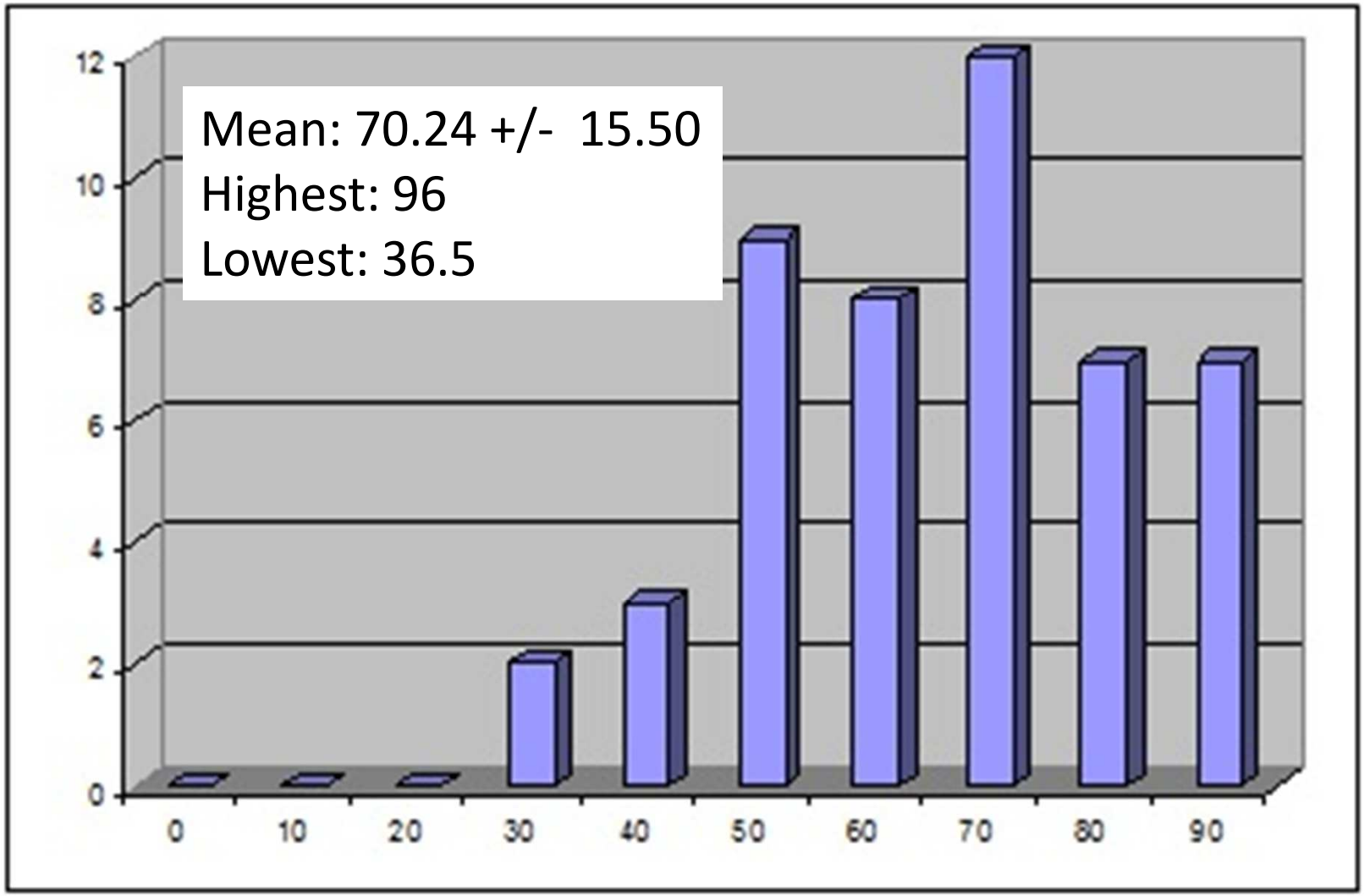
Lecture 12: Local Descriptors, SIFT & Single Object Recognition

Professor Fei-Fei Li

Stanford Vision Lab

Midterm overview



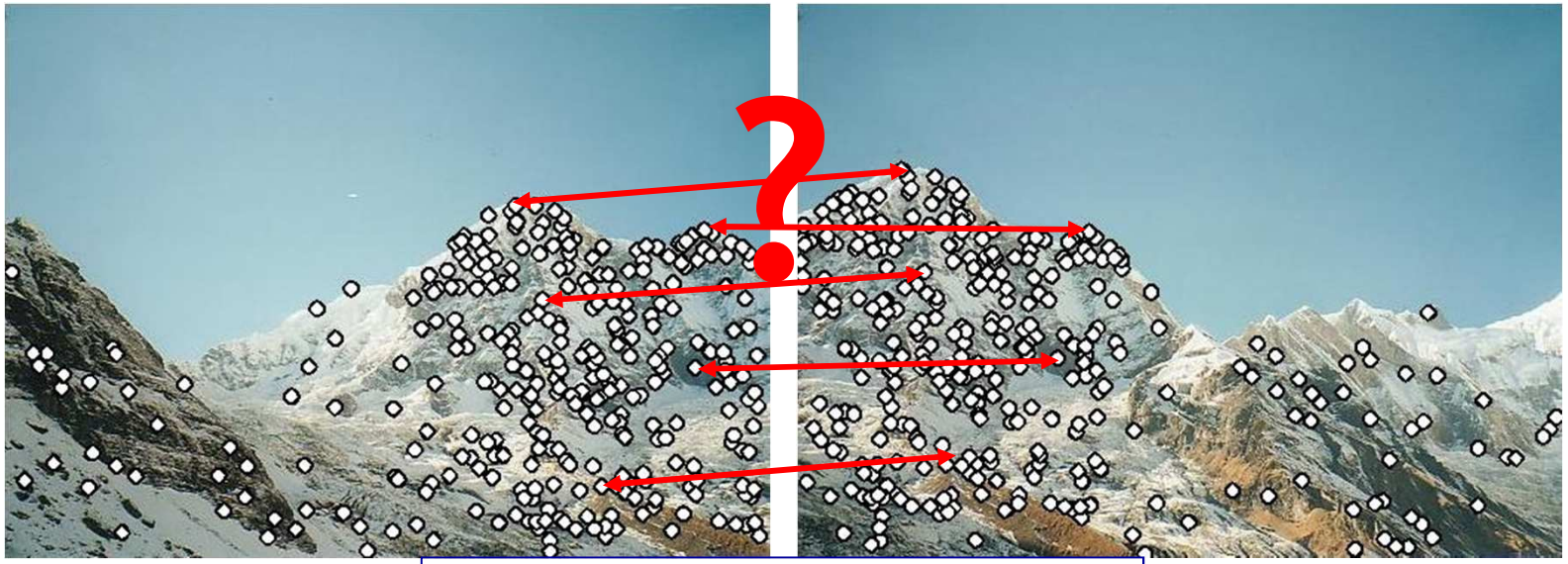
What we will learn today

- Local descriptors
  - SIFT (Problem Set 3 (Q2))
  - An assortment of other descriptors (Problem Set 3 (Q4))
- Recognition and matching with local features (Problem Set 3 (Q3))
  - Matching objects with local features: David Lowe
  - Panorama stitching: Brown & Lowe
  - Applications

Local Descriptors

- We know how to detect points Last lecture (#11)
- Next question:

How to *describe* them for matching?



This lecture (#12)

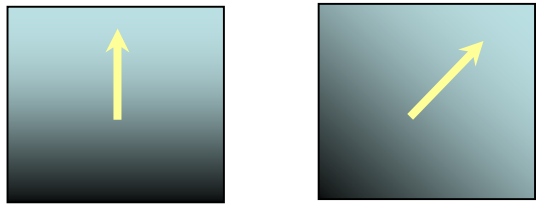
Point descriptor should be:

1. **Invariant**
2. **Distinctive**

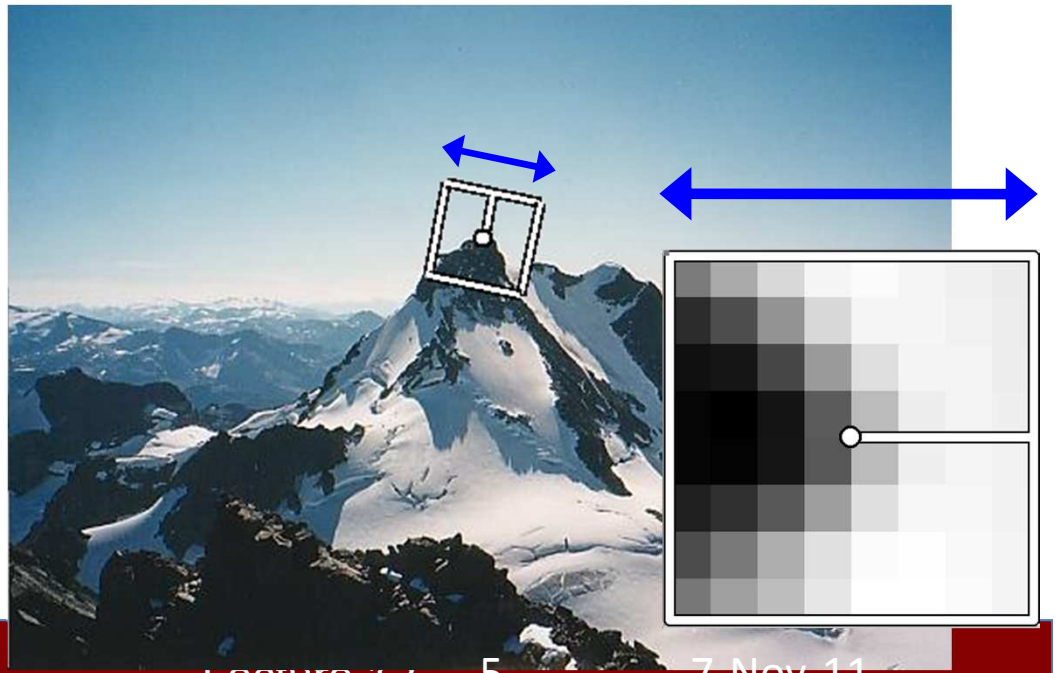
Slide credit: Kristen Grauman

Rotation Invariant Descriptors

- Find local orientation
  - Dominant direction of gradient for the image patch



- Rotate patch according to this angle
  - This puts the patches into a canonical orientation.

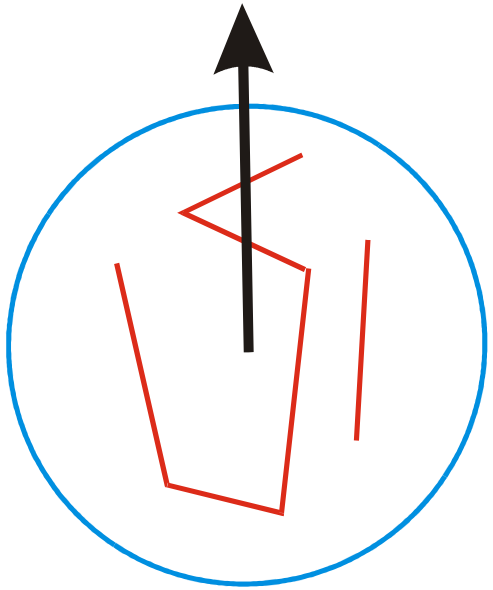


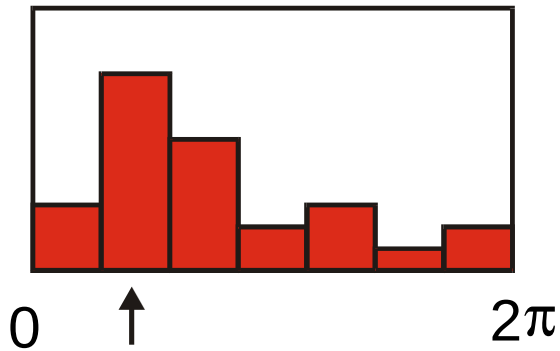
Slide credit: Svetlana Lazebnik, Matthew Brown

Orientation Normalization: Computation

[Lowe, SIFT, 1999]

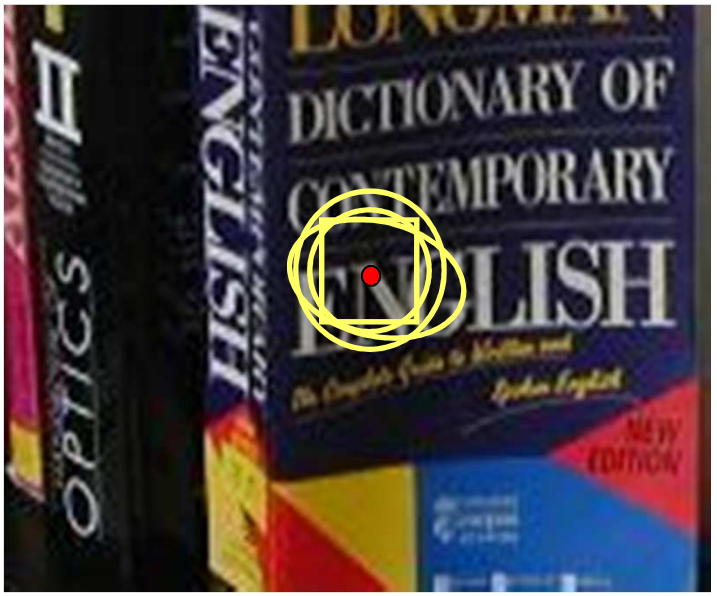
- Compute orientation histogram
- Select dominant orientation
- Normalize: rotate to fixed orientation





Slide adapted from David Lowe

The Need for Invariance

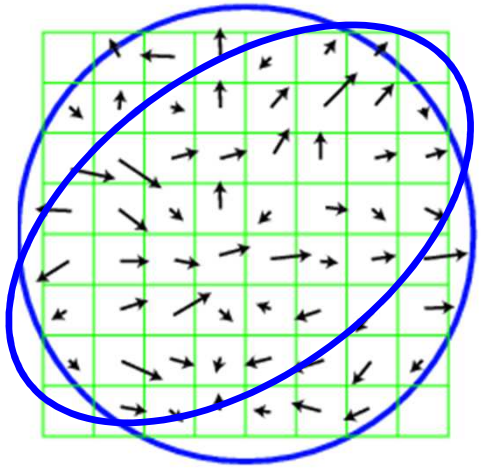


- Up to now, we had invariance to
  - Translation
  - Scale
  - Rotation
- Not sufficient to match regions under viewpoint changes
  - For this, we need also affine adaptation

Slide credit: Tinne Tuytelaars

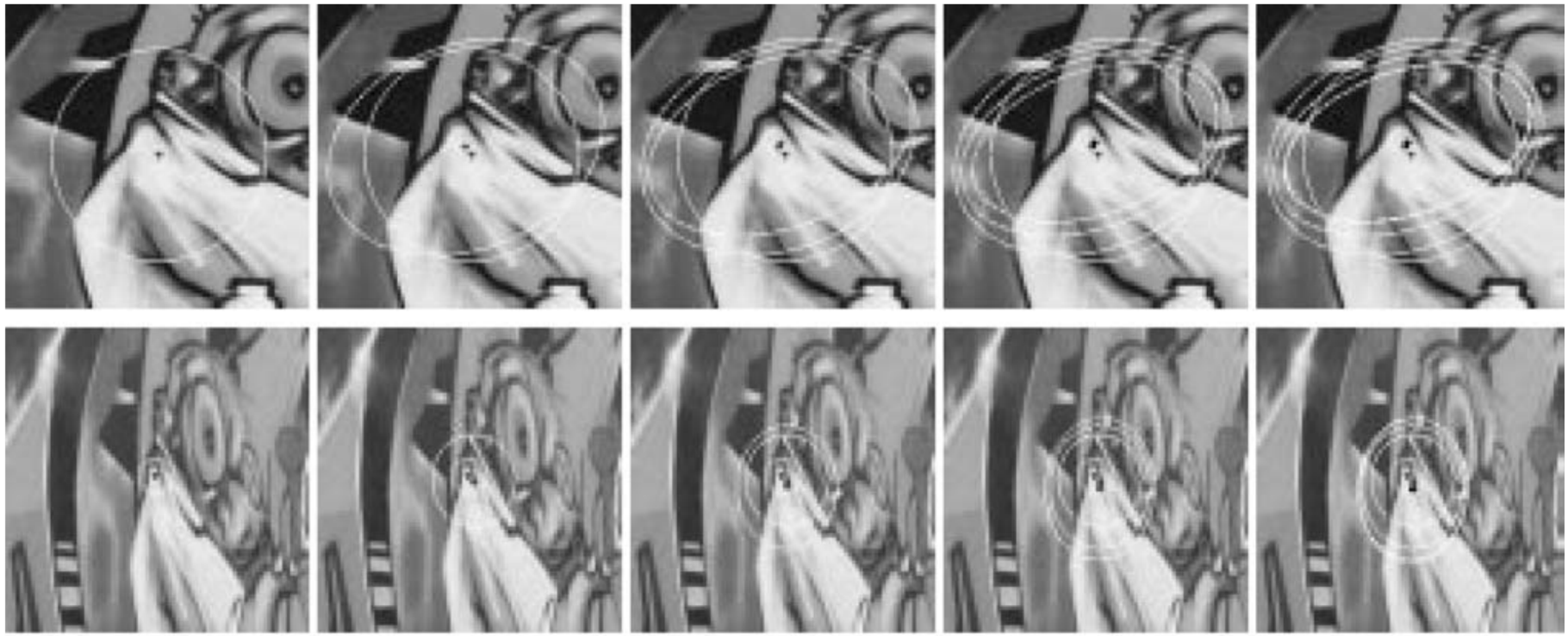
Affine Adaptation

- Problem:
  - Determine the characteristic shape of the region.
  - Assumption: shape can be described by “local affine frame”.
- Solution: iterative approach
  - Use a circular window to compute second moment matrix.
  - Compute eigenvectors to adapt the circle to an ellipse.
  - Recompute second moment matrix using new window and iterate…



Slide adapted from Svetlana Lazebnik

Iterative Affine Adaptation



1. Detect keypoints, e.g. multi-scale Harris
2. Automatically select the scales
3. Adapt affine shape based on second order moment matrix
4. Refine point location

K. Mikolajczyk and C. Schmid, Scale and affine invariant interest point detectors, IJCV 60(1):63-86, 2004.

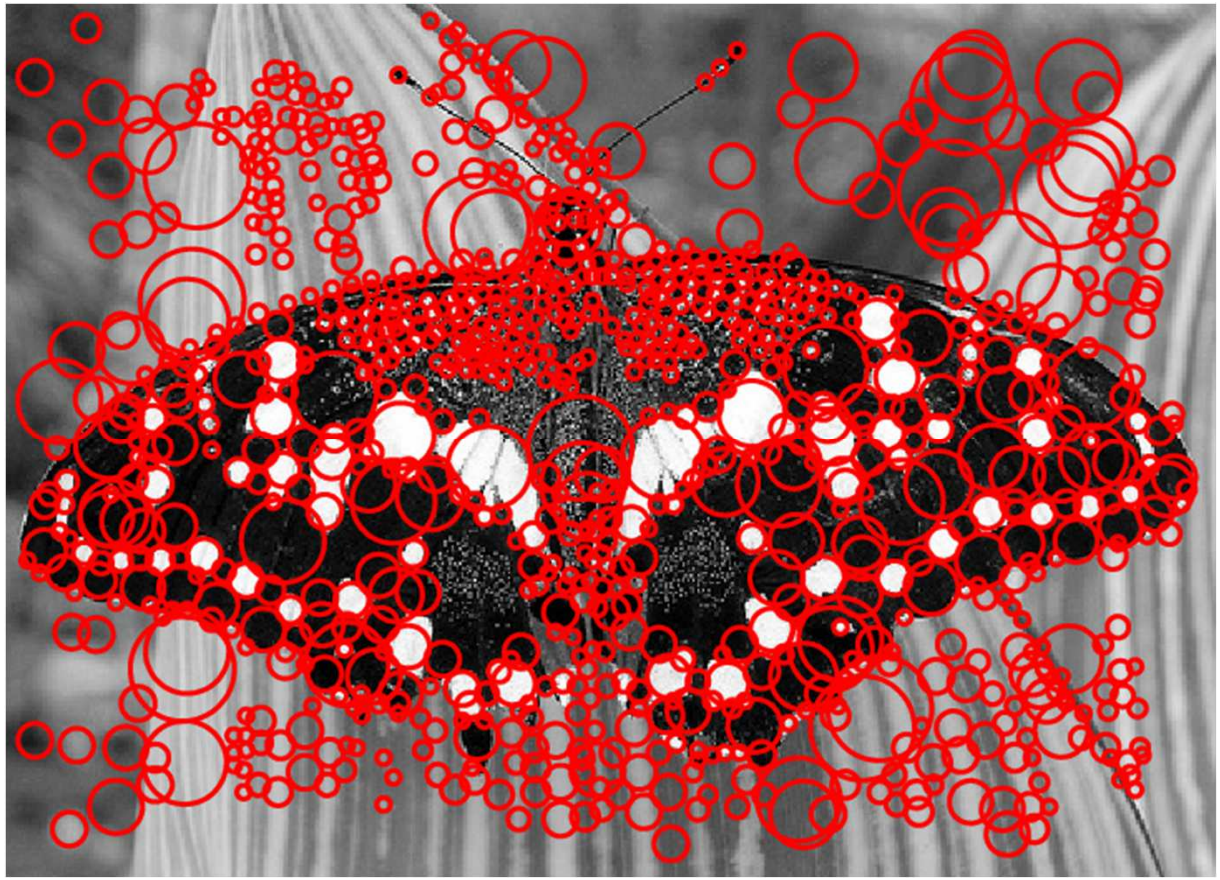
Slide credit: Tinne Tuytelaars

Affine Normalization/Deskewing

- Steps
  - Rotate the ellipse's main axis to horizontal
  - Scale the x axis, such that it forms a circle

Slide credit: Tinne Tuytelaars

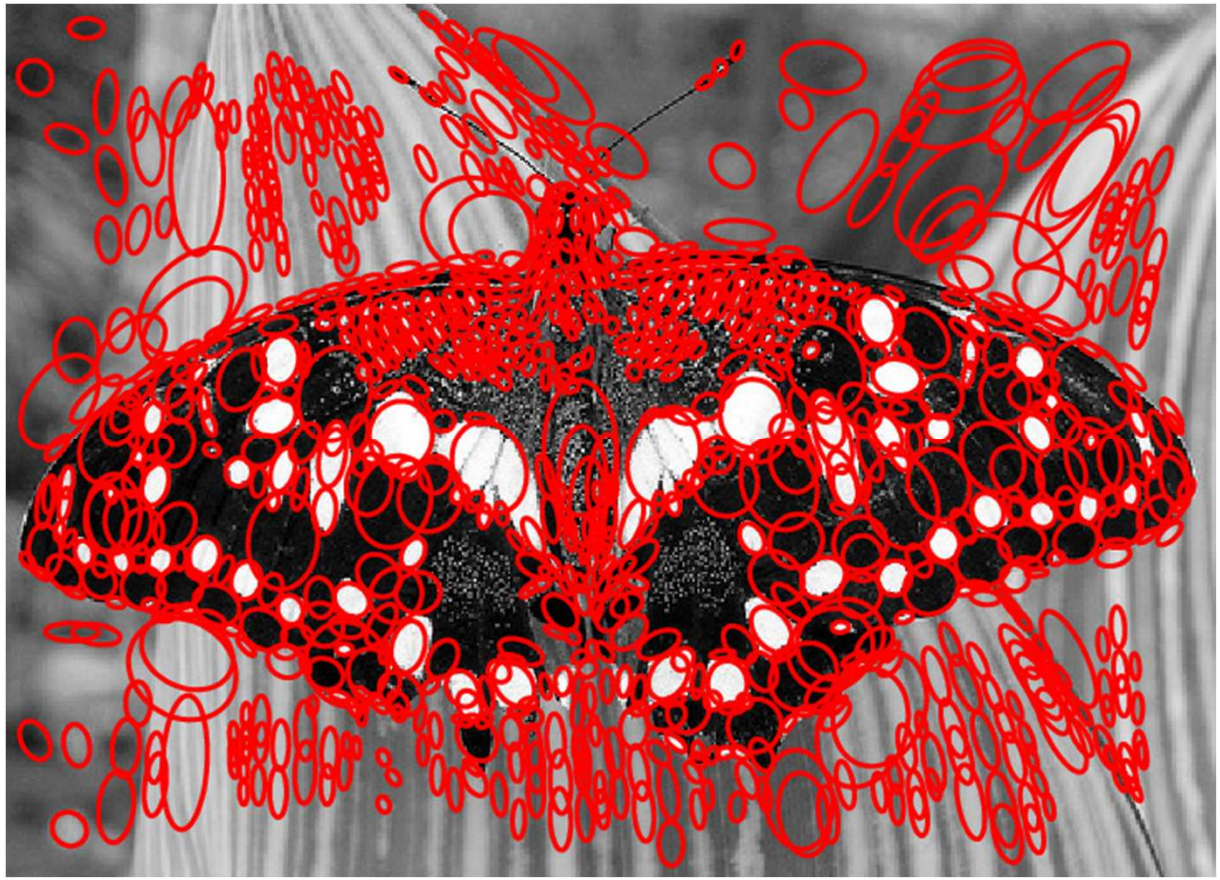
Affine Adaptation Example



Scale-invariant regions (blobs)

Slide credit: Svetlana Lazebnik

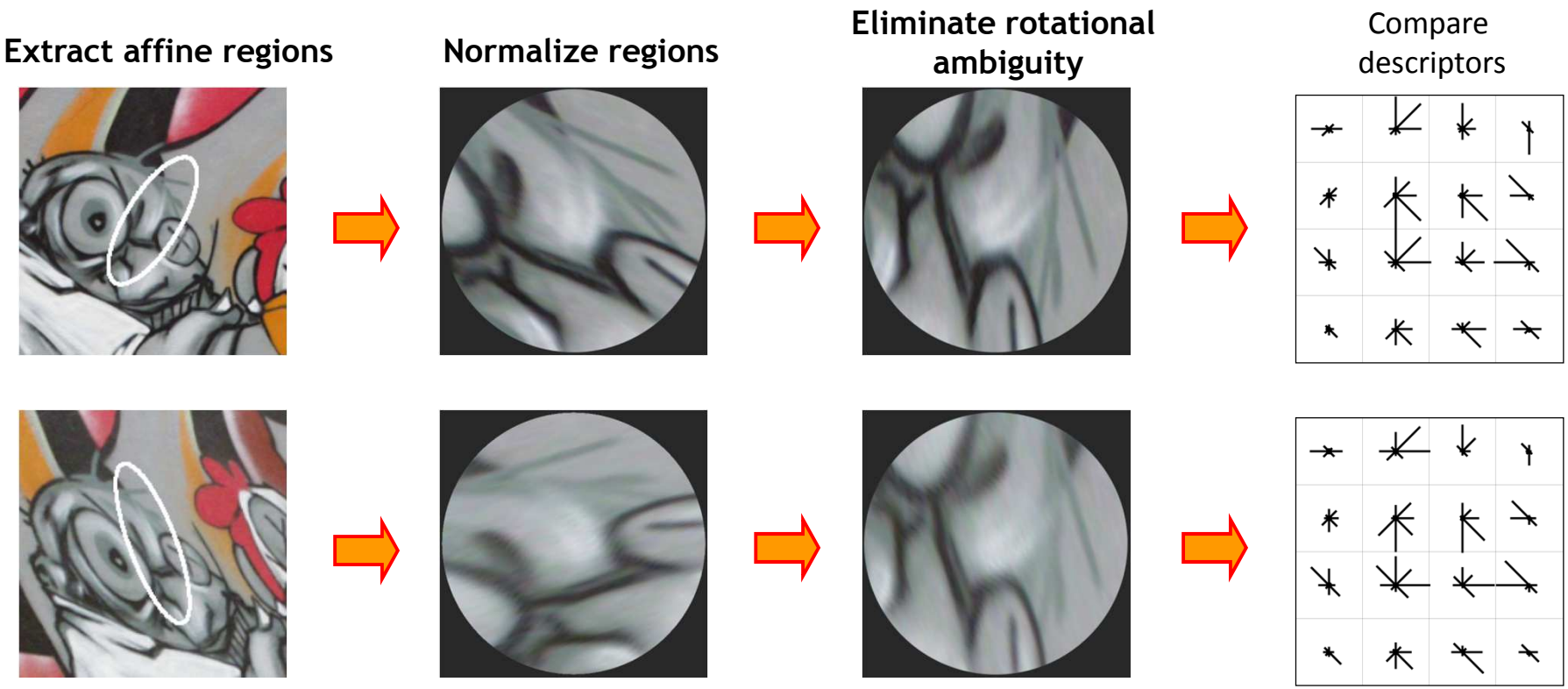
Affine Adaptation Example



Affine-adapted blobs

Slide credit: Svetlana Lazebnik

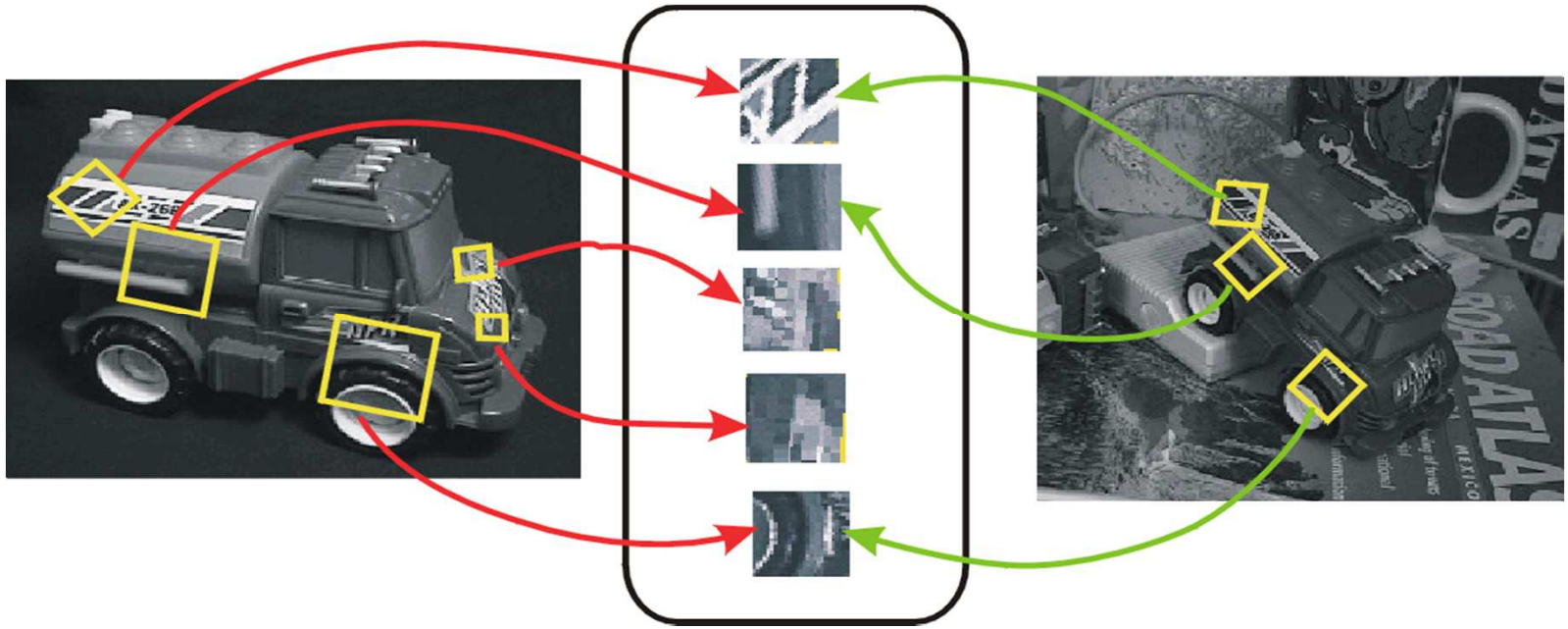
Summary: Affine-Inv. Feature Extraction



Slide credit: Svetlana Lazebnik

Invariance vs. Covariance

- Invariance:
  - features(transform(image)) = features(image)
- Covariance:
  - features(transform(image)) = transform(features(image))



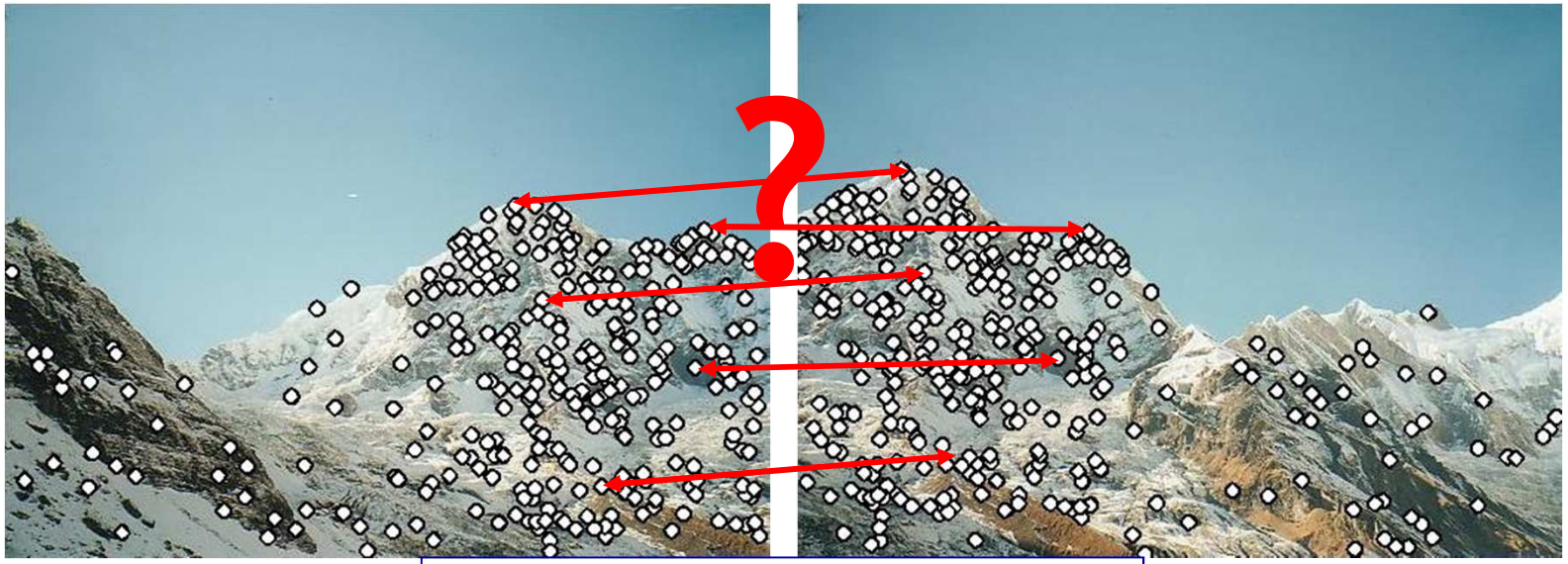
Covariant detection ⇒ invariant description

Slide credit: Svetlana Lazebnik, David Lowe

Local Descriptors

- We know how to detect points
- Next question:

How to *describe* them for matching?



Point descriptor should be:

1. **Invariant**
2. **Distinctive**

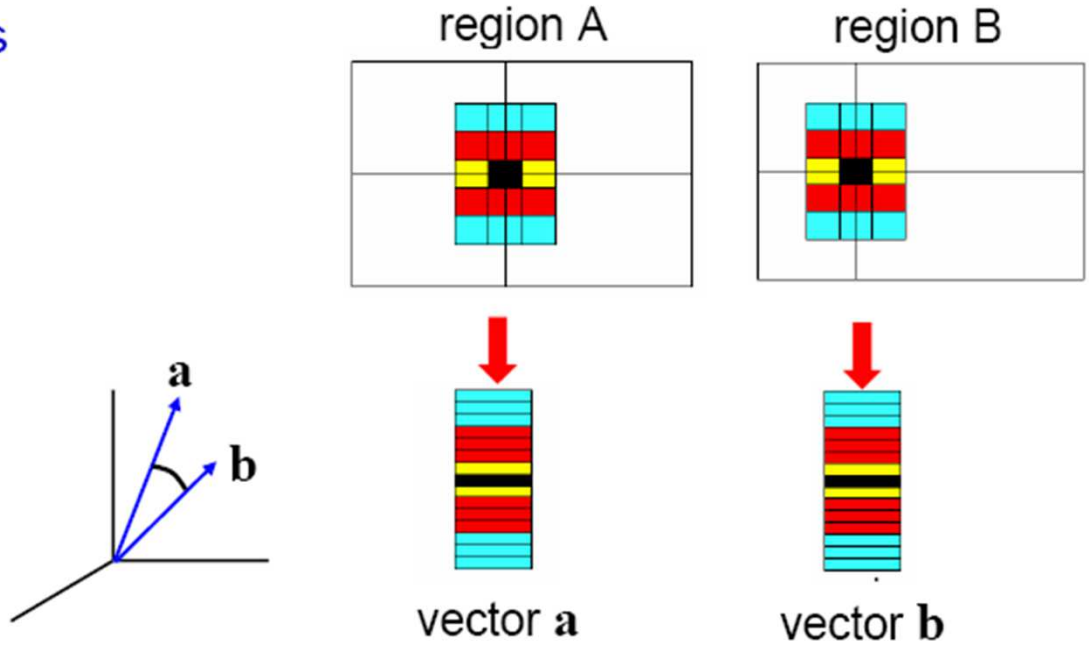
Slide credit: Kristen Grauman

Local Descriptors

- Simplest descriptor: list of intensities within a patch.
- What is this going to be invariant to?

Write regions as vectors

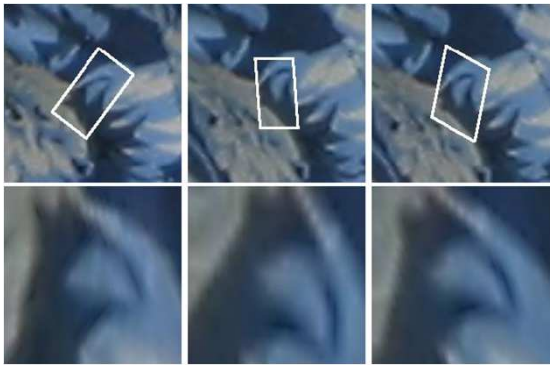
$$A \rightarrow \mathbf{a}, \quad B \rightarrow \mathbf{b}$$



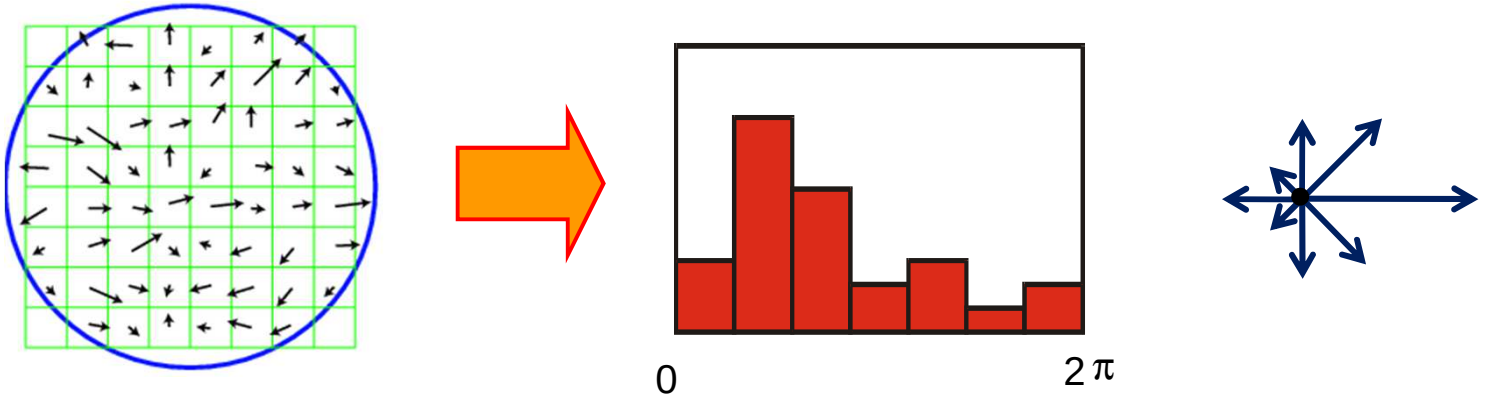
Slide credit: Kristen Grauman

Feature Descriptors

- Disadvantage of patches as descriptors:
  - Small shifts can affect matching score a lot



- Solution: histograms

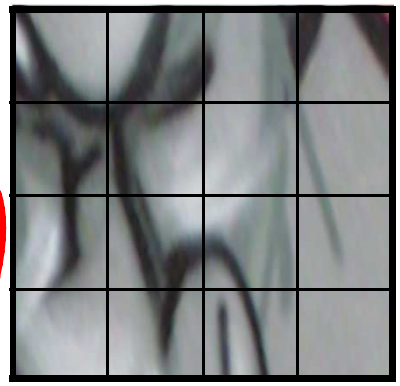


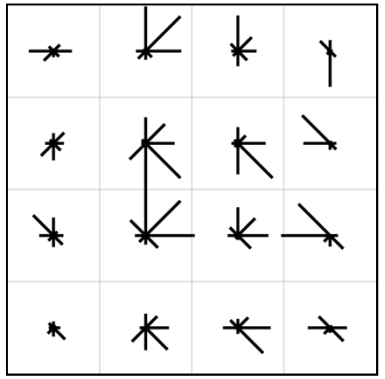
Slide credit: Svetlana Lazebnik

Feature Descriptors: SIFT

- Scale Invariant Feature Transform
- Descriptor computation:
  - Divide patch into 4x4 sub-patches: 16 cells
  - Compute histogram of gradient orientations (8 reference angles) for all pixels inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions

(optimal) subtract from dominant orientation (normalize orient.)



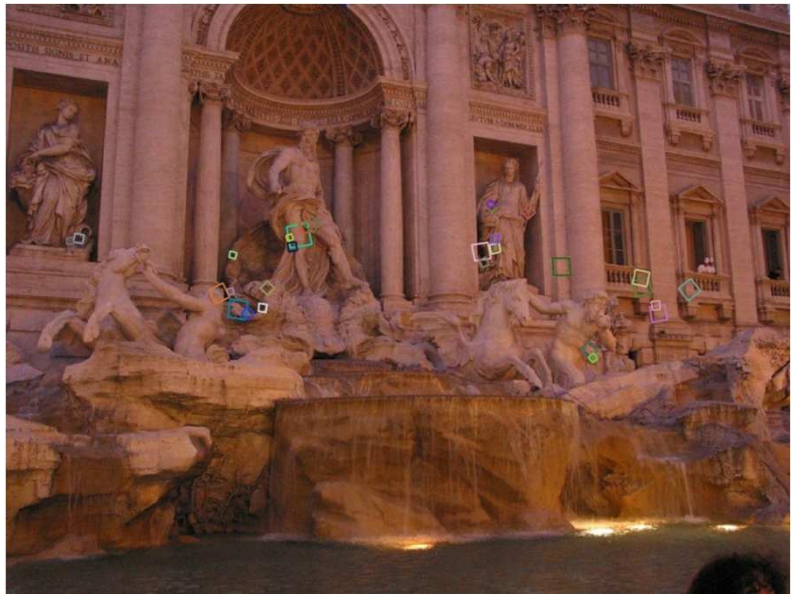


David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Slide credit: Svetlana Lazebnik

Overview: SIFT

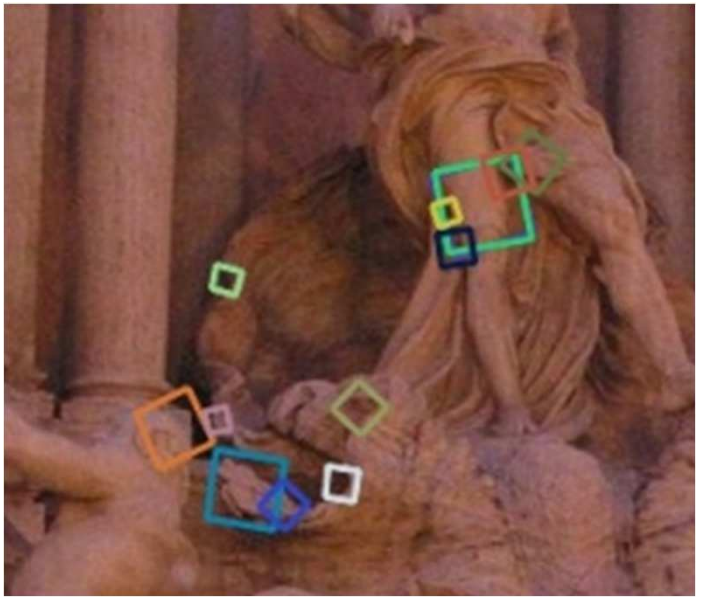
- Extraordinarily robust matching technique
  - Can handle changes in viewpoint up to ~60 deg. out-of-plane rotation
  - Can handle significant changes in illumination
    - Sometimes even day vs. night (below)
  - Fast and efficient—can run in real time
  - Lots of code available
    - http://people.csail.mit.edu/albert/ladypack/wiki/index.php/Known_implementations_of_SIFT



Slide credit: Steve Seitz

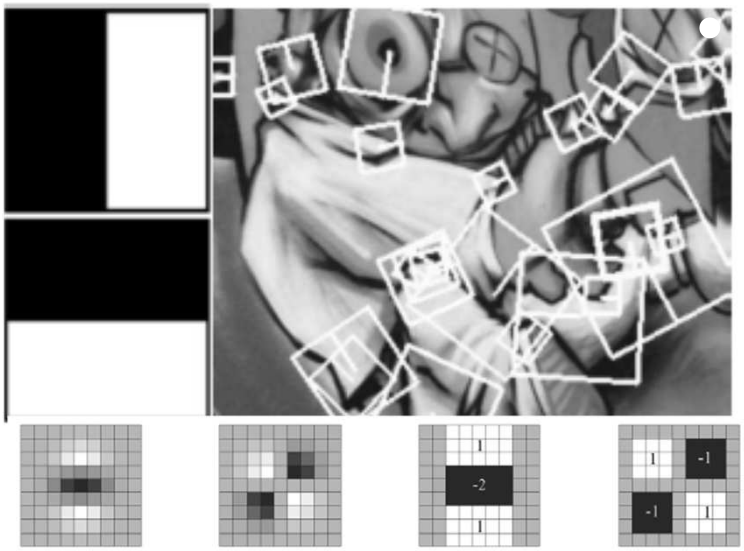
Working with SIFT Descriptors

- One image yields:
  - n 128-dimensional descriptors: each one is a histogram of the gradient orientations within a patch
    - [n x 128 matrix]
  - n scale parameters specifying the size of each patch
    - [n x 1 vector]
  - n orientation parameters specifying the angle of the patch
    - [n x 1 vector]
  - n 2D points giving positions of the patches
    - [n x 2 matrix]



Slide credit: Steve Seitz

Local Descriptors: SURF



Fast approximation of SIFT idea

- Efficient computation by 2D box filters & integral images
  ⇒ 6 times faster than SIFT
- Equivalent quality for object identification
- http://www.vision.ee.ethz.ch/~surf

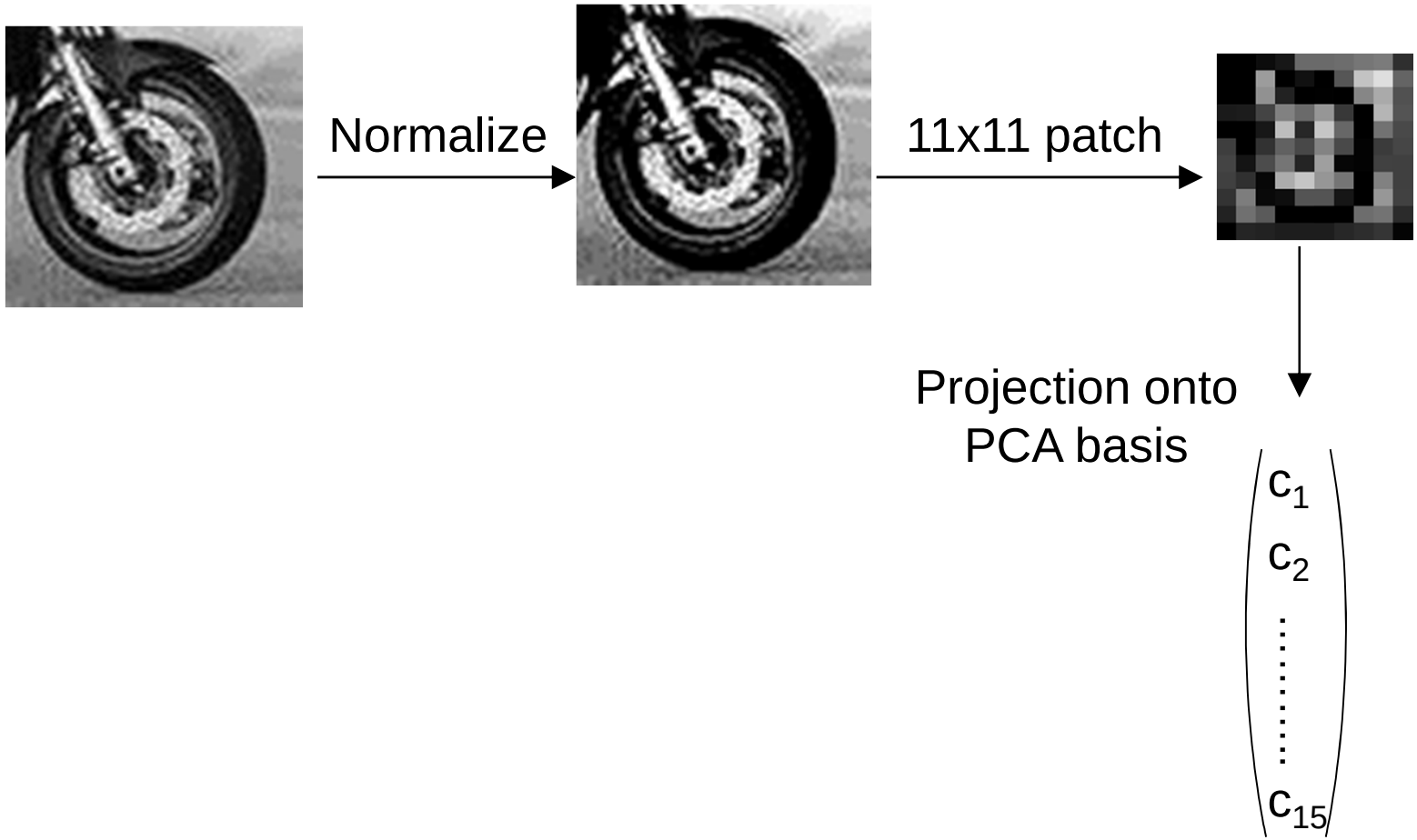
GPU implementation available

- Feature extraction @ 100Hz (detector + descriptor, 640×480 img)
- http://homes.esat.kuleuven.be/~ncorneli/gpusurf/

[Bay, ECCV'06], [Cornelis, CVGPU'08]

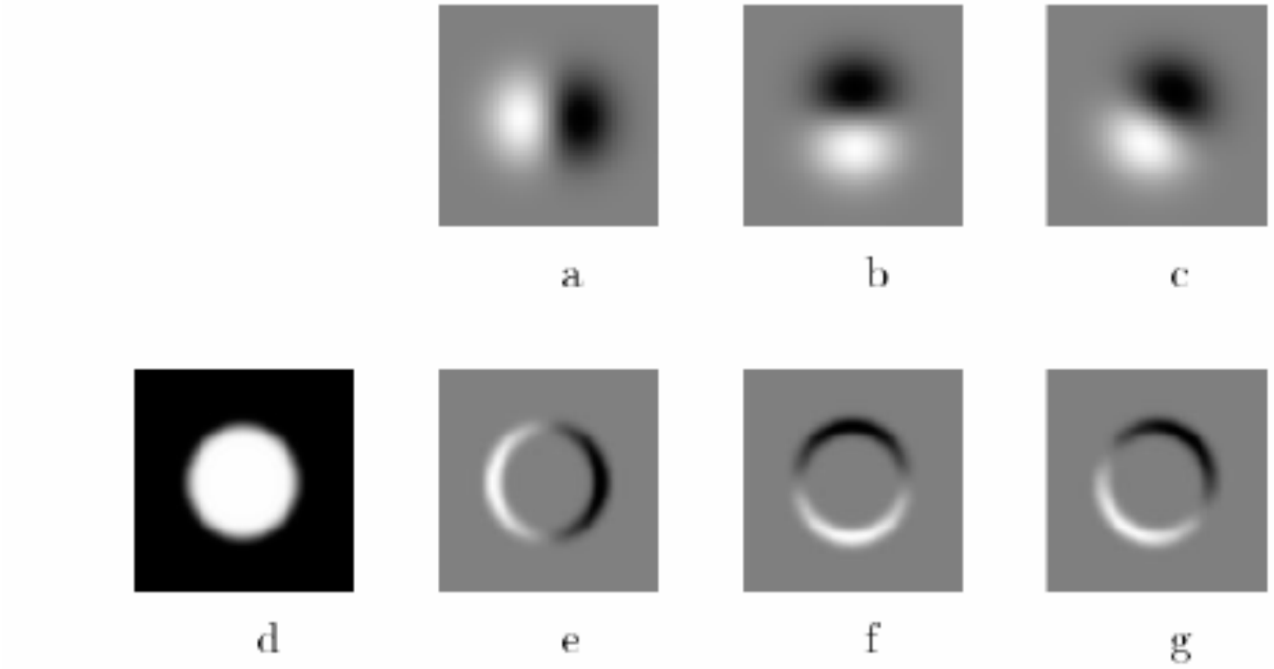
http://www.vision.ee.ethz.ch/~surf

Other local descriptors: Gray-scale intensity



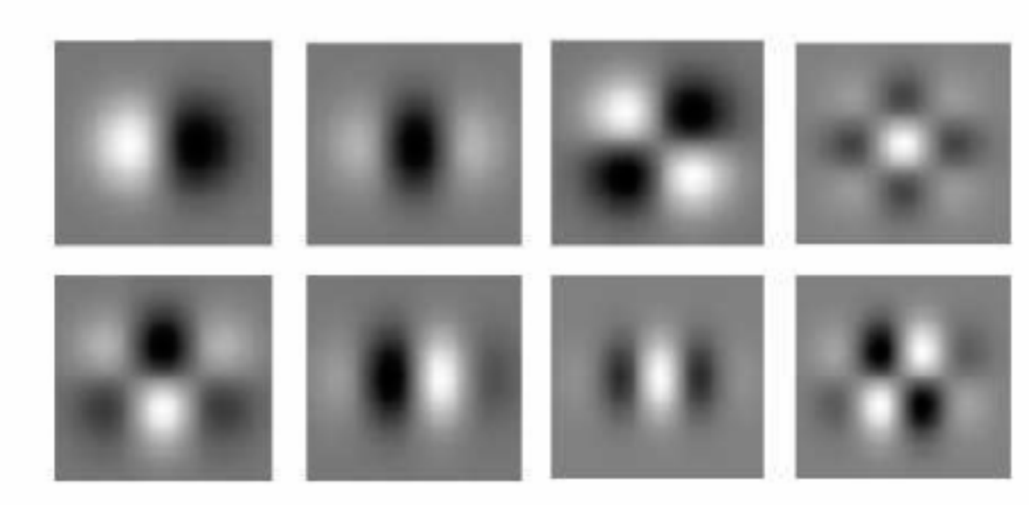
Other local descriptors: Steerable filters

$$
\begin{aligned}
R_1^{0^\circ} &= G_1^0 * I \\
R_1^{90^\circ} &= G_1^{90} * I \\
\text{then} & \\
R_1^{\theta} &= \cos(\theta) R_1^{0^\circ} + \sin(\theta) R_1^{90^\circ}
\end{aligned}
$$



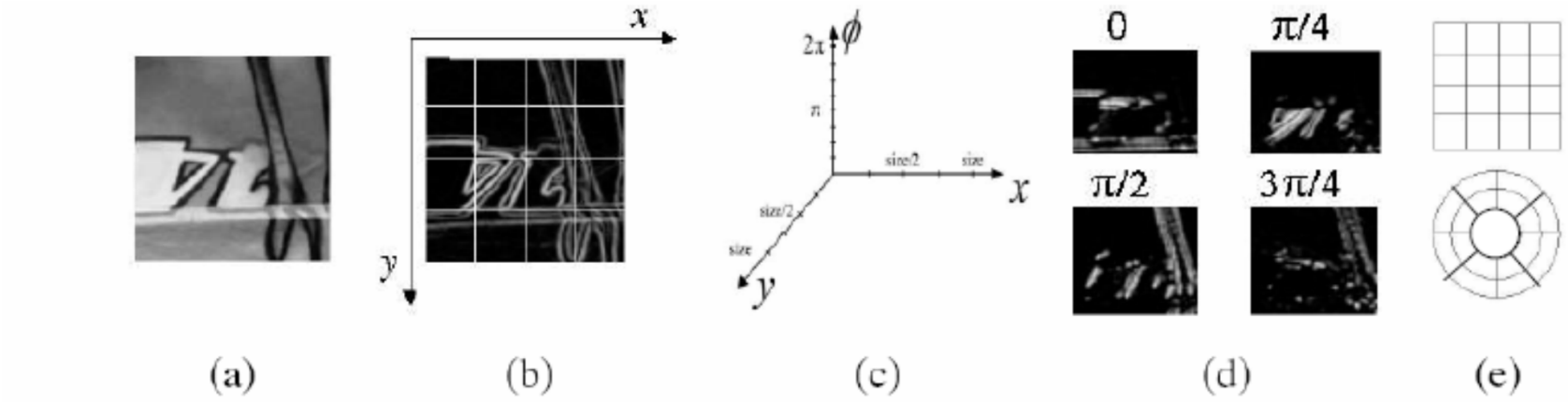
Other local descriptors: Gray-scale intensity

Gaussian derivatives up to 4^{th} order. The remaining derivatives can be computed by rotation of 90 degrees.



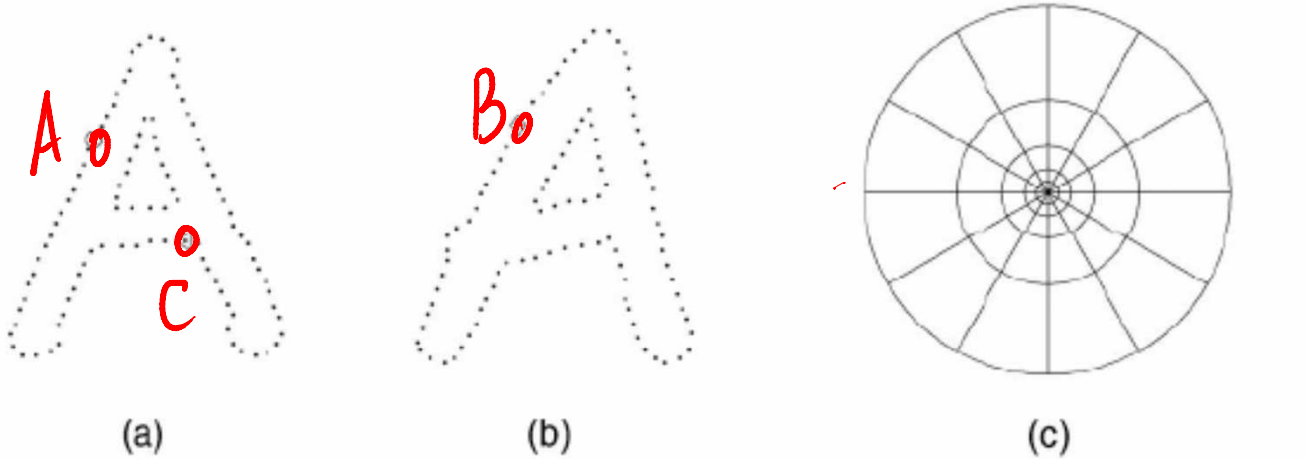
Other local descriptors: GLOH

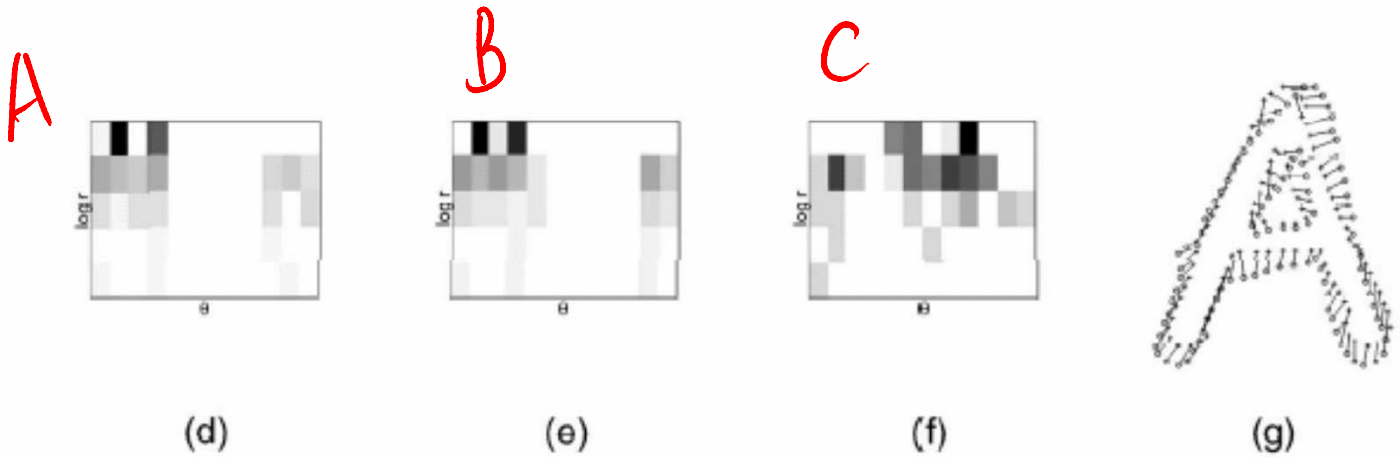
- GLOH: Gradient Location and Orientation Histogram(Miko04)
  - Very similar to SIFT.
  - Log-polar location grid:
    - 3 bins in radial direction;
    - 8 bins in angular direction
    - Gradient orientation quantized in 16 bins.
  - Total: (2x8+1)*16=272 bins→PCA dimension reduction.



Other local descriptors: Shape context

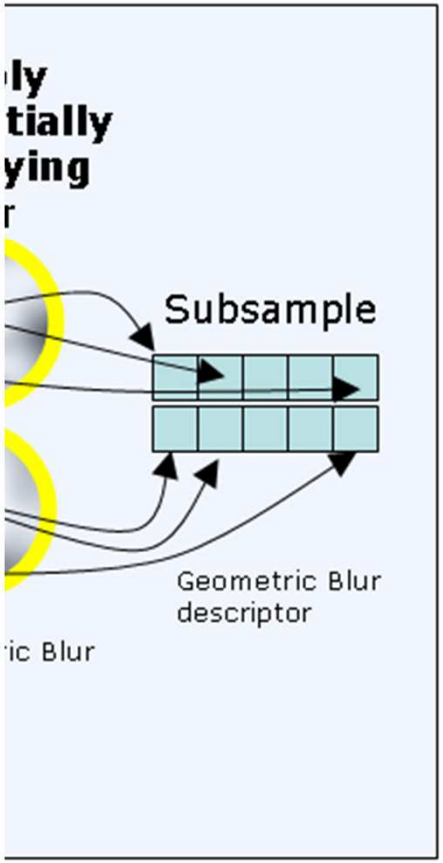
[Belongie et al. 2002]



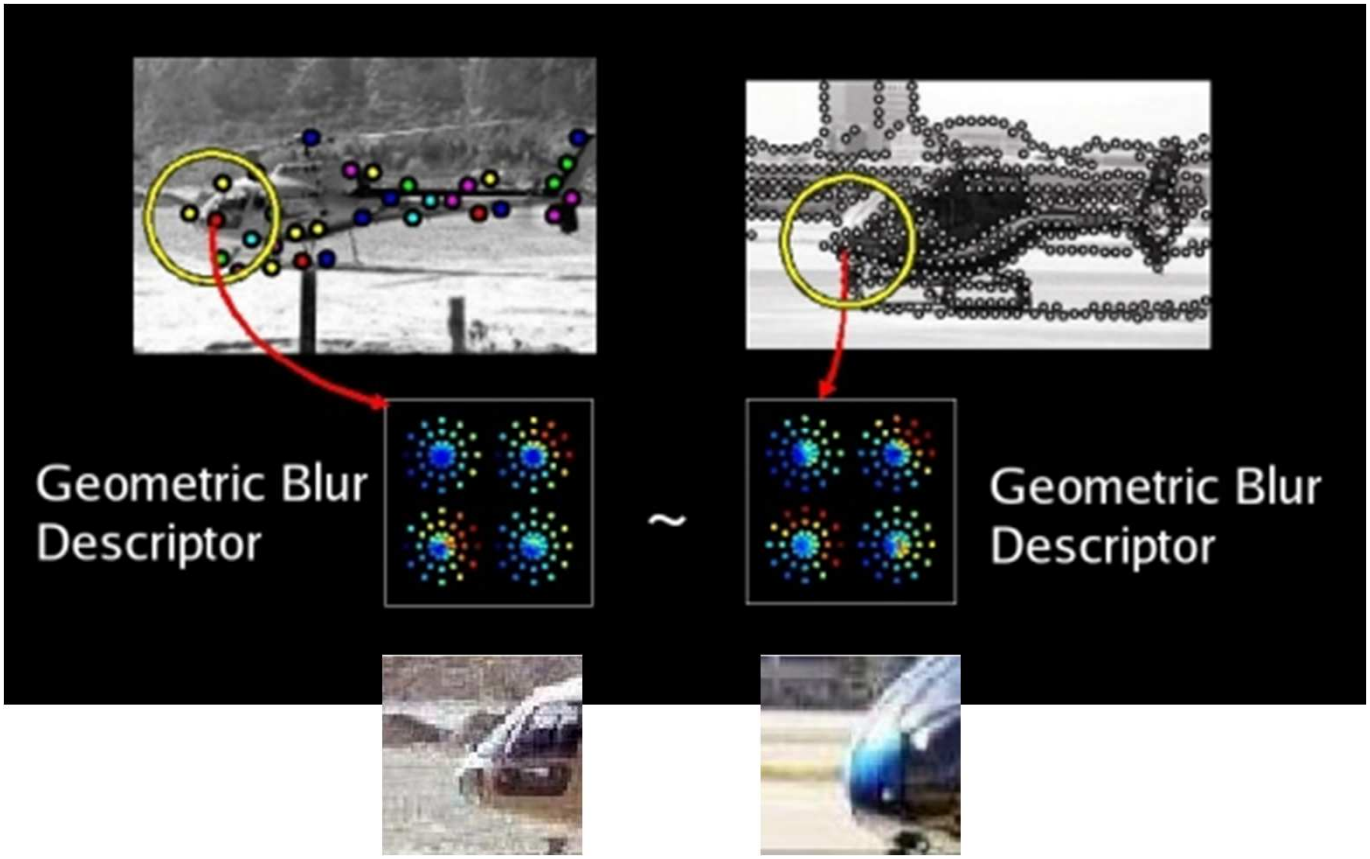


Other local descriptors: Geometric Blur [Berg et al. 2001]

In practice compute discrete blur levels for whole image and sample as needed for each feature location.



Other local descriptors: Geometric Blur



Affine Covariant Features

Collaborative work between: the Visual Geometry Group, Katholieke Universiteit Leuven, Inria Rhone-Alpes and the Center for Machine Perception.

Affine Covariant Region Detectors

Input image

Region detector

Detector output

format:

1.0

m

$u_1\ v_1\ a_1\ b_1\ c_1$

:

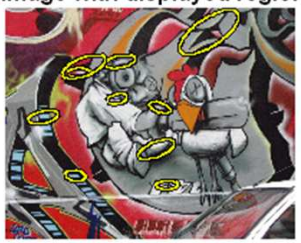
:

$u_m\ v_m\ a_m\ b_m\ c_m$

output example: img1.haraff

display_features.m

Image with displayed regions



Parameters defining an affine region

u,v,a,b,c in $a(x-u)(x-u)+2b(x-u)(y-v)+c(y-v)(y-v)=1$

with (0,0) at image top left corner

Code

- provided by the authors, see publications for details and links to authors web sites.

Linux binaries	Example of use	Displaying r
Harris-Affine & Hessian-Affine	prompt>./h_affine.ln -haraff -i img1.ppm -o img1.haraff -thres 1000	matlab>> d
	prompt>./h_affine.ln -hesaff -i img1.ppm -o img1.hesaff -thres 500	matlab>> d
MSER - Maximaly stable extremal regions (also Windows)	prompt>./mser.ln -t 2 -es 2 -i img1.ppm -o img1.mser	matlab>> d
IBR - Intensity extrema based detector	prompt>./ibr.ln img1.ppm img1.ibr -scalefactor 1.0	matlab>> d
EBR - Edge based detector	prompt> ./ebr.ln img1.ppm img1.ebr	matlab>> d
Salient region detector	prompt>./salient.ln img1.ppm img1.sal	matlab>> d

http://www.robots.ox.ac.uk/~vgg/research/affine/detectors.html#binaries

Value of Local Features

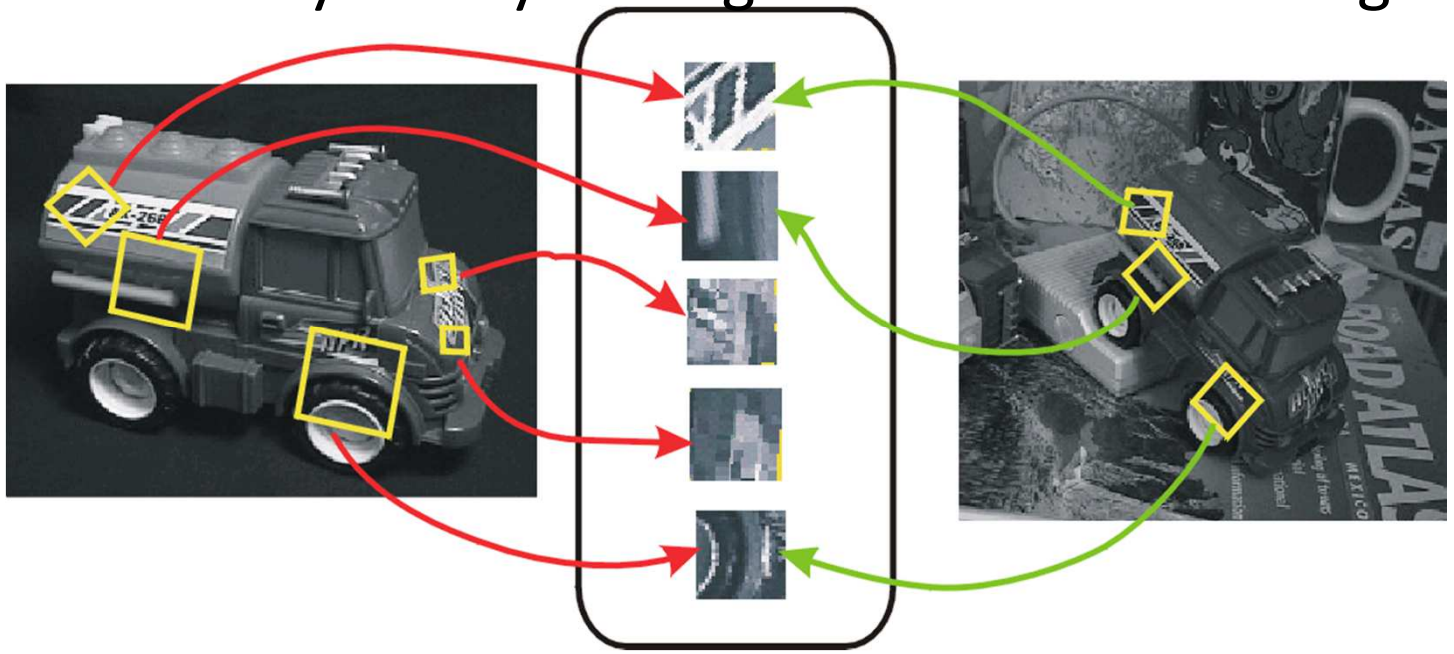
- Advantages
  - Critical to find distinctive and repeatable local regions for multi-view matching.
  - Complexity reduction via selection of distinctive points.
  - Describe images, objects, parts without requiring segmentation; robustness to clutter & occlusion.
  - Robustness: similar descriptors in spite of moderate view changes, noise, blur, etc.

What we will learn today

- Local descriptors
  - SIFT
  - An assortment of other descriptors
- Recognition and matching with local features (Problem Set 3 (Q3))
  - Matching objects with local features: David Lowe
  - Panorama stitching: Brown & Lowe
  - Applications

Recognition with Local Features

- Image content is transformed into local features that are invariant to translation, rotation, and scale
- Goal: Verify if they belong to a consistent configuration



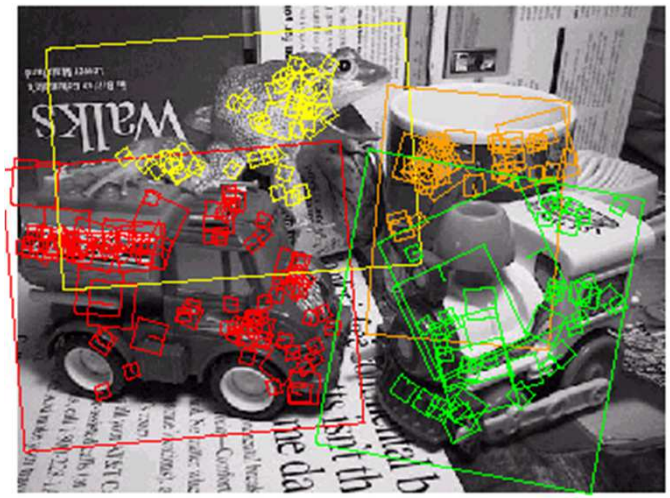
Local Features, e.g. SIFT

Slide credit: David Lowe

Finding Consistent Configurations

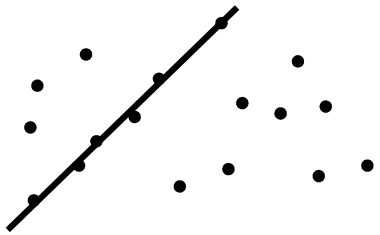
- Global spatial models
  - Generalized Hough Transform [Lowe99]
  - RANSAC [Obdrzalek02, Chum05, Nister06]
  - Basic assumption: object is planar

- Assumption is often justified in practice
  - Valid for many structures on buildings
  - Sufficient for small viewpoint variations on 3D objects

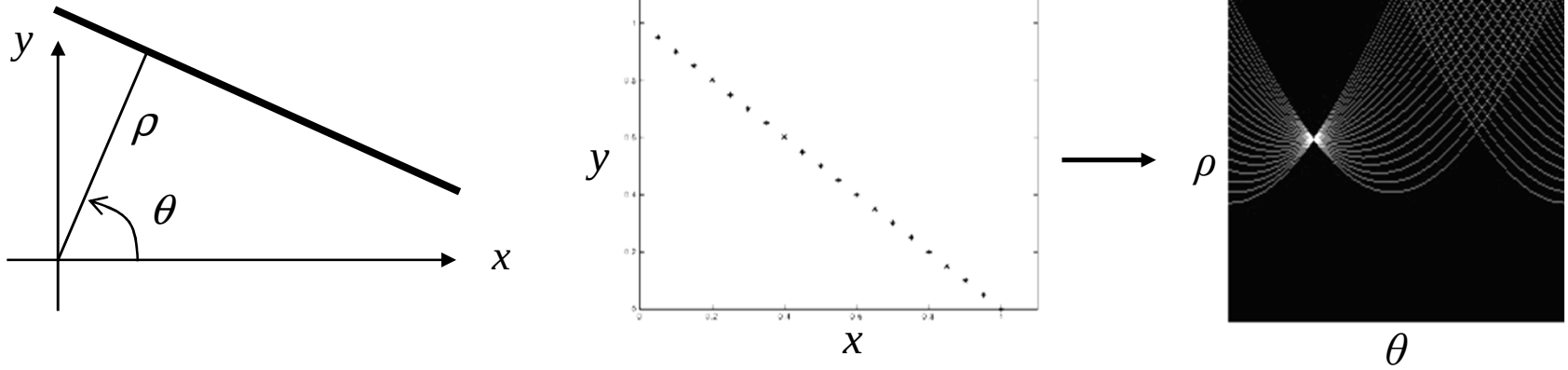


Reminder: Hough Transform

- Origin: Detection of straight lines in clutter
  - Basic idea: each candidate point votes for all lines that it is consistent with.
  - Votes are accumulated in quantized array
  - Local maxima correspond to candidate lines

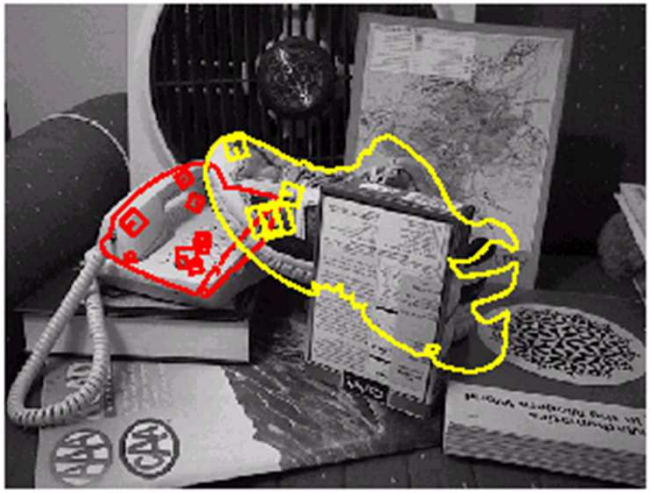


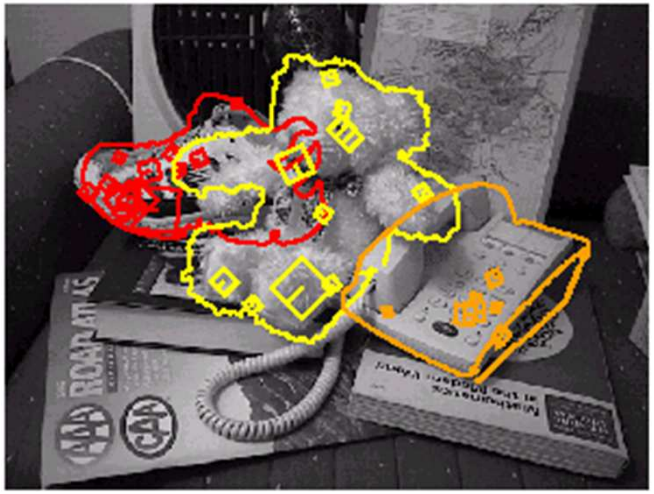
- Representation of a line
  - Usual form $y = a\,x + b$ has a singularity around 90º.
  - Better parameterization: $x\cos(\theta) + y\sin(\theta) = \rho$



3D Object Recognition

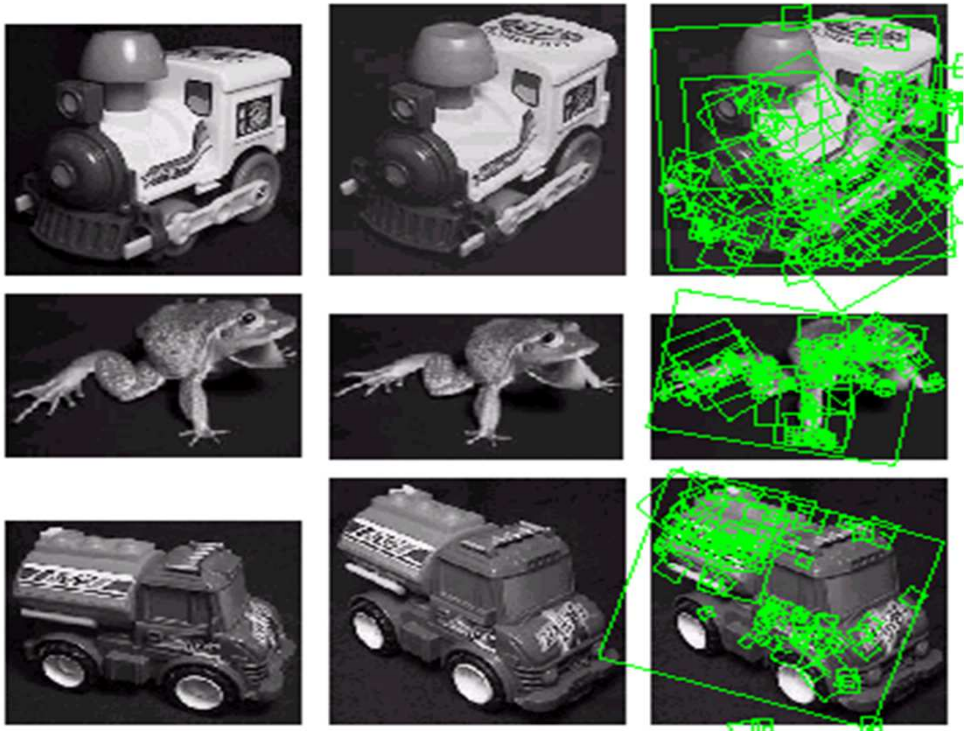
- Gen. Hough T. for Recognition
  - Typically only 3 feature matches needed for recognition
  - Extra matches provide robustness
  - Affine model can be used for planar objects





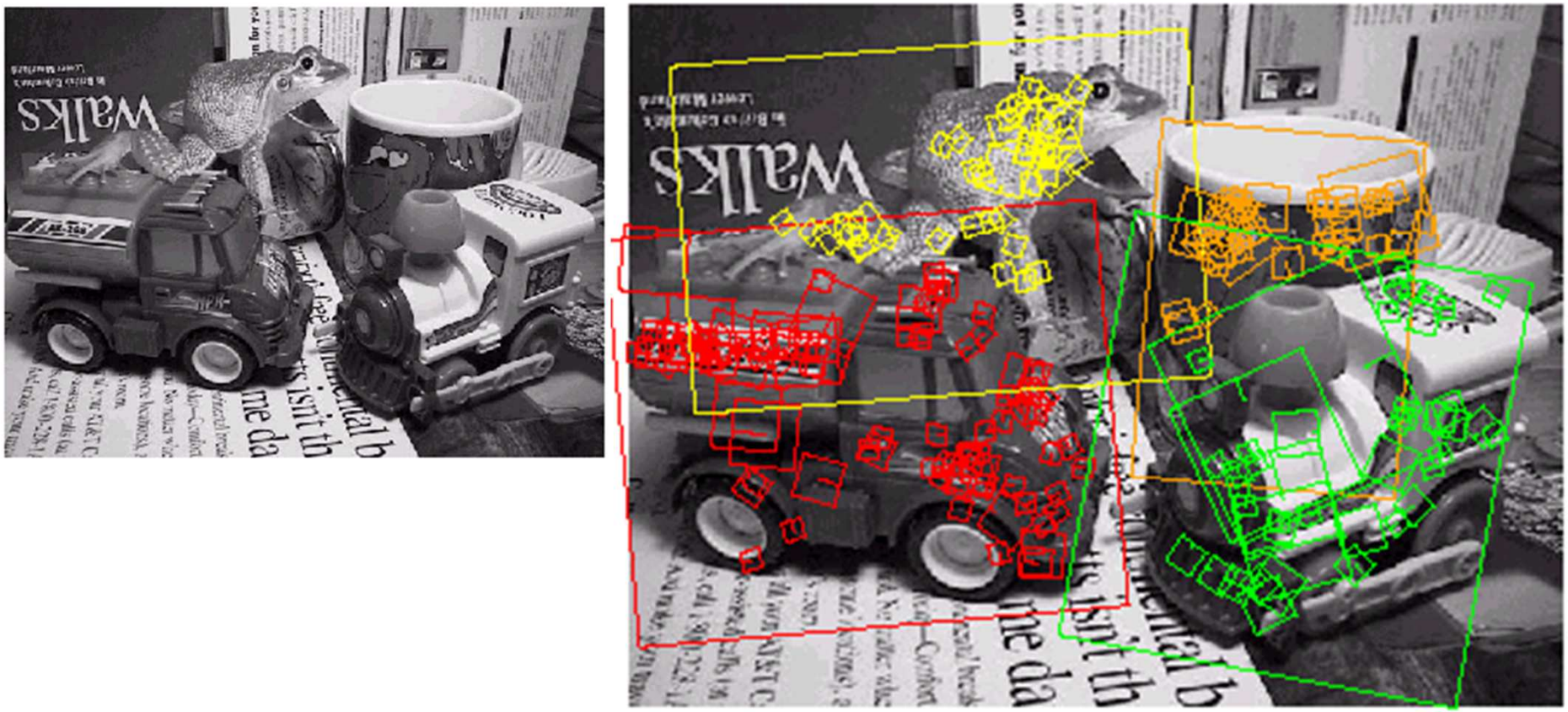
View Interpolation

- Training
  - Training views from similar viewpoints are clustered based on feature matches.
  - Matching features between adjacent views are linked.
- Recognition
  - Feature matches may be spread over several training viewpoints.

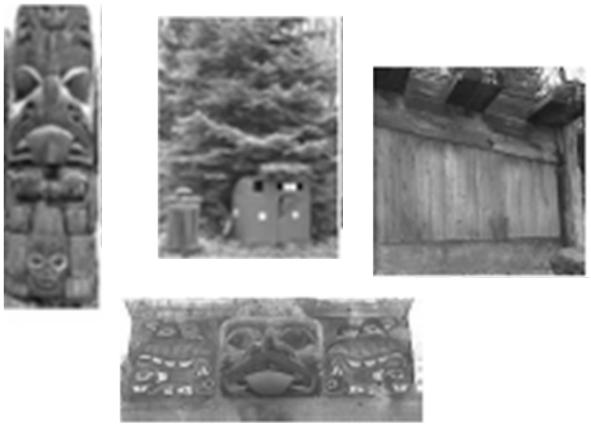


⇒ Use the known links to “transfer votes” to other viewpoints.

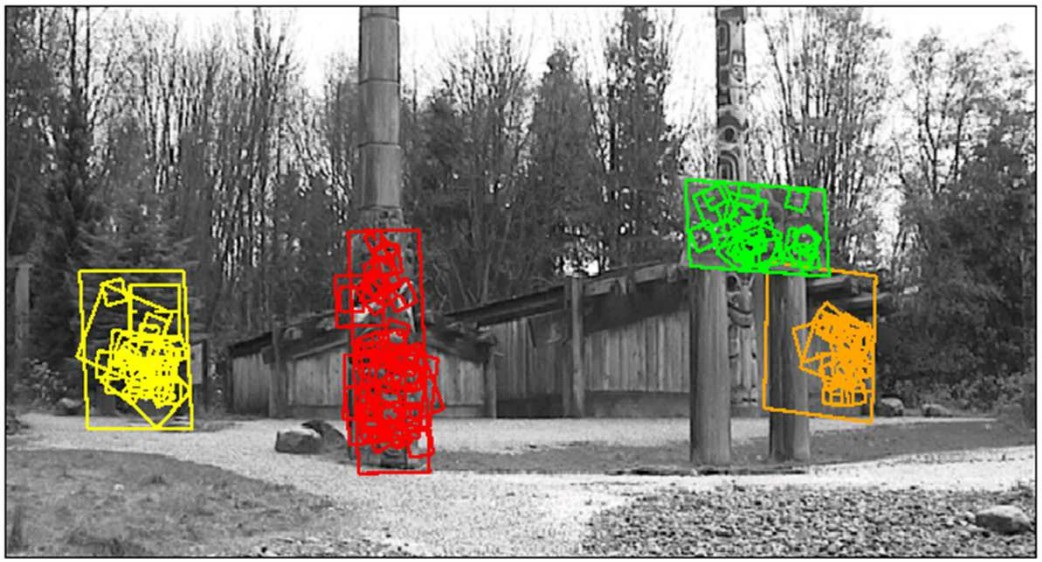
Recognition Using View Interpolation



Location Recognition



Training

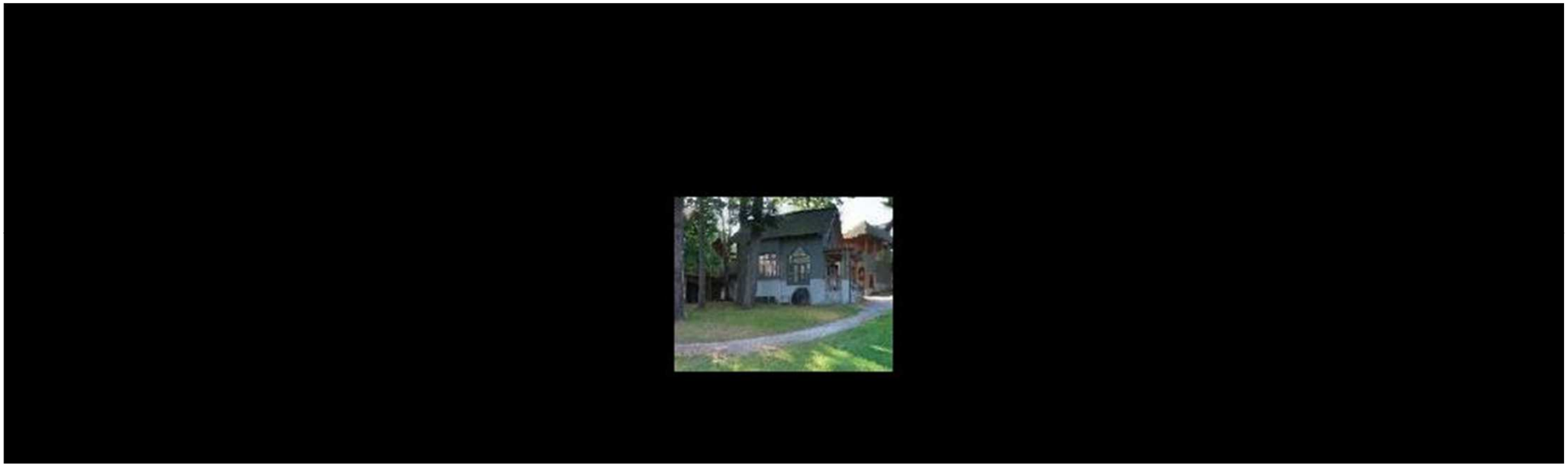


What we will learn today

- Local descriptors
  - SIFT
  - An assortment of other descriptors
- Recognition and matching with local features (Problem Set 3 (Q3))
  - Matching objects with local features: David Lowe
  - Panorama stitching: Brown & Lowe
  - Applications

Introduction

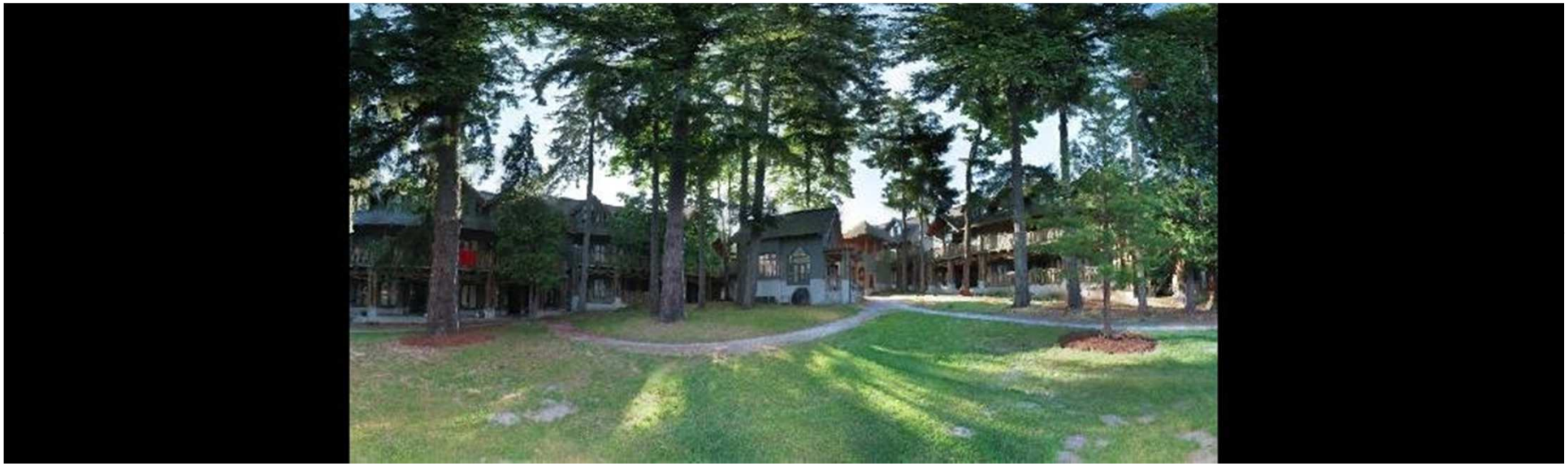
- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°



Slide credit: Matthew Brown

Introduction

- Are you getting the whole picture?
  - Compact Camera FOV = 50 x 35°
  - Human FOV = 200 x 135°



Slide credit: Matthew Brown

Introduction

- Are you getting the whole picture?
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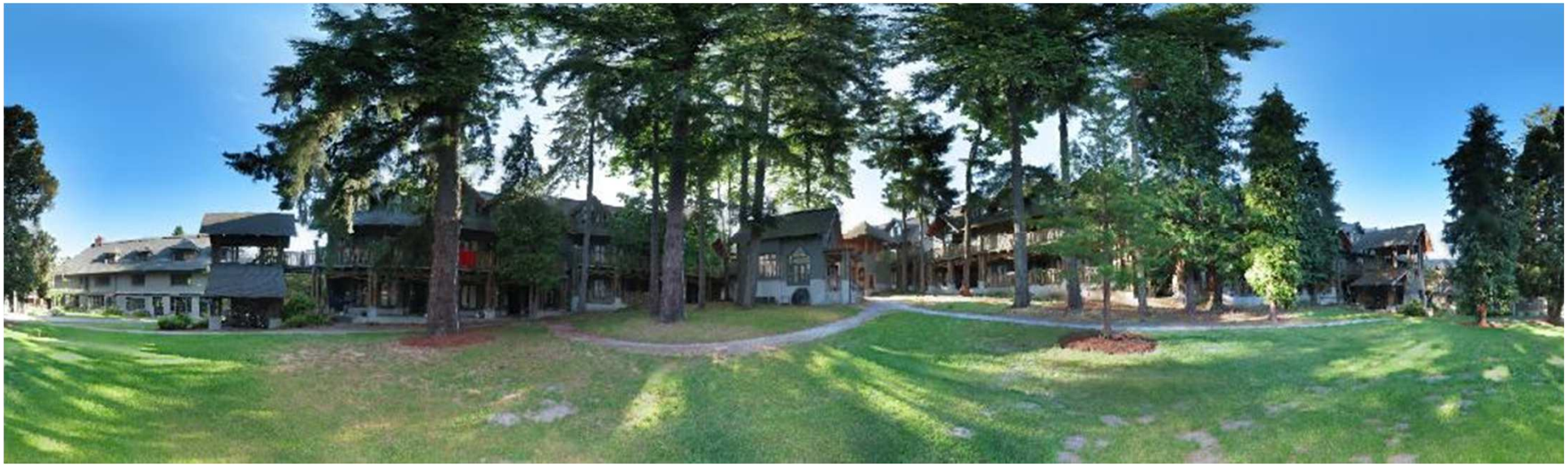

Slide credit: Matthew Brown

Why “Recognising Panoramas”?

- 1D Rotations (θ)
  - Ordering $\Rightarrow$ matching images

Slide credit: Matthew Brown

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- 2D Rotations (θ, ϕ)
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Slide credit: Matthew Brown

Why "Recognising Panoramas"?

- 1D Rotations (θ)
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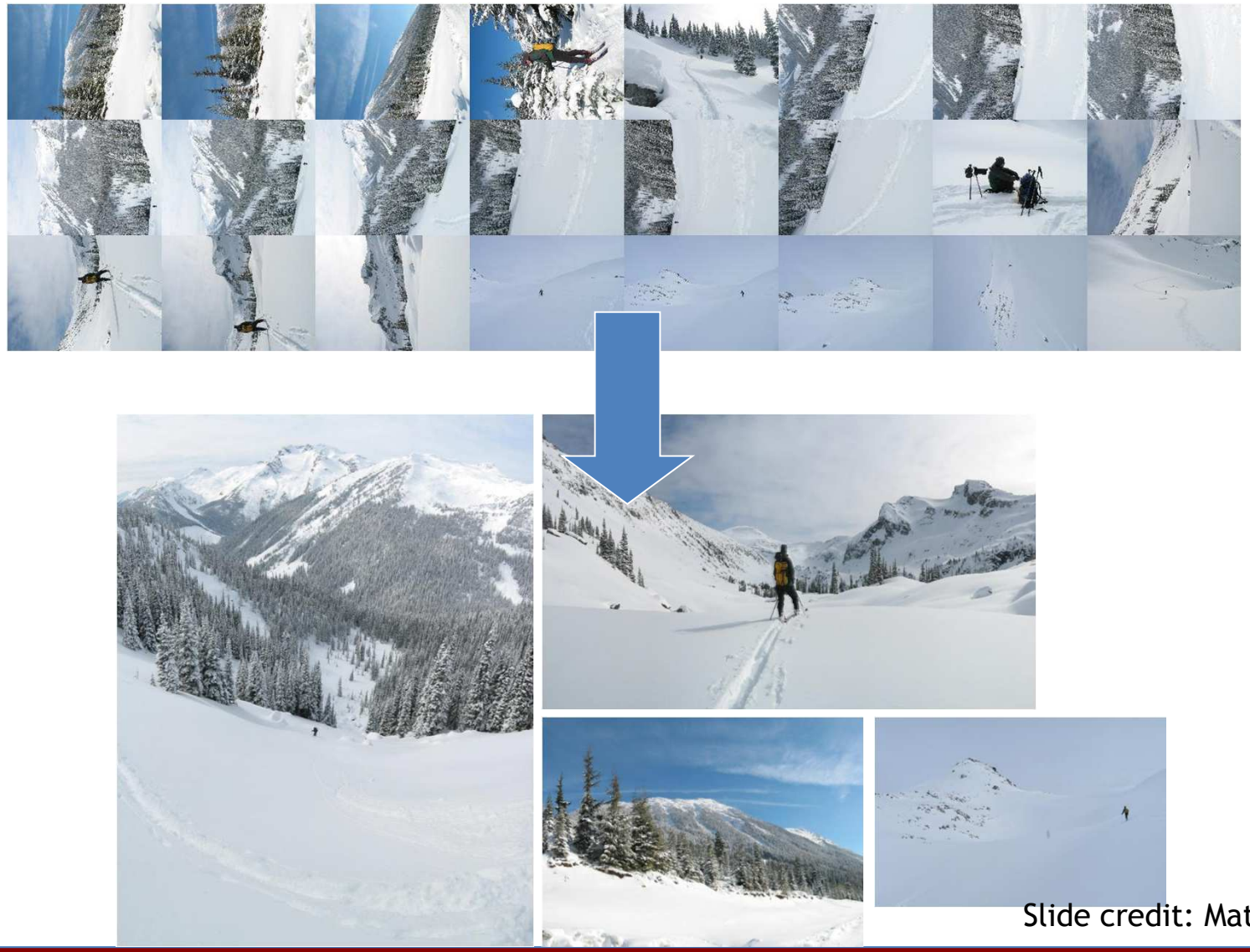
Why “Recognising Panoramas”?

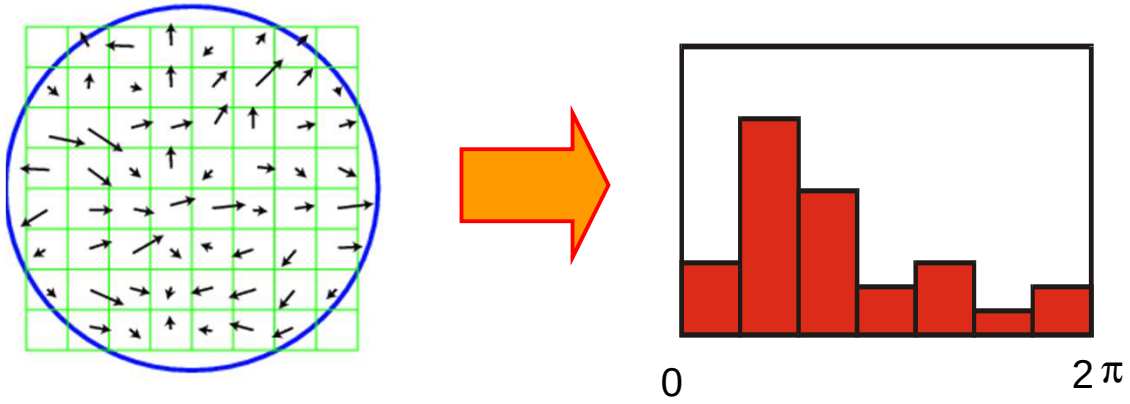


Slide credit: Matthew Brown

Algorithm (Brown & Lowe, 2003)

1. Feature Matching
   a) SIFT Features
   b) Nearest Neighbor Matching
2. Image Matching
3. Bundle Adjustment
4. Multi-band Blending

Nearest Neighbor Matching of SIFT Features



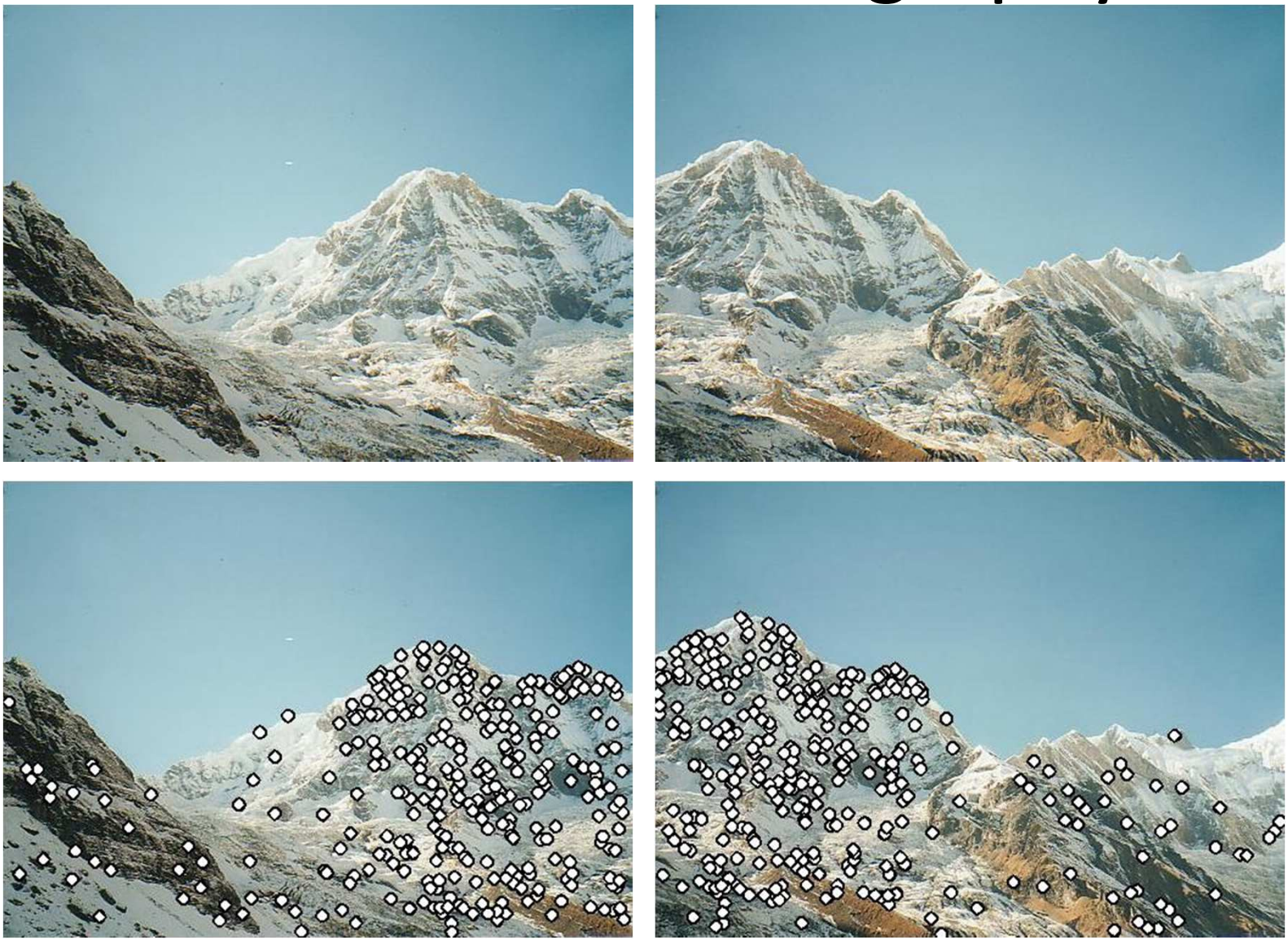
- Find k-NN for each feature
  - k ≈ number of overlapping images (we use k = 4)
- Use k-d tree
  - k-d tree recursively bi-partitions data at mean in the dimension of maximum variance
  - Approximate nearest neighbors found in O(nlogn)

Slide credit: Matthew Brown

Algorithm (Brown & Lowe, 2003)

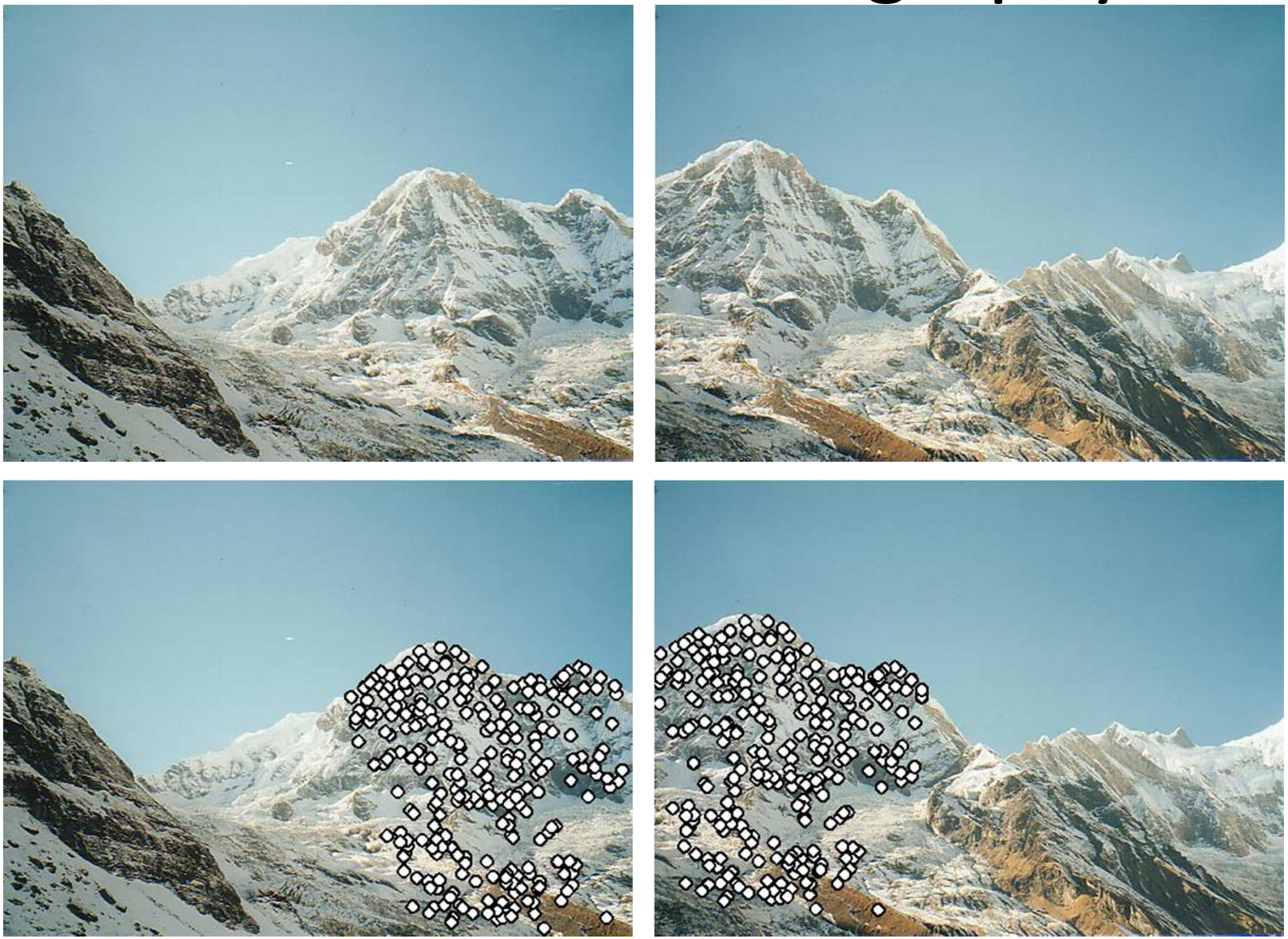
1. Feature Matching
2. Image Matching
   a) RANSAC for Homography
   b) Probabilistic model for verification
3. Bundle Adjustment
4. Multi-band Blending

RANSAC for Homography



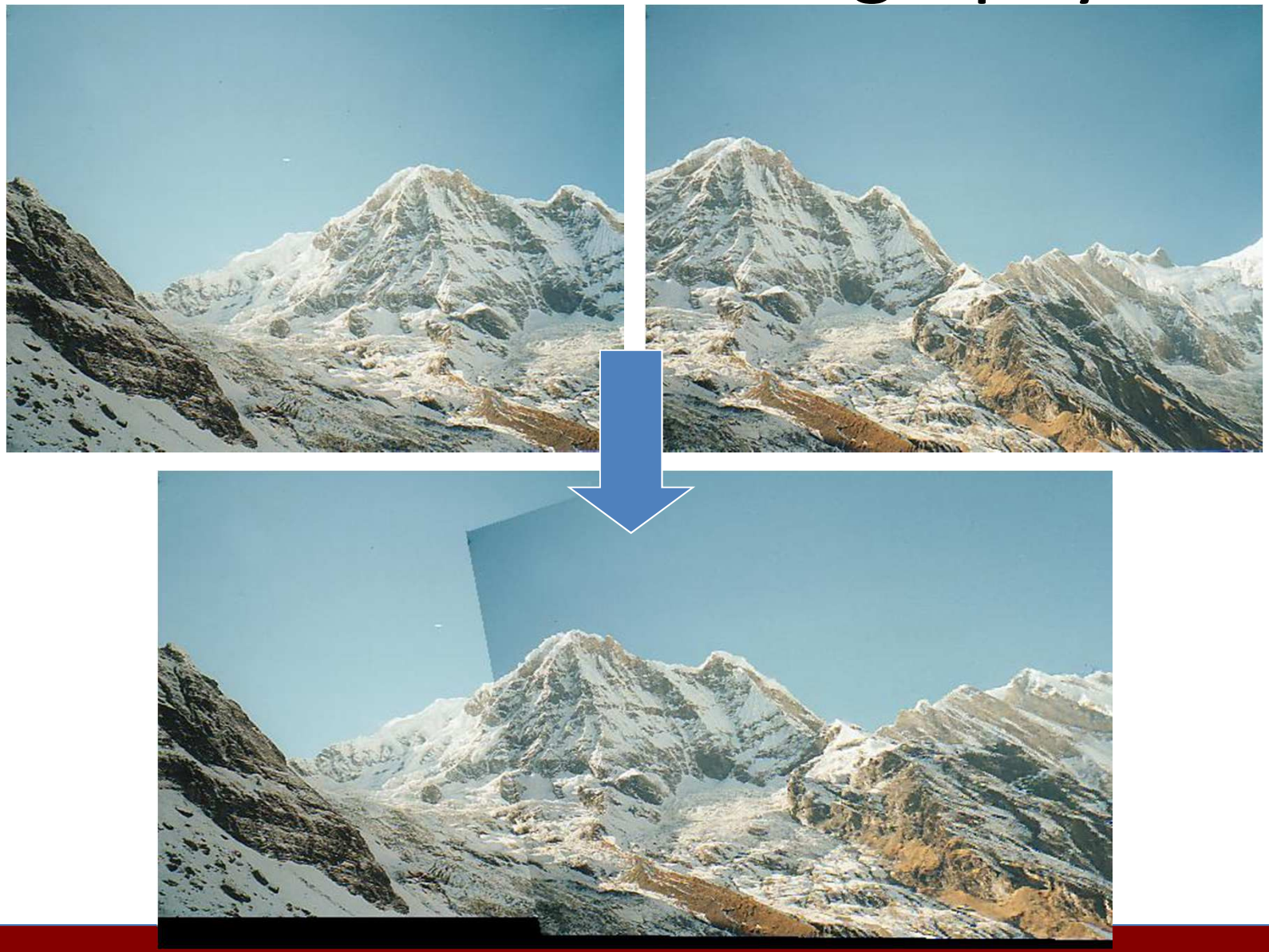
Slide credit: Matthew Brown

RANSAC for Homography



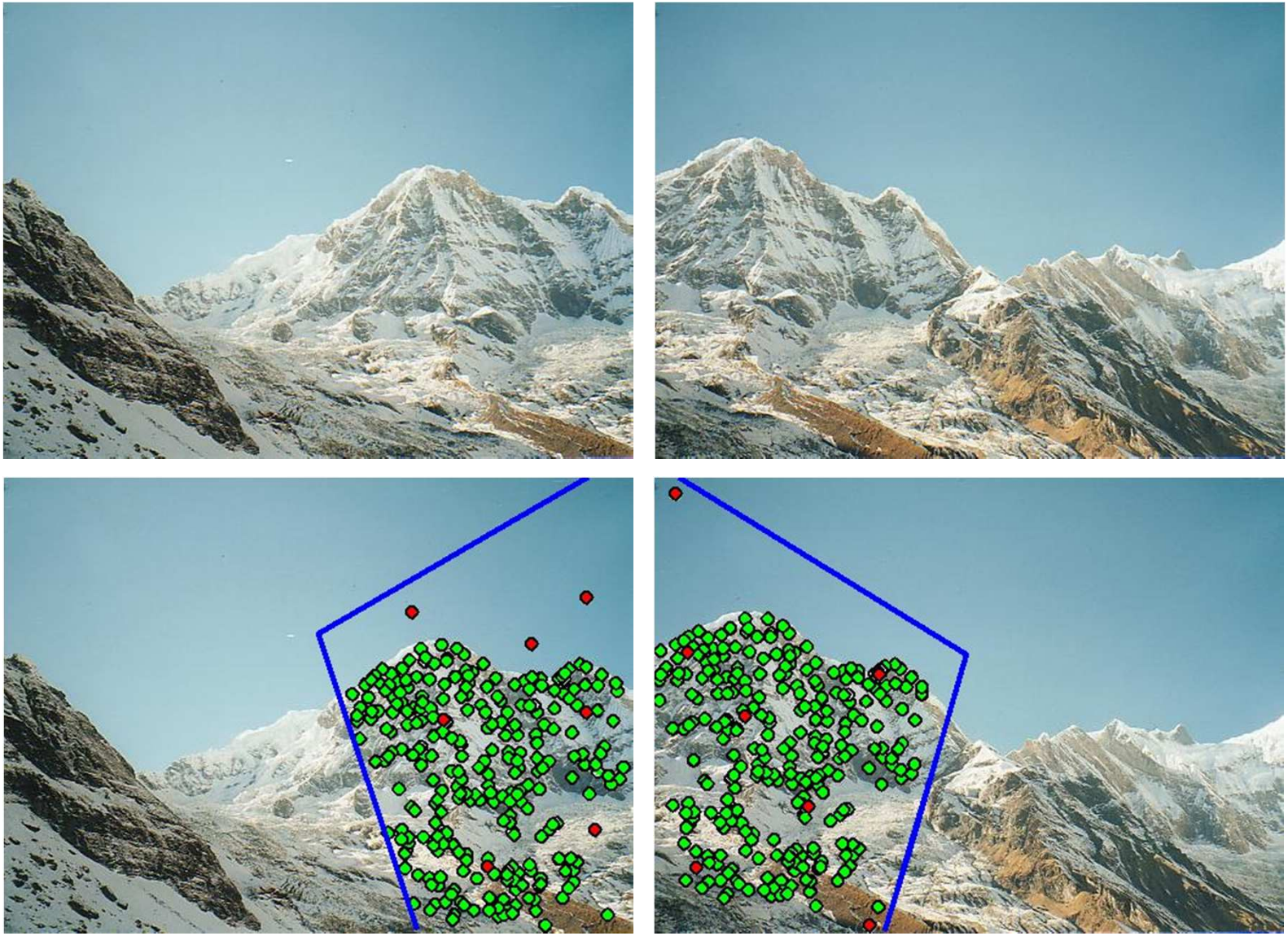
Slide credit: Matthew Brown

RANSAC for Homography



Slide credit: Matthew Brown

Probabilistic model for verification



Slide credit: Matthew Brown

Probabilistic model for verification

- Compare probability that this set of RANSAC inliers/outliers was generated by a correct/false image match

$$\frac{B(n_i; n_f, p_1)}{B(n_i; n_f, p_0)} \underset{reject}{\overset{accept}{\gtrless}} \frac{1}{\frac{1}{p_{min}} - 1}$$

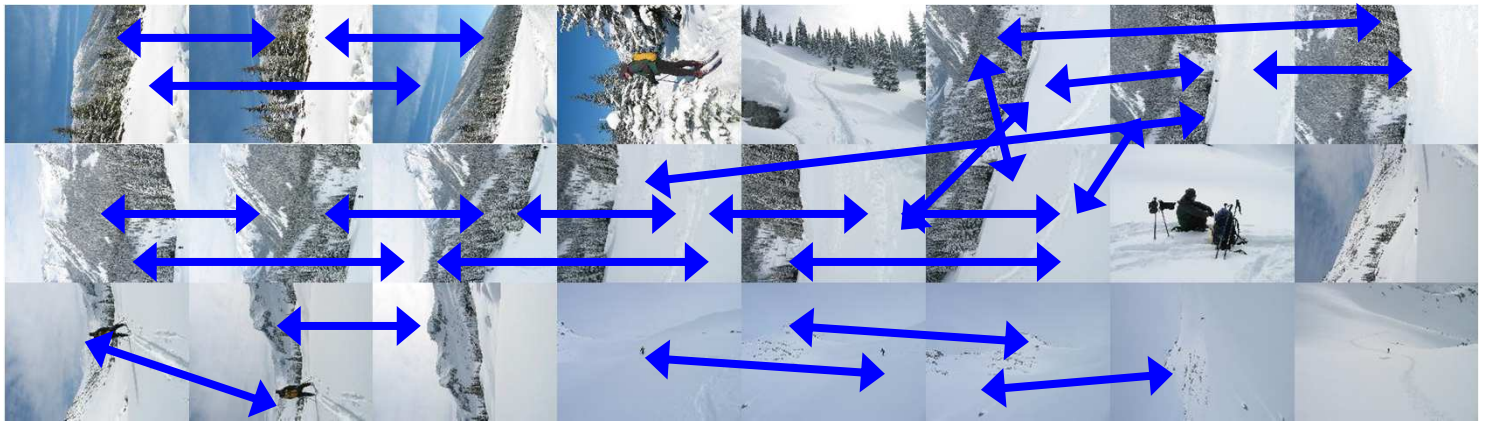
  - n_i = #inliers, n_f = #features
  - p_1 = p(inlier | match), p_0 = p(inlier | ~match)
  - p_{min} = acceptance probability

- Choosing values for p_1, p_0 and p_{min}

$$n_i > 5.9 + 0.22 n_f$$

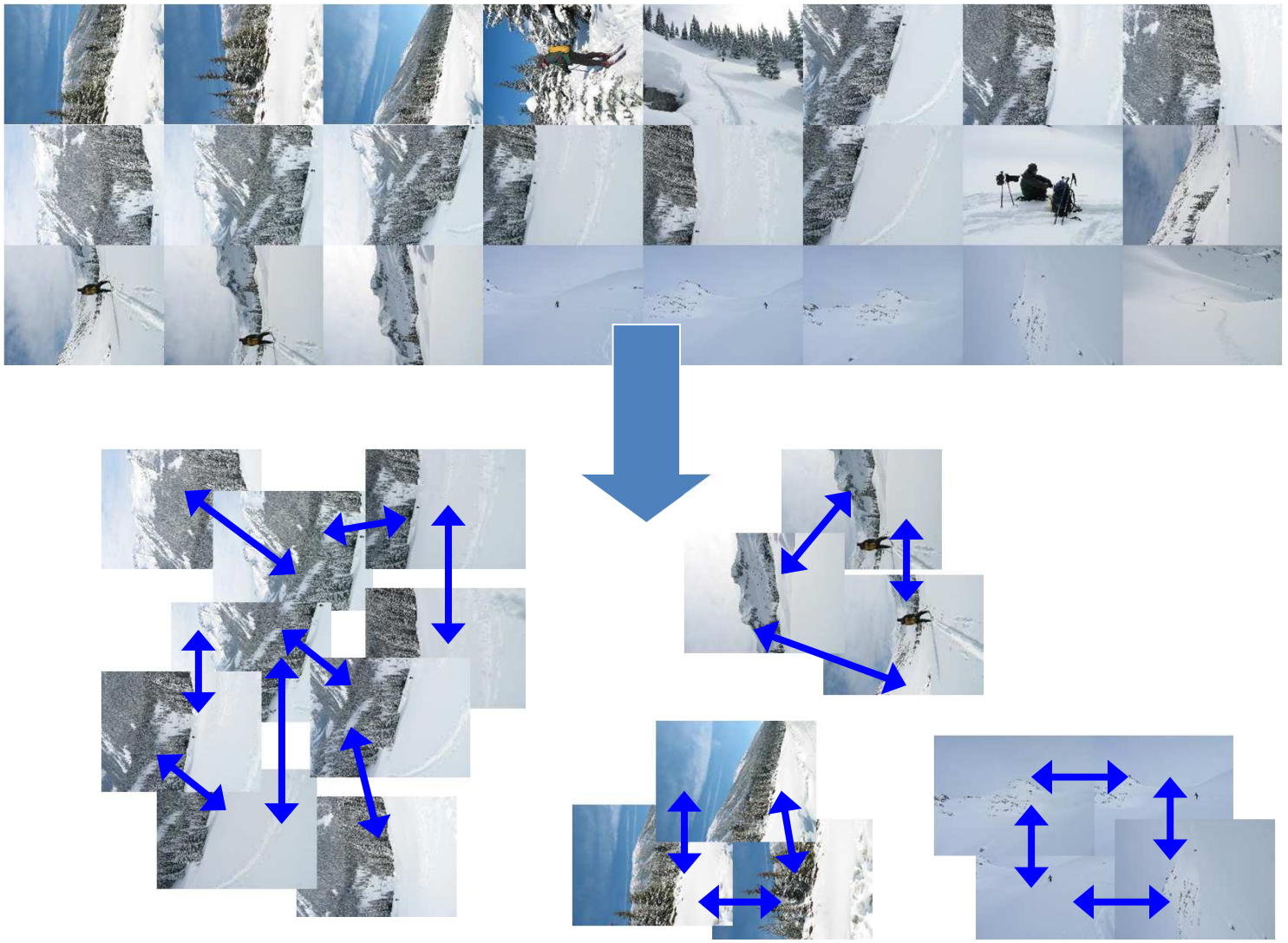
Slide credit: Matthew Brown

Finding the panoramas



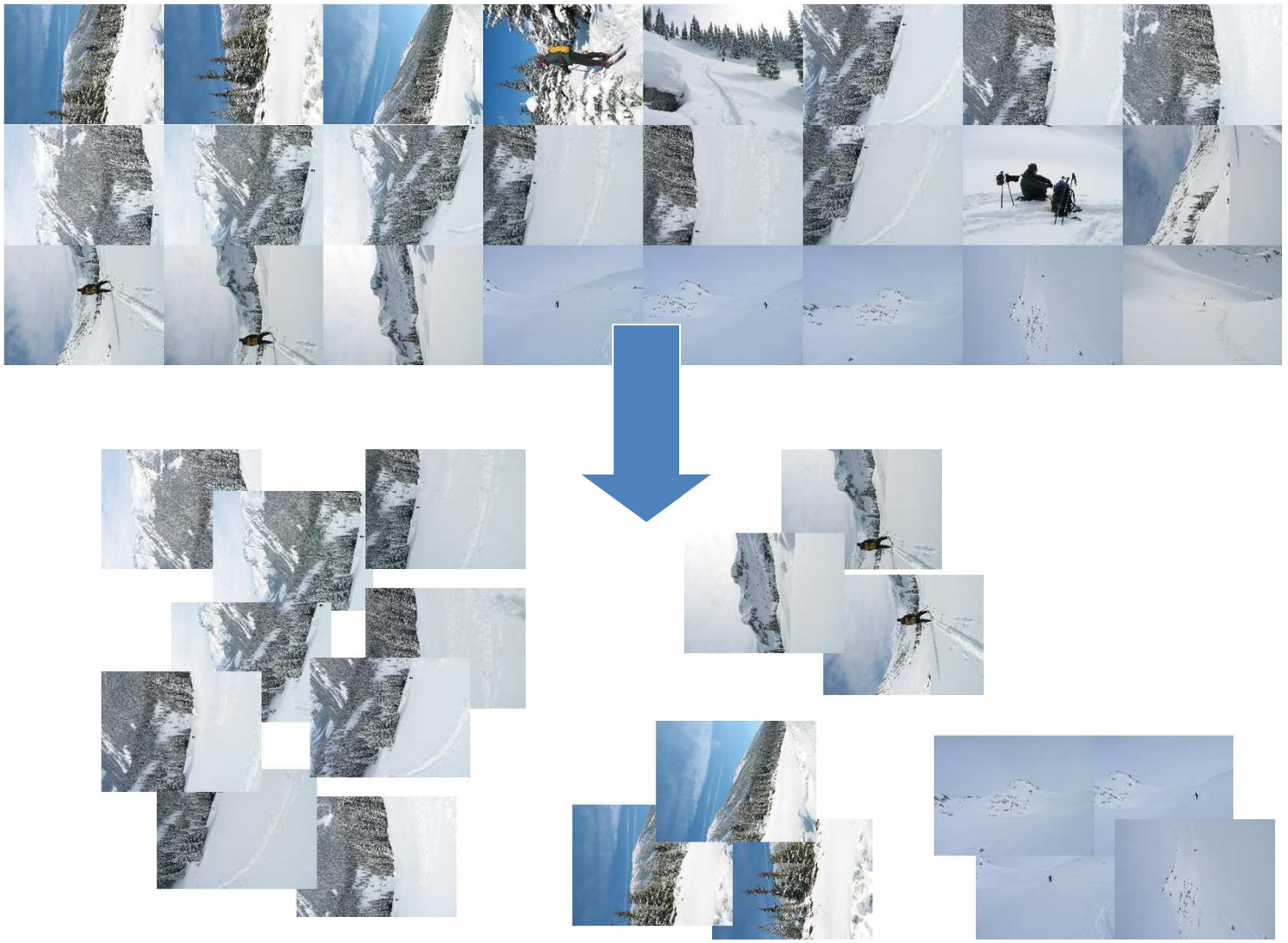
Slide credit: Matthew Brown

Finding the panoramas



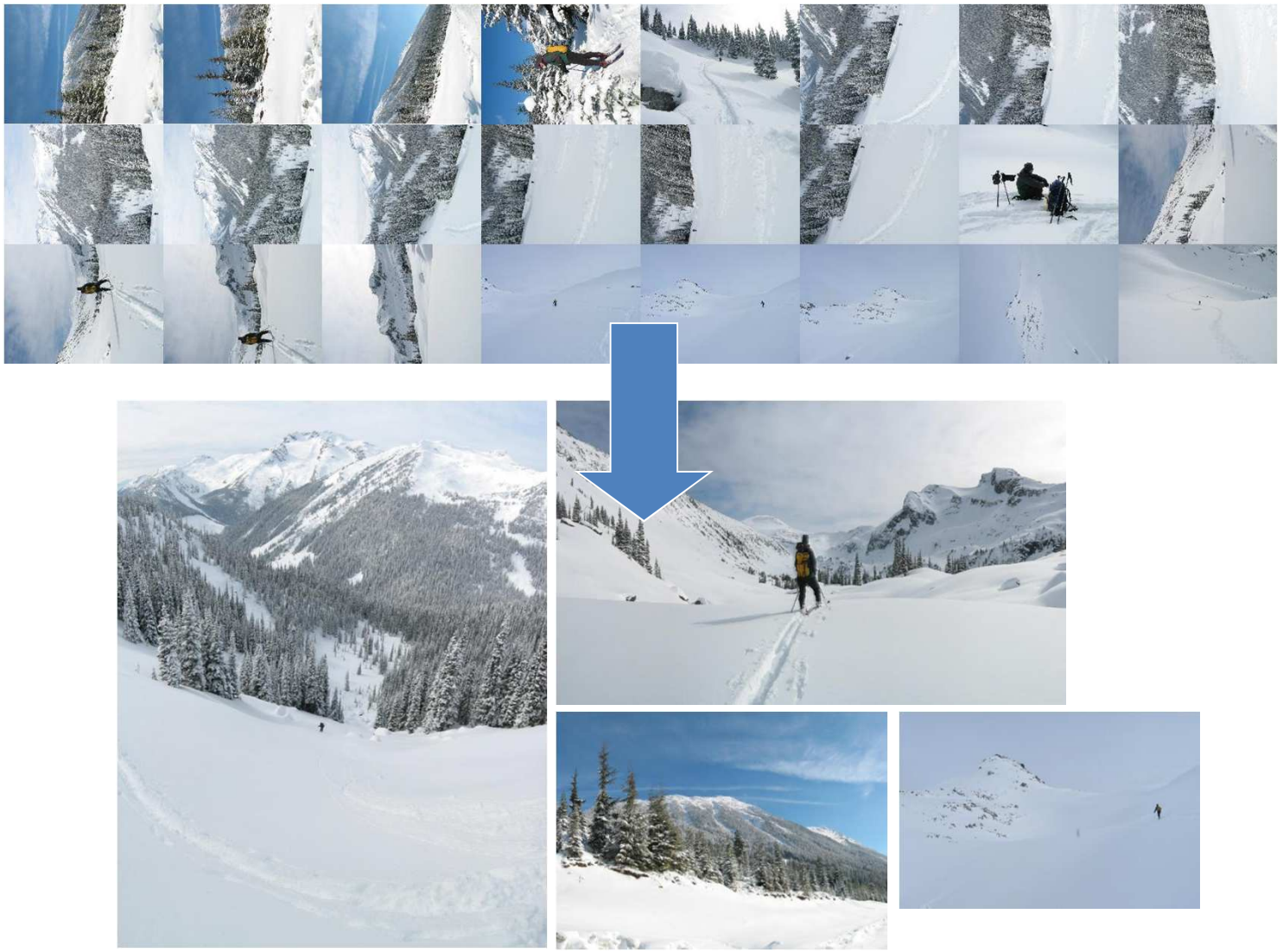
Slide credit: Matthew Brown

Finding the panoramas



Slide credit: Matthew Brown

Finding the panoramas



Slide credit: Matthew Brown

Algorithm (Brown & Lowe, 2003)

1. Feature Matching
2. Image Matching
3. Bundle Adjustment (extra reading: H & Z, Chp 18)
   a) Error function
4. Multi-band Blending

Error function

- Sum of squared projection errors

$$e = \sum_{i=1}^{n} \sum_{j \epsilon \mathcal{I}(i)} \sum_{k \epsilon \mathcal{F}(i,j)} f(\mathbf{r}_{ij}^{k})^2$$

  - n = #images
  - I(i) = set of image matches to image i
  - F(i, j) = set of feature matches between images i,j
  - r_{ij}^{k} = residual of k^{th} feature match between images i,j

- Robust error function

$$f(\mathbf{x}) = \begin{cases} |\mathbf{x}|, & \text{if } |\mathbf{x}| < x_{max} \\ x_{max}, & \text{if } |\mathbf{x}| \geq x_{max} \end{cases}$$

Slide credit: Matthew Brown

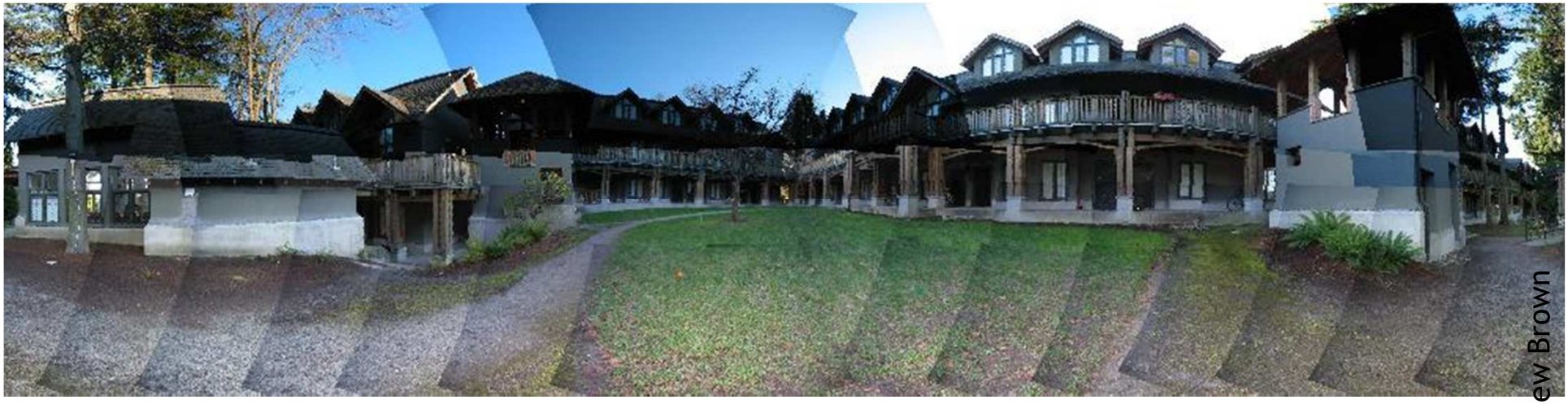
Bundle Adjustment

- New images initialised with rotation, focal length of best matching image

Slide credit: Matthe

Bundle Adjustment

- New images initialised with rotation, focal length of best matching image



Slide credit: Matthew Brown

Algorithm (Brown & Lowe, 2003)

1. Feature Matching
2. Image Matching
3. Bundle Adjustment
4. Multi-band Blending

Multi-band Blending

- Burt & Adelson 1983
  - Blend frequency bands over range $\propto \lambda$

Slide credit: Matthew Brown

2-band Blending

Low frequency (λ > 2 pixels)

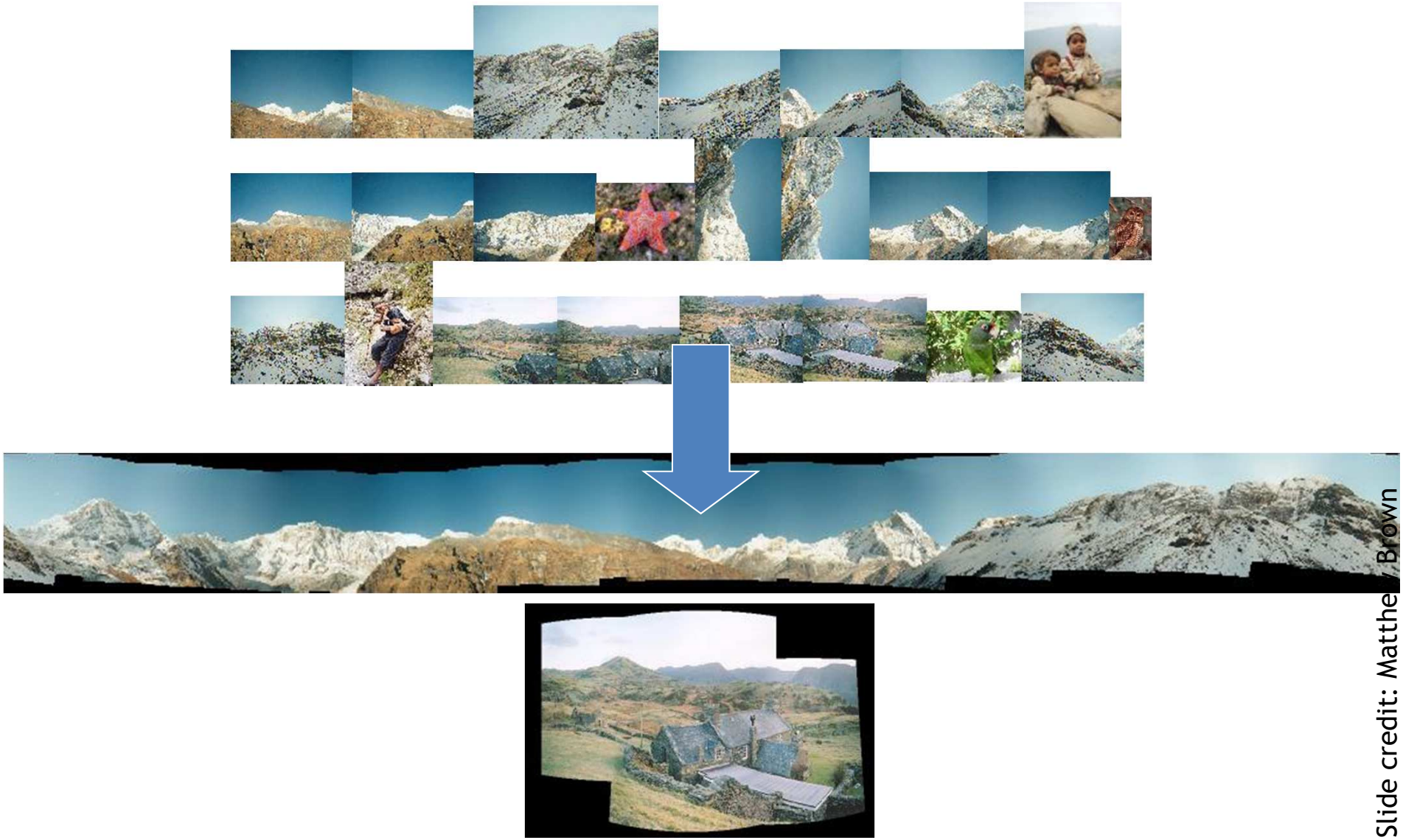
High frequency (λ < 2 pixels)

Slide credit: Matthew Brown

Linear Blending

2-band Blending

Results



Slide credit: Matthew Brown

Results

(a) Matier data set (7 images)

iPhone version available

(b) Matier final stitch

[Brown, Szeliski, and Winder, 2005]

http://www.cs.ubc.ca/~mbrown/autostitch/autostitch.html

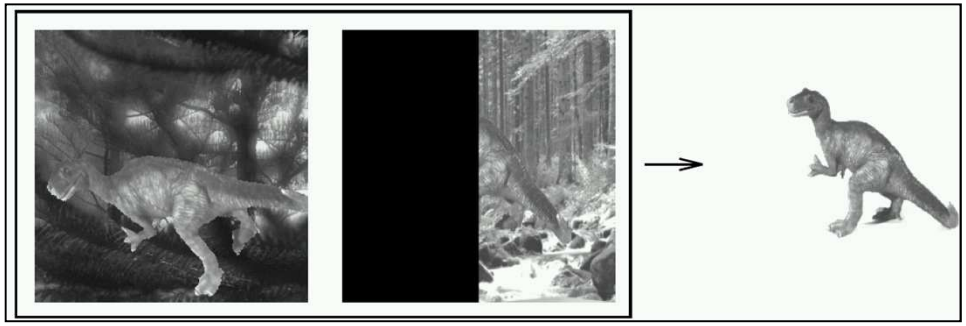
Applications of Local Invariant Features

- Wide baseline stereo
- Motion tracking
- Mobile robot navigation
- 3D reconstruction
- Recognition
  - Specific objects
  - Textures
  - Categories
- …

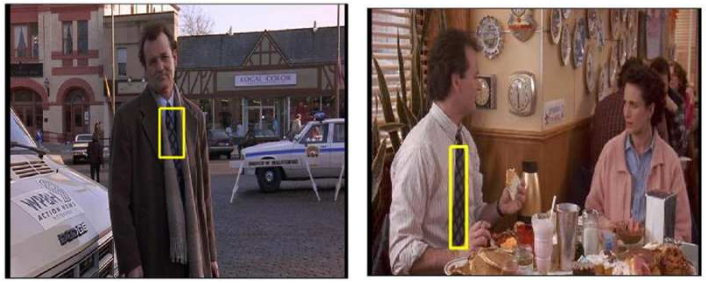
Wide-Baseline Stereo

Image from T. Tuytelaars ECCV 2006 tutorial

Recognition of Specific Objects, Scenes



Schmid and Mohr 1997



Sivic and Zisserman, 2003

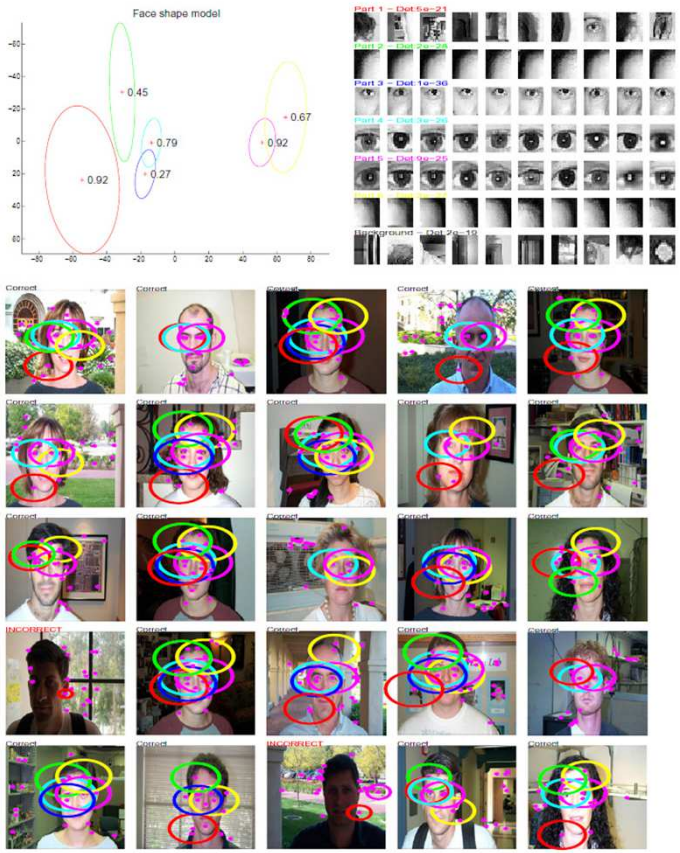
Rothganger et al. 2003

Lowe 2002

Slide credit: Kristen Grauman

Recognition of Categories

Constellation model



Weber et al. (2000)
Fergus et al. (2003)
Fei-Fei et al. (2003)

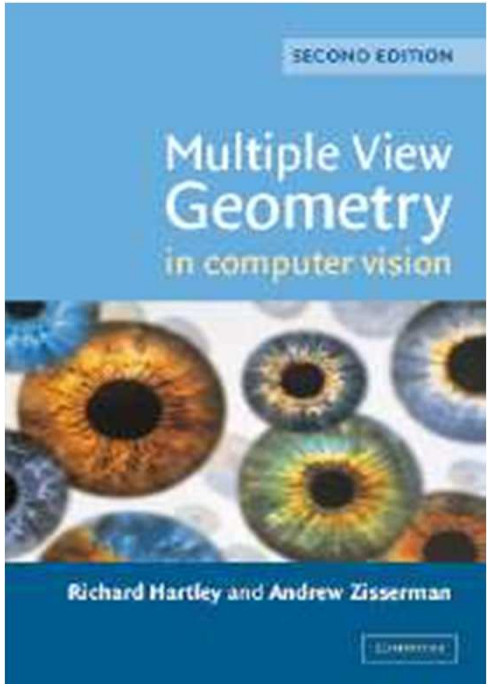
Bags of words

Csurka et al. (2004)
Fei-Fei & Perona (2005)
Dorko & Schmid (2005)
Sivic et al. (2005)
Lazebnik et al. (2006), …

Slide credit: Svetlana Lazebnik

References and Further Reading

- More details on homography estimation can be found in Chapter 4.7 of
  - R. Hartley, A. Zisserman
    Multiple View Geometry in Computer Vision
    2nd Ed., Cambridge Univ. Press, 2004



- Details about the DoG detector and the SIFT descriptor can be found in
  - D. Lowe, Distinctive image features from scale-invariant keypoints, *IJCV* 60(2), pp. 91-110, 2004

- Try the available local feature detectors and descriptors
  - http://www.robots.ox.ac.uk/~vgg/research/affine/detectors.html#binaries

What we have learned today

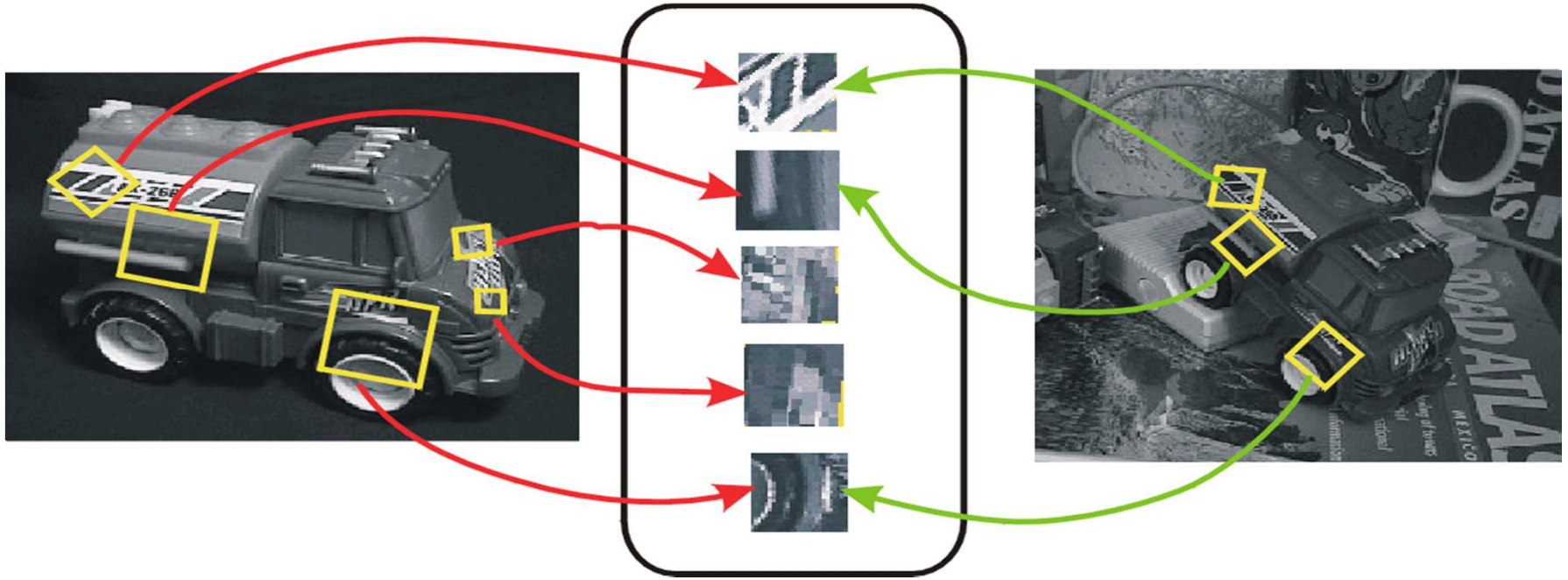
- Local descriptor
  - SIFT (Problem Set 3 (Q2))
  - An assortment of other descriptors (Problem Set 3 (Q4))
- Recognition and matching with local features (Problem Set 3 (Q3))
  - Matching objects with local features: David Lowe
  - Panorama stitching: Brown & Lowe
  - Applications

Supplementary materials

- Matching images
  - Affine Transformation
  - Projective Transformation (Homography)

Recognition with Local Features

- Image content is transformed into local features that are invariant to translation, rotation, and scale
- Goal: Verify if they belong to a consistent configuration

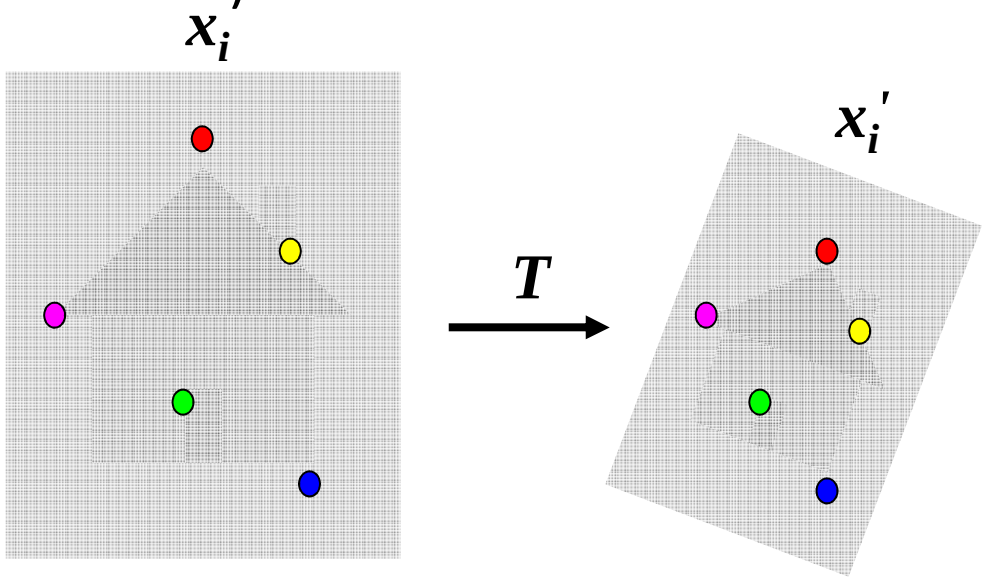


Local Features, e.g. SIFT

Slide credit: David Lowe

Alignment Problem

- We have previously considered how to fit a model to image evidence
  - e.g., a line to edge points
- In alignment, we will fit the parameters of some transformation according to a set of matching feature pairs ("correspondences").



Slide credit: Kristen Grauman

Basic 2D Transformations

- Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shearing

Slide credit: Alyosha Efros

What Can be Represented by a 2×2 Matrix?

- 2D Scaling?

$$x' = s_x * x$$
$$y' = s_y * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- 2D Rotation around (0,0)?

$$x' = \cos\theta * x - \sin\theta * y$$
$$y' = \sin\theta * x + \cos\theta * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- 2D Shearing?

$$x' = x + sh_x * y$$
$$y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Slide credit: Alyosha Efros

What Can be Represented by a 2×2 Matrix?

- 2D Mirror about y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- 2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- 2D Translation?

$$x' = x + t_x$$
$$y' = y + t_y$$

NO!

Slide credit: Alyosha Efros

2D Linear Transforms

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Only linear 2D transformations can be represented with a 2x2 matrix.
- Linear transformations are combinations of …
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

Slide credit: Alyosha Efros

Homogeneous Coordinates

- Q: How can we represent translation as a 3x3 matrix using homogeneous coordinates?

$$x' = x + t_x$$

$$y' = y + t_y$$

- A: Using the rightmost column:

$$\text{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Slide credit: Alyosha Efros

2D Affine Transformation

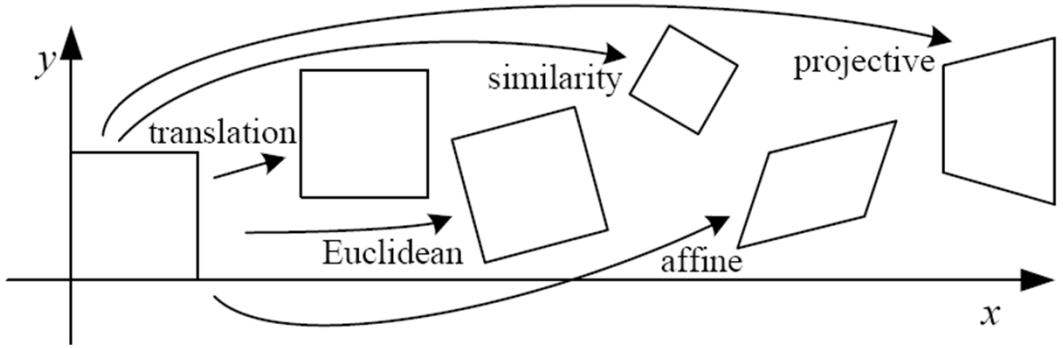
$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Affine transformations:
  - Linear transformations
  - Translations
- Parallel lines remain parallel

Projective Transformation

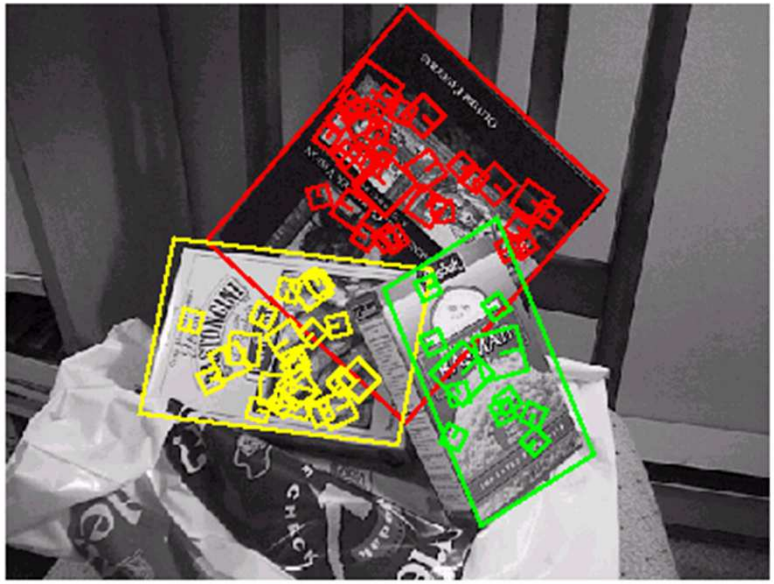
$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Projective transformations:
  - Affine transformations, and
  - Projective warps
- Parallel lines do not necessarily remain parallel



Let's Start with Affine Transformations

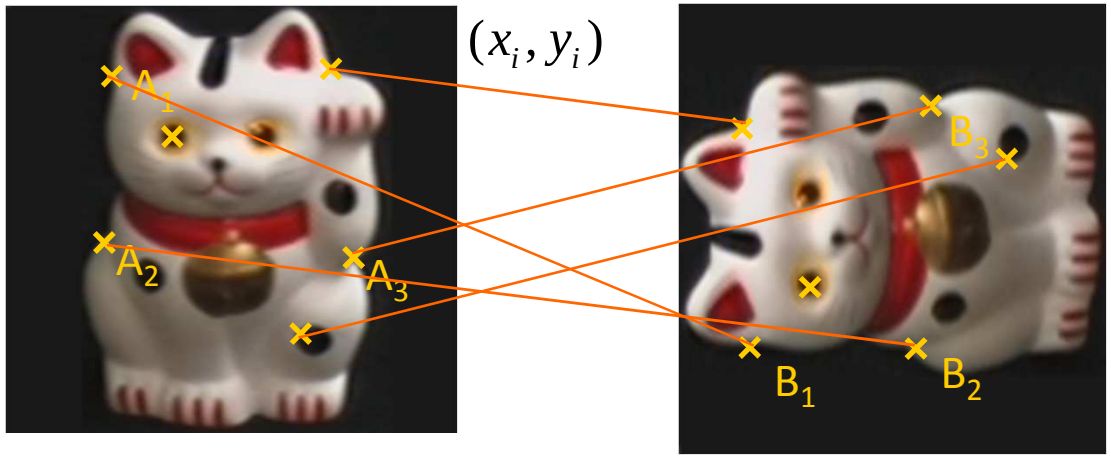
- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
- Can be used to initialize fitting for more complex models



Slide credit: Svetlana Lazebnik, David Lowe

Fitting an Affine Transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Slide credit: Bastian Leibe

Fitting an Affine Transformation

- Assuming we know the correspondences, how do we get the transformation?

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \cdots & & & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ \cdots & & & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

- How many matches (correspondence pairs) do we need to solve for the transformation parameters?
- Once we have solved for the parameters, how do we compute the coordinates of the corresponding point for (x_{new}, y_{new})?

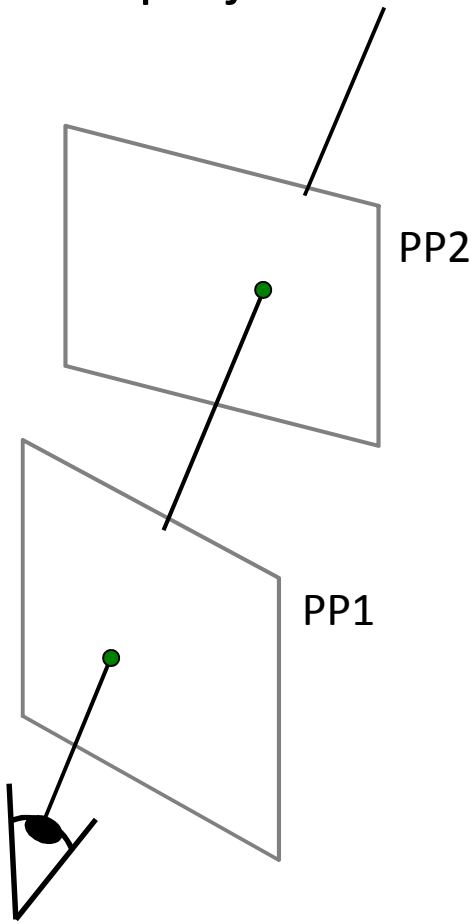
Slide credit: Bastian Leibe

Fitting a Projective Transformation (i.e. Homography)

- A projective transform is a mapping between any two perspective projections with the same center of projection.
  - I.e. two planes in 3D along the same sight ray
- Properties
  - Rectangle should map to arbitrary quadrilateral
  - Parallel lines aren't
  - but must preserve straight lines
- This is called a homography

$$\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

p' H p



Slide credit: Alyosha Efros

Fitting a Projective Transformation (i.e. Homography)

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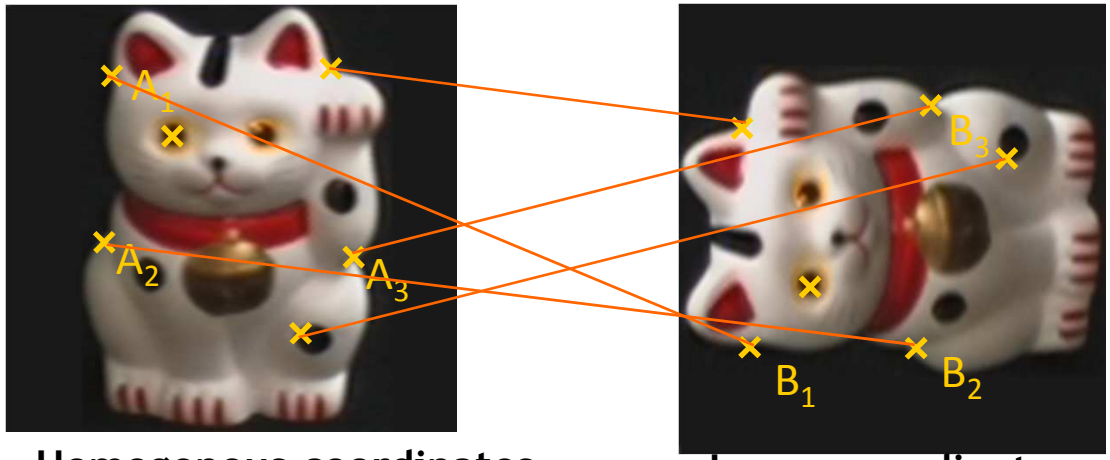
$$\underbrace{\begin{bmatrix} wx' \\ wy' \\ w \end{bmatrix}}_{p'} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{p}$$

Set scale factor to 1
$\Rightarrow$ 8 parameters left.

Slide credit: Alyosha Efros

Fitting a Projective Transformation (i.e. Homography)

- Estimating the transformation



Homogenous coordinates Image coordinates

$$\mathbf{x}_{A_1} \leftrightarrow \mathbf{x}_{B_1}$$
$$\mathbf{x}_{A_2} \leftrightarrow \mathbf{x}_{B_2}$$
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$$\vdots$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = \frac{1}{z'} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Matrix notation

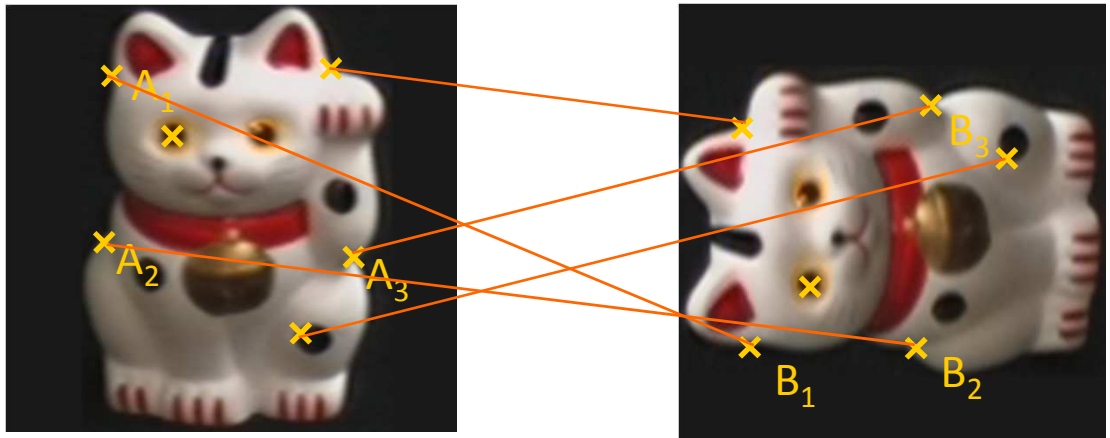
$$x' = Hx$$

$$x'' = \frac{1}{z'} x'$$

Slide credit: Krystian Mikolajczyk

Fitting a Projective Transformation (i.e. Homography)

- Estimating the transformation



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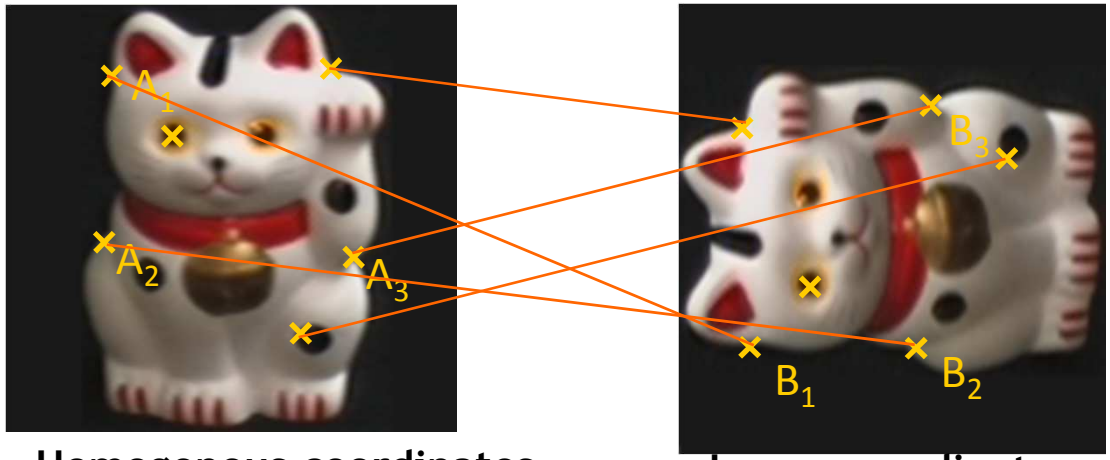
Fitting a Projective Transformation (i.e. Homography)

- Estimating the transformation



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Matrix notation

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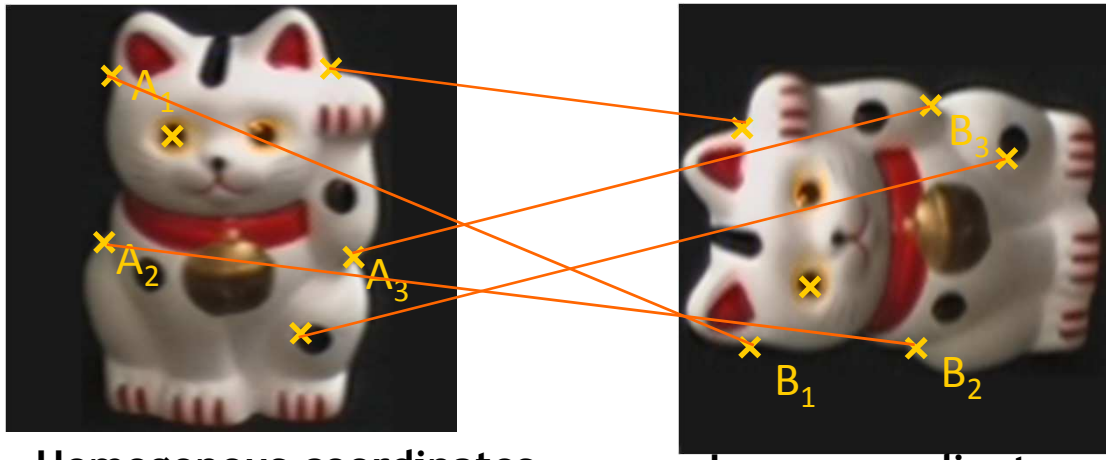
Fitting a Projective Transformation (i.e. Homography)

- Estimating the transformation



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Fitting a Projective Transformation (i.e. Homography)

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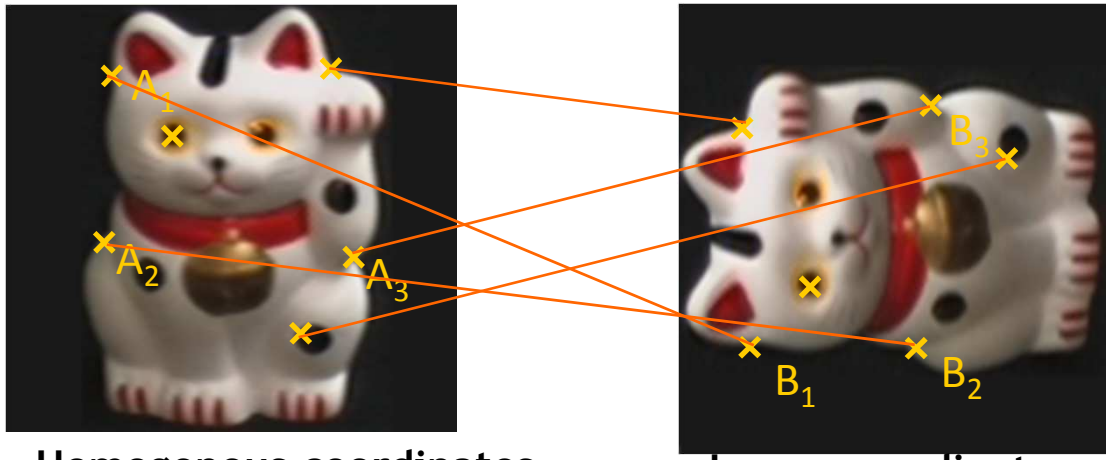

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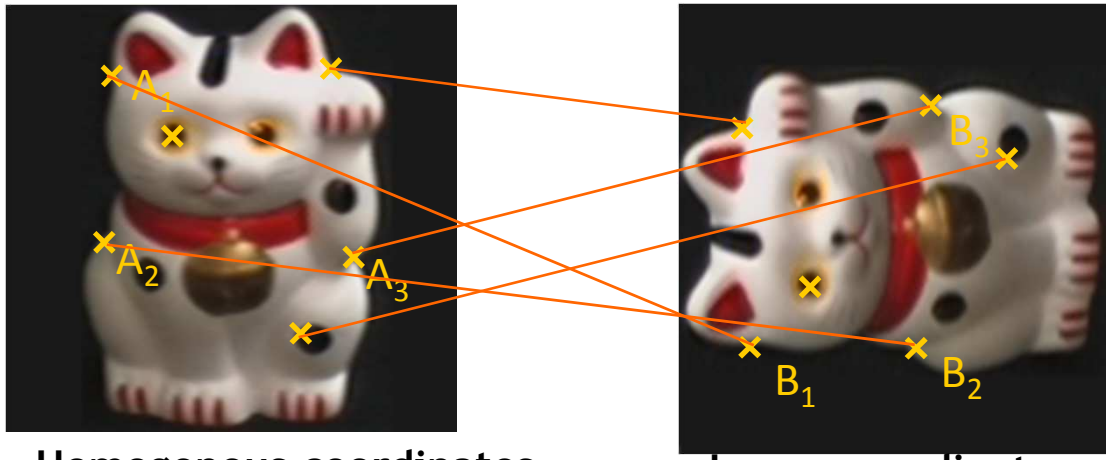
Slide credit: Krystian Mikolajczyk

Fitting a Projective Transformation (i.e. Homography)

- Estimating the transformation



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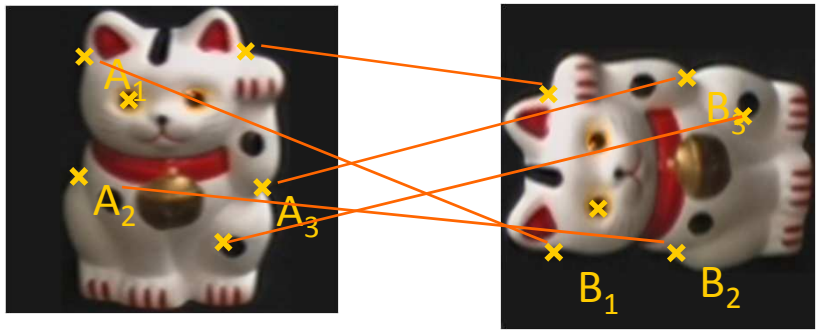
$$h_{11}\, x_{B_1} + h_{12} y_{B_1} + h_{13} - x_{A_1} h_{31}\, x_{B_1} - x_{A_1} h_{32} y_{B_1} - x_{A_1} = 0$$
$$h_{21}\, x_{B_1} + h_{22} y_{B_1} + h_{23} - y_{A_1} h_{31}\, x_{B_1} - y_{A_1} h_{32} y_{B_1} - y_{A_1} = 0$$

Slide credit: Krystian Mikolajczyk

Fitting a Projective Transformation (i.e. Homography)

- Estimating the transformation

$$h_{11}\,x_{B_1} + h_{12}y_{B_1} + h_{13} - x_{A_1}h_{31}\,x_{B_1} - x_{A_1}h_{32}y_{B_1} - x_{A_1} = 0$$
$$h_{21}\,x_{B_1} + h_{22}y_{B_1} + h_{23} - y_{A_1}h_{31}\,x_{B_1} - y_{A_1}h_{32}y_{B_1} - y_{A_1} = 0$$



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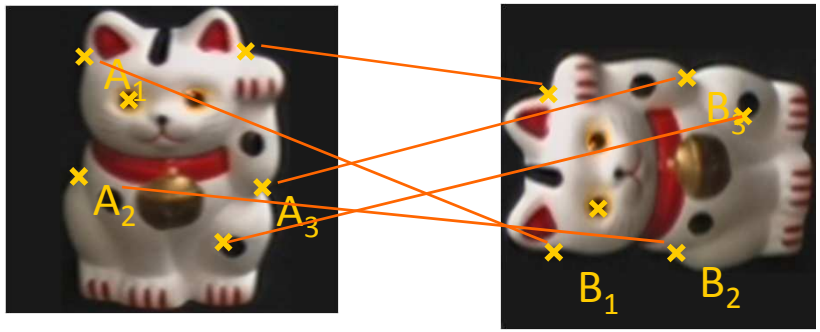
$$\begin{bmatrix} x_{B_1} & y_{B_1} & 1 & 0 & 0 & 0 & -x_{A_1}x_{B_1} & -x_{A_1}y_{B_1} & -x_{A_1} \\ 0 & 0 & 0 & x_{B_1} & y_{B_1} & 1 & -y_{A_1}x_{B_1} & -y_{A_1}y_{B_1} & -y_{A_1} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \end{bmatrix}$$

$$Ah = 0$$

Slide credit: Krystian Mikolajczyk

Fitting a Projective Transformation (i.e. Homography)

- Estimating the transformation
- Solution:
  - Null-space vector of A
  - Corresponds to smallest eigenvector



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$$\vdots$$

$$Ah = 0$$

SVD

$$\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^T = \mathbf{U}\begin{bmatrix} d_{11} & \cdots & d_{19} \\ \vdots & \ddots & \vdots \\ d_{91} & \cdots & d_{99} \end{bmatrix}\begin{bmatrix} v_{11} & \cdots & v_{19} \\ \vdots & \ddots & \vdots \\ v_{91} & \cdots & v_{99} \end{bmatrix}^T$$

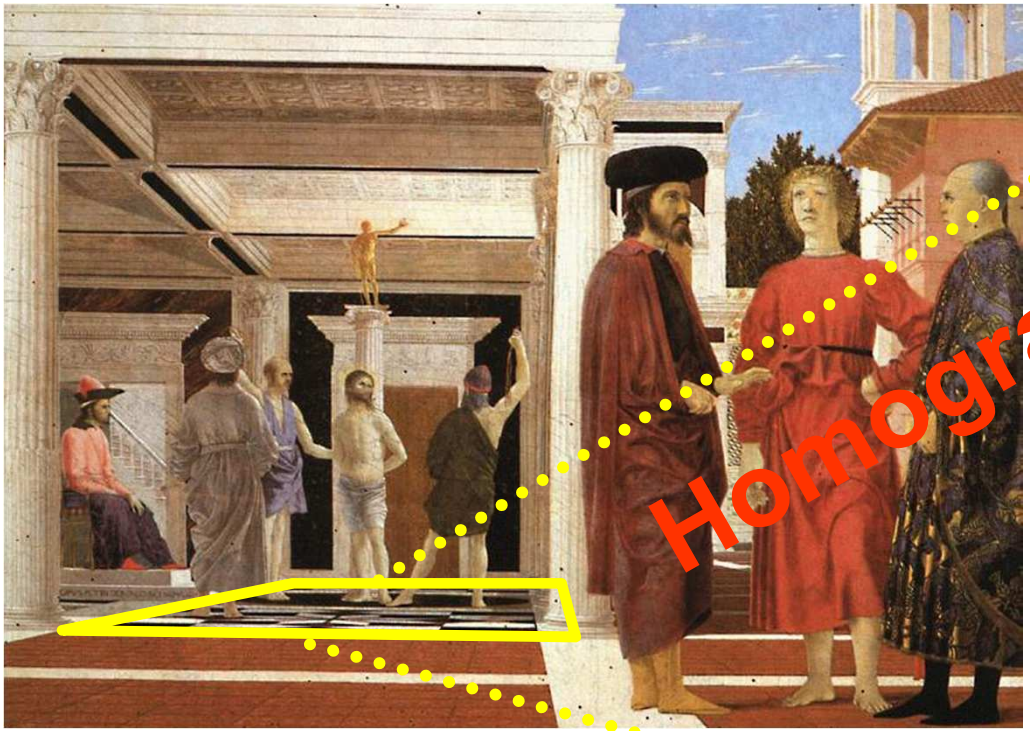
$$\mathbf{h} = \frac{[v_{19}, \cdots, v_{99}]}{v_{99}}$$

Minimizes least square error

Slide credit: Krystian Mikolajczyk

Uses: Analyzing Patterns and Shapes

- What is the shape of the b/w floor pattern?



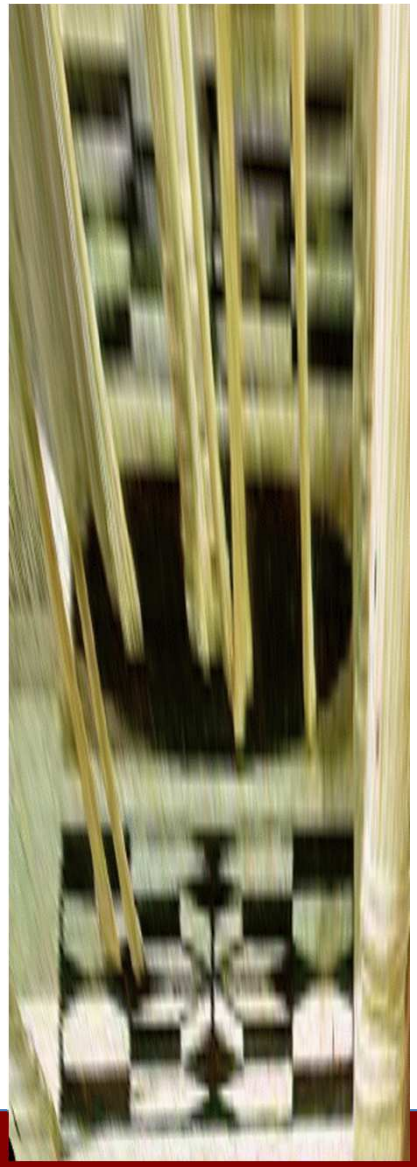
Homography

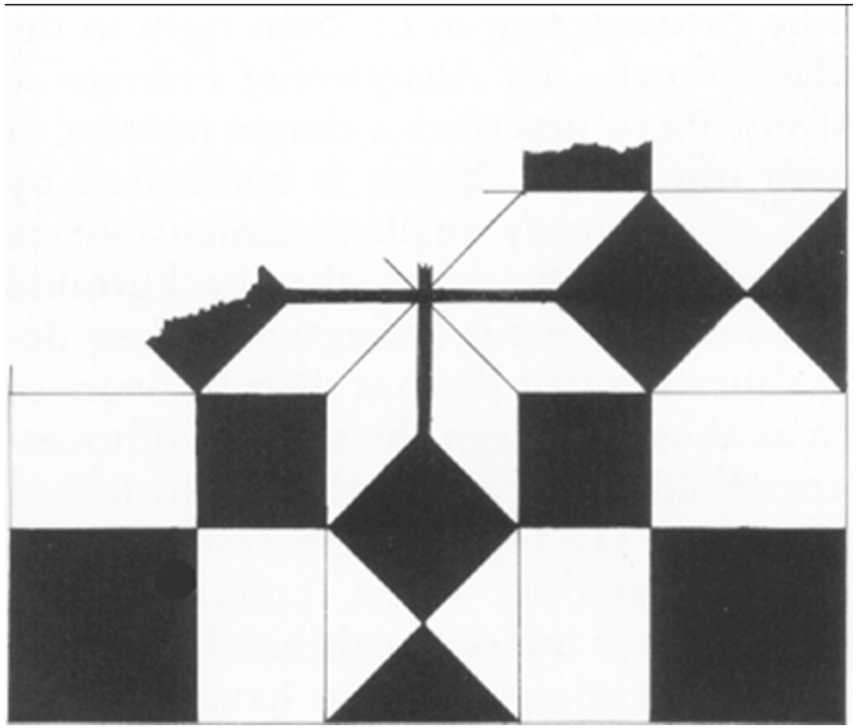
The floor (enlarged)

Slide credit: Antonio Criminisi

Analyzing Patterns and Shapes

Automatic rectification





From Martin Kemp *The Science of Art*
(manual reconstruction)

Slide credit: Antonio Criminisi