Small Sample Properties of Tests for Spatial Dependence in Regression Models: Some Further Results

by

Luc Anselin

and

Raymond Florax

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Luc Anselin
Regional Research Institute
West Virginia University
Morgantown, WV 26506-6825 USA
luc@lambik2.rri.wvu.edu

Raymond Florax
Department of General Economics
Wageningen Agricultural University
6700 EW Wageningen
The Netherlands
Raymond.Florax@ALG.SHHK.WAU.NL

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ABSTRACT: This paper extends earlier work on the properties of tests for spatial dependence in regression models by means of a series of Monte Carlo experiments for both irregular and regular spatial configurations. Eight tests are considered that are all based on the results of an ordinary least squares regression: *Moran's I*; four Lagrange Multiplier (LM) tests (for spatial error dependence; spatial lag dependence; second order spatial error dependence; and for a first order spatial autoregressive moving average or SARMA process); two robust forms of the Lagrange Multiplier test; and a recently suggested robust test. The empirical size of the tests is assessed and their power is compared against three one-directional alternatives (spatial AR error dependence; spatial MA error dependence; and spatial AR lag) and three two-directional alternatives (second order spatial AR errors; second order spatial MA errors; and a SARMA process). The results of the simulations confirm the power of *Moran's I* as a misspecification test against various forms of spatial dependence and provide additional evidence on the power of LM tests against spatial lag alternatives. The use of LM tests, in combination with their robust variants is recommended as the most informative diagnostic to guide a spatial model specification search.

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SMALL SAMPLE PROPERTIES OF TESTS FOR SPATIAL DEPENDENCE IN REGRESSION MODELS: SOME FURTHER RESULTS

1. Introduction

It has now been more than two decades since Cliff and Ord (1972) and Hordijk (1974) applied the principle of Moran's *I* test for spatial autocorrelation to the residuals of regression models for cross-sectional data. To date, Moran's *I* statistic is still the most widely applied diagnostic for spatial dependence in regression models [e.g., Johnston (1984); King (1987); Case (1991)]. However, in spite of the well know consequences of ignoring spatial dependence for inference and estimation [for a review, see Anselin (1988a)], testing for this type of misspecification remains rare in applied empirical work, as illustrated in the surveys of Anselin and Griffith (1988) and Anselin and Hudak (1992). In part, this may be due to the rather complex expressions for the moments of Moran's *I*, and the difficulties encountered in implementing them in econometric software [for detailed discussion, see Cliff and Ord (1981); Anselin (1992); Tiefelsdorf and Boots (1994)]. Recently, a number of alternatives to Moran's *I* have been developed, such as the tests of Burridge (1980) and Anselin (1988b, 1994), which are based on the Lagrange Multiplier (LM) principle, and the robust tests of Kelejian and Robinson (1992) and Bera and Yoon (1992). These tests are all asymptotic and distributed as χ^2 variates. Since they do not require the computation of specific moments of the statistic, they are easy to implement and straightforward to interpret. However, they are all large sample tests and evidence on their finite sample properties is still limited.

In general, the study of finite sample properties of tests for spatial dependence in regression models has not received much attention in the literature. In Bartels and Hordijk (1977) and Brandsma and Ketellapper (1979), the power of Moran's *I* was compared for a number of different estimates for regression residuals. In both simulation studies, the conclusion was that standard ordinary least squares residuals yielded the highest power. In Anselin and Rey (1991), an extensive set of simulation experiments was carried out, comparing Moran's *I* to two Lagrange Multiplier tests for a wide range of regular lattice sizes and error distributions, and for both error and lag forms of spatial dependence. Their conclusion confirmed the theoretical findings on the power of Moran's *I* [King (1981)], but also indicated a tendency for this test to have power against several types of alternatives, including non-normality, heteroskedasticity and different forms of spatial dependence. In contrast, the use of Lagrange Multiplier tests seemed to provide a better basis for the indication of the proper alternative hypothesis, while being fairly close in power to Moran's *I* for spatial error dependence and superior for spatial lag dependence.

In this paper, we provide some further evidence on the finite sample properties of tests for spatial dependence in linear regression models. We extend the existing studies in two important respects. First, we study the properties of some tests for which no results on small sample performance exist to date. This includes the robust Kelejian-Robinson and Bera-Yoon tests, and the Lagrange Multiplier tests for higher order forms of spatial dependence, i.e., second order spatial error dependence and spatial autoregressive moving average (SARMA) processes. Secondly, we compare the power of the tests for both autoregressive and moving average forms of spatial dependence. The latter has not yet been considered.

In all, we consider eight tests for spatial dependence that are based on the results of a classical ordinary least squares regression: Cliff and Ord's (1972) Moran's I (Moran); the Lagrange Multiplier test for spatial error dependence (LM-ERR) due to Burridge (1980); the Kelejian and Robinson (1992) robust test for spatial error dependence (K-R); the test for spatial error dependence robust to the presence of spatial lag dependence (LM-EL) of Bera and Yoon (1992); the LM test for second order spatial error dependence (LM-ERR(2)) of Anselin (1994); the LM test for a spatial autoregressive moving average process (SARMA) of Anselin (1994); the LM test for spatial lag dependence (LM-LAG) of Anselin (1988b); and the test for a spatial lag, robust to the presence of spatial error dependence (LM-LE) of Bera and Yoon (1992). The Monte Carlo experiments are carried out for two irregular and two regular lattice configurations, for both normal and lognormal error terms, and for six alternative hypotheses: three one-dimensional alternatives, i.e., spatial autoregressive error dependence, spatial moving average error dependence and spatial autoregressive lag dependence; and three two-dimensional alternatives, second order spatial autoregressive error dependence, second order spatial moving average error dependence, and a SARMA process.

In the remainder of the paper, we first outline the various tests in more formal terms. This is followed by a description of the experimental design. The results of the Monte Carlo simulations are discussed next. We close with some concluding remarks and recommendations on strategies to carry out specification testing for spatial dependence in practice.

2. Tests for Spatial Dependence

2.1. Null and alternative hypotheses

All tests considered in this paper are based on estimation under the null hypothesis of no spatial dependence, i.e., by means of ordinary least squares. The specification of this linear regression equation is, for $r = 1, ..., R$ spatial units:

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon, \tag{1}$$

with $\mathbf{y}$ as a R by 1 vector of observations on the dependent variable, $\mathbf{X}$ as a R by K matrix of observations on the explanatory variables, ε as a R by 1 vector of uncorrelated (independent) error terms with zero mean and variance matrix $\sigma^2\mathbf{I}$, and β as a K by 1 vector of regression coefficients. In this paper, we will assume the error terms to be homoskedastic. An extension to heteroskedastic errors can be obtained in a straightforward manner [see Anselin (1988a)].

The most general alternative of spatial dependence is a spatial autoregressive moving average or SARMA process [Huang (1984)]. The first order form of such a SARMA process, or, a SARMA(1, 1) process can be expressed as:

$$y = \rho \mathbf{W}_1 \mathbf{y} + \mathbf{X}\beta + \theta_1 \mathbf{W}_1 \mu + \mu, \tag{2}$$

where **y**, **X** and β are as before, $\mathbf{W}_1$ is a R by R spatial weights matrix (typically standardized such that each row sums to one), and μ is an R by 1 vector of error terms.[1] The first term in equation (2), $\rho \mathbf{W}_1 \mathbf{y}$ is a spatial lag (or, spatially lagged dependent variable), with associated autoregressive parameter ρ. The next to last term, $\theta_1 \mathbf{W}_1 \mu$ represents the lag in a spatial moving average, with associated parameter θ_1. A test on the absence of any spatial dependence in (2) then becomes a test on the joint null hypothesis H_0: $\rho = 0$ and $\theta_1 = 0$. Typically, only tests for one type of dependence are carried out, and the other type is assumed to be absent. In other words, the null hypothesis in a test for a spatial autoregressive process is H_0: $\rho = 0$, conditional upon $\theta_1 = 0$. Similarly, the null hypothesis in a test for a spatial moving average error process is H_0: $\theta_1 = 0$, conditional upon $\rho = 0$. When these conditions are not satisfied, i.e., when spatial dependence of the other form is present, a test can no longer be based on OLS regression results, but must be computed from the maximum likelihood estimates of the appropriate spatial model, such as a spatial autoregressive lag model [see Anselin (1988a)]. Alternatively, tests must be constructed that are robust to the presence of the other form of dependence, as in the approach suggested by Bera and Yoon (1992).

The SARMA(1, 1) process can easily be extended to higher orders of spatial dependence. For example, a second order moving average process for the error terms can be specified as:

$$\varepsilon = \theta_1 \mathbf{W}_1 \mu + \theta_2 \mathbf{W}_2 \mu + \mu, \tag{3}$$

where $\mathbf{W}_2$ is a second spatial weights matrix, different from $\mathbf{W}_1$ (e.g., reflecting second order contiguity), with associated parameter θ_2. A test for a second order spatial error MA process is then a test on the joint null hypothesis H_0: $\theta_1 = 0$ and $\theta_2 = 0$. If one of these parameters is assumed to be nonzero, a test for the other one can still be constructed, but must be based on estimates in the appropriate spatial model (typically estimated by maximum likelihood) and no longer on OLS results. Again, robust forms may be constructed based on the principles outlined in Bera and Yoon (1992) [for details, see Anselin (1994)].

Tests for spatial error dependence are typically couched in terms of an alternative hypothesis that is a spatial autoregressive rather than a moving average process. Such a SAR error process is specified as:

$$\varepsilon = \lambda_1 \mathbf{W}_1 \varepsilon + \mu, \tag{4}$$

for a first order process, with λ_1 as the autoregressive parameter, or as:

$$\varepsilon = \lambda_1 \mathbf{W}_1 \varepsilon + \lambda_2 \mathbf{W}_2 \varepsilon + \mu, \tag{5}$$

1 A recently suggested alternative is a form of spatial error components, in which the spatial dependence in the error term is as $\varepsilon = \mathbf{W}\nu + \mu$, where ν and μ are uncorrelated error terms. See Kelejian and Robinson (1993) for further details.

for a second order process, with λ_2 as the autoregressive parameter associated with the second weights matrix, $\mathbf{W}_2$. Tests for spatial error dependence are then tests on the null hypothesis H_0: $\lambda_1 = 0$, or on the joint null hypothesis H_0: $\lambda_1 = 0$ and $\lambda_2 = 0$. In practice, the distinction between autoregressive and moving average error processes is mostly irrelevant, since tests for either form that are based on OLS estimation results are identical.[2] However, the distinction is important with respect to the comprehensive model (2) in the sense that a spatial autoregressive process with spatial autoregressive error terms is not identified, unless the weights matrices for the lag dependence and the error dependence are different. Typically, these weights will be the same, so that the SARMA specification is the only allowable one that combines the two forms of spatial dependence [see Anselin (1988a), for details].

2.2. *Tests for spatial error dependence*

The tests for spatial error dependence considered in this paper are summarized in the first five rows of Table 1. The most familiar among these is Moran's I (Moran), which, for row-standardized weights is defined as:[3]

$$I = \mathbf{e}'\mathbf{W}_1\mathbf{e}/\mathbf{e}'\mathbf{e}, \tag{6}$$

where $\mathbf{e}$ is a R by 1 vector of regression residuals from OLS estimation of equation (1). Inference for this test is carried out on the basis of an asymptotically normal standardized z-value, obtained by subtracting the expected value and dividing by the standard deviation. The detailed moments are derived and discussed at length in Cliff and Ord (1972) and are not repeated here [see also Anselin (1988a); Anselin and Rey (1991); Anselin and Hudak (1992)]. It is important to note that, in contrast to the tests based on the Lagrange Multiplier principle, Moran's I test does not have a direct correspondence with a particular alternative hypothesis.

The second test, LM-ERR, is based on the Lagrange Multiplier principle and was originally suggested in Burridge (1980). The test is identical for spatial autoregressive and spatial moving average errors. It is defined as:

$$\text{LM-ERR} = (\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2/T_1, \tag{7}$$

where $s^2 = \mathbf{e}'\mathbf{e}/R$, and $T_1 = \mathrm{tr}\ (\mathbf{W}_1'\mathbf{W}_1 + \mathbf{W}_1^2)$, with tr as the matrix trace operator. This statistic is distributed as χ^2 with one degree of freedom.

The third test, K-R, is the robust large sample test suggested by Kelejian and Robinson (1992). This test does not assume normality, nor linearity, and is derived from an auxiliary regression using cross products of residuals of observations that are potentially spatially correlated, and cross products of the corresponding explanatory variables. Specifically, the dependent variable in the auxiliary regression is:

2 Note that this property is not specific to spatial dependence, but a general property of Lagrange Multiplier tests. See Bera and Ullah (1991), for a review.

3 Since all simulations are carried out for row-standardized weights, only the expressions of the tests for this case are listed. The general form of Moran's I is given in a number of sources, most notably Cliff and Ord (1972).

TABLE 1: Tests for Spatial Dependence in Regression Models[a]

Test	Formulation	Distribution	Source
Moran	$\mathbf{e}'\mathbf{W}_1\mathbf{e}/\mathbf{e}'\mathbf{e}$	N(0,1)	Cliff and Ord (1972)
LM-ERR	$(\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2/\ T_1$	$\chi^2(1)$	Burridge (1980)
K-R	$(\gamma'\mathbf{Z}'\mathbf{Z}\gamma)/(\alpha'\alpha/h_R)$	$\chi^2(K)$	Kelejian and Robinson (1992)
LM-EL	$[\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2 - T_1(R\tilde{J}_{\rho\cdot\beta})^{-1}(\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2)]^2/[T_1 - T_1^2(R\tilde{J}_{\rho\cdot\beta})^{-1}]$	$\chi^2(1)$	Bera and Yoon (1992)
LM-ERR(2)	$(\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2/\ T_1 + (\mathbf{e}'\mathbf{W}_2\mathbf{e}/s^2)^2/\ T_2$	$\chi^2(2)$	Anselin (1994)
SARMA	$(\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2 - \mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2/[R\tilde{J}_{\rho\cdot\beta} - T_1] + (\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2/\ T_1$	$\chi^2(2)$	Anselin (1988b, 1994)
LM-LAG	$(\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2)^2/(R\tilde{J}_{\rho\cdot\beta})$	$\chi^2(1)$	Anselin (1988b)
LM-LE	$(\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2 - \mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2/[R\tilde{J}_{\rho\cdot\beta} - T_1]$	$\chi^2(1)$	Bera and Yoon (1992)

a. Notation as defined in main text.

$$C_h = e_i e_j, \tag{8}$$

where h is an index for each cross product, e is a residual term and i, j are contiguous observations. The explanatory variables in the auxiliary regression, Z_h are formed as cross products of X_i and X_j. With γ as the coefficient vector obtained from OLS estimation in a regression of **C** on **Z**, and α as the associated vector of residuals, the K-R statistic results as:

$$\text{K-R} = (\gamma'\mathbf{Z}'\mathbf{Z}\gamma)/(\alpha'\alpha/h_R), \tag{9}$$

where h_R is the number of observations in the auxiliary vector (8). The statistic is distributed as χ^2 with K degrees of freedom, where K is the number of explanatory variables in **Z**.

The fourth test, LM-EL, is the adjusted Lagrange Multiplier test of Bera and Yoon (1992) that is robust to local misspecification in the form of a spatial lag term. The test is computed as:

$$\text{LM-EL} = [\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2 - T_1(R\tilde{J}_{\rho\cdot\beta})^{-1}(\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2)]^2/[T_1 - T_1^2(R\tilde{J}_{\rho\cdot\beta})^{-1}], \tag{10}$$

with

$$(R\tilde{J}_{\rho\cdot\beta})^{-1} = [T_1 + (\mathbf{W}_1\mathbf{X}\beta)'\mathbf{M}(\mathbf{W}_1\mathbf{X}\beta)/s^2]^{-1}, \tag{11}$$

where $\mathbf{W}_1\mathbf{X}\beta$ is a spatial lag of the predicted values from an OLS regression of (1), $\mathbf{M} = \mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ is the familiar projection matrix, and the other notation is as before. Just like its LM-ERR counterpart, this statistic is also distributed as χ^2 with one degree of freedom.

The final test for spatial error dependence is a Lagrange Multiplier test for a second order spatial dependence, LM-ERR(2). In general, as shown in Anselin (1994), tests for higher order error dependence are simply the sum of the corresponding one-directional tests, distributed as χ^2 with degrees of freedom equal to the number of terms in the sum. The second order test is thus:

$$\text{LM-ERR(2)} = (\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2 / T_1 + (\mathbf{e}'\mathbf{W}_2\mathbf{e}/s^2)^2 / T_2, \tag{12}$$

with $T_2 = \text{tr}\,(\mathbf{W}_2{}'\mathbf{W}_2 + \mathbf{W}_2^2)$, and the rest of the notation as before.

2.3. *Tests for spatial lag dependence*

The bottom three rows in Table 1 give a summary of three tests for spatial lag dependence. The first, SARMA, is a Lagrange Multiplier test for a joint spatial lag and spatial moving average error, as in equation (2). As shown in Anselin (1994), it is identical to the LM test for a joint spatial lag and spatial autoregressive error of Anselin (1988b), except that the restriction on the weights matrices is relaxed (a process with identical weights matrices for the lag and the error is not identified). The corresponding statistic is:

$$\text{SARMA} = (\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2 - \mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2 / [R\tilde{J}_{\rho\cdot\beta} - T_1] + (\mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2 / T_1, \tag{13}$$

in the same notation as before. This statistic is distributed as χ^2 with two degrees of freedom.

The second test, LM-LAG, is the Lagrange Multiplier test for spatial lag dependence of Anselin (1988b):

$$\text{LM-LAG} = (\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2)^2 / (R\tilde{J}_{\rho\cdot\beta}), \tag{14}$$

distributed as χ^2 with one degrees of freedom. Note that the SARMA test (13) is not simply the sum of the two one-directional tests (7) and (14), which distinguishes this case from the results in Jarque and Bera (1980) and Bera and Jarque (1982).

The final test considered, LM-LE, is the counterpart of LM-EL, i.e., a test for a spatial lag robust to local misspecification in the form of a spatial moving average error process by Bera and Yoon (1992). This test is defined as: [4]

$$\text{LM-LE} = (\mathbf{e}'\mathbf{W}_1\mathbf{y}/s^2 - \mathbf{e}'\mathbf{W}_1\mathbf{e}/s^2)^2 / [R\tilde{J}_{\rho\cdot\beta} - T_1], \tag{15}$$

distributed as χ^2 with one degrees of freedom. Note that this is exactly the first term in the sum (13). In other words, LM-LE (15) and LM-ERR (7) add up to SARMA (13).

4 Note that the original formulation by Bera and Yoon (1992) is for spatial autoregressive error dependence. However, as shown in Anselin (1994), the results are identical for spatial MA errors. Given the identification problems with spatial AR errors in a spatial AR process, MA errors only are considered here.

3. Experimental Design

The experimental design for the Monte Carlo simulations is based on a format extensively used in earlier studies, e.g., in Anselin and Griffith (1988), Anselin and Rey (1991), and Florax and Folmer (1992, 1994). The model under the null hypothesis of no spatial dependence is the classical regression model, as in equation (1). Using standard practice for Monte Carlo simulations, the R observations on the dependent variables are generated from a vector of standard normal random variates ε and a R by 3 matrix of explanatory variables $\mathbf{X}$, consisting of a constant term and two variates drawn from a uniform (0, 10) distribution. In addition to a normal error, a lognormal error term is generated as well, with mean and variance equal to that of the normal variates.[5] In each experiment, 5000 data sets are generated. The tests are evaluated at their theoretical (asymptotic) critical values for $\alpha = 0.05$ and the proportion of rejections (i.e., the proportion of times the computed test statistic exceeded its asymptotic critical value) is reported. For a Type I error of 0.05, the 5000 replications yield a sample standard deviation of 0.0031, which is judged sufficiently precise for our purposes.[6]

The configurations used to generate spatial dependence are formally expressed in four weights matrices. These correspond to sample sizes 40, 81 and 127. The weights matrices of size 40 and 127 are for two actual irregularly shaped regionalizations of the Netherlands, illustrated in Figures 1 and 2. The weights matrices for $R = 81$ correspond to a regular square 9 by 9 grid, with contiguity defined by both the rook (4 neighbors) and the queen (8 neighbors) criterion. The relative merits of using regular versus irregular spatial configurations are discussed at length in Haining (1986) and Anselin (1986). In this experiment, we included both types, the irregular ones to reflect the types of economic regions often encountered in empirical work, and the regular ones to focus on the effect of the characteristics of the connectivity structure on the properties of the tests. In contrast to what holds for time series applications, in space the sample size (R) is not the only variable important in achieving convergence to asymptotic properties of tests and estimators. As shown in Anselin (1988a), the degree of interconnectedness between observations (locations) is also an important factor in determining the extent to which the central limit theorems on dependent spatial processes hold.

A number of different criteria have been suggested to quantify the connectedness structure in a spatial weights matrix, three of which are illustrated in Table 2 for the configurations considered here. The percent zero cells gives and indication of the sparseness of the weights matrix: as R increases, everything else being the same, this percentage should decrease, as it does for the sequence $R = 40$, $R = 81$ (rook) and $R = 127$. Note the qualitative difference between the rook and queen weights for $R = 81$, the latter being

5 The lognormal variates e_i are obtained from the standard normal variates u_i by means of the transformation $e_i = \exp(0.694u_i - 1.272)$.

6 The sample standard deviation is obtained as the square root of $p(1-p)/R$. However, this is based on an assumption of independence between the rejection frequencies. Since all tests are computed for the same generated data, the actual sample standard deviation will be smaller, due to the positive covariance between the statistics. For further discussion of this issue, see, e.g., Davidson and MacKinnon (1993, pp. 738–55).

TABLE 2: Characteristics of Weights Matrices

	$R = 40$	$R = 81$ (queen)	$R = 81$ (rook)	$R = 127$
% nonzero cells	10.77	8.40	4.44	3.76
average links	4.20	6.72	3.56	4.73
largest root	4.90	7.42	3.80	6.25

much less sparse and more interconnected. The average number of links per observation should not increase with R. Again, for the $R = 40$, $R = 81$ (rook) and $R = 127$ sequence this variable is around 4 to 5, while for the queen contiguity it is much higher, at 6.72. The difference between the averages for the rook and queen weights and their theoretical values of 4 and 8 reflects the influence of border and corner cells (with fewer neighboring grid cells). A final characteristic is represented by the largest root of the (unstandardized) weights matrix.[7] As shown in the third row of Table 1, both irregular lattice structures achieve a higher value than the rook criterion for $R = 81$, but are lower than the queen criterion. It is often argued that this reflects a stronger overall spatial interconnectedness for the queen grid cells, and thus a greater potential for nonzero spatial covariances.[8] In our simulation experiments, all weights matrices are used in row-standardized form.

In total, we considered six alternative hypotheses of spatial dependence. Three of these are one-directional, i.e., a function of a single spatial parameter, and three are two-directional, i.e., a function of two spatial parameters. The spatially dependent observations are generated by means of an appropriate spatial transformation applied to a vector of errors or "observations" of uncorrelated values, as follows:

(a) *Spatial autoregressive error*:

$$\varepsilon = (\mathbf{I} - \lambda_1 \mathbf{W}_1)^{-1} \mu ,$$

where μ is a vector of standard normal variates, and the other notation is as before. The resulting vector of spatially autocorrelated errors ε is added to the $\mathbf{X}\beta$ vector to generate a vector of observations on the dependent variable $\mathbf{y}$.

(b) *Spatial moving average error*:

$$\varepsilon = (\mathbf{I} + \theta_1 \mathbf{W}_1) \mu ,$$

with the spatially autocorrelated errors ε added to the explanatory variables in the same way as for (a).

(c) *Spatial autoregressive lag*:

7 For row-standardized weights matrices, the largest root is always 1 and thus not informative in this respect.

8 For a more extensive discussion of the interpretation of the maximum eigenvalue as a characteristic of a spatial network, see, e.g., Boots (1984) and Boots and Royle (1991).

$$y = (\mathbf{I} - \rho \mathbf{W}_1)^{-1} (\mathbf{X}\beta + \varepsilon) ,$$

where ε is a vector of standard normal variates.

(d) *Second order spatial autoregressive error*:

$$\varepsilon = (\mathbf{I} - \lambda_1 \mathbf{W}_1 - \lambda_2 \mathbf{W}_2)^{-1} \mu ,$$

and proceeding in the same way as for (a).

(e) *Second order spatial moving average error*:

$$\varepsilon = (\mathbf{I} + \theta_1 \mathbf{W}_1 + \theta_2 \mathbf{W}_2) \mu ,$$

and proceeding in the same way as for (a).

(f) *SARMA process*:

$$y = (\mathbf{I} - \rho \mathbf{W}_1)^{-1} [\mathbf{X}\beta + (\mathbf{I} + \theta_1 \mathbf{W}_1) \mu] .$$

For the one-directional alternative hypotheses, the spatial parameters take on values from 0.1 to 0.9. For ease of interpretation, negative parameter values are excluded [see Anselin and Rey (1991) for a discussion of the complications caused by negative parameter values]. The maximum value of 0.9 reflects the constraint on the Jacobian term $|\mathbf{I} - \zeta \mathbf{W}|$ for the autoregressive processes and $|\mathbf{I} + \zeta \mathbf{W}|$ for the moving average processes, where ζ represents a spatial parameter ρ, λ_1, or θ_1. As is well known, the Jacobian term simplifies to an expression in the roots of the weights matrix, as shown in Ord (1975) for autoregressive processes:

$$\ln |\mathbf{I} - \zeta \mathbf{W}| = \sum_i \ln (1 - \zeta \omega_i) ,$$

where the ω_i are the eigenvalues of the weights matrix. Consequently, the restriction on the parameter is of the form $\zeta < 1/\omega_i, \forall i$. The resulting acceptable parameter space for autoregressive processes is:[9]

$$1/\omega_{min} < \zeta < 1/\omega_{max} , \tag{16}$$

where the subscripts indicate the minimum and maximum eigenvalues in real terms [see Anselin (1982)]. For moving average processes, ζ should be replaced by $-\zeta$ in expression (16). For row-standardized weights, the largest eigenvalue is 1, and $1/\omega_{min} \leq -1$, which effectively constrains the positive parameter values to $\zeta < 1$. The combinations of parameter value, spatial configuration and error distribution yield a total of 216 cases for the one-directional alternatives.

9 For a different perspective, see Kelejian and Robinson (1994), where the parameter space is defined over the entire range of real values, with the exception of R singularity points.

For the two-directional alternatives, the constraints on the parameter space are slightly more complex. In a SARMA process the constraint (16) holds separately for the autoregressive and the moving average parameter, yielding 81 parameter combinations (positive values only). For the second order autoregressive and moving average processes, the Jacobian terms are, respectively, $|\mathbf{I} - \lambda_1 \mathbf{W}_1 - \lambda_2 \mathbf{W}_2|$ and $|\mathbf{I} + \theta_1 \mathbf{W}_1 + \theta_2 \mathbf{W}_2|$, yielding constraints on the parameters of the form $\lambda_1 \omega_{1,i} + \lambda_2 \omega_{2,i} < 1$ for the AR parameters, and $-(\theta_1 \omega_{1,i} + \theta_2 \omega_{2,i}) < 1$ for the MA parameters. For positive parameter values, this implies that the sum of λ_1 and λ_2 should be less than 1, yielding 36 allowable parameter combinations. For spatial moving average errors, all 81 combinations are allowed. In total, this yields 1,584 cases for the two-directional alternatives.

7. Results of Monte Carlo Experiments

The Monte Carlo experiments were designed to assess a number of properties of the various tests for spatial dependence in different circumstances. In addition to providing insight into the bias and power of the five tests for which no prior finite sample results exist, i.e., KR, LM-EL, LM-LE, LM-ERR(2) and SARMA, the focus of the investigation was on three issues of a more general concern: (a) a comparison of the relative power of the tests against error dependence between spatial autoregressive and spatial moving average errors; (b) a comparison of the relative power of the tests against higher order forms of dependence, and the effect of higher order dependence on the power of one-directional tests; and (c) the power trade-offs involved in using the robust LM-EL and LM-LE tests, both in the presence and in the absence of local misspecification. These issues form the common theme in the discussion of detailed results that follows.

7.1. *Empirical size of tests*

The proportion of rejections of the null hypothesis of no spatial dependence, when none is present, is given in Table 3, for each test, four spatial weights, and for both normal and lognormal error terms. Since the specified critical values were for $\alpha = 0.05$, a significant deviation from this rejection proportion would indicate a bias of the tests in finite samples. For 5000 replications and under a normal approximation to the binomial, a 95% confidence interval centered on $p = 0.05$ would include rejection frequencies between 0.044 and 0.056. It is encouraging to note that for $R = 127$, with normal error terms, all eight tests yield rejection frequencies within this range. Also, even for $R = 40$, this is the case for all but the LM-ERR(2) test. This indicates a correct size for even moderately sized and small data sets. Moreover, four tests, LM-LAG, LM-ERR, LM-EL and LM-LE yield rejection frequencies within the 95% confidence interval in all four samples. This is in general agreement with the results for LM-LAG and LM-ERR in Anselin and Rey (1991). The LM-ERR(2) test significantly under-rejects the null hypothesis for $R = 40$ and the queen case, but not for the others.

The poorest performance results for the rook case (relative to the queen configuration for the same number of observations), where Moran, SARMA and K-R all over-reject the null hypothesis, only margin-

TABLE 3: Empirical Size of Tests[a]

Test	$R = 40$	$R = 81$ (queen)	$R = 81$ (rook)	$R = 127$
Normal Distribution				
Moran	0.051	0.054	0.057	0.051
LM-ERR	0.046	0.046	0.056	0.049
K-R	0.049	0.045	0.065	0.051
LM-EL	0.046	0.049	0.053	0.051
LM-ERR(2)	0.036	0.039	0.048	0.048
SARMA	0.051	0.045	0.057	0.048
LM-LAG	0.052	0.054	0.054	0.051
LM-LE	0.055	0.052	0.055	0.054
Lognormal Distribution				
Moran	0.041	0.044	0.049	0.044
LM-ERR	0.033	0.034	0.047	0.041
K-R	0.073	0.063	0.079	0.062
LM-EL	0.038	0.041	0.046	0.043
LM-ERR(2)	0.028	0.033	0.039	0.042
SARMA	0.042	0.043	0.047	0.050
LM-LAG	0.049	0.051	0.052	0.048
LM-LE	0.050	0.052	0.053	0.056

a. A 95% confidence interval for $p = 0.05$ with 5000 replications is $0.044 < p < 0.056$.

ally for the former two, but significantly so for K-R. A similar result occurred in Anselin and Rey (1991), where differences in empirical size were also found when different weights matrices were used for the same number of observations. It is not clear why the rook case would stand out in this respect. The only indication as to how it differs from the other layouts is that it yields the smallest maximum eigenvalue of the four configurations (but its rejection frequencies are always higher). To some extent, this influence of the choice of the weights matrix is counterintuitive, since there is no spatial dependence present. It further highlights the difference between the two-dimensional spatial dependence and serial dependence in time series analysis, which is one-dimensional (and one-directional). In one dimension, first order dependence (first order autocorrelation) is defined unambiguously, while this is not the case in two dimensions. As shown in Anselin and Rey (1991, Table 4), this is particularly an issue in small samples and is much less pronounced as the number of observations increases (in the limit, the size of R dominates the effect of the connectedness structure).

A misspecification in the form of a lognormal error term seems to affect the size of the tests more for the error tests than for the lag tests, as was the case in Anselin and Rey (1991). For LM-LAG and LM-LE, the rejection frequency remains in the 95% interval for the four cases, while the SARMA test significantly under-rejects for $R = 40$ and the queen case. Of the error tests, LM-ERR and LM-EL significantly under-reject in three configurations (for $R = 40$ and 127, and for the queen case), while LM-ERR(2) under-rejects in all four cases, and Moran only for $R = 40$. In practice, an under-rejection of the null hypothesis when no spatial dependence is present does not have any consequences, since the standard estimation results are interpreted as they should be (without taking spatial effects into account). On the other hand, an over-rejection would tend to result in unnecessary estimation of spatial models and cause problems with pre-testing [see Florax and Folmer (1992)]. This is the case for the K-R test with lognormal errors, which significantly over-rejects the null hypothesis in all four configurations. There are two possible explanations for the apparent bias of this test. One is that the test may not be robust to lognormal errors. Another, and more likely explanation is that the K-R test does not yet achieve its "large sample" robust properties for the sample sizes considered in the experiments.

7.2. *Power of tests against first order spatial error dependence*

The rejection frequencies of the tests against an alternative of spatial autoregressive errors are listed in Table 4 for normal disturbance terms, and in Table A.1. in the Appendix for lognormal errors. To more specifically illustrate the relative performance of the five error tests, their power functions are shown in Figure 3, for $R = 40$, and in Figure 4, for $R = 127$. Of all the tests, Moran always achieves highest power, with LM-ERR as a close second, becoming virtually indistinguishable for $R = 127$ and $\lambda_1 \geq 0.5$. This confirms earlier findings on the superiority of Moran in Anselin and Rey (1991), but now relative to a wider range of competitors. Of the other error tests, the power of K-R is always considerably lower, particularly for small values of λ_1, but even in the largest sample. LM-EL entails a small loss of power relative to LM-ERR, and is slightly inferior to LM-ERR(2) in the smaller samples. However, for $R = 127$, the power functions of LM-EL and LM-ERR(2) are virtually identical (and cross at $\lambda_1 = 0.5$) and approach the one for LM-ERR, as Figure 4 illustrates. In other words, even in moderately sized samples, the penalty for the correction for a potential lag in LM-EL is almost negligible when no lag is actually present. On the other hand, LM-ERR(2) has high power, even when no second order error dependence is present, though it is always less than LM-ERR. The poor performance of K-R may be in part due to its higher degrees of freedom (3 in the current experiments). Note that all error tests perform rather poorly in the smallest sample, achieving the 95% rejection mark only for $\lambda > 0.7$ for Moran, LM-ERR and LM-ERR(2), and for $\lambda > 0.8$ for LM-EL and K-R.

Two of the three lag tests, SARMA and LM-LAG, achieve considerable power against AR error dependence, the former at levels comparable to LM-EL and LM-ERR(2), but always less than LM-ERR. In other words, SARMA has higher power against AR error dependence, compared to LM-LAG, which makes it slightly less suitable in a specification search aimed at distinguishing lag from error dependence. On the other hand, a comparison of LM-ERR and LM-LAG, as suggested in Anselin and Rey (1991), provides an indication of the proper alternative.[10]

10 Note that a specification search for a spatial regression model often occurs in contexts of multiple comparisons and pre-testing, as shown in Florax and Folmer (1992, 1994). Nevertheless, they found that the simple decision rule formulated in Anselin and Rey (1991) performs well.

TABLE 4: Power of Tests Against First Order Spatial Autoregressive Errors — Normal Distribution

R	λ_1	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)	SARMA	LM-LAG	LM-LE
40	0.1	0.090	0.064	0.067	0.066	0.058	0.071	0.067	0.067
	0.2	0.173	0.125	0.109	0.109	0.111	0.122	0.089	0.076
	0.3	0.313	0.242	0.194	0.207	0.215	0.222	0.125	0.081
	0.4	0.491	0.401	0.318	0.333	0.355	0.365	0.180	0.096
	0.5	0.687	0.612	0.487	0.524	0.552	0.564	0.253	0.122
	0.6	0.835	0.790	0.670	0.689	0.742	0.753	0.379	0.141
	0.7	0.931	0.910	0.828	0.830	0.878	0.885	0.540	0.154
	0.8	0.984	0.974	0.931	0.923	0.961	0.962	0.724	0.166
	0.9	0.997	0.996	0.987	0.972	0.993	0.994	0.899	0.171
81 (queen)	0.1	0.100	0.066	0.071	0.065	0.066	0.072	0.063	0.063
	0.2	0.222	0.161	0.137	0.146	0.139	0.158	0.091	0.075
	0.3	0.403	0.312	0.258	0.285	0.279	0.276	0.119	0.082
	0.4	0.612	0.533	0.445	0.490	0.489	0.494	0.174	0.105
	0.5	0.811	0.758	0.658	0.697	0.706	0.707	0.279	0.122
	0.6	0.928	0.898	0.833	0.866	0.873	0.871	0.399	0.159
	0.7	0.980	0.973	0.949	0.958	0.961	0.965	0.579	0.199
	0.8	0.998	0.995	0.989	0.991	0.991	0.992	0.767	0.264
	0.9	1.000	1.000	0.999	0.999	0.999	0.999	0.933	0.356
81 (rook)	0.1	0.098	0.072	0.065	0.070	0.060	0.063	0.052	0.048
	0.2	0.260	0.208	0.152	0.179	0.151	0.164	0.079	0.056
	0.3	0.508	0.431	0.307	0.389	0.326	0.350	0.107	0.057
	0.4	0.755	0.691	0.541	0.645	0.587	0.603	0.153	0.063
	0.5	0.918	0.889	0.773	0.839	0.816	0.831	0.271	0.066
	0.6	0.981	0.974	0.922	0.952	0.948	0.953	0.402	0.083
	0.7	0.998	0.997	0.984	0.991	0.992	0.993	0.595	0.093
	0.8	1.000	1.000	0.998	0.999	0.999	0.999	0.811	0.118
	0.9	1.000	1.000	1.000	1.000	1.000	1.000	0.965	0.159
127	0.1	0.145	0.118	0.097	0.112	0.099	0.108	0.068	0.058
	0.2	0.403	0.353	0.237	0.318	0.292	0.300	0.107	0.073
	0.3	0.728	0.683	0.496	0.628	0.605	0.609	0.184	0.084
	0.4	0.922	0.900	0.755	0.869	0.861	0.867	0.301	0.103
	0.5	0.990	0.986	0.931	0.975	0.975	0.978	0.464	0.123
	0.6	0.999	0.998	0.990	0.996	0.997	0.998	0.651	0.157
	0.7	1.000	1.000	0.999	1.000	1.000	1.000	0.844	0.187
	0.8	1.000	1.000	1.000	1.000	1.000	1.000	0.964	0.221
	0.9	1.000	1.000	0.999	1.000	1.000	1.000	0.999	0.247

The robust LM-LE test performs remarkably well, yielding low rejection frequencies even for $\lambda_1 = 0.9$ (e.g., 25% for $R = 127$). The correction for error dependence in LM-LE thus seems to work in the right direction when no lag dependence is present, especially for small values of λ_1.

The effect of lognormal errors on the power of the tests is small. As indicated by the results in Table A.1, the power of the error tests is slightly less than for the normal case for small values of λ_1, but

TABLE 5: Power of Tests Against First Order Spatial Moving Average Errors — Normal Distribution

R	θ_1	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)	SARMA	LM-LAG	LM-LE
40	0.1	0.090	0.061	0.066	0.063	0.065	0.072	0.063	0.058
	0.2	0.152	0.103	0.098	0.097	0.099	0.104	0.086	0.075
	0.3	0.256	0.190	0.157	0.163	0.171	0.178	0.098	0.078
	0.4	0.389	0.307	0.237	0.259	0.281	0.279	0.135	0.085
	0.5	0.529	0.445	0.333	0.370	0.411	0.389	0.178	0.101
	0.6	0.663	0.570	0.435	0.479	0.536	0.522	0.219	0.113
	0.7	0.761	0.682	0.533	0.582	0.649	0.630	0.266	0.118
	0.8	0.844	0.779	0.627	0.682	0.762	0.729	0.313	0.143
	0.9	0.902	0.854	0.716	0.766	0.850	0.816	0.353	0.142
81 (queen)	0.1	0.094	0.062	0.073	0.063	0.064	0.071	0.061	0.061
	0.2	0.192	0.134	0.126	0.122	0.120	0.127	0.078	0.067
	0.3	0.329	0.250	0.206	0.226	0.227	0.227	0.106	0.085
	0.4	0.490	0.401	0.317	0.370	0.366	0.360	0.138	0.085
	0.5	0.633	0.550	0.448	0.502	0.511	0.496	0.179	0.096
	0.6	0.754	0.686	0.566	0.638	0.649	0.630	0.223	0.107
	0.7	0.852	0.795	0.684	0.749	0.769	0.749	0.272	0.117
	0.8	0.917	0.875	0.773	0.839	0.864	0.836	0.312	0.131
	0.9	0.960	0.937	0.859	0.905	0.931	0.909	0.379	0.133
81 (rook)	0.1	0.099	0.071	0.072	0.069	0.056	0.065	0.055	0.050
	0.2	0.233	0.176	0.131	0.161	0.118	0.140	0.069	0.047
	0.3	0.478	0.400	0.272	0.362	0.292	0.303	0.092	0.045
	0.4	0.712	0.646	0.452	0.582	0.517	0.531	0.131	0.051
	0.5	0.868	0.823	0.642	0.771	0.741	0.730	0.177	0.051
	0.6	0.950	0.924	0.798	0.889	0.885	0.864	0.234	0.053
	0.7	0.986	0.973	0.898	0.954	0.968	0.944	0.290	0.055
	0.8	0.997	0.995	0.957	0.984	0.996	0.984	0.342	0.057
	0.9	0.999	0.997	0.981	0.994	0.999	0.994	0.411	0.058
127	0.1	0.141	0.114	0.094	0.106	0.097	0.103	0.066	0.067
	0.2	0.366	0.322	0.215	0.285	0.275	0.267	0.104	0.069
	0.3	0.639	0.583	0.401	0.538	0.517	0.509	0.154	0.074
	0.4	0.858	0.825	0.627	0.781	0.768	0.771	0.217	0.080
	0.5	0.956	0.937	0.797	0.910	0.909	0.903	0.293	0.103
	0.6	0.989	0.983	0.909	0.970	0.974	0.968	0.385	0.111
	0.7	0.998	0.997	0.968	0.993	0.995	0.992	0.466	0.113
	0.8	1.000	1.000	0.988	0.998	0.999	0.999	0.532	0.124
	0.9	1.000	1.000	0.997	1.000	1.000	1.000	0.618	0.133

equal to or slightly larger for $\lambda_1 > 0.5$. The relative position of the performance of the tests is not affected, and K-R in particular is not distinguished by a higher degree of "robustness."

The rejection frequencies of the eight tests against spatial moving average errors are reported in Table 5 for normal errors, and in Table A.2 of the Appendix for lognormal errors. The power functions for the five specific error tests are illustrated for $R = 40$ in Figure 5, and for $R = 127$ in Figure 6.

A striking feature of the results is that, for $\theta_1 > 0.1$, and for similar parameter values, the power of all tests is considerably lower than that against AR errors.[11] This is clearly illustrated by Figure 5, where even Moran never reaches a rejection frequency higher than 90%. This raises serious doubt about the usefulness of these tests against spatial MA errors in small samples such as the $R = 40$ used here. The power is more acceptable for $R = 127$, but still remains considerably lower than against AR errors for similar parameter values. On the other hand, the relative ranking of tests is not affected, and K-R continues to perform poorly, even in the largest sample, as illustrated in Figure 6. As in the AR case, there is a loss of power involved in using the robust LM-EL test compared to the LM-ERR tests, particularly in the smallest configuration, but this is much less the case for $R = 127$. Note that LM-EL moves from well below the power function for LM-ERR in Figure 5, to slightly above the one for LM-ERR(2), and much closer to LM-ERR in Figure 6.

Another interesting aspect of the results in Table 5 is that LM-LAG no longer has very good power against error dependence. For example, for $R = 40$, the null hypothesis is rejected by LM-LAG in only 35% of the cases for $\theta_1 = 0.9$, compared to 90% for $\lambda_1 = 0.9$ in Table 4. Even in the largest configuration, the rejection frequency is only about 62% for LM-LAG (with $\theta_1 = 0.9$). To some extent, this is to be expected, given the much greater similarity between a lag process and a spatial AR error process (e.g., in the form of a spatial Durbin model), while such a similarity does not exist with MA errors. On the other hand, the clear superiority of LM-ERR compared to LM-LAG in this context would tend to strengthen the decision rule of Anselin and Rey (1991).

Of the other two lag tests, SARMA has good power against MA errors, less than LM-ERR (except in one instance, for $\theta_1 = 0.1$ and $R = 40$), but comparable to LM-EL. Again, in practice a SARMA test should be compared to the one-directional LM-ERR test in order to aid in identifying the proper alternative hypothesis. Finally, the robust LM-LE test performs even better (i.e., has lower power) than in the AR case. For example, for the rook configuration with $R = 81$, the rejection frequencies of LM-LE are practically within the 95% confidence interval around 0.05 for all values of θ_1.

The effect of lognormal errors is similar to that in the AR error case.

7.3. *Power of tests against spatial autoregressive lag dependence*

The rejection frequencies of the eight tests against a first order spatial autoregressive lag are reported in Table 6 for normal disturbances and in Table A.3 of the Appendix for lognormal errors. The power functions of the three lag tests are illustrated in Figure 7, for $R = 40$.

The LM-LAG test is clearly the most powerful test against this alternative, achieving a 95% rejection level for $\rho > 0.3$ in the smallest sample, and for $\rho > 0.1$ in the rook ($R = 81$) and $R = 127$ configurations. The two other lag tests have only slightly less power and are almost indistinguishable in the largest data

11 In a strict sense, the parameter values for an AR error process and a MA error process are not equivalent, since each process implies a different range for the spatial interaction between observations. For an AR process, all observations interact, while for a MA process, only the first and second order neighbors interact, as shown in Anselin (1994). In other words, the same parameter values imply a stronger interaction for an AR process than for a MA process.

TABLE 6: Power of Tests Against First Order Spatial Autoregressive Lag — Normal Distribution

R	ρ	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)	SARMA	LM-LAG	LM-LE
	0.1	0.092	0.067	0.071	0.048	0.060	0.150	0.193	0.183
	0.2	0.206	0.147	0.132	0.040	0.123	0.458	0.554	0.501
	0.3	0.423	0.331	0.259	0.033	0.263	0.783	0.858	0.797
	0.4	0.697	0.609	0.479	0.026	0.506	0.966	0.978	0.959
40	0.5	0.877	0.822	0.703	0.018	0.746	0.996	0.998	0.994
	0.6	0.975	0.956	0.887	0.010	0.914	1.000	1.000	0.999
	0.7	0.997	0.993	0.976	0.002	0.984	1.000	1.000	1.000
	0.8	1.000	1.000	0.997	0.001	0.998	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.006	1.000	1.000	1.000	1.000
	0.1	0.119	0.084	0.089	0.052	0.076	0.234	0.299	0.276
	0.2	0.342	0.260	0.241	0.062	0.232	0.734	0.810	0.777
	0.3	0.692	0.610	0.551	0.118	0.557	0.980	0.992	0.984
	0.4	0.936	0.904	0.865	0.248	0.868	1.000	1.000	1.000
81 (queen)	0.5	0.993	0.990	0.983	0.441	0.986	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.632	0.999	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.802	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.860	1.000	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.351	1.000	1.000	1.000	1.000
	0.1	0.096	0.073	0.081	0.057	0.063	0.372	0.463	0.458
	0.2	0.210	0.162	0.164	0.063	0.174	0.931	0.967	0.955
	0.3	0.395	0.321	0.320	0.061	0.438	1.000	1.000	1.000
	0.4	0.630	0.557	0.566	0.061	0.766	1.000	1.000	1.000
81 (r)	0.5	0.858	0.806	0.805	0.051	0.953	1.000	1.000	1.000
	0.6	0.981	0.969	0.964	0.027	0.998	1.000	1.000	1.000
	0.7	1.000	0.999	0.997	0.010	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.001	1.000	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.1	0.177	0.142	0.123	0.045	0.125	0.549	0.653	0.607
	0.2	0.601	0.547	0.460	0.053	0.507	0.992	0.996	0.993
	0.3	0.943	0.927	0.870	0.092	0.925	1.000	1.000	1.000
	0.4	0.999	0.998	0.991	0.116	0.998	1.000	1.000	1.000
127	0.5	1.000	1.000	1.000	0.077	1.000	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.027	1.000	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.002	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000

set. In other words, the penalty in terms of power for the robustness against error dependence in LM-LE, when none is present, is almost negligible. On the other hand, there is hardly any power difference between this test and the SARMA test that explicitly takes error dependence into account.

Overall, the power functions of the three lag tests compare very favorably to the ones for tests against error dependence, as illustrated by the much steeper slope in Figure 7, even relative to the slopes for $R = 127$ in Figures 4 and 6. This reliability of the lag tests is encouraging, since the consequences of ignoring a spatial lag (as an omitted variable) when one should be included (i.e., inconsistent estimates) are much more serious than the ones of ignoring spatially correlated errors (less efficient estimates).

Four of the five error tests also have power against a spatial lag (Moran the most), but much less than the lag tests. The behavior of LM-EL is very interesting. Except somewhat for $\rho = 0.7$ and 0.8 in the queen case ($R = 81$), this test has no power against lag dependence, as it should. Moreover, its power function tends to decrease with increasing values of ρ. For small values of ρ, the rejection frequency of LM-EL is very close to its expected value of 0.05, but for large values, it becomes almost negligible (except for the queen case). Since the LM-EL test is robust to "local" misspecification, this is not surprising, although it may be a bit disconcerting in practice. On the other hand, a clear discrepancy between the indication of LM-ERR and LM-EL, while both LM-LAG and LM-LE are significant would provide strong evidence for lag dependence as opposed to error dependence. The extent to which such a decision rule would hold in a specification search characterized by pre-testing remains to be further investigated.

Relatively speaking, the effect of a misspecification in the form of lognormal errors is much less on the lag tests than in the case of error dependence. The robustness of LM-LAG which was found in Anselin and Rey (1991) is thus extended to SARMA and LM-LE as well. Interestingly, for small values of ρ in Table A.3, the power of the tests is slightly higher in the presence of lognormal errors, while for larger values it is lower (although the latter effect is marginal, given the strong power of the tests for large values of ρ).

7.4. *Power of tests against second order spatial error dependence*

The results on the rejection frequencies of the five tests for error autocorrelation used against higher order alternatives are reported in Tables 7 to 10 for second order autoregressive errors, and in Tables 11 to 14 for second order moving average errors. These results are for normal disturbances only. To conserve space, results for lognormal errors are not reported.[12] The power function for the LM-ERR(2) test is further illustrated in Figures 8 and 9, for the rook configuration ($R = 81$). Since two parameters vary, this should be a three dimensional power surface. However, given the problems with interpreting such figures, a series of cross sections is presented instead, showing the variation of power in function of the second order parameter (respectively, λ_2 for AR, and θ_2 for MA), for selected values of the first order parameter (respectively, λ_1 for AR, and θ_1 for MA).

The LM-ERR(2) test turns out to have acceptable power against both autoregressive and moving average errors, particularly whenever $\lambda_2 \geq \lambda_1$ and $\theta_2 \geq \theta_1$, but not in the smallest data set. As in the tests against one-directional alternatives, the power is higher for AR than for MA errors, for the same parameter values, as illustrated in Figures 8 and 9. Note that the power curve for $\lambda_1 = 0$ in Figure 8 is actually the power of LM-ERR(2) against first order AR error dependence (as in Figures 3 and 4), but for the weights

12 The effect of lognormal errors is in general fairly small and qualitatively similar to their effect for the one-directional alternatives. The detailed results are available from the authors.

TABLE 7: Power of Tests Against Second Order Spatial Autoregressive Errors
$R = 40$ — Normal Distribution

λ_1	λ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.1	0.1	0.097	0.071	0.081	0.071	0.067
	0.2	0.117	0.087	0.097	0.085	0.100
	0.3	0.142	0.107	0.117	0.099	0.150
	0.4	0.183	0.151	0.159	0.137	0.234
	0.5	0.233	0.194	0.202	0.182	0.341
	0.6	0.299	0.258	0.267	0.227	0.469
	0.7	0.411	0.373	0.368	0.330	0.626
	0.8	0.532	0.491	0.487	0.441	0.759
0.2	0.1	0.194	0.141	0.130	0.129	0.136
	0.2	0.224	0.170	0.159	0.147	0.172
	0.3	0.247	0.201	0.191	0.186	0.238
	0.4	0.343	0.283	0.269	0.249	0.357
	0.5	0.406	0.353	0.343	0.315	0.471
	0.6	0.516	0.465	0.449	0.422	0.614
	0.7	0.632	0.592	0.570	0.534	0.762
0.3	0.1	0.339	0.268	0.217	0.232	0.237
	0.2	0.386	0.316	0.280	0.280	0.308
	0.3	0.461	0.393	0.339	0.341	0.401
	0.4	0.524	0.462	0.423	0.412	0.510
	0.5	0.618	0.569	0.529	0.514	0.638
	0.6	0.720	0.680	0.648	0.628	0.763
0.4	0.1	0.552	0.473	0.377	0.402	0.425
	0.2	0.595	0.531	0.446	0.461	0.494
	0.3	0.656	0.596	0.522	0.524	0.587
	0.4	0.741	0.690	0.633	0.627	0.703
	0.5	0.821	0.787	0.756	0.739	0.819
0.5	0.1	0.733	0.672	0.558	0.581	0.611
	0.2	0.774	0.721	0.634	0.648	0.685
	0.3	0.830	0.790	0.729	0.726	0.776
	0.4	0.892	0.869	0.826	0.820	0.862
0.6	0.1	0.867	0.825	0.737	0.742	0.780
	0.2	0.900	0.871	0.805	0.799	0.843
	0.3	0.942	0.924	0.884	0.876	0.913
0.7	0.1	0.949	0.930	0.874	0.875	0.908
	0.2	0.974	0.964	0.930	0.935	0.952
0.8	0.1	0.990	0.986	0.964	0.961	0.978

TABLE 8: Power of Tests Against Second Order Spatial Autoregressive Errors
$R = 81$ (queen) — Normal Distribution

λ_1	λ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.1	0.1	0.111	0.079	0.086	0.080	0.080
	0.2	0.131	0.094	0.097	0.095	0.114
	0.3	0.165	0.128	0.130	0.119	0.186
	0.4	0.208	0.165	0.169	0.157	0.300
	0.5	0.279	0.235	0.227	0.220	0.448
	0.6	0.356	0.310	0.308	0.284	0.586
	0.7	0.486	0.441	0.429	0.410	0.756
	0.8	0.617	0.575	0.559	0.552	0.874
0.2	0.1	0.237	0.181	0.166	0.165	0.162
	0.2	0.289	0.224	0.203	0.206	0.236
	0.3	0.320	0.259	0.247	0.247	0.308
	0.4	0.407	0.345	0.324	0.317	0.449
	0.5	0.498	0.447	0.410	0.412	0.587
	0.6	0.606	0.559	0.527	0.531	0.739
	0.7	0.722	0.681	0.667	0.659	0.859
0.3	0.1	0.436	0.358	0.303	0.328	0.331
	0.2	0.498	0.421	0.362	0.381	0.407
	0.3	0.561	0.501	0.442	0.460	0.517
	0.4	0.649	0.591	0.541	0.556	0.645
	0.5	0.730	0.681	0.643	0.648	0.762
	0.6	0.825	0.792	0.760	0.759	0.873
0.4	0.1	0.649	0.574	0.495	0.530	0.522
	0.2	0.716	0.656	0.574	0.610	0.629
	0.3	0.776	0.724	0.667	0.686	0.727
	0.4	0.843	0.805	0.751	0.771	0.823
	0.5	0.900	0.875	0.845	0.858	0.903
0.5	0.1	0.842	0.793	0.723	0.749	0.758
	0.2	0.873	0.835	0.773	0.804	0.810
	0.3	0.919	0.896	0.852	0.871	0.882
	0.4	0.962	0.949	0.921	0.931	0.944
0.6	0.1	0.949	0.927	0.881	0.902	0.906
	0.2	0.968	0.954	0.926	0.936	0.939
	0.3	0.980	0.972	0.961	0.967	0.968
0.7	0.1	0.990	0.984	0.964	0.976	0.978
	0.2	0.994	0.992	0.985	0.988	0.988
0.8	0.1	0.999	0.998	0.995	0.996	0.997

TABLE 9: Power of Tests Against Second Order Spatial Autoregressive Errors
$R = 81$ (rook) — Normal Distribution

λ_1	λ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.1	0.1	0.124	0.095	0.089	0.097	0.099
	0.2	0.167	0.137	0.127	0.127	0.196
	0.3	0.225	0.192	0.181	0.179	0.366
	0.4	0.282	0.247	0.245	0.224	0.556
	0.5	0.336	0.303	0.318	0.281	0.757
	0.6	0.424	0.393	0.418	0.358	0.905
	0.7	0.534	0.509	0.538	0.472	0.974
	0.8	0.648	0.631	0.674	0.588	0.997
0.2	0.1	0.317	0.261	0.208	0.241	0.222
	0.2	0.388	0.324	0.265	0.300	0.340
	0.3	0.456	0.395	0.335	0.361	0.505
	0.4	0.559	0.516	0.464	0.481	0.703
	0.5	0.663	0.618	0.571	0.578	0.861
	0.6	0.762	0.731	0.702	0.704	0.954
	0.7	0.846	0.828	0.820	0.798	0.992
0.3	0.1	0.588	0.520	0.400	0.465	0.445
	0.2	0.674	0.617	0.504	0.572	0.592
	0.3	0.763	0.719	0.623	0.671	0.748
	0.4	0.838	0.803	0.735	0.766	0.874
	0.5	0.905	0.887	0.846	0.854	0.951
	0.6	0.952	0.944	0.927	0.931	0.989
0.4	0.1	0.824	0.782	0.651	0.723	0.705
	0.2	0.885	0.858	0.750	0.809	0.812
	0.3	0.932	0.916	0.856	0.886	0.913
	0.4	0.963	0.951	0.918	0.938	0.966
	0.5	0.987	0.985	0.974	0.977	0.991
0.5	0.1	0.955	0.938	0.863	0.907	0.897
	0.2	0.972	0.961	0.918	0.942	0.944
	0.3	0.992	0.988	0.970	0.978	0.981
	0.4	0.997	0.996	0.992	0.994	0.995
0.6	0.1	0.993	0.988	0.964	0.976	0.976
	0.2	0.997	0.995	0.982	0.988	0.990
	0.3	0.999	0.999	0.998	0.998	0.999
0.7	0.1	0.999	0.999	0.995	0.998	0.998
	0.2	1.000	1.000	0.999	1.000	1.000
0.8	0.1	1.000	1.000	1.000	1.000	1.000

TABLE 10: Power of Tests Against Second Order Spatial Autoregressive Errors
$R = 127$ — Normal Distribution

λ_1	λ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.1	0.1	0.169	0.136	0.110	0.134	0.143
	0.2	0.221	0.192	0.152	0.174	0.287
	0.3	0.288	0.251	0.211	0.229	0.482
	0.4	0.357	0.323	0.273	0.305	0.706
	0.5	0.499	0.467	0.396	0.440	0.868
	0.6	0.638	0.607	0.550	0.579	0.963
	0.7	0.816	0.793	0.742	0.765	0.993
	0.8	0.938	0.934	0.912	0.915	1.000
0.2	0.1	0.450	0.402	0.281	0.368	0.363
	0.2	0.537	0.489	0.365	0.453	0.535
	0.3	0.621	0.581	0.462	0.546	0.708
	0.4	0.724	0.686	0.571	0.652	0.857
	0.5	0.842	0.822	0.726	0.793	0.953
	0.6	0.921	0.908	0.860	0.891	0.992
	0.7	0.979	0.976	0.958	0.969	0.999
0.3	0.1	0.771	0.731	0.564	0.676	0.679
	0.2	0.826	0.799	0.644	0.752	0.804
	0.3	0.888	0.867	0.751	0.835	0.902
	0.4	0.942	0.931	0.845	0.905	0.966
	0.5	0.981	0.977	0.937	0.966	0.995
	0.6	0.994	0.994	0.982	0.991	0.999
0.4	0.1	0.944	0.931	0.817	0.900	0.907
	0.2	0.971	0.962	0.890	0.946	0.954
	0.3	0.984	0.980	0.937	0.969	0.983
	0.4	0.995	0.994	0.981	0.990	0.997
	0.5	0.999	0.999	0.993	0.998	1.000
0.5	0.1	0.994	0.992	0.960	0.987	0.989
	0.2	0.996	0.996	0.981	0.991	0.993
	0.3	0.999	0.998	0.993	0.998	0.998
	0.4	1.000	1.000	0.999	0.999	1.000
0.6	0.1	1.000	1.000	0.996	0.999	1.000
	0.2	1.000	1.000	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	1.000	1.000
0.7	0.1	1.000	1.000	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	1.000	1.000
0.8	0.1	1.000	1.000	1.000	1.000	1.000

matrix $\mathbf{W}_2$, i.e., the second order contiguity matrix. Similarly, the power curve for $\theta_1 = 0$ in Figure 9 illustrates the power of LM-ERR(2) against first order MA error dependence for the second order contiguity matrix (similar to Figures 5 and 6). In both Figures 8 and 9, the power functions for values of the first order coefficient of 0.0 and 0.1 are almost identical. In those instances, the LM-ERR(2) test is inferior to Moran and LM-ERR.

A striking feature of these results is the lack of symmetry between the power functions for the first and second order parameters. For example, in Table 10, the rejection frequency for LM-ERR(2) with $\lambda_1 = 0.1$ and $\lambda_2 = 0.5$ is only 0.868, while for the reverse case ($\lambda_1 = 0.5$ and $\lambda_2 = 0.1$) it is 0.989. Again, this is due to the different degree of connectedness implied by the second order weights matrix $\mathbf{W}_2$, compared to the first order contiguity matrix, $\mathbf{W}_1$.

The power functions also show a qualitatively different pattern between Figures 8 and 9, similar to what was found for first order error dependence. For AR errors, power clearly increases with higher values of both parameters. However, this is not the case for MA errors, where for high values of θ_1 (t1 in Figure 9), power first decreases with θ_2 and then increases, but only very slowly. The differential pattern between AR and MA errors is also found for the other tests. Whereas for AR errors, all five tests have power increasing with both λ_1 and λ_2, for MA errors the power for all but the LM-ERR(2) test first increases with θ_2 for low values of θ_1, but then actually decreases with θ_2 for high values of θ_1. This coincides with a clear power superiority of both Moran and LM-ERR whenever $\theta_1 > \theta_2$, and a superiority of LM-ERR(2) in the reverse case. K-R and LM-EL always have lowest power (but very similar), though LM-EL achieves higher rejection frequencies than LM-ERR(2) in some cases, similar to LM-ERR. Clearly, the power functions of the one-directional tests are driven by the value of the first order parameter, while LM-ERR(2) has good relative power when the second parameter is large.

In practice, the similarity between the powers of the one-directional Moran and LM-ERR tests on the one hand, and the two-directional LM-ERR(2) test on the other hand complicates the specification search for the proper spatial lag length.[13] In this respect, it would be interesting to assess the properties of a Bera-Yoon like correction to the LM-ERR(2) test, as outlined in Anselin (1994). As we found for the one-directional alternatives, the comparison of the uncorrected to the corrected LM tests may provide insight into the presence of the potential misspecification.

13 In many ways, the identification of the proper spatial lag length is similar to the selection of the proper weights matrix. Often, higher order spatial dependence will be implemented when the first order weights are incapable of capturing sufficient interaction.

TABLE 11: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 40$ — Normal Distribution

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
	0.1	0.095	0.070	0.078	0.072	0.065
	0.2	0.101	0.076	0.089	0.079	0.078
	0.3	0.111	0.087	0.100	0.084	0.109
	0.4	0.125	0.106	0.117	0.100	0.150
0.1	0.5	0.137	0.119	0.130	0.108	0.209
	0.6	0.152	0.131	0.144	0.124	0.281
	0.7	0.176	0.154	0.164	0.137	0.364
	0.8	0.187	0.170	0.185	0.161	0.448
	0.9	0.199	0.187	0.183	0.166	0.517
	0.1	0.157	0.112	0.112	0.108	0.103
	0.2	0.169	0.125	0.119	0.118	0.118
	0.3	0.186	0.142	0.136	0.129	0.151
	0.4	0.187	0.147	0.150	0.133	0.187
0.2	0.5	0.184	0.152	0.159	0.135	0.232
	0.6	0.212	0.171	0.178	0.159	0.299
	0.7	0.225	0.188	0.193	0.172	0.366
	0.8	0.233	0.200	0.206	0.180	0.438
	0.9	0.253	0.221	0.221	0.198	0.526
	0.1	0.262	0.190	0.164	0.171	0.167
	0.2	0.264	0.200	0.175	0.182	0.176
	0.3	0.274	0.220	0.188	0.198	0.216
	0.4	0.293	0.233	0.209	0.212	0.258
0.3	0.5	0.296	0.234	0.210	0.209	0.293
	0.6	0.305	0.258	0.234	0.218	0.351
	0.7	0.308	0.260	0.241	0.237	0.402
	0.8	0.313	0.263	0.257	0.242	0.473
	0.9	0.324	0.281	0.271	0.246	0.532
	0.1	0.383	0.310	0.250	0.262	0.266
	0.2	0.393	0.313	0.255	0.275	0.277
	0.3	0.399	0.325	0.273	0.292	0.294
	0.4	0.392	0.321	0.265	0.289	0.322
0.4	0.5	0.381	0.317	0.280	0.276	0.350
	0.6	0.396	0.334	0.294	0.294	0.393
	0.7	0.398	0.340	0.312	0.299	0.446
	0.8	0.404	0.344	0.324	0.305	0.496
	0.9	0.412	0.356	0.331	0.318	0.557
	0.1	0.533	0.448	0.338	0.379	0.385
	0.2	0.531	0.449	0.344	0.379	0.383
0.5	0.3	0.520	0.442	0.356	0.380	0.389
	0.4	0.528	0.455	0.367	0.392	0.424
	0.5	0.516	0.443	0.373	0.389	0.443
	0.6	0.509	0.436	0.374	0.390	0.466

TABLE 11: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 40$ — Normal Distribution (Continued)

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.5	0.7	0.501	0.437	0.380	0.398	0.501
	0.8	0.501	0.440	0.386	0.387	0.549
	0.9	0.496	0.440	0.396	0.392	0.601
0.6	0.1	0.659	0.578	0.439	0.491	0.517
	0.2	0.644	0.564	0.448	0.493	0.504
	0.3	0.647	0.574	0.458	0.488	0.508
	0.4	0.631	0.561	0.454	0.486	0.509
	0.5	0.626	0.555	0.455	0.480	0.532
	0.6	0.617	0.543	0.465	0.489	0.548
	0.7	0.627	0.559	0.473	0.503	0.583
	0.8	0.608	0.544	0.474	0.485	0.613
	0.9	0.603	0.537	0.477	0.487	0.649
0.7	0.1	0.765	0.691	0.543	0.600	0.643
	0.2	0.749	0.679	0.549	0.592	0.621
	0.3	0.746	0.677	0.551	0.595	0.612
	0.4	0.752	0.682	0.562	0.602	0.621
	0.5	0.714	0.651	0.543	0.579	0.607
	0.6	0.708	0.642	0.547	0.578	0.620
	0.7	0.708	0.644	0.552	0.582	0.648
	0.8	0.693	0.634	0.547	0.572	0.666
	0.9	0.689	0.625	0.551	0.565	0.690
0.8	0.1	0.849	0.788	0.643	0.692	0.756
	0.2	0.838	0.776	0.630	0.693	0.727
	0.3	0.830	0.767	0.641	0.684	0.712
	0.4	0.823	0.769	0.647	0.682	0.710
	0.5	0.810	0.756	0.649	0.691	0.706
	0.6	0.799	0.745	0.633	0.676	0.706
	0.7	0.792	0.738	0.650	0.666	0.722
	0.8	0.783	0.732	0.637	0.657	0.733
	0.9	0.777	0.728	0.635	0.657	0.751
0.9	0.1	0.901	0.849	0.706	0.770	0.827
	0.2	0.896	0.854	0.726	0.766	0.814
	0.3	0.896	0.852	0.723	0.778	0.805
	0.4	0.884	0.845	0.724	0.766	0.800
	0.5	0.882	0.834	0.717	0.763	0.787
	0.6	0.870	0.828	0.717	0.755	0.782
	0.7	0.860	0.819	0.719	0.750	0.782
	0.8	0.856	0.815	0.709	0.742	0.793
	0.9	0.846	0.802	0.718	0.739	0.802

TABLE 12: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 81$ (queen) — Normal Distribution

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
	0.1	0.100	0.076	0.078	0.071	0.071
	0.2	0.114	0.088	0.095	0.081	0.097
	0.3	0.130	0.096	0.106	0.101	0.151
	0.4	0.146	0.116	0.123	0.111	0.205
0.1	0.5	0.153	0.131	0.127	0.125	0.285
	0.6	0.172	0.142	0.138	0.144	0.368
	0.7	0.192	0.166	0.167	0.162	0.466
	0.8	0.211	0.188	0.186	0.175	0.560
	0.9	0.224	0.208	0.210	0.191	0.650
	0.1	0.204	0.146	0.132	0.139	0.139
	0.2	0.199	0.144	0.142	0.139	0.146
	0.3	0.221	0.173	0.158	0.154	0.196
	0.4	0.226	0.180	0.170	0.165	0.252
0.2	0.5	0.222	0.176	0.171	0.167	0.312
	0.6	0.252	0.205	0.205	0.194	0.399
	0.7	0.266	0.222	0.214	0.199	0.475
	0.8	0.281	0.241	0.244	0.223	0.571
	0.9	0.292	0.249	0.255	0.239	0.652
	0.1	0.332	0.259	0.221	0.248	0.226
	0.2	0.331	0.257	0.230	0.233	0.238
	0.3	0.335	0.266	0.241	0.248	0.269
	0.4	0.343	0.281	0.259	0.252	0.314
0.3	0.5	0.354	0.286	0.271	0.272	0.368
	0.6	0.375	0.315	0.288	0.290	0.457
	0.7	0.371	0.316	0.300	0.291	0.519
	0.8	0.379	0.321	0.307	0.300	0.588
	0.9	0.398	0.345	0.332	0.316	0.670
	0.1	0.468	0.386	0.316	0.347	0.346
	0.2	0.481	0.400	0.329	0.367	0.355
	0.3	0.473	0.391	0.341	0.361	0.371
	0.4	0.487	0.410	0.351	0.377	0.411
0.4	0.5	0.493	0.425	0.366	0.392	0.470
	0.6	0.487	0.417	0.376	0.388	0.508
	0.7	0.483	0.418	0.387	0.394	0.567
	0.8	0.491	0.435	0.409	0.393	0.632
	0.9	0.506	0.447	0.412	0.417	0.693
	0.1	0.604	0.523	0.430	0.484	0.480
	0.2	0.617	0.538	0.454	0.502	0.481
0.5	0.3	0.625	0.547	0.469	0.508	0.505
	0.4	0.619	0.548	0.478	0.506	0.528
	0.5	0.620	0.545	0.480	0.508	0.556
	0.6	0.614	0.546	0.493	0.510	0.596

TABLE 12: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 81$ (queen) — Normal Distribution (Continued)

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.5	0.7	0.612	0.546	0.494	0.508	0.635
	0.8	0.609	0.547	0.493	0.510	0.684
	0.9	0.616	0.554	0.509	0.525	0.740
0.6	0.1	0.746	0.668	0.573	0.622	0.624
	0.2	0.755	0.678	0.568	0.630	0.620
	0.3	0.729	0.665	0.570	0.619	0.610
	0.4	0.732	0.661	0.577	0.619	0.627
	0.5	0.733	0.668	0.588	0.627	0.647
	0.6	0.730	0.667	0.593	0.628	0.679
	0.7	0.726	0.664	0.595	0.626	0.706
	0.8	0.716	0.663	0.604	0.624	0.738
	0.9	0.705	0.650	0.598	0.616	0.778
0.7	0.1	0.853	0.793	0.688	0.749	0.753
	0.2	0.843	0.785	0.686	0.742	0.731
	0.3	0.828	0.773	0.690	0.740	0.722
	0.4	0.819	0.764	0.683	0.730	0.725
	0.5	0.823	0.769	0.685	0.735	0.736
	0.6	0.814	0.763	0.691	0.719	0.744
	0.7	0.814	0.770	0.695	0.729	0.775
	0.8	0.806	0.758	0.688	0.729	0.787
	0.9	0.806	0.761	0.697	0.730	0.820
0.8	0.1	0.907	0.870	0.787	0.831	0.844
	0.2	0.906	0.870	0.788	0.835	0.834
	0.3	0.907	0.870	0.785	0.833	0.824
	0.4	0.897	0.859	0.789	0.830	0.821
	0.5	0.893	0.857	0.780	0.826	0.820
	0.6	0.888	0.853	0.786	0.826	0.833
	0.7	0.872	0.835	0.766	0.808	0.826
	0.8	0.873	0.840	0.786	0.804	0.849
	0.9	0.865	0.825	0.762	0.798	0.854
0.9	0.1	0.952	0.925	0.863	0.900	0.914
	0.2	0.944	0.915	0.845	0.887	0.894
	0.3	0.944	0.920	0.860	0.898	0.897
	0.4	0.947	0.920	0.860	0.895	0.890
	0.5	0.928	0.902	0.845	0.876	0.877
	0.6	0.933	0.908	0.848	0.882	0.883
	0.7	0.928	0.901	0.845	0.877	0.884
	0.8	0.924	0.897	0.842	0.875	0.890
	0.9	0.911	0.883	0.833	0.862	0.893

TABLE 13: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 81$ (rook) — Normal Distribution

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.1	0.1	0.122	0.096	0.093	0.091	0.095
	0.2	0.136	0.108	0.108	0.102	0.155
	0.3	0.148	0.128	0.136	0.118	0.249
	0.4	0.189	0.160	0.166	0.154	0.392
	0.5	0.218	0.189	0.197	0.177	0.537
	0.6	0.239	0.212	0.224	0.206	0.689
	0.7	0.272	0.248	0.282	0.236	0.793
	0.8	0.288	0.268	0.293	0.253	0.873
	0.9	0.313	0.298	0.322	0.281	0.926
0.2	0.1	0.275	0.218	0.158	0.194	0.174
	0.2	0.288	0.238	0.193	0.213	0.247
	0.3	0.302	0.250	0.217	0.229	0.335
	0.4	0.322	0.272	0.240	0.249	0.463
	0.5	0.343	0.302	0.275	0.280	0.587
	0.6	0.335	0.295	0.291	0.274	0.697
	0.7	0.373	0.333	0.327	0.300	0.802
	0.8	0.368	0.331	0.350	0.308	0.872
	0.9	0.394	0.361	0.375	0.340	0.932
0.3	0.1	0.492	0.423	0.305	0.382	0.324
	0.2	0.502	0.435	0.327	0.394	0.382
	0.3	0.525	0.463	0.360	0.420	0.464
	0.4	0.524	0.465	0.381	0.431	0.557
	0.5	0.533	0.475	0.404	0.442	0.664
	0.6	0.524	0.467	0.421	0.437	0.749
	0.7	0.524	0.473	0.446	0.443	0.835
	0.8	0.521	0.483	0.452	0.452	0.895
	0.9	0.538	0.496	0.472	0.456	0.940
0.4	0.1	0.708	0.641	0.478	0.586	0.519
	0.2	0.724	0.663	0.517	0.613	0.578
	0.3	0.723	0.667	0.528	0.618	0.623
	0.4	0.706	0.662	0.547	0.604	0.682
	0.5	0.710	0.655	0.563	0.619	0.750
	0.6	0.701	0.650	0.558	0.609	0.813
	0.7	0.699	0.653	0.592	0.613	0.871
	0.8	0.678	0.632	0.581	0.600	0.918
	0.9	0.674	0.635	0.607	0.601	0.946
0.5	0.1	0.865	0.825	0.661	0.774	0.730
	0.2	0.863	0.825	0.686	0.783	0.738
	0.3	0.867	0.827	0.696	0.779	0.764
	0.4	0.859	0.823	0.714	0.782	0.798
	0.5	0.864	0.829	0.729	0.788	0.841
	0.6	0.844	0.813	0.731	0.777	0.875

TABLE 13: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 81$ (rook) — Normal Distribution (Continued)

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
0.5	0.7	0.832	0.797	0.729	0.767	0.910
	0.8	0.817	0.781	0.729	0.749	0.942
	0.9	0.803	0.772	0.727	0.737	0.962
0.6	0.1	0.953	0.927	0.807	0.892	0.871
	0.2	0.952	0.931	0.824	0.905	0.875
	0.3	0.952	0.932	0.844	0.904	0.881
	0.4	0.949	0.933	0.853	0.902	0.899
	0.5	0.944	0.922	0.847	0.902	0.911
	0.6	0.936	0.918	0.848	0.887	0.927
	0.7	0.918	0.896	0.845	0.874	0.944
	0.8	0.912	0.891	0.839	0.864	0.962
	0.9	0.903	0.880	0.835	0.848	0.975
0.7	0.1	0.990	0.980	0.917	0.966	0.965
	0.2	0.988	0.980	0.928	0.964	0.958
	0.3	0.985	0.977	0.934	0.961	0.956
	0.4	0.985	0.977	0.923	0.963	0.952
	0.5	0.981	0.971	0.926	0.959	0.959
	0.6	0.978	0.968	0.927	0.953	0.967
	0.7	0.972	0.962	0.927	0.948	0.973
	0.8	0.959	0.950	0.915	0.935	0.977
	0.9	0.961	0.948	0.916	0.933	0.984
0.8	0.1	0.998	0.995	0.966	0.987	0.993
	0.2	0.997	0.994	0.968	0.987	0.988
	0.3	0.999	0.996	0.972	0.988	0.987
	0.4	0.997	0.995	0.975	0.990	0.988
	0.5	0.997	0.993	0.974	0.987	0.986
	0.6	0.992	0.988	0.969	0.980	0.984
	0.7	0.992	0.989	0.971	0.982	0.987
	0.8	0.989	0.986	0.964	0.977	0.991
	0.9	0.986	0.982	0.961	0.973	0.994
0.9	0.1	0.999	0.998	0.985	0.994	0.997
	0.2	1.000	0.999	0.989	0.996	0.998
	0.3	1.000	1.000	0.992	0.998	0.997
	0.4	1.000	0.999	0.992	0.998	0.997
	0.5	0.999	0.999	0.990	0.998	0.995
	0.6	0.999	0.999	0.989	0.997	0.995
	0.7	0.998	0.997	0.986	0.992	0.994
	0.8	0.997	0.996	0.987	0.992	0.995
	0.9	0.995	0.994	0.985	0.990	0.997

TABLE 14: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 127$ — Normal Distribution

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
	0.1	0.152	0.127	0.109	0.117	0.125
	0.2	0.165	0.140	0.117	0.124	0.211
	0.3	0.183	0.159	0.139	0.155	0.336
	0.4	0.201	0.174	0.153	0.163	0.491
0.1	0.5	0.218	0.190	0.169	0.181	0.657
	0.6	0.248	0.220	0.193	0.207	0.779
	0.7	0.267	0.243	0.208	0.229	0.886
	0.8	0.288	0.262	0.226	0.242	0.951
	0.9	0.312	0.290	0.256	0.279	0.974
	0.1	0.358	0.315	0.226	0.291	0.281
	0.2	0.391	0.342	0.244	0.314	0.366
	0.3	0.398	0.354	0.262	0.322	0.466
	0.4	0.419	0.380	0.284	0.353	0.612
0.2	0.5	0.431	0.393	0.308	0.371	0.722
	0.6	0.442	0.400	0.311	0.371	0.816
	0.7	0.469	0.430	0.357	0.397	0.899
	0.8	0.457	0.426	0.362	0.402	0.945
	0.9	0.481	0.447	0.387	0.430	0.973
	0.1	0.631	0.581	0.410	0.542	0.513
	0.2	0.640	0.587	0.425	0.549	0.558
	0.3	0.641	0.599	0.445	0.559	0.642
	0.4	0.647	0.604	0.463	0.557	0.719
0.3	0.5	0.661	0.625	0.475	0.584	0.798
	0.6	0.662	0.624	0.491	0.581	0.869
	0.7	0.656	0.621	0.498	0.581	0.916
	0.8	0.664	0.634	0.509	0.591	0.960
	0.9	0.663	0.632	0.530	0.598	0.973
	0.1	0.848	0.821	0.629	0.770	0.754
	0.2	0.849	0.816	0.627	0.774	0.765
	0.3	0.851	0.824	0.646	0.779	0.805
	0.4	0.833	0.804	0.644	0.760	0.838
0.4	0.5	0.835	0.808	0.649	0.774	0.883
	0.6	0.829	0.800	0.650	0.766	0.920
	0.7	0.832	0.806	0.667	0.769	0.943
	0.8	0.833	0.806	0.676	0.765	0.970
	0.9	0.832	0.806	0.686	0.765	0.984
	0.1	0.948	0.933	0.792	0.905	0.894
	0.2	0.948	0.933	0.807	0.911	0.902
0.5	0.3	0.946	0.931	0.801	0.902	0.909
	0.4	0.940	0.924	0.798	0.903	0.923
	0.5	0.935	0.923	0.801	0.888	0.938
	0.6	0.940	0.925	0.802	0.893	0.957

TABLE 14: Power of Tests Against Second Order Spatial Moving Average Errors
$R = 127$ — Normal Distribution (Continued)

θ_1	θ_2	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)
	0.7	0.927	0.914	0.798	0.881	0.970
0.5	0.8	0.920	0.902	0.805	0.879	0.979
	0.9	0.914	0.898	0.801	0.877	0.987
	0.1	0.989	0.983	0.909	0.973	0.969
	0.2	0.990	0.985	0.910	0.970	0.968
	0.3	0.986	0.978	0.899	0.967	0.967
	0.4	0.984	0.979	0.902	0.965	0.969
0.6	0.5	0.983	0.978	0.897	0.967	0.977
	0.6	0.979	0.973	0.899	0.959	0.981
	0.7	0.976	0.971	0.902	0.955	0.986
	0.8	0.975	0.967	0.899	0.951	0.990
	0.9	0.972	0.964	0.896	0.950	0.994
	0.1	0.998	0.996	0.966	0.993	0.992
	0.2	0.998	0.997	0.962	0.993	0.993
	0.3	0.998	0.996	0.965	0.990	0.992
	0.4	0.996	0.995	0.959	0.990	0.991
0.7	0.5	0.995	0.993	0.958	0.989	0.992
	0.6	0.995	0.992	0.954	0.987	0.993
	0.7	0.994	0.993	0.953	0.986	0.996
	0.8	0.992	0.990	0.948	0.984	0.996
	0.9	0.990	0.988	0.952	0.981	0.997
	0.1	0.999	0.999	0.987	0.998	0.998
	0.2	0.999	0.999	0.991	0.999	0.998
	0.3	0.999	0.999	0.987	0.998	0.998
	0.4	0.999	0.999	0.987	0.997	0.998
0.8	0.5	0.999	0.998	0.985	0.997	0.997
	0.6	0.999	0.999	0.985	0.996	0.999
	0.7	0.998	0.997	0.982	0.996	0.997
	0.8	0.999	0.998	0.982	0.995	0.998
	0.9	0.998	0.997	0.980	0.995	0.998
	0.1	1.000	1.000	0.998	1.000	1.000
	0.2	1.000	1.000	0.996	1.000	0.999
	0.3	1.000	1.000	0.996	1.000	1.000
	0.4	1.000	1.000	0.996	1.000	1.000
0.9	0.5	1.000	1.000	0.996	0.999	0.999
	0.6	1.000	1.000	0.994	0.999	0.999
	0.7	1.000	0.999	0.993	0.999	1.000
	0.8	1.000	1.000	0.993	0.998	0.999
	0.9	1.000	1.000	0.993	0.999	1.000

7.5. *Power of tests against a SARMA (1, 1) process*

The rejection frequencies against a SARMA alternative are listed in Tables 15 to 18. We focus in particular on the properties of SARMA, LM-EL and LM-LE, for which the power functions are illustrated in Figures 10 to 15, for the queen ($R = 81$) configuration, and using the same format as for the second order processes. The four other one-directional tests are included in the tables for the sake of completeness.[14]

The SARMA test has excellent power against this alternative, achieving 95% rejection frequencies for $\rho \geq 0.4, \forall \theta_1$, even in the smallest sample. For R = 127, a 95% rejection is obtained for all but three parameter combinations (ρ = 0.1 and $\theta_1 \leq 0.3$). This is also illustrated by the steep slope of the power curves with increasing values of ρ in Figure 10, and the horizontal power curves with θ_1 for $\rho \geq 0.3$ in Figure 11. Except for a few instances, SARMA also has the highest power of all tests. Interestingly, in the three smaller configurations and for small values of ρ, Moran and LM-LAG achieve slightly superior power, but this pattern weakens with sample size and disappears by $R = 127$.

The power functions in Figures 12 and 13 illustrate the extent to which the LM-EL test is robust to the presence of lag dependence. For small values of ρ in Figure 12, the power function mimics that of the tests against first order moving average error dependence (e.g., as in Figure 5), with slowly increasing power with values of θ_1. The curves for ρ = 0.0 and ρ = 0.1 in particular are very close, confirming the proper behavior of this test against "local" misspecification. For higher values of ρ, this pattern is not maintained however. The power functions in Figure 13 illustrate the strange behavior of LM-EL with increasing values of ρ. As shown in the discussion of one-directional alternatives, the power function is almost horizontal for θ_1 = 0.0 and small values of ρ, as it should, but this is not the case for higher values of ρ, where power first goes up and then decreases. For $R = 127$ in particular, power becomes negligible for high ρ, even for high values of θ_1.

In contrast, the power functions of LM-LE seem almost unaffected by the value of θ_1. In Figure 14, the power curves for the five values of θ_1 are virtually identical and illustrate the good power of this test against spatial autoregressive alternatives. Similarly, in Figure 15, the power curves are more or less horizontal with θ_1, even for high values. The difference in properties between lag and error tests is thus maintained for their robust forms, in the presence of misspecification. Overall, LM-LE comes across as much more reliable than LM-EL, and very similar in power to SARMA. The robust tests thus seem more appropriate to test for lag dependence in the presence of error correlation than for the reverse case.

14 Results for lognormal errors are very similar and not reported here (they are available from the authors). Also, the results for LM-ERR(2) are not reported, since this test is inappropriate for a mixed process, whereas the included one-directional tests attempt to capture one aspect of the mixed process.

TABLE 15: Power of Tests Against Spatial ARMA Process
$R = 40$ — Normal Distribution

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
0.1	0.1	0.173	0.119	0.108	0.060	0.201	0.237	0.178
	0.2	0.275	0.203	0.165	0.090	0.273	0.299	0.189
	0.3	0.408	0.318	0.246	0.158	0.357	0.353	0.201
	0.4	0.544	0.461	0.348	0.238	0.491	0.416	0.202
	0.5	0.666	0.583	0.443	0.342	0.583	0.471	0.207
	0.6	0.772	0.695	0.551	0.445	0.690	0.518	0.223
	0.7	0.852	0.797	0.644	0.564	0.778	0.576	0.235
	0.8	0.911	0.861	0.736	0.641	0.848	0.617	0.240
	0.9	0.942	0.912	0.808	0.729	0.897	0.656	0.236
0.2	0.1	0.339	0.259	0.207	0.060	0.532	0.602	0.481
	0.2	0.475	0.385	0.293	0.095	0.603	0.644	0.480
	0.3	0.606	0.515	0.395	0.151	0.664	0.688	0.462
	0.4	0.712	0.627	0.495	0.222	0.751	0.739	0.465
	0.5	0.808	0.741	0.601	0.318	0.801	0.771	0.465
	0.6	0.870	0.815	0.691	0.420	0.857	0.802	0.460
	0.7	0.920	0.883	0.768	0.497	0.911	0.828	0.442
	0.8	0.951	0.922	0.819	0.567	0.934	0.839	0.454
	0.9	0.972	0.952	0.877	0.640	0.954	0.850	0.423
0.3	0.1	0.565	0.474	0.371	0.047	0.838	0.881	0.784
	0.2	0.691	0.602	0.464	0.076	0.856	0.905	0.785
	0.3	0.767	0.690	0.567	0.134	0.893	0.911	0.751
	0.4	0.844	0.778	0.655	0.196	0.913	0.931	0.747
	0.5	0.899	0.858	0.739	0.273	0.941	0.931	0.718
	0.6	0.930	0.897	0.806	0.338	0.955	0.947	0.708
	0.7	0.961	0.940	0.862	0.420	0.969	0.952	0.679
	0.8	0.976	0.963	0.892	0.480	0.980	0.956	0.658
	0.9	0.985	0.975	0.923	0.540	0.986	0.958	0.651
0.4	0.1	0.770	0.691	0.571	0.041	0.964	0.984	0.947
	0.2	0.839	0.780	0.658	0.063	0.972	0.985	0.929
	0.3	0.899	0.850	0.738	0.104	0.973	0.983	0.914
	0.4	0.928	0.897	0.804	0.150	0.984	0.985	0.891
	0.5	0.959	0.935	0.847	0.200	0.988	0.986	0.877
	0.6	0.970	0.953	0.887	0.244	0.990	0.988	0.854
	0.7	0.986	0.975	0.923	0.304	0.994	0.989	0.843
	0.8	0.991	0.984	0.940	0.353	0.994	0.991	0.823
	0.9	0.994	0.990	0.959	0.408	0.997	0.989	0.799
0.5	0.1	0.926	0.887	0.792	0.029	0.997	0.998	0.988
	0.2	0.946	0.914	0.824	0.044	0.997	0.999	0.983
	0.3	0.964	0.939	0.870	0.065	0.997	0.999	0.976
	0.4	0.974	0.958	0.903	0.097	0.998	0.998	0.966
	0.5	0.982	0.970	0.923	0.117	0.998	0.998	0.954
	0.6	0.991	0.985	0.950	0.163	0.998	0.999	0.944

TABLE 15: Power of Tests Against Spatial ARMA Process
$R = 40$ — Normal Distribution (Continued)

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
	0.7	0.995	0.992	0.961	0.209	1.000	0.998	0.926
0.5	0.8	0.996	0.993	0.971	0.218	0.998	0.999	0.911
	0.9	0.998	0.995	0.980	0.273	1.000	0.999	0.892
	0.1	0.981	0.965	0.918	0.014	1.000	1.000	0.998
	0.2	0.989	0.978	0.938	0.020	1.000	1.000	0.997
	0.3	0.990	0.982	0.950	0.032	1.000	1.000	0.995
	0.4	0.996	0.991	0.963	0.046	1.000	1.000	0.986
0.6	0.5	0.997	0.992	0.976	0.063	0.999	1.000	0.984
	0.6	0.998	0.996	0.980	0.087	1.000	1.000	0.975
	0.7	0.999	0.998	0.988	0.103	1.000	1.000	0.967
	0.8	1.000	0.998	0.989	0.131	1.000	1.000	0.953
	0.9	0.999	0.999	0.993	0.151	1.000	1.000	0.947
	0.1	0.998	0.996	0.982	0.005	1.000	1.000	1.000
	0.2	0.998	0.996	0.985	0.008	1.000	1.000	0.999
	0.3	0.998	0.998	0.987	0.013	1.000	1.000	0.997
	0.4	1.000	0.999	0.992	0.024	1.000	1.000	0.995
0.7	0.5	0.999	0.998	0.993	0.031	1.000	1.000	0.991
	0.6	1.000	0.999	0.997	0.041	1.000	1.000	0.987
	0.7	1.000	1.000	0.997	0.049	1.000	1.000	0.985
	0.8	1.000	1.000	0.998	0.062	1.000	1.000	0.975
	0.9	1.000	1.000	0.998	0.076	1.000	1.000	0.966
	0.1	1.000	0.999	0.998	0.003	1.000	1.000	1.000
	0.2	1.000	1.000	0.998	0.003	1.000	1.000	0.999
	0.3	1.000	1.000	0.998	0.005	1.000	1.000	0.997
	0.4	1.000	1.000	0.998	0.009	1.000	1.000	0.997
0.8	0.5	1.000	1.000	0.999	0.013	1.000	1.000	0.994
	0.6	1.000	1.000	0.999	0.014	1.000	1.000	0.989
	0.7	1.000	1.000	0.999	0.024	1.000	1.000	0.988
	0.8	1.000	1.000	0.999	0.025	1.000	1.000	0.980
	0.9	1.000	1.000	1.000	0.037	1.000	1.000	0.973
	0.1	1.000	1.000	1.000	0.004	1.000	1.000	0.999
	0.2	1.000	1.000	1.000	0.004	1.000	1.000	0.998
	0.3	1.000	1.000	1.000	0.005	1.000	1.000	0.996
	0.4	1.000	1.000	1.000	0.009	1.000	1.000	0.993
0.9	0.5	1.000	1.000	1.000	0.008	1.000	1.000	0.987
	0.6	1.000	1.000	1.000	0.016	1.000	1.000	0.982
	0.7	1.000	1.000	1.000	0.015	1.000	1.000	0.974
	0.8	1.000	1.000	1.000	0.020	1.000	1.000	0.971
	0.9	1.000	1.000	1.000	0.018	1.000	1.000	0.957

TABLE 16: Power of Tests Against Spatial ARMA Process
$R = 81$ (queen) — Normal Distribution

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
	0.1	0.225	0.167	0.144	0.080	0.298	0.357	0.285
	0.2	0.366	0.286	0.239	0.146	0.395	0.429	0.292
	0.3	0.515	0.431	0.349	0.245	0.512	0.482	0.304
	0.4	0.677	0.592	0.488	0.393	0.634	0.556	0.304
0.1	0.5	0.793	0.724	0.617	0.542	0.741	0.589	0.302
	0.6	0.878	0.824	0.727	0.669	0.824	0.630	0.309
	0.7	0.928	0.893	0.817	0.768	0.887	0.690	0.316
	0.8	0.960	0.942	0.882	0.853	0.936	0.720	0.315
	0.9	0.984	0.972	0.932	0.914	0.968	0.767	0.320
	0.1	0.497	0.405	0.342	0.115	0.783	0.849	0.766
	0.2	0.636	0.551	0.473	0.204	0.827	0.874	0.759
	0.3	0.763	0.686	0.582	0.319	0.868	0.888	0.731
	0.4	0.845	0.786	0.686	0.457	0.904	0.904	0.731
0.2	0.5	0.915	0.875	0.789	0.595	0.946	0.918	0.722
	0.6	0.950	0.924	0.861	0.709	0.965	0.924	0.695
	0.7	0.976	0.956	0.908	0.802	0.974	0.938	0.682
	0.8	0.985	0.975	0.943	0.865	0.986	0.950	0.671
	0.9	0.993	0.988	0.965	0.922	0.992	0.956	0.680
	0.1	0.793	0.724	0.648	0.209	0.983	0.993	0.979
	0.2	0.869	0.815	0.742	0.311	0.988	0.991	0.973
	0.3	0.928	0.889	0.814	0.430	0.992	0.993	0.968
	0.4	0.953	0.928	0.874	0.563	0.994	0.994	0.955
0.3	0.5	0.974	0.960	0.920	0.675	0.995	0.995	0.949
	0.6	0.988	0.977	0.950	0.766	0.997	0.995	0.936
	0.7	0.992	0.984	0.965	0.839	0.997	0.995	0.923
	0.8	0.997	0.994	0.982	0.891	0.999	0.994	0.907
	0.9	0.998	0.997	0.989	0.934	0.999	0.995	0.901
	0.1	0.960	0.938	0.895	0.341	0.999	1.000	0.999
	0.2	0.978	0.959	0.926	0.465	1.000	1.000	0.998
	0.3	0.984	0.974	0.950	0.563	0.999	1.000	0.998
	0.4	0.993	0.985	0.967	0.671	1.000	1.000	0.997
0.4	0.5	0.997	0.994	0.978	0.754	1.000	1.000	0.994
	0.6	0.998	0.996	0.989	0.808	1.000	1.000	0.992
	0.7	0.999	0.998	0.991	0.870	1.000	1.000	0.988
	0.8	0.999	0.999	0.996	0.911	1.000	1.000	0.985
	0.9	1.000	1.000	0.998	0.938	1.000	1.000	0.979
	0.1	0.997	0.995	0.990	0.518	1.000	1.000	1.000
	0.2	0.998	0.997	0.993	0.610	1.000	1.000	1.000
0.5	0.3	0.999	0.997	0.991	0.682	1.000	1.000	1.000
	0.4	1.000	0.999	0.996	0.752	1.000	1.000	1.000
	0.5	1.000	0.999	0.997	0.809	1.000	1.000	1.000
	0.6	1.000	0.999	0.998	0.863	1.000	1.000	1.000

TABLE 16: Power of Tests Against Spatial ARMA Process
$R = 81$ (queen) — Normal Distribution (Continued)

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
	0.7	1.000	1.000	0.998	0.892	1.000	1.000	0.998
0.5	0.8	1.000	1.000	1.000	0.925	1.000	1.000	0.998
	0.9	1.000	1.000	0.999	0.944	1.000	1.000	0.997
	0.1	1.000	1.000	1.000	0.688	1.000	1.000	1.000
	0.2	1.000	1.000	0.999	0.744	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.786	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.825	1.000	1.000	1.000
0.6	0.5	1.000	1.000	1.000	0.868	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.886	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.911	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.927	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.943	1.000	1.000	1.000
	0.1	1.000	1.000	1.000	0.809	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.835	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.861	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.873	1.000	1.000	1.000
0.7	0.5	1.000	1.000	1.000	0.899	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.906	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.916	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.933	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.946	1.000	1.000	1.000
	0.1	1.000	1.000	1.000	0.866	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.866	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.878	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.890	1.000	1.000	1.000
0.8	0.5	1.000	1.000	1.000	0.895	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.891	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.903	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.897	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.900	1.000	1.000	1.000
	0.1	1.000	1.000	1.000	0.351	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.357	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.375	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.369	1.000	1.000	1.000
0.9	0.5	1.000	1.000	1.000	0.361	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.359	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.372	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.365	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.377	1.000	1.000	1.000

TABLE 17: Power of Tests Against Spatial ARMA Process
$R = 81$ (rook) — Normal Distribution

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
0.1	0.1	0.225	0.167	0.144	0.080	0.298	0.357	0.285
	0.2	0.366	0.286	0.239	0.146	0.395	0.429	0.292
	0.3	0.515	0.431	0.349	0.245	0.512	0.482	0.304
	0.4	0.677	0.592	0.488	0.393	0.634	0.556	0.304
	0.5	0.793	0.724	0.617	0.542	0.741	0.589	0.302
	0.6	0.878	0.824	0.727	0.669	0.824	0.630	0.309
	0.7	0.928	0.893	0.817	0.768	0.887	0.690	0.316
	0.8	0.960	0.942	0.882	0.853	0.936	0.720	0.315
	0.9	0.984	0.972	0.932	0.914	0.968	0.767	0.320
0.2	0.1	0.497	0.405	0.342	0.115	0.783	0.849	0.766
	0.2	0.636	0.551	0.473	0.204	0.827	0.874	0.759
	0.3	0.763	0.686	0.582	0.319	0.868	0.888	0.731
	0.4	0.845	0.786	0.686	0.457	0.904	0.904	0.731
	0.5	0.915	0.875	0.789	0.595	0.946	0.918	0.722
	0.6	0.950	0.924	0.861	0.709	0.965	0.924	0.695
	0.7	0.976	0.956	0.908	0.802	0.974	0.938	0.682
	0.8	0.985	0.975	0.943	0.865	0.986	0.950	0.671
	0.9	0.993	0.988	0.965	0.922	0.992	0.956	0.680
0.3	0.1	0.793	0.724	0.648	0.209	0.983	0.993	0.979
	0.2	0.869	0.815	0.742	0.311	0.988	0.991	0.973
	0.3	0.928	0.889	0.814	0.430	0.992	0.993	0.968
	0.4	0.953	0.928	0.874	0.563	0.994	0.994	0.955
	0.5	0.974	0.960	0.920	0.675	0.995	0.995	0.949
	0.6	0.988	0.977	0.950	0.766	0.997	0.995	0.936
	0.7	0.992	0.984	0.965	0.839	0.997	0.995	0.923
	0.8	0.997	0.994	0.982	0.891	0.999	0.994	0.907
	0.9	0.998	0.997	0.989	0.934	0.999	0.995	0.901
0.4	0.1	0.960	0.938	0.895	0.341	0.999	1.000	0.999
	0.2	0.978	0.959	0.926	0.465	1.000	1.000	0.998
	0.3	0.984	0.974	0.950	0.563	0.999	1.000	0.998
	0.4	0.993	0.985	0.967	0.671	1.000	1.000	0.997
	0.5	0.997	0.994	0.978	0.754	1.000	1.000	0.994
	0.6	0.998	0.996	0.989	0.808	1.000	1.000	0.992
	0.7	0.999	0.998	0.991	0.870	1.000	1.000	0.988
	0.8	0.999	0.999	0.996	0.911	1.000	1.000	0.985
	0.9	1.000	1.000	0.998	0.938	1.000	1.000	0.979
0.5	0.1	0.997	0.995	0.990	0.518	1.000	1.000	1.000
	0.2	0.998	0.997	0.993	0.610	1.000	1.000	1.000
	0.3	0.999	0.997	0.991	0.682	1.000	1.000	1.000
	0.4	1.000	0.999	0.996	0.752	1.000	1.000	1.000
	0.5	1.000	0.999	0.997	0.809	1.000	1.000	1.000
	0.6	1.000	0.999	0.998	0.863	1.000	1.000	1.000

TABLE 17: Power of Tests Against Spatial ARMA Process
$R = 81$ (rook) — Normal Distribution (Continued)

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
	0.7	1.000	1.000	0.998	0.892	1.000	1.000	0.998
0.5	0.8	1.000	1.000	1.000	0.925	1.000	1.000	0.998
	0.9	1.000	1.000	0.999	0.944	1.000	1.000	0.997
	0.1	1.000	1.000	1.000	0.688	1.000	1.000	1.000
	0.2	1.000	1.000	0.999	0.744	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.786	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.825	1.000	1.000	1.000
0.6	0.5	1.000	1.000	1.000	0.868	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.886	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.911	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.927	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.943	1.000	1.000	1.000
	0.1	1.000	1.000	1.000	0.809	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.835	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.861	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.873	1.000	1.000	1.000
0.7	0.5	1.000	1.000	1.000	0.899	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.906	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.916	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.933	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.946	1.000	1.000	1.000
	0.1	1.000	1.000	1.000	0.866	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.866	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.878	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.890	1.000	1.000	1.000
0.8	0.5	1.000	1.000	1.000	0.895	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.891	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.903	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.897	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.900	1.000	1.000	1.000
	0.1	1.000	1.000	1.000	0.351	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.357	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.375	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.369	1.000	1.000	1.000
0.9	0.5	1.000	1.000	1.000	0.361	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.359	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.372	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.365	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.377	1.000	1.000	1.000

TABLE 18: Power of Tests Against Spatial ARMA Process
$R = 127$ — Normal Distribution

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
0.1	0.1	0.411	0.358	0.258	0.121	0.673	0.727	0.590
	0.2	0.696	0.642	0.470	0.311	0.806	0.805	0.590
	0.3	0.876	0.846	0.672	0.561	0.911	0.851	0.580
	0.4	0.964	0.955	0.840	0.787	0.965	0.883	0.569
	0.5	0.992	0.988	0.928	0.910	0.992	0.917	0.573
	0.6	0.998	0.998	0.974	0.968	0.998	0.939	0.547
	0.7	1.000	1.000	0.996	0.991	1.000	0.948	0.528
	0.8	1.000	1.000	0.998	0.998	1.000	0.961	0.519
	0.9	1.000	1.000	1.000	1.000	1.000	0.975	0.515
0.2	0.1	0.801	0.760	0.635	0.164	0.994	0.998	0.991
	0.2	0.931	0.913	0.809	0.379	0.998	0.998	0.984
	0.3	0.983	0.976	0.909	0.604	0.999	0.999	0.979
	0.4	0.992	0.989	0.960	0.780	0.999	0.999	0.968
	0.5	0.998	0.998	0.988	0.902	1.000	0.999	0.965
	0.6	1.000	1.000	0.997	0.966	1.000	1.000	0.950
	0.7	1.000	1.000	0.999	0.989	1.000	0.999	0.944
	0.8	1.000	1.000	1.000	0.996	1.000	0.999	0.922
	0.9	1.000	1.000	1.000	0.999	1.000	0.999	0.917
0.3	0.1	0.985	0.978	0.940	0.209	1.000	1.000	1.000
	0.2	0.995	0.992	0.972	0.421	1.000	1.000	1.000
	0.3	0.999	0.999	0.990	0.619	1.000	1.000	1.000
	0.4	1.000	1.000	0.997	0.789	1.000	1.000	1.000
	0.5	1.000	1.000	0.999	0.888	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.947	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.976	1.000	1.000	0.999
	0.8	1.000	1.000	1.000	0.990	1.000	1.000	0.997
	0.9	1.000	1.000	1.000	0.994	1.000	1.000	0.995
0.4	0.1	1.000	1.000	0.997	0.227	1.000	1.000	1.000
	0.2	1.000	1.000	0.999	0.407	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.559	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.697	1.000	1.000	1.000
	0.5	1.000	1.000	1.000	0.823	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.895	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.941	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.970	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.979	1.000	1.000	1.000
0.5	0.1	1.000	1.000	1.000	0.176	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.277	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.400	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.540	1.000	1.000	1.000
	0.5	1.000	1.000	1.000	0.651	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.723	1.000	1.000	1.000

TABLE 18: Power of Tests Against Spatial ARMA Process
$R = 127$ — Normal Distribution (Continued)

ρ	θ_1	Moran	LM-ERR	K-R	LM-EL	SARMA	LM-LAG	LM-LE
0.5	0.7	1.000	1.000	1.000	0.792	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.853	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.891	1.000	1.000	1.000
0.6	0.1	1.000	1.000	1.000	0.056	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.093	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.164	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.246	1.000	1.000	1.000
	0.5	1.000	1.000	1.000	0.325	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.393	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.462	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.553	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.595	1.000	1.000	1.000
0.7	0.1	1.000	1.000	1.000	0.003	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.011	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.016	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.036	1.000	1.000	1.000
	0.5	1.000	1.000	1.000	0.055	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.090	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.126	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.163	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.207	1.000	1.000	1.000
0.8	0.1	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.001	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.001	1.000	1.000	1.000
	0.5	1.000	1.000	1.000	0.003	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.003	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.011	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.013	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.023	1.000	1.000	1.000
0.9	0.1	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.2	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.3	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.4	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.5	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.000	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.001	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.001	1.000	1.000	1.000

8. Conclusions

As in any Monte Carlo experiment, the generality of the results is limited in scope by the design. However, by using irregular lattices for two sample sizes and two different weights for the same regular lattice size, we feel we captured some of the important determinants of the properties of tests for spatial dependence.

Some of our results confirm the earlier findings of Anselin and Rey (1991), in particular with respect to the power of Moran as a test against any form of spatial dependence, and regarding the good properties of the LM tests against lag dependence. We also provided more insight into the role of the connectedness structure of the spatial weights matrix as a determinant of the properties of the tests, highlighted by the differences between the rook and queen cases for $R = 81$. For all tests, power was higher in the rook case. A new finding was the distinction between alternatives with autoregressive error dependence and moving average dependence. Clearly, tests against the former achieve higher power faster than tests against the latter. However, it should be kept in mind that the two forms of dependence are not mirror images, as illustrated by the different acceptable range for the parameters and the difference in the extent of the implied spatial interaction. In practice, it is not possible to distinguish between AR and MA error dependence based solely on the tests considered here. Instead, such a specification search should be based on non-nested tests. The extent to which the differences in power of the tests affect the properties of the resulting pre-test estimators also must be further investigated.

Another useful confirmation of earlier findings in Anselin and Rey (1991) was the higher power of lag tests in smaller samples, relative to tests against error dependence. Given the more serious consequences of ignoring a spatial lag for estimation and inference, this is a reassuring property.

Of the new tests considered, the power of K-R was rather disappointing in the samples considered. Since this is a large sample test (i.e., based on some large sample approximations), the configurations considered here may not be sufficiently large to achieve its theoretical properties. Similar conclusions may be drawn for the robustness of K-R to lognormal errors, which was not impressive in the samples considered. It is possible that the power of this test depends on the degree of spatial autocorrelation in the explanatory variables $\mathbf{X}$, for which the cross-products $\mathbf{Z}$ are used in equation (9). In the experimental design used here, these observations are independent, while in most realistic contexts they may be spatially correlated as well [e.g., Florax and Folmer (1992)]. This remains to be investigated. In practice, it would seem that the usefulness of K-R as an additional test (in addition to the LM tests) would be rather limited and restricted to large samples. The performance of K-R in larger samples and for other forms of error misspecification remains to be further investigated. This is also the case for the other tests, since the effect of lognormality on their power was rather marginal.

The robust LM-EL and LM-LE tests performed remarkably well against one-directional alternatives, which suggests that they may be usefully combined with the LM-ERR and LM-LAG tests to indicate which of the two forms of dependence (lag or error) is the proper alternative. In other words, this may result in an augmented decision rule, similar to the one formulated in Anselin and Rey (1991): when LM-LAG is more significant than LM-ERR and LM-LE is significant while LM-EL is not, a lag dependence is the likely alternative; and when LM-ERR is more significant than LM-LAG and LM-EL is significant while LM-LE is not, an error dependence is the likely alternative. While the simple rule was found to perform well in an actual specification search [Florax and Folmer (1992, 1994)], the extent to which this is the case for the augmented rule remains to be assessed.

However, matters are a bit more complex against SARMA alternatives. Here, there is a clear distinction between LM-LE, which performs well, and LM-EL, which is only reliable for truly "local" misspecification.

The results on the tests against higher order forms of spatial dependence were less reassuring. While the new LM-ERR(2) and SARMA tests had good power properties, so did the one-directional tests, including Moran. While this is a useful finding for misspecification testing, in the sense that problems with the original model are clearly indicated, it is a less insightful guide for an effective specification search. In this respect, the application of a Bera-Yoon like correction to the tests should be further investigated. In practice, it would seem that a one-directional test against a spatial lag (LM-LAG or LM-LE) should usefully preceed a test against higher order dependence. The rejection of lag dependence allows the proper use of LM-ERR(2) to assess higher order dependence. On the other hand, if lag dependence is not rejected, SARMA is the proper test to check for remaining error dependence, given the strange performance of LM-EL for non-local alternatives. For these higher order alternatives, a remaining issue is the extent to which the tests based on the null hypothesis (e.g., the SARMA) compare to tests carried out for a one-directional alternative, such as a test for error dependence in a first order spatial lag model. Similarly, the effect of using multiple tests on the properties of the resulting pre-test estimators needs to be considered.

Overall, while some important issues remain to be investigated, our study reaffirmed the usefulness of Monte Carlo experimentation to assess the properties of tests against spatial dependence. As these tests are becoming more used in empirical practice, it is necessary to gain insight into their properties for typical sample sizes, such as the U.S. states (48) and the number of counties in a state (around 100). In this respect, the scant evidence for the spatial case pales in comparison to the wealth of studies on serial dependence in the time domain. While the rather poor performance of the error tests in the smallest sample (R = 40) was a bit disconcerting, it was encouraging to see the asymptotic properties of the Lagrange Multiplier tests being closely approximated for the largest sample (R = 127). Given the ease by which these tests can be implemented and interpreted, they should become a standard element in the set of diagnostic tools used by empirical researchers who apply regression models to cross-sectional data.

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FIGURE 1: Regionalization of the Netherlands, $R = 40$

FIGURE 2: Regionalization of the Netherlands, $R = 127$ (2 unconnected regions excluded)

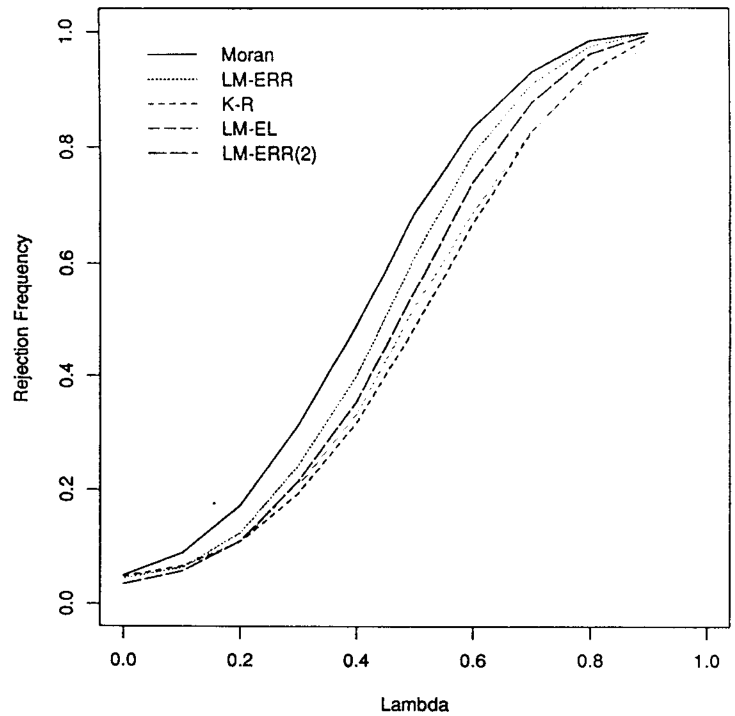

FIGURE 3: Power Against Spatial AR Error, $R = 40$

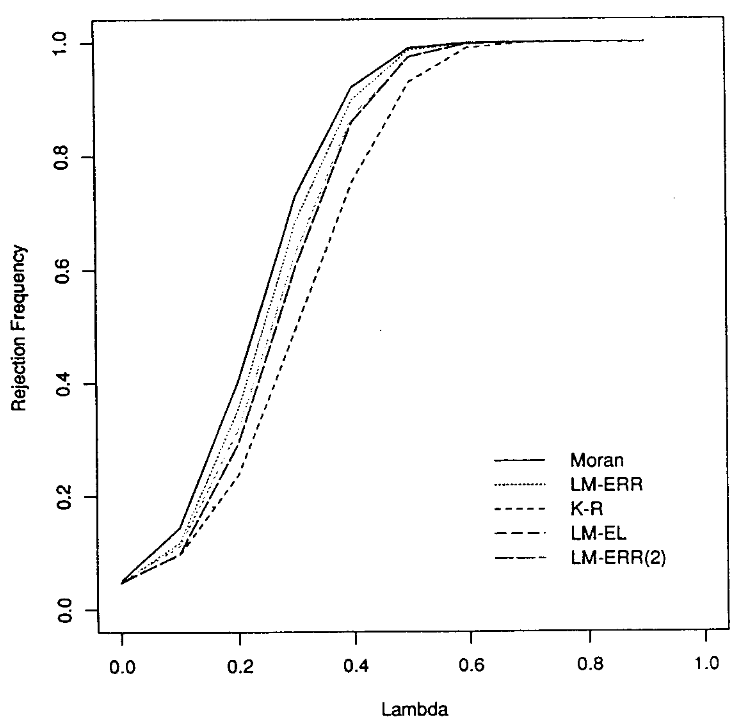

FIGURE 4: Power Against Spatial AR Error, $R = 127$

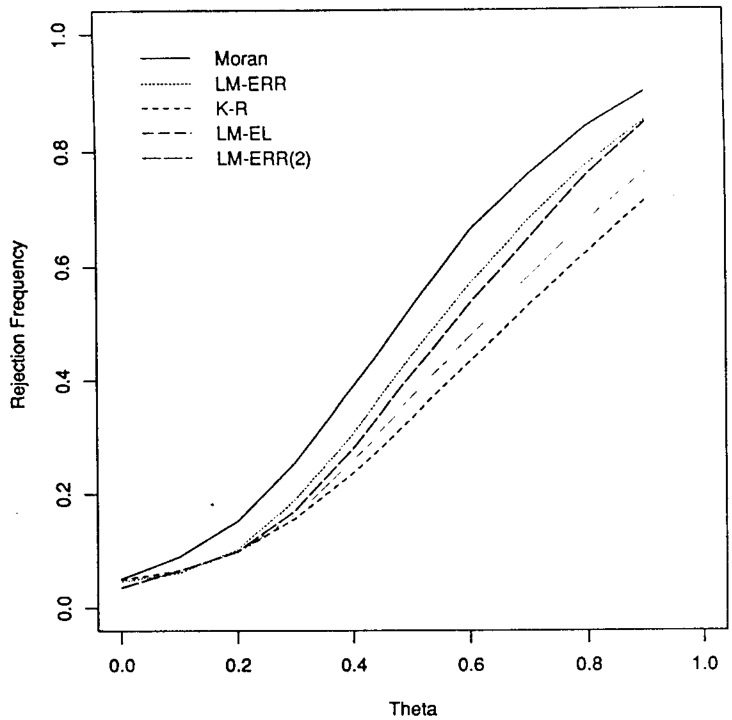

FIGURE 5: Power Against Spatial MA Error, $R = 40$

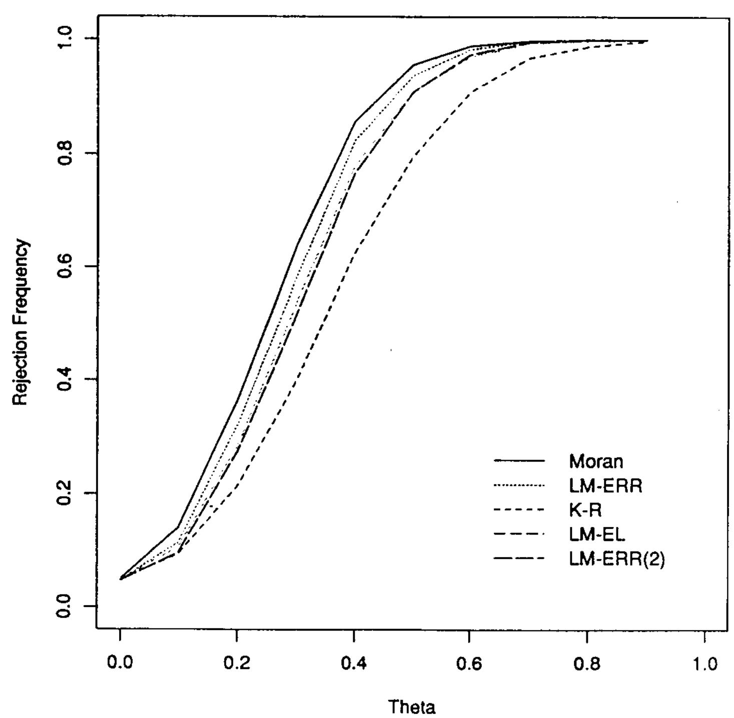

FIGURE 6: Power Against Spatial MA Error, $R = 127$

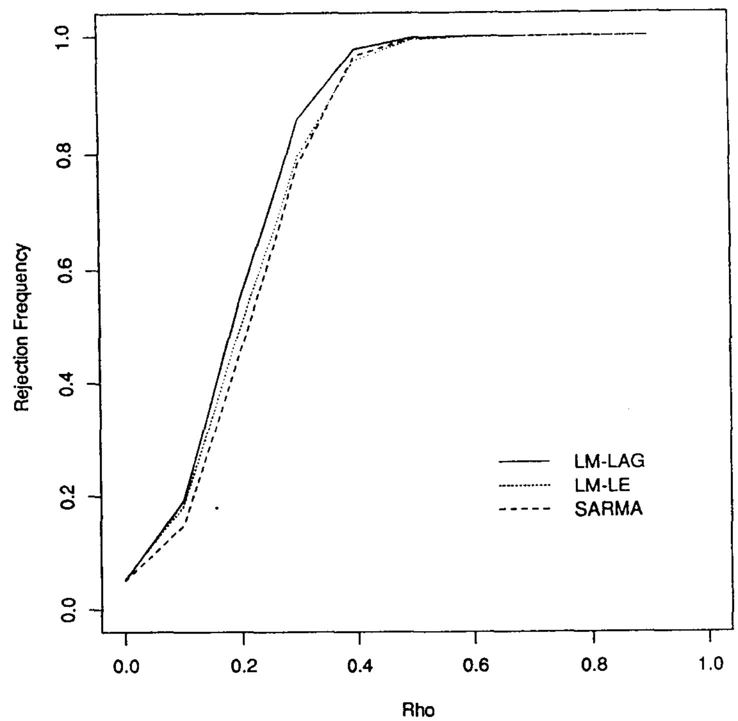

FIGURE 7: Power Against Spatial AR Lag, $R = 40$

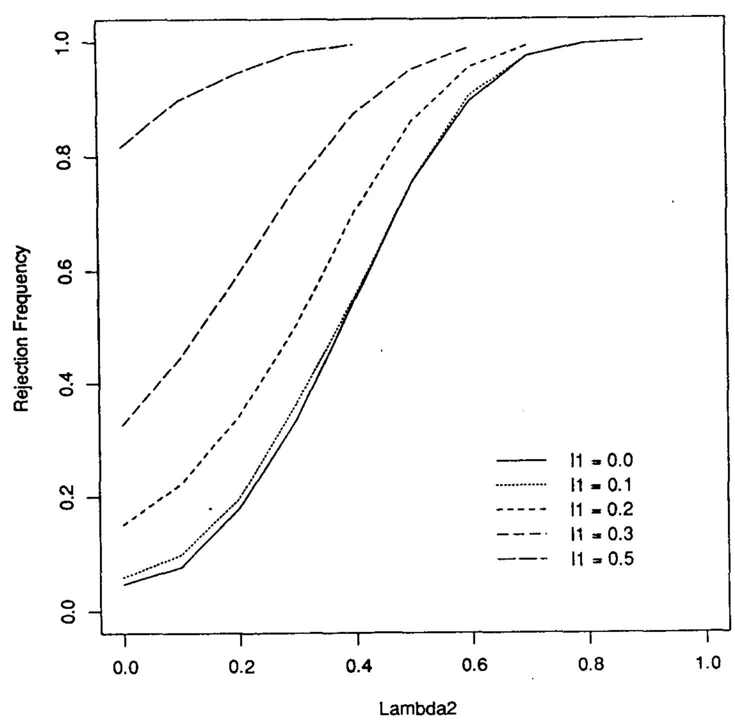

FIGURE 8: Power of LM-ERR(2) Against Second Order AR Errors, $R = 81$ (rook)

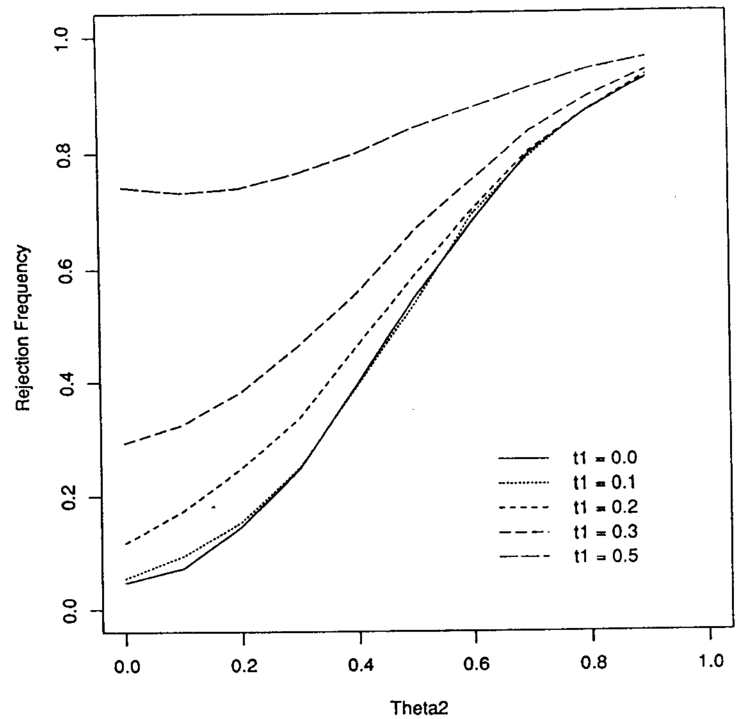

FIGURE 9: Power of LM-ERR(2) Against Second Order MA Errors, $R = 81$ (rook)

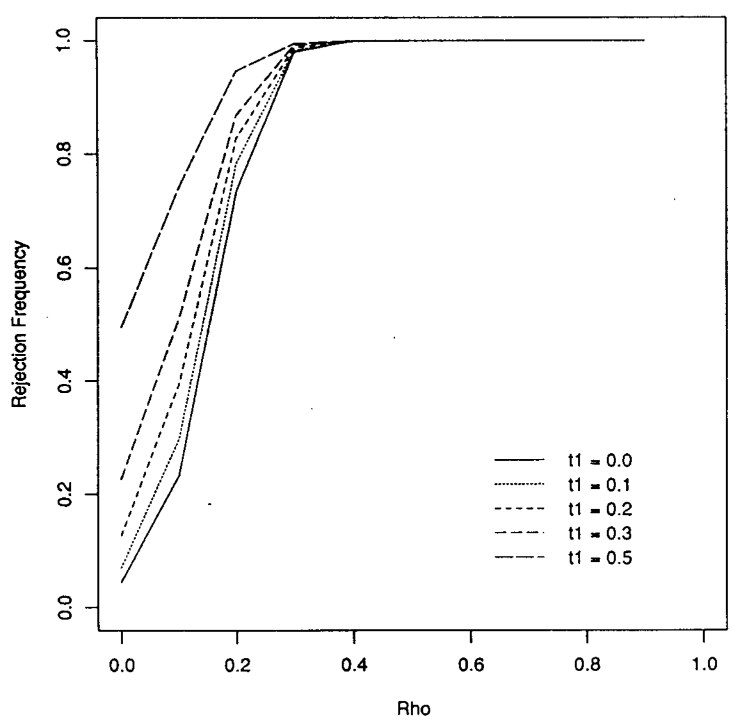

FIGURE 10: Power of SARMA Against SARMA (1, 1), AR Dimension, $R = 81$ (queen)

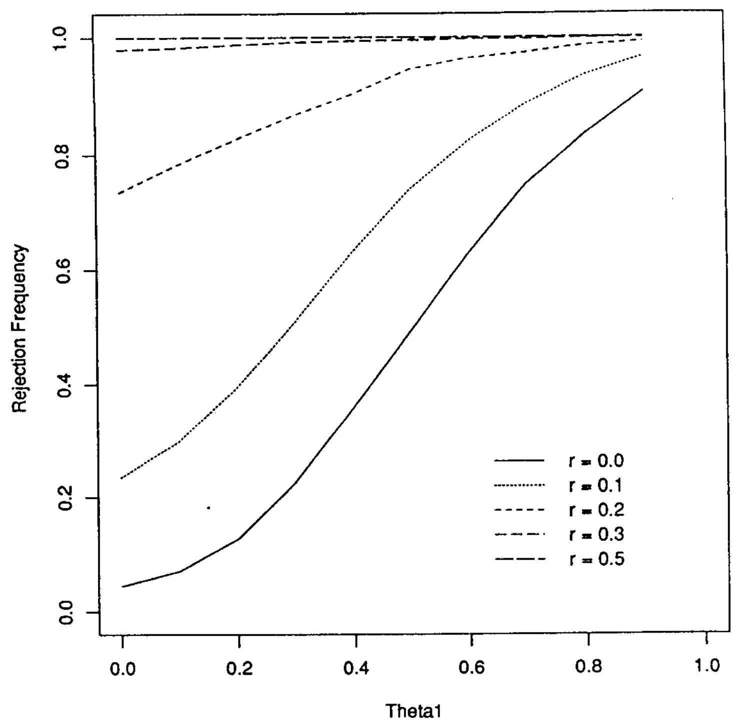

FIGURE 11: Power of SARMA Against SARMA (1, 1), MA Dimension, $R = 81$ (queen)

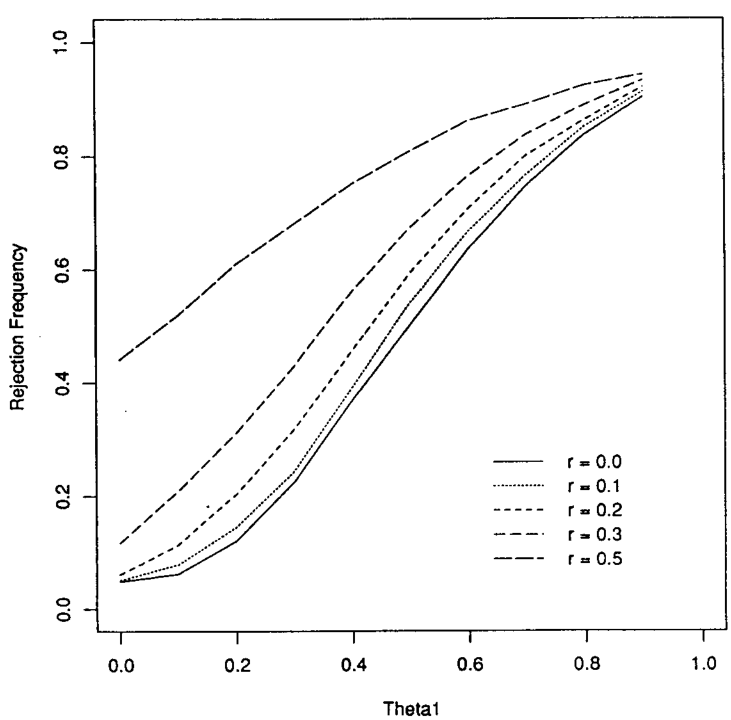

FIGURE 12: Power of LM-EL Against SARMA (1, 1), MA Dimension, $R = 81$ (queen)

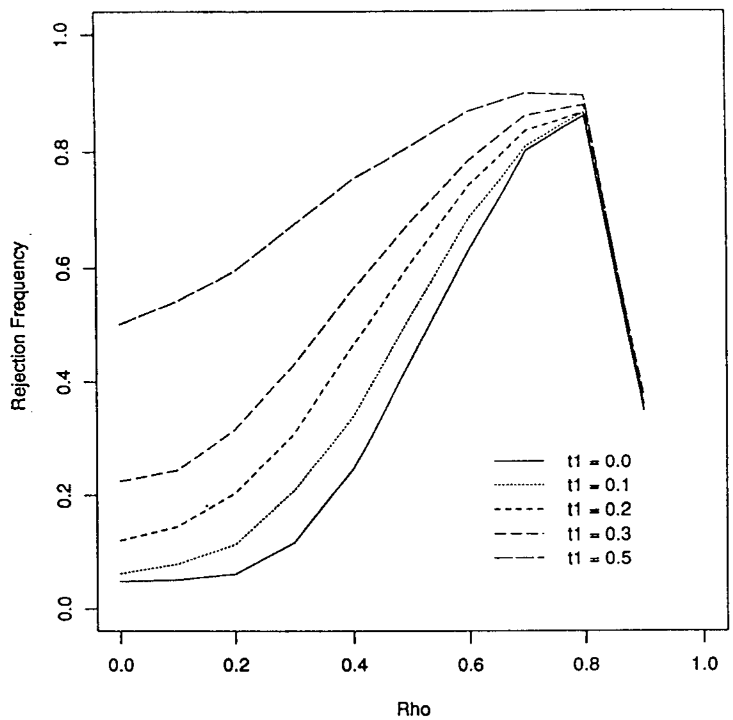

FIGURE 13: Power of LM-EL Against SARMA (1, 1), AR Dimension, $R = 81$ (queen)

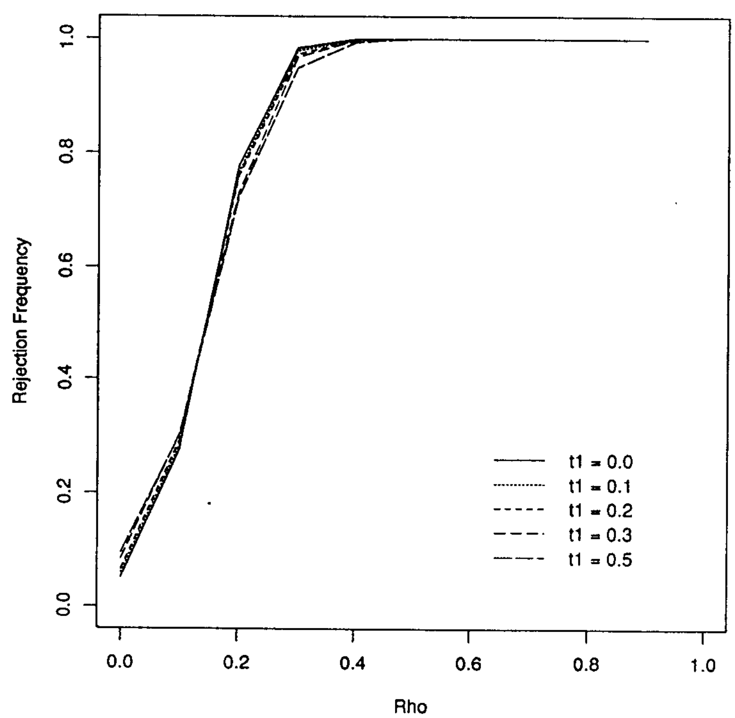

FIGURE 14: Power of LM-LE Against SARMA (1, 1), AR Dimension, $R = 81$ (queen)

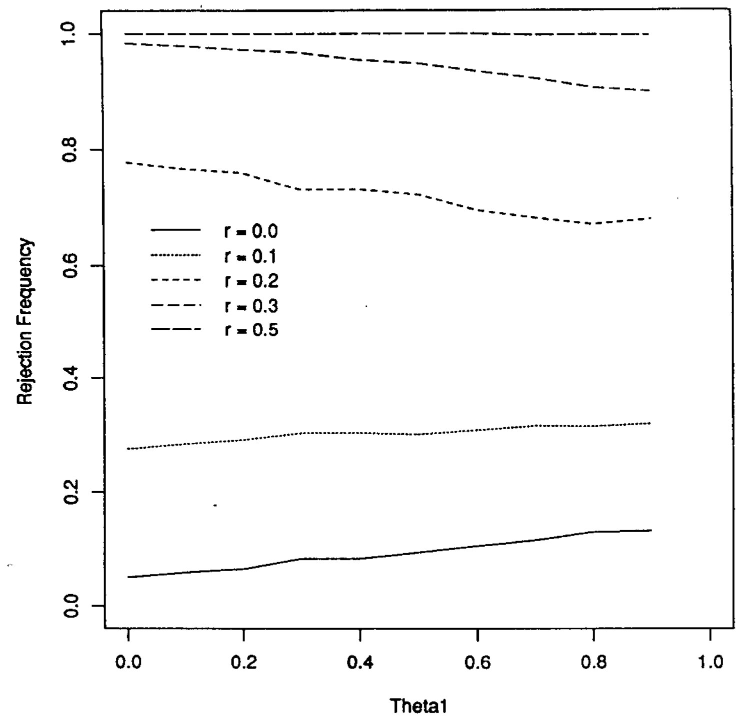

FIGURE 15: Power of LM-LE Against SARMA (1, 1), MA Dimension, $R = 81$ (queen)

TABLE A.1: Power of Tests Against First Order Spatial Autoregressive Errors – Lognormal Distribution

R	λ_1	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)	SARMA	LM-LAG	LM-LE
40	0.1	0.077	0.052	0.071	0.054	0.046	0.063	0.066	0.069
	0.2	0.156	0.107	0.110	0.101	0.092	0.111	0.085	0.076
	0.3	0.296	0.225	0.197	0.201	0.188	0.215	0.116	0.085
	0.4	0.483	0.393	0.330	0.333	0.331	0.362	0.171	0.096
	0.5	0.692	0.607	0.530	0.520	0.537	0.567	0.249	0.122
	0.6	0.848	0.794	0.723	0.699	0.740	0.762	0.368	0.135
	0.7	0.946	0.921	0.868	0.840	0.889	0.898	0.531	0.149
	0.8	0.988	0.980	0.956	0.926	0.969	0.971	0.718	0.163
	0.9	0.999	0.997	0.992	0.963	0.996	0.995	0.891	0.161
81 (queen)	0.1	0.088	0.057	0.068	0.061	0.055	0.071	0.061	0.062
	0.2	0.206	0.150	0.127	0.138	0.129	0.149	0.085	0.075
	0.3	0.399	0.308	0.250	0.286	0.265	0.277	0.118	0.084
	0.4	0.618	0.528	0.466	0.484	0.463	0.485	0.169	0.104
	0.5	0.821	0.757	0.702	0.699	0.698	0.707	0.272	0.124
	0.6	0.942	0.912	0.880	0.870	0.882	0.884	0.391	0.154
	0.7	0.988	0.979	0.968	0.963	0.967	0.968	0.559	0.201
	0.8	0.998	0.997	0.996	0.993	0.994	0.995	0.751	0.262
	0.9	1.000	1.000	1.000	0.999	1.000	1.000	0.929	0.368
81 (rook)	0.1	0.091	0.067	0.064	0.062	0.050	0.057	0.055	0.053
	0.2	0.246	0.189	0.133	0.173	0.136	0.148	0.071	0.053
	0.3	0.498	0.420	0.310	0.373	0.308	0.342	0.104	0.057
	0.4	0.769	0.706	0.571	0.644	0.581	0.607	0.152	0.059
	0.5	0.929	0.899	0.820	0.849	0.820	0.842	0.253	0.069
	0.6	0.988	0.980	0.950	0.958	0.956	0.963	0.383	0.084
	0.7	0.999	0.999	0.991	0.994	0.995	0.995	0.579	0.103
	0.8	1.000	1.000	0.999	0.999	1.000	0.999	0.796	0.115
	0.9	1.000	1.000	1.000	1.000	1.000	1.000	0.959	0.163
127	0.1	0.129	0.104	0.085	0.104	0.089	0.101	0.063	0.055
	0.2	0.381	0.326	0.222	0.302	0.270	0.280	0.103	0.067
	0.3	0.717	0.674	0.504	0.617	0.584	0.589	0.183	0.082
	0.4	0.931	0.907	0.795	0.870	0.849	0.858	0.292	0.095
	0.5	0.991	0.988	0.952	0.972	0.973	0.976	0.444	0.125
	0.6	1.000	0.999	0.991	0.996	0.998	0.998	0.621	0.153
	0.7	1.000	1.000	0.998	1.000	1.000	1.000	0.808	0.182
	0.8	1.000	1.000	0.998	1.000	1.000	1.000	0.948	0.213
	0.9	1.000	1.000	0.992	1.000	1.000	1.000	0.997	0.240

TABLE A.2: Power of Tests Against First Order Spatial Moving Average Errors – Lognormal Distribution

R	θ_1	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)	SARMA	LM-LAG	LM-LE
	0.1	0.066	0.048	0.071	0.047	0.042	0.059	0.057	0.061
	0.2	0.135	0.088	0.087	0.091	0.082	0.099	0.082	0.076
	0.3	0.245	0.180	0.151	0.163	0.151	0.164	0.093	0.072
	0.4	0.375	0.289	0.236	0.261	0.246	0.263	0.127	0.091
40	0.5	0.518	0.432	0.343	0.368	0.377	0.384	0.175	0.101
	0.6	0.656	0.562	0.466	0.474	0.512	0.502	0.209	0.102
	0.7	0.779	0.699	0.577	0.596	0.656	0.643	0.259	0.114
	0.8	0.863	0.794	0.676	0.691	0.771	0.747	0.319	0.134
	0.9	0.915	0.870	0.761	0.763	0.855	0.824	0.345	0.148
	0.1	0.084	0.053	0.069	0.055	0.053	0.064	0.059	0.062
	0.2	0.173	0.119	0.110	0.121	0.104	0.122	0.077	0.066
	0.3	0.307	0.227	0.184	0.214	0.201	0.207	0.103	0.086
	0.4	0.484	0.390	0.324	0.355	0.341	0.345	0.129	0.087
81 (queen)	0.5	0.646	0.555	0.485	0.503	0.501	0.501	0.175	0.100
	0.6	0.771	0.692	0.612	0.631	0.646	0.631	0.214	0.102
	0.7	0.876	0.817	0.744	0.752	0.777	0.765	0.266	0.119
	0.8	0.930	0.891	0.838	0.843	0.875	0.849	0.303	0.129
	0.9	0.969	0.945	0.908	0.909	0.940	0.923	0.364	0.139
	0.1	0.098	0.070	0.066	0.071	0.052	0.066	0.055	0.056
	0.2	0.230	0.176	0.119	0.159	0.119	0.139	0.067	0.051
	0.3	0.467	0.383	0.262	0.344	0.263	0.289	0.090	0.051
	0.4	0.715	0.634	0.472	0.575	0.497	0.522	0.121	0.051
81 (rook)	0.5	0.885	0.834	0.697	0.774	0.741	0.737	0.170	0.056
	0.6	0.962	0.939	0.828	0.896	0.907	0.887	0.225	0.057
	0.7	0.992	0.984	0.923	0.963	0.976	0.962	0.274	0.057
	0.8	0.999	0.998	0.955	0.989	0.996	0.990	0.324	0.055
	0.9	1.000	0.999	0.974	0.994	0.999	0.996	0.387	0.065
	0.1	0.130	0.104	0.078	0.107	0.091	0.100	0.069	0.065
	0.2	0.345	0.297	0.200	0.270	0.241	0.250	0.104	0.068
	0.3	0.631	0.576	0.395	0.522	0.478	0.488	0.148	0.078
	0.4	0.854	0.819	0.652	0.775	0.738	0.749	0.212	0.080
127	0.5	0.950	0.936	0.824	0.901	0.900	0.897	0.279	0.095
	0.6	0.990	0.984	0.937	0.970	0.972	0.973	0.366	0.111
	0.7	0.998	0.997	0.972	0.992	0.993	0.993	0.438	0.108
	0.8	1.000	1.000	0.988	0.998	0.999	0.999	0.511	0.126
	0.9	1.000	1.000	0.994	0.999	0.999	0.999	0.586	0.124

TABLE A.3: Power of Tests Against First Order Spatial Autoregressive Lag – Lognormal Distribution

R	ρ	Moran	LM-ERR	K-R	LM-EL	LM-ERR(2)	SARMA	LM-LAG	LM-LE
40	0.1	0.081	0.053	0.066	0.036	0.045	0.163	0.224	0.211
	0.2	0.205	0.139	0.128	0.031	0.113	0.502	0.599	0.548
	0.3	0.439	0.345	0.283	0.026	0.269	0.805	0.866	0.809
	0.4	0.734	0.642	0.526	0.021	0.532	0.954	0.975	0.942
	0.5	0.908	0.856	0.761	0.014	0.776	0.990	0.996	0.979
	0.6	0.978	0.961	0.913	0.008	0.928	0.999	1.000	0.992
	0.7	0.997	0.995	0.983	0.002	0.990	1.000	1.000	0.996
	0.8	1.000	1.000	0.997	0.003	0.999	1.000	1.000	0.998
	0.9	1.000	1.000	1.000	0.006	1.000	1.000	1.000	0.998
81 (queen)	0.1	0.112	0.076	0.082	0.043	0.068	0.249	0.321	0.310
	0.2	0.339	0.258	0.237	0.053	0.229	0.750	0.823	0.790
	0.3	0.720	0.638	0.610	0.129	0.578	0.971	0.984	0.971
	0.4	0.942	0.916	0.896	0.277	0.877	0.999	0.999	0.997
	0.5	0.994	0.991	0.990	0.487	0.985	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.660	1.000	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.810	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.858	1.000	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.373	1.000	1.000	1.000	1.000
81 (rook)	0.1	0.092	0.068	0.075	0.051	0.057	0.399	0.499	0.490
	0.2	0.190	0.142	0.155	0.059	0.171	0.928	0.962	0.946
	0.3	0.368	0.299	0.331	0.055	0.460	0.996	0.999	0.998
	0.4	0.628	0.555	0.584	0.056	0.786	1.000	1.000	1.000
	0.5	0.867	0.816	0.832	0.049	0.959	1.000	1.000	1.000
	0.6	0.980	0.970	0.971	0.028	0.996	1.000	1.000	1.000
	0.7	1.000	0.999	0.998	0.009	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.001	1.000	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000
127	0.1	0.160	0.131	0.112	0.041	0.123	0.560	0.665	0.621
	0.2	0.617	0.556	0.474	0.065	0.515	0.988	0.994	0.989
	0.3	0.949	0.929	0.889	0.106	0.921	1.000	1.000	0.999
	0.4	0.999	0.998	0.995	0.126	0.997	1.000	1.000	1.000
	0.5	1.000	1.000	1.000	0.095	1.000	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	0.031	1.000	1.000	1.000	1.000
	0.7	1.000	1.000	1.000	0.003	1.000	1.000	1.000	1.000
	0.8	1.000	1.000	1.000	0.001	1.000	1.000	1.000	1.000
	0.9	1.000	1.000	1.000	0.000	1.000	1.000	1.000	1.000