Distance decay functions for modelling cycling uptake

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2 Introduction

Trips of short distances tend to be more frequent than trips of long distances. This is a fundamental feature of transport behaviour, observed in the majority of transport systems worldwide. The concept is neatly captured by the term *distance decay* (*dd*), meaning simply that the proportion of trips of any given type tends to zero as distance tends to infinity.

Distance decay — along with the synonymous *deterrence function* (Simini et al. 2012) — builds on older terms expressing the same fundamental truth, such as the 'friction of distance' and the 'gravity model' (described in the subsequent section). Work focussed explicitly on *dd* explores *how* the proportion of trips (p) declines with distance (d) (Iacono, Krizek, and El-Geneidy 2008), beyond the simple observation that the slope is negative beyond a certain threshold distance. A resergence of interest in the distance-frequency relationship has refined and, in some cases, sought to overturn pre-existing (and often context-specific) formulations, such as $p =$
$alphae^{betad}$ (A. Wilson 1971) and $p =$
$alphad^{beta}$ (Fotheringham 1981). Much *dd* research asks: what is the best function for linking the proportion (different types of) trip to distance?

Mathematically, what is f of x

$$p = f(x)$$

where x is distance and p is the proportion of trips made in any distance band? Distance decay models can be empirically tested, and therefore ranked, by comparing observed travel behaviour with the levels expected based on different distance decay functions.

Equation 1.1 can clearly be refined in many directions. Subscripts can be added to the terms and new variables can be added to illustrate how distance decay changes with a range of additional factors such as type of bicycle and hillness of the route. how distance decay changes in response to a range of variables. The type of trip (e.g. time of day, purpose), characteristics of the people making the trip (e.g. age and sex) and the location and physical surroundings of the trip (e.g. hilliness, transport infrastructure) have all been found to affect the shape of distance decay curves [@Fotheringham1981;Fingleton (2005)]. Critically for understanding transport systems is the *mode*, or 'method' of transport. Mode refers to the choice of 'vehicle' (e.g. walking, cycling, car driving) and has been found, unsurprisingly, to dramatically affect the speed travelled during and amount of effort required for trips of different distances. The *dd* concept is especially applicable to active travel modes due to physiological limits on human mechanical power output and the slow speed of these modes. The next section shows that the concept has a long history in the academic literature but that interest in the term from the perspective of walking and cycling is relatively recent.

3 The distance decay literature

A variety of terms have been used to express the idea underlying the *dd* concept outlined in this paper. These including the 'first law of geography,' the 'friction of distance' and 'the gravity model.' It is worth reviewing these terms before describing work refering to *dd* directly, to put the term in its wider historical context. The final part of this section reviews recent literature explicitly using the term 'distance decay' to explore the concept

3.1 The gravity model

The 'gravity model' of movement patterns helped to quantify and generalise early incarnations if *dd* (Zipf et al. 1946). This rule (it is sometimes referred to as the 'gravity law') suggests that travel incentives are roughly analogous to Newtonian gravitation (Ravenstein 1885). The resulting formulae imply that the total movement between two places (T_ij between origin i and destination j) is proportional to the product of their populations (m_i and n_j), divided by a power of the distance between them:

$$T_{ij} = \frac{m_i n_j}{d^\beta}$$

as it has sometimes been called has been a rich source of theoretical and methodological advance in many fields, primarily urban modelling but also in fields as diverse as highway planning (Jung, Wang, and Stanley 2008), national transport of minerals (Zuo et al. 2013) and spatial epidemiology (Balcan et al. 2009).

3.2 The radiation model

Despite dissenting voices — including the statement that "a strict gravity model simply did not work" for modelling urban systems and that some subsequent refinements to the gravity model were "fudge factors" (A. A. A. G. Wilson 1998) — the gravity model has been one of the dominant tools for understanding urban mobility over the past 100 years (Masucci et al. 2013). A recent development in this field has been the 'radiation model,' which has been found to fit travel to work and other flow data well (Simini et al. 2012). This new formula for estimating flow rates between geographic zones is interesting in its ommission of distance as an explicit explanatory variable. Instead, the radiation model uses the number 'intervening opportunities' (IO) as a proxy for *dd* the denominator to estimate flow:

$$ddapprox(m_i + s\_ij)(m_i + n_j + s\_ij)$$

A recent study compared the parameter-free radiation model against the gravity model on a large intra-city (London) dataset on commuting. It was found that neither model produced a satisfactory fit with the data, leading to the conclusion that "commuting at the city scale still lacks a valid model and that further research is required to understand the mechanism behind urban mobility" (Masucci et al. 2013). The 'first law of geography' and the related concept of the 'friction of distance' are alternative yet closely related (i.e. not mutually exclusive) terms for exploring *dd* th

3.3 The first law of geography

Tobler's famous **first law of geography** states that "everything is related to everything else, but near things are more related than distant things" (Tobler 1970). The phrase implies that interaction between things declines with distance without specifying how or why. Clearly, the increased frequency of communications and transport between places that are close to each other can help explain spatial autocorrelation (Miller et al. 2004). However, unlike *dd*, the 'first law' says little about the way in which relatedness between entities declines with distace. Moreover, in a world of accelerating globalisation under the auspices of the ongoing 'digital revolution,' the relevance of the 'first law' to the system-level processes it was proposed to explain has come under scrutiny (Westlund 2013).

To overcome this issue, the scope of this paper is limited to transportation of people and goods, opposed to immaterial communication. This simplifies the issue and makes it more tangible. Due to fundament physical limits on the efficiency with which matter can be moved (MacKay 2013), and a limited supply of energy (especially pertinent in an era of resource over-extraction and climate change), such physical transport will always be limited to some degree by distance. This notion of travel frequency tending to zero as distance tends to infinity, present in the 'friction of distance' terminology, is also encapsulated by the more recent phrase 'distance decay' used in this paper.

3.4 The friction of distance

The concept of the 'friction of distance' has been in use for over 100 enjoying steady (albeit slowing) growth in use until the early 2000s (Fig. 1). The term helps to explain why Tobler's Law is true, implying that it is more *difficult* (e.g. in terms of energy use) for distant things to interact. 'Friction' is thus defined as "the difficulty of moving a volume (passengers or goods)" (Muvingi 2012). The friction of distance is predominantly used as a synonym for *dd*, and to some extent the slowing use of the term illustrated in Fig. 1 can be seen as a consequence of the latter simply replacing the former. One study, for example, refers to "the friction of distance parameter in gravity models" (Cliff, Martin, and Ord, n.d.) whereas others refer to 'distance decay' in spatial interaction and gravity models respectively: the terms are largely interchangeable [Griffith and Jones (1980);McCall2006]. However, the question of precisely *how* the metaphorical friction increases with distance has been tackled to a lesser extent in the (generally older) literature using the term. The recent *dd* literature, by contrast, is largely focussed this question, as described in the subsequent section.

3.5 Recent distance decay literature

As illustrated in Figure \ref{fcitations}, *dd* has grown rapidly as a term in the academic literature over the past 50 years, compared with the well-established "gravity model" terminology. By the 1970s, it seems that *dd* overtook the "friction of distance" amongst transport researchers. Although Tobler's 'First Law' of geography has gained rapid acceptance since its inception in the 1970s, it has a far lower (by a factor of 5) rate of use than *dd* in the transport literature. In other words, a quantitative review of the literature demonstrates that *dd* is an up-and-coming term in transport studies. This section does not attempt a full overview of the 10,000+ articles using the *dd* terminology in the academic literature, instead focussing on a few key and highly cited works that helped set the agenda for *dd* research.

"friction of distance" terms. (which tends to assign importance to interaction between places rather than the impact of distance)

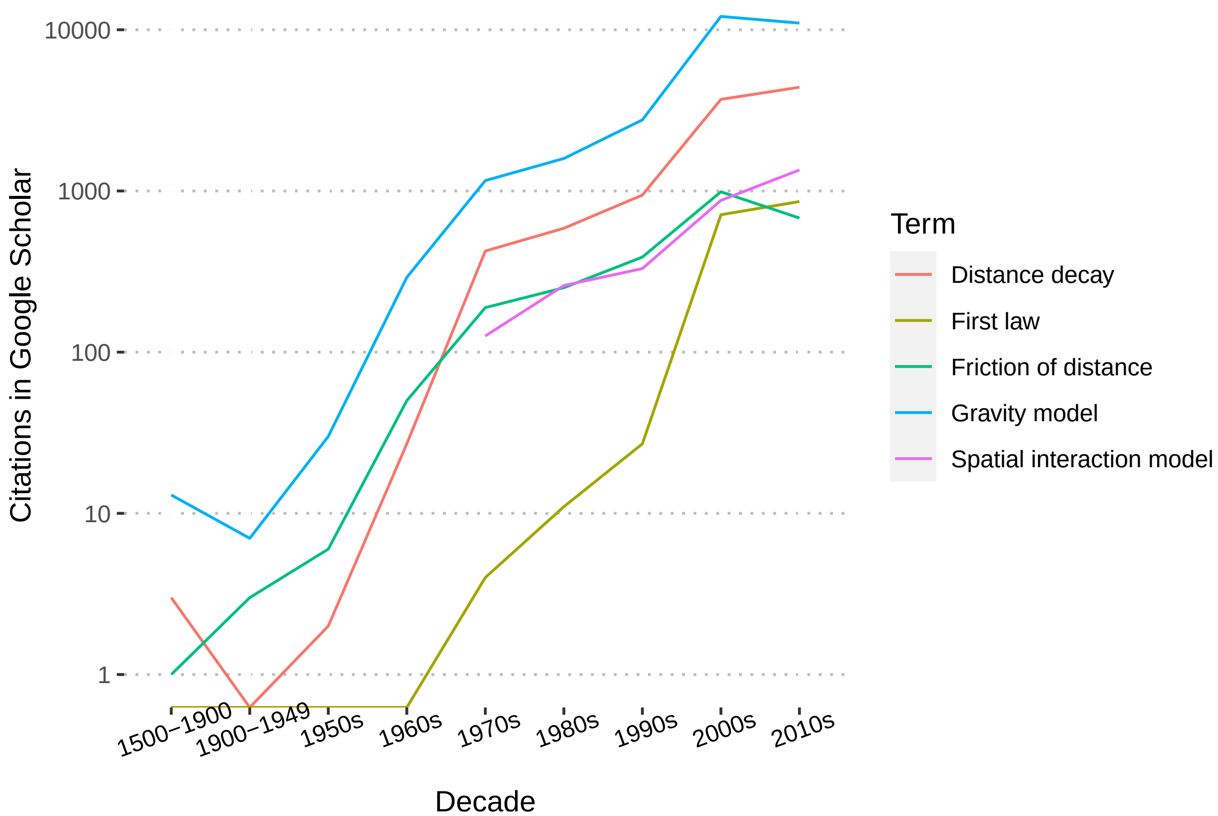

Figure 1: (#fcitationsunnamed-chunk-7)Number of citations per decade for different terms for ‘distance decay’ from Google Scholar.

Within the general understanding that trips become less frequent with distance (beyond some threshold limit) there are various ways of describing the dependent variable that *dd* functions seek to explain. *dd* can be understood as:

- the absolute number of trips expected for any given distance band (*db*): $f(d) = T\_db$
- the proportion of *all* trips that are made for a given distance band: $f(d) = T\_db/T$
- the proportion of trips *within a given distance band* made by a particular trip type (e.g. walking): \newline $f(d) = Twalk\_db/T\_db$

Of these definitions the second is the most generalisable, being a proportion (0
geqp
*leq*1) that tends to zero with increasing distance for any mode of transport. The third definition of *dd* also estimates a proportion with the same constraints, but is less generalisable: the result will not always tend to zero (the proportion of trips by air travel, for example, will tend to one for large trip distances). The first and simplest definition is the least generalisable as it is highly dependent on the total amount of travel.

4 Functional forms of distance decay

Four functional forms commonly used in the literature to characterise distance decay curves were described in a recent paper (Mart'ınez and Viegas 2013) as:

- Exponential functions, e^{betax} (A. Wilson 1971).
- Power functions, x^{beta} (Fotheringham 1981).
- Tanner functions, $x^{beta_1}e^{beta_2 x}$ (Openshaw 1977).
- Box-Cox functions, $exp($
  beta
  $fracx^{gamma} - 1gamma)$ when the parameter
  gamma
  *neq*0 and x^{beta} when
  $gamma = 0$

In addition to these, we have added the additional functions:

- Modified beta distributions, which have been used to model distance decay in ecological research (**nekola_scale_2014?**)
- A modified version of the generalised logistic or 'Richards' function

$$f(x) = \frac{1}{(1 + Qe^{-B(x-M)})^{1/v}}$$

Each of the 4 parameters in this model can be altered to ensure optimal fit and each has a separate meaning:

- B is analogous to β in the other functions
- Q represents the intercept
- v affects the skewness of the curve
- M is the x value of maximum growth

4.1 A linear model

The simplest model to fit to the data is a linear model. For illustrative purposes we will fit a linear model to the data presented in Figure 1, with different intercepts and gradients for each of the groups (Figure 2):

The intercepts and gradients for each group are presented in table !!!

The above numbers are equations that describe the relationship between distance and *clc* for each group. In the `Mal_Young_NC` group, for example,

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