Spoilt for Choice? An Investigation Into Creating Gastner and Newman-style Cartograms

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Summary

A number of choices are encountered when creating cartograms using the Gastner and Newman algorithm. Two important ones are the starting map projection, and the resolution of the grid size used to compute the cartogram transform. We experiment with a number of projection and grid size combinations, and define a measure of 'cartogram success', and use this, together with a more descriptive assessment, to identify best practice in choosing resolution and initial projection.

KEYWORDS: Cartogram, Rasterisation, Visualisation, Cartography, Map Projection

1 Introduction

Cartograms are well established map projections used for the creation of statistical maps that take into account the underlying population of geographical reporting units (GRUs). They have the property that the projected area of each areal unit is proportional to its population[1]. There are a number of standard algorithms to compute cartograms - see for example Tobler (1973), Dorling (1996) or Sagar (2013). Most of these begin with a standard map projection, with populations supplied for each GRU, and then compute a 'warp' - of the map drawn in this projection to obtain the cartogram. One particular example of this approach is the Gastner and Newman algorithm (Gastner and Newman, 2004). This solves the *diffusion equation* - Equation 1 below, using a pixel-based approximation.

$$\frac{\partial \rho(x,y,t)}{\partial t} = \nabla^2 \rho(x,y,t) \tag{1}$$

This equation describes the flow of fluids with varying density ρ at time t and at each location (x, y). For cartograms, it is assumed that ρ is proportional to population density. Study of Equation 1 reveals that fluids

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[1]At least approximately

will diffuse towards an asymptotic state of uniform density. Thus, for each initial location the mapping onto a particle's asymptotic location provides a cartogram transformation.

1.1 Spoilt for choice?

A number of observations can be made about this algorithm.

1. The method is based on approximation.
2. It depends on a grid-based estimation of population density.
3. This estimation is derived from a map using a conventional projection. Several such projections exist.

These raise a number of issues - observation 2 suggests that a number of choices must be made prior to running the cartogram algorithm - namely, at what resoluton should the grid be created, and what method should be used to estimate the density values in the grid. Observation 3 implies a further choice - that of the starting map projection - and observation 1 leads to an over-arching issue that, although the cartogram algorithm is often presented as a unique process, there are in fact a number of degrees of freedom. The aim of this paper is to investigate the effect of varying these, and hopefully uncover some 'best practice' recommendations for the creation of cartograms.

2 Choices for Initial Conditions

The two key choices relate to the initial map projection, and the resolution of the grid used in the numerical solution of the diffusion equation. The Gastner and Newman algorithm begins with a set of initial densities - typically these are obtained from a set of polygons in a 'conventional' map projection with associated population counts, and assigning population density estimates to each polygon by dividing the count by the corresponding polygon areas. The results are then converted to a raster grid before applying the algorithm - each pixel is assigned to a polygon (approximately) and the density associated with that polygon is then assigned to the pixel – at which point the algorithm is applied.

2.1 Choice of Initial Map Projection

There are several 'conventional' map projections[2], and several possible starting configurations for the algorithm. Map projections can be classified in a number of ways, but one helpful approach here is to use the following classes:

- **Equal Area** - These are projections that give polygons whose area is the same as that on the surface of the Earth. One cost of achieving this is that shapes of areas are prone to distortion.
- **Conformal** - These projections preserve local angles, so that the projected angles where curves meet agree with those on the Earth's surface.

[2] actually an infinite number if parameters such as the location of parallels are allowed to vary

- **Equidistant** - These preserve distances from some fixed point, or line on the Earth's surface.
- **Compromise** - These do not attempt to preserve area, distance or local angles, but instead aim to strike a balance between the distortions, in order to produce æsthetically pleasing results.

Projections may also be classified by the developable surface onto which the Earth's surface is projected: cones, cylinders and planes are typical examples (Bugayevskiy and Snyder, 1995). This leads to a cross classification of surface and property, which often features in the name: for example, *Lambert's Azimuthal Equal Area* is an area preserving projection based on a plane.

The projections we have chosen for the experiments described in this paper cover a range of types and developable surfaces. They are listed, together with their properties in Table 1.

Table 1: Projections used

Developable Surface	Projection Name	Type	Abbreviation
Cone	Lambert Conformal Conic	Conformal	LCC
	Albers Equal Area	Equal Area	AEA
	Equidistant Conic	Equidistant	EqDC
Cylinder	Robinson Pseudocylindrical	Compromise	Robin
	Mercator	Conformal	Merc
	Eckert Type VI	Equal Area	Eck6
	Mollweide	Equal Area	Moll
	Equidistant Cylindrical	Equidistant	EQC
Plane	Van der Grinten	Compromise	VanDG
	Stereographic	Conformal	Stere
	Lambert Azimuthal Equal Area	Equal Area	LAEA
	Azimuthal Equidistant	Equidistant	AEqD

It could be argued that equal area projections are the most appropriate choice, since these will give the most accurate estimates of density for the Gaster and Newman algorithm. We therefore include a large number of these. However two questions we wish to address are

- How do results differ between different equal area projections?
- How robust are the results to the use of other kinds of projection?

and on this basis we also include a variety of other projections, representing all combinations of type and developable surface.

2.2 Resolution of Raster Representation

As discussed above, once a projection is chosen, the next step is to rasterise the density estimates. A second choice influencing the outcome is the resolution of the raster used to do this. Clearly, the greater this is, the more accurate the result – recall that the algorithm is based on a differential equation representing a continuous system. On the other hand, computation time will increase with resolution - and it will be useful to identify a point at which no notable improvements are achieved, so that unneccesarily long program runs are avoided. Thus, for each map projection, cartograms are created at a number of resolutions. In each case the raster is square, and of size $n \times n$ where n is one of $\{512, 768, 1024, 1280, 1536\}$.

3 Evaluation

The set of map projections and resolutions outlined above were used to compute cartograms of European economic regions. The software used was Brunsdon's *getcartr* package in R[3]. Cartograms are assessed in two ways here. Firstly, an objective scoring system is used. Although based on approximation, the transformed areas in a cartogram should ideally be proportional to their underlying populations. Thus, if a cartogram algorithm has worked effectively, then

$$A_i = kP_i \tag{2}$$

where A_i is the area of a zone i (in cartogram space), P_i is the corresponding population, and k is some constant value. Thus, for any given cartogram, fitting a least squares regression line *without an intercept* should give an estimate of k, say $\hat{k}$. Perhaps more usefully, looking at the size of the residuals - that is the values of $A_i - \hat{k}P_i$ gives an indication of how well this linear fit has worked for each area. Squaring these residuals and summing gives an overall measure of the degree of disagreement in the proportionality between area and population and hence a measure of the success of the cartogram. As a further enhancement, the measurement can be standardised by computing $1 - R^2$ for the fitted, intercept-free model, allowing for different scale metrics in cartogram space. If we call this measure γ, then it may be seen that

$$\gamma = \frac{\sum_i r_i^2}{\sum_i \hat{k}^2 P_i^2 + \sum_i r_i^2} \text{ where } r_i = A_i - \hat{k}P_i \tag{3}$$

For an ideal[4] cartogram, this value will be zero. Thus, this quantity will be computed for each cartogram created.

A second approach to assessment will be to assess the cartograms visually - although it is possible that several cartograms may have γ values near to zero, there could be æsthetic reasons why some may be preferable to others. Although this is a subjective matter, visualisations identifying cartograms with different characteristics will be used to investigate any qualitative traits (for example, excessively stretching certain countries) that may be associated with particular initial projection characteristics.

3.1 Assessment via Objective Scoring

Firstly, γ was computed for each of the twelve projections at the five different resolutions stated above - the results are tabulated in Table 2.

[3]`https://github.com/chrisbrunsdon/getcartr`

[4]only in the sense defined above.

Table 2: γ values for Cartograms

Resolution	512	768	1024	1280	1536
LCC	0.936	0.028	0.017	0.012	0.008
AEA	0.929	0.043	0.026	0.016	0.013
EqDC	0.706	0.741	0.017	0.011	0.007
Robin	0.946	0.035	0.740	0.014	0.009
Merc	0.826	0.874	0.018	0.012	0.009
Eck6	0.911	0.032	0.018	0.012	0.008
Moll	0.939	0.035	0.019	0.012	0.009
EQC	0.845	0.871	0.027	0.019	0.013
VanDG	0.955	0.790	0.019	0.012	0.008
Stere	0.911	0.024	0.015	0.010	0.007
LAEA	0.918	0.708	0.020	0.013	0.009
AEqD	0.939	0.027	0.017	0.010	0.008

Patterns are more clearly identified when using graphical approaches. In Figure 1 the value of γ is plotted on a log scale against pixel resolution for each projection. It can be seen generally that after some very poor performances for low resolutions, γ decreases exponentially - the reductions are linear when using the logarithmic scale. One interesting observation is that for the Robinson Pseudocylindrical projection, at one resolution (1024) performance is worse than the next lowest one (768).

In Figure 2 the same relationships are shown, shaded by development surface. Here, it seems there is no clear winner.

In Figure 3 the relationships are shown yet again, shaded by projection class. Here, it seems there is no clear winner - although the conformal projections seem to fair better, with none of them in the worst performing projections. Although in theory one would expect equal area projections to be the most appropriate, they do not seem to be outstanding.

3.2 Visual Assessment

In Figure 4 each of the original maps of Europe are shown beside the corresponding cartograms. The presentation exploits Edward Tufte's notion of 'small multiples' and possibly owes an apology to Andy Warhol's estate[5]. The colouring of the maps corresponds to the developable surface (background), and the class of projection (foreground). The colourings are given in Table 3.

In addition, the layout of the individual projections is given in Table 4.

Visually, some cartograms appear more 'squashed' along the north/south axis, although this doesn't seem to be associated with any particular developable surface, or projection type. In particular, the Albers Equal Area and Equidistant Cylindrical projections are very squashed - and the Mercator notably elongated. It is

[5] `http://www.oilpainting-repro.com/prod/marilyn-monroe-3x3-warhol-1925,217.html`

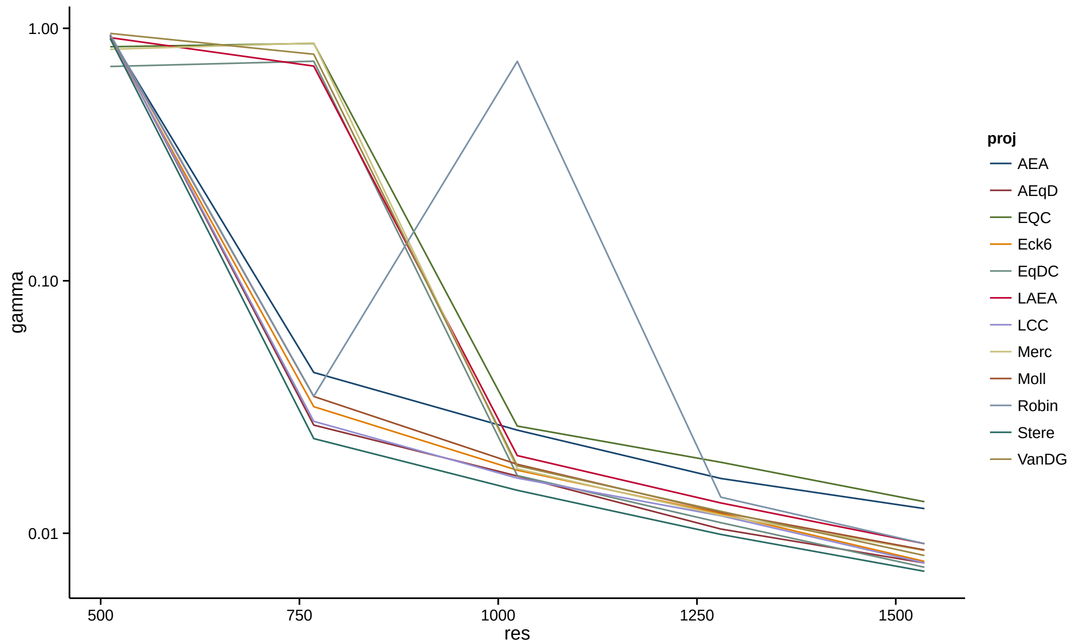

Figure 1: γ vs. Resolution Showing Projection

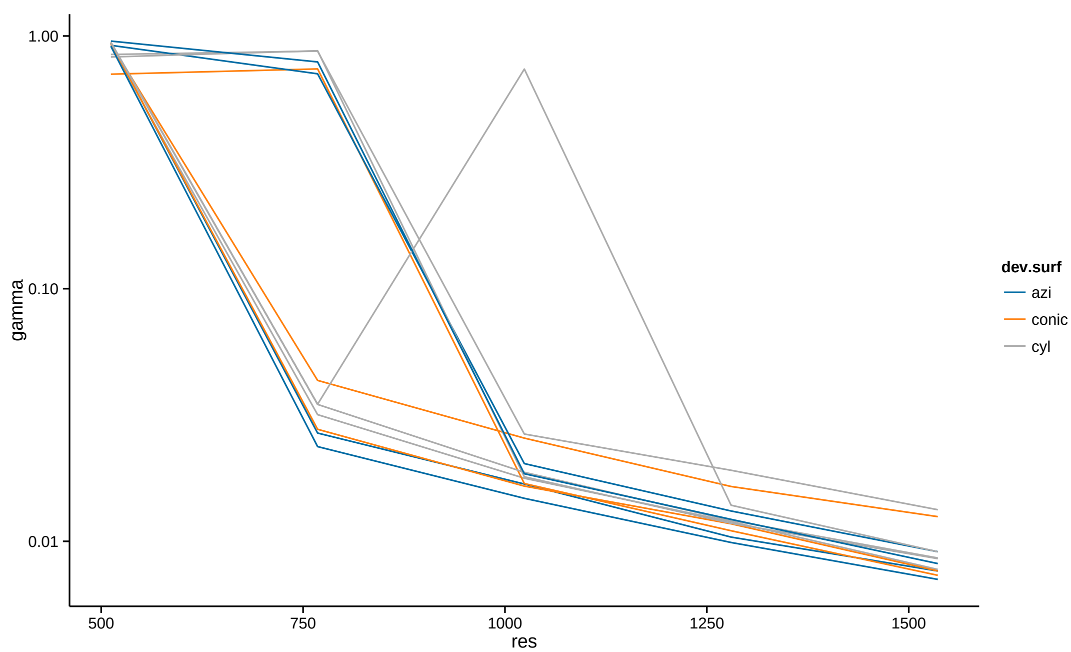

Figure 2: γ vs. Resolution Showing Development Surface

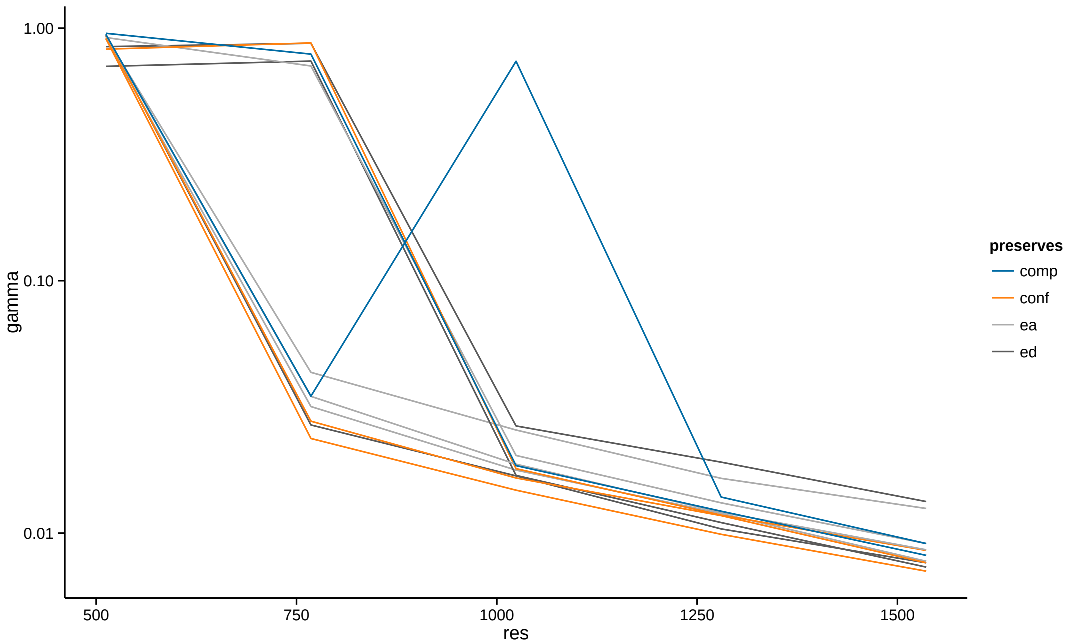

Figure 3: γ vs. Resolution Showing Projection Class

Table 3: Colouring for Figure 4

Characteristic	Colour Feature
Developable Surfaces	
Cone	Purple Background
Cylinder	Orange Background
Plane	Green Background
Projection Type	
Conformal	Blue Foreground
Equal Area	Green Foreground
Equidistant	Purple Foreground
Compromise	Red Foreground

Table 4: Projection Arrangements

	1	2	3	4
1	LCC	AEA	EqDC	Robin
2	Merc	Eck6	Moll	EQC
3	VanDG	Stere	LAEA	AEqD

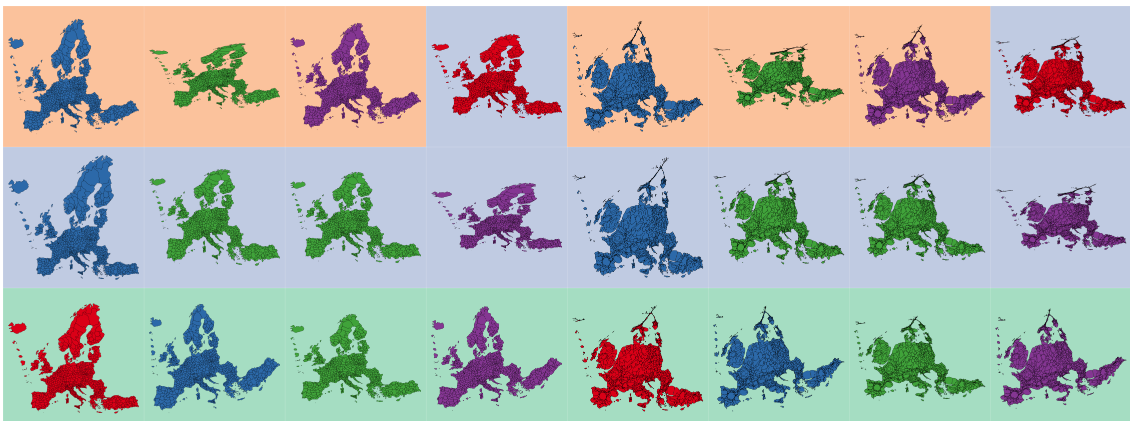

Figure 4: Original Projections (LHS) and Cartograms (RHS)

perhaps an unsurprising fact, but the cartograms seem to take on the squashed/elongated characteristics of the conventional projections used to produce them.

4 Conclusions

In relation to the questions asked earlier, it seems that the cartogram algorithm is relatively robust to the initial choice of map projection - despite expectations that equal area projections should outperform others. Of the equal area projections, Eckert Type VI performs best, but is outshone by several other projections. One possible explanation, at least at the Europe-wide scale, is that other factors, such as the inaccuracies due to rasterisation come into play, and their influence outways that of projection type. For work in the near future, world maps will be considered - and presented at the talk we propose here.

In terms of 'best practice', choice of projection seems less important than resolution of the raster approximation. Roughly speaking, once some very poor results for low resolutions have been encountered, deviation from a 'perfect cartogram' as measured by γ reduces exponentially with resolution. We would suggest checking γ as a way of deciding appropriate resolution, and choosing a squashed or elongated starting projection according to æsthetic preference, bearing in mind the aspect ratio of the resultant cartogram is influenced by this.

5 Acknowledgements

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6 Biography

Chris Brunsdon is Professor of Geocomputation and Director of Maynooth University National Centre for Geocomputation. His research interests involve spatial statistics, visualisation and geocomputation applied to a number of areas, including crime pattern analysis, and the analysis of environmental data.

Martin Charlton is a Senior Research Fellow and Deputy Director of Maynooth University National Centre for Geocomputation. His research interests involve geographical information systems, data analysis and geocomputation applied to a number of areas, including health data, and the analysis of housing data data.

Both Chris and Martin played rôles in the development of Geographically Weighted Regression, and are actively involved in developing and implementing tools for this and related techniques in the R programming language.

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