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Name - Deepak

IIT2021182

Q. Suppose that $X_1, X_2, \ldots X_n$ form a random Sample from distribution for which the p.d.f. $f(X/\theta)$ is follows.

$$f(x/\theta) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1 \\ 0 & \text{o.w.} \end{cases}$$

Also suppose that the value of θ is unknown $(\theta > 0)$. Find MLE of θ.

Sol^n Let $$L(\theta) = \prod_{i=1}^{n} \theta x_i^{\theta-1} = \theta^n \prod_{i=1}^{n} x_i^{\theta-1}$$

Taking log both side

$$\log(L(\theta)) = n \log \theta + (\theta-1) \ln\left(\prod_{i=1}^{n} x_i\right)$$

diff. both side w.r.t θ

$$\frac{1}{L(\theta)} L'(\theta) = \frac{n}{\theta} + \ln\left(\prod_{i=1}^{n} x_i\right)$$

For maximizing we know $L'(\theta) = 0$

$$\Rightarrow \frac{n}{\theta} + \ln\left(\prod_{i=1}^{n} x_i\right) = 0$$

$$\theta = -n / \ln\left(\prod_{i=1}^{n} x_i\right)$$

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$$\theta_0 = \frac{-n}{\sum_{i=1}^{n} \log x_i}$$

θ_0 is the critical point of L. For $\theta < \infty$ $\ln(L)$ is increasing & for $\theta > \theta_0$ $\ln(L)$ is decresing.

Thus θ_0 is global maximum & the MLE of θ

MLE of θ is $\boxed{\theta = \frac{-n}{\sum_{i=1}^{n} \ln(x_i)}}$