true (but unknown) demand function is nonlinear”
- ▶ Takes a program evaluation/treatment effects approach to understanding the “causal effect” of price on quantity demanded.
- ▶ Aside: Is there even such a thing as the causal effect of price on quantity demanded?

# Four Cases

Ranked in increasing complexity

1. Linear system with constant coefficients

$$\begin{aligned}q_t^d(p, z, x) &= \alpha_0 + \alpha_1 p + \alpha_2 z + \alpha_3 x + \epsilon_t \\q_t^s(p, z, x) &= \beta_0 + \beta_1 p + \beta_2 z + \beta_3 x + \eta_t\end{aligned}$$

2. Linear system with non-constant coefficients

$$\begin{aligned}q_t^d(p, z, x) &= \alpha_{0t} + \alpha_{1t}p + \alpha_{2t}z + \alpha_{3t}x + \epsilon_t \\q_t^s(p, z, x) &= \beta_{0t} + \beta_{1t}p + \beta_{2t}z + \beta_{3t}x + \eta_t\end{aligned}$$

3. Nonlinear system with constant shape (separable)

$$\begin{aligned}q_t^d(p, z, x) &= q^d(p, z, x) + \epsilon_t \\q_t^s(p, z, x) &= q^s(p, z, x) + \eta_t\end{aligned}$$

4. Nonlinear system with time-varying shape (non-separable)

$$\begin{aligned}q_t^d(p, z, x) &= q^d(p, z, x, \epsilon_t) \\q_t^s(p, z, x) &= q^s(p, z, x, \eta_t)\end{aligned}$$



Two kinds:

1. Heterogeneity depending on value of  $p$  fixing  $t$  (only relevant in nonlinear models)
2. Heterogeneity across  $t$ , fixing  $p$  (cases 2 and 4).
  - ▶ The problem is that we don't generally know which kind of heterogeneity we face.
  - ▶ Is case (4) hopeless? Or what can we expect to learn?
  - ▶ Even econometricians struggle with non-linear non-separable models (!)

# AIG: Assumptions

Assume binary instrument  $z_t \in \{0, 1\}$  to make things easier.

1. Regularity conditions on  $q_t^d, q_t^s, p_t, z_t, w_t$  first and second moment and is stationary, etc.
  - ▶  $q_t^d(p, z, x)$ ,  $q_t^s(p, z, x)$  are continuously differentiable in  $p$ .
2.  $z_t$  is a valid instrument in  $q_t^d$ 
  - ▶ Exclusion: for all  $p, t$

$$q_t^d(p, z = 1, x_t) = q_t^d(p, z = 0, x_t) \equiv q_t^d(p, x_t)$$

ie: conditioning on  $p_t$  means no dependence on  $z_t$

- ▶ Relevance: for some period  $t$ :  $q_t^s(p_t, 1, x_t) \neq q_t^s(p_t, 0, x_t)$ .

ie:  $z_t$  actually shifts supply somewhere!

- ▶ Independence:  $\epsilon_t, \eta_t, z_t$  are mutually independent conditional on  $x_t$ .

# Wald Estimator

Focus on the simple case:

- ▶  $z \in \{0, 1\}$  where 1 denotes “stormy at sea” and 0 denotes “calm at sea”
- ▶ Idea is that offshore weather makes fishing more difficult but doesn’t change onshore demand.
- ▶ Ignore  $x$  (for now at least) or assume we condition on each value of  $x$ .

$$\hat{\alpha}_{1,0} \xrightarrow{p} \frac{\mathbb{E}[q_t | z_t = 1] - \mathbb{E}[q_t | z_t = 0]}{\mathbb{E}[p_t | z_t = 1] - \mathbb{E}[p_t | z_t = 0]} \equiv \alpha_{1,0}$$

- ▶ If we are in case (1) then we are good. In fact, any IV gives us a consistent estimate of  $\alpha_1$
- ▶ If we are in case (4) then  $\alpha_{1,0}$  the object we recover, is not an estimator of a structural parameter.
- ▶ Moreover, this is at best a LATE, and thus it differs depending on which instrument we use!

# AGI: Negative Results

- ▶ Should we divorce structural estimation from estimating “deep” population parameters (as suggested by Lucas critique)?
- ▶ Authors make the point that IV estimator identifies something about relationship between  $p$  and  $q$ , without identifying deep structural parameters?
- ▶ In IO this is a somewhat heretical idea (especially to start the course with).

# AGI: Structural Interpretation

In order to interpret the Wald estimator  $\alpha_{1,0}$  we make some additional **economic** assumptions on the structure of the problem:

1. Observed price is market clearing price  $q_t^d(p_t) = q_t^s(p_t, z_t)$  for all  $t$ . (This means no frictions!).
2. “Potential prices”: for each value of  $z$  there is a unique market clearing price

$$\forall z, t : \tilde{p}(z, t) \text{ s.t. } q_t^d(\tilde{p}(z, t)) = q_t^s(\tilde{p}(z, t), z).$$

$\tilde{p}(z, t)$  is the potential price under any counterfactual  $(z, t)$

# AGI: Structural Interpretation

- ▶ Just like in IV we need denominator to be nonzero so that  $E[p_t|z_t = 1] \neq E[p_t|z_t = 0]$ .
- ▶ Other key assumption is the familiar **monotonicity** assumption
  - ▶  $\tilde{p}(z, t)$  is weakly increasing in  $z$ .
  - ▶ Just like in program evaluation this is the key assumption. There it rules out “defiers” here it allows us to interpret the **average slope** as  $\alpha_{1,0}$ .
  - ▶ Assumption is untestable because you do not observe both potential outcomes  $\tilde{p}(0, t)$  and  $\tilde{p}(1, t)$  (same as in program evaluation).
  - ▶ Any story about IV is just a story! (Always the case!) unless we have repeated observations on the same individual.

## AGI: Lemma 1

The key result establishes that the numerator of  $\alpha_{1,0}$ :

$$\mathbb{E}[q_t | z_t = 1] - \mathbb{E}[q_t | z_t = 0] = \mathbb{E}_t \left[ \int_{\tilde{p}(0,t)}^{\tilde{p}(1,t)} \frac{\partial q_t^d(s)}{\partial s} ds \right]$$

- ▶ For each  $t$  we average over the slope of demand curve among the two potential prices:  
 $\int_{\tilde{p}(0,t)}^{\tilde{p}(1,t)} \frac{\partial q_t^d(s)}{\partial s} ds$
- ▶ This range could differ for each  $t$ .
- ▶ Then we average this average over all  $t$ .

# AGI: Figure

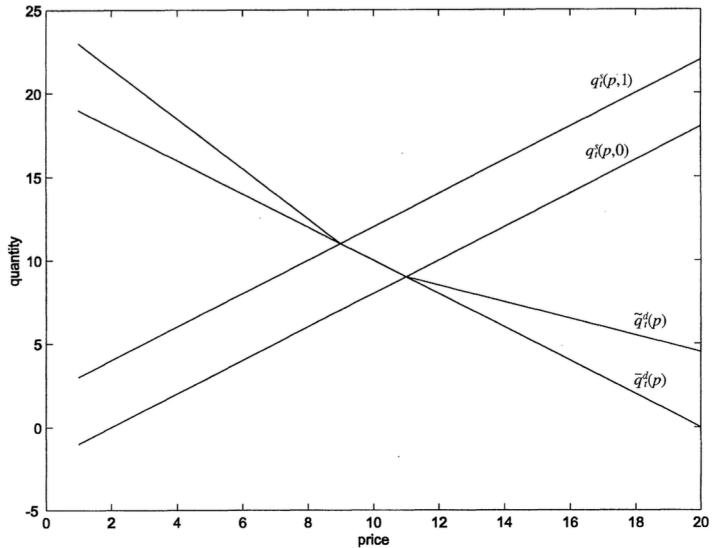


FIGURE 1



# AGI: Takeaways

What did we learn?

- ▶  $\alpha_{1,0}$  only provides information about demand curve in range of potential price variation induced by the instrument.
- ▶ Don't know anything about demand curve outside this range!
- ▶ For different instruments  $z$ ,  $\alpha_{1,0}$  has a different interpretation like the LATE does. (Different from the linear model where anything works!).
- ▶ This is a bit weird: different cost shocks could trace out different paths along the demand curve—why do we care if price change came from a tax change or an input price change? Are they tracing out different subpopulations?
- ▶ We need monotonicity so that we know the range of integration  $\tilde{p}(0, t) \rightarrow \tilde{p}(1, t)$  instead of  $\tilde{p}(1, t) \rightarrow \tilde{p}(0, t)$
- ▶ Observations where  $\tilde{p}(0, t) = \tilde{p}(1, t)$  don't factor into the average but we don't know what these observations are because potential prices are unobserved! What is the relevant sub-sample?

$$\alpha_{1,0} = \frac{\mathbb{E}\left[\int_{\tilde{p}(0,t)}^{\tilde{p}(1,t)} \frac{\partial q_t^d(s)}{\partial s} ds\right]}{\mathbb{E}\tilde{p}(1,t) - \mathbb{E}\tilde{p}(0,t)}$$

$$\rightarrow \int_0^\infty E\left[\frac{\partial q_t^d(s)}{\partial s} | s \in [\tilde{p}(0,t), \tilde{p}(1,t)]\right] \omega(s) ds$$

- ▶ given  $t$  average the slope of  $q_t^d$  from  $\tilde{p}(0,t)$  to  $\tilde{p}(1,t)$
- ▶ given price  $s \in [\tilde{p}(0,t), \tilde{p}(1,t)]$  average  $q_t^d(s)$  across  $t$ . (randomness is due to  $\epsilon_t$ ).
- ▶ Weight  $\omega(s)$  is not a function of  $t$  but it is largest for prices most likely to fall between  $\tilde{p}(0,t)$  and  $\tilde{p}(1,t)$ .
- ▶ Case (2):  $q_t^d(p) = \alpha_{0t} + \alpha_{1t}p + \epsilon_t$ .

$$\alpha_{1,0} = \frac{\mathbb{E}[\alpha_{1t}(\tilde{p}(1,t) - \tilde{p}(0,t))]}{\mathbb{E}\tilde{p}(1,t) - \mathbb{E}\tilde{p}(0,t)} \neq \mathbb{E}\alpha_{1,t}$$

We need mean independence

- ▶ Suppose we had a continuous  $z$  instead, now we can do a full nonparametric IV estimator.

$$\alpha(z) = \lim_{\nu \rightarrow 0} \frac{\mathbb{E}(q_t|z) - \mathbb{E}(q_t|z - \nu)}{\mathbb{E}(p_t|z) - \mathbb{E}(p_t|z - \nu)}$$

- ▶ Use a kernel to estimate  $\hat{q}|z$  and  $\hat{p}|z$

$$\alpha'(z) = \frac{\hat{q}'(z)}{\hat{p}'(z)} \approx \frac{\hat{q}'(z + h) - \hat{q}(z)}{\hat{p}'(z + h) - \hat{p}(z)}$$

# AGI: Takeaways

- ▶ When you have a parametric model, you don't need these results because we can define whatever (nonlinear) parametric functional form we want.
- ▶ There we will focus on parsimonious and realistic parametric functional forms. (this is the rest of the course)
- ▶ If we don't have a parametric model, then these show us that linear IV estimators give us some average (a particular one!) of slopes.
- ▶ Caveat: this only works for a single product. In the multi-product case things are a lot more complicated
  - ▶ For multiproduct oligopoly it is much harder to satisfy the **monotonicity** condition. Why?