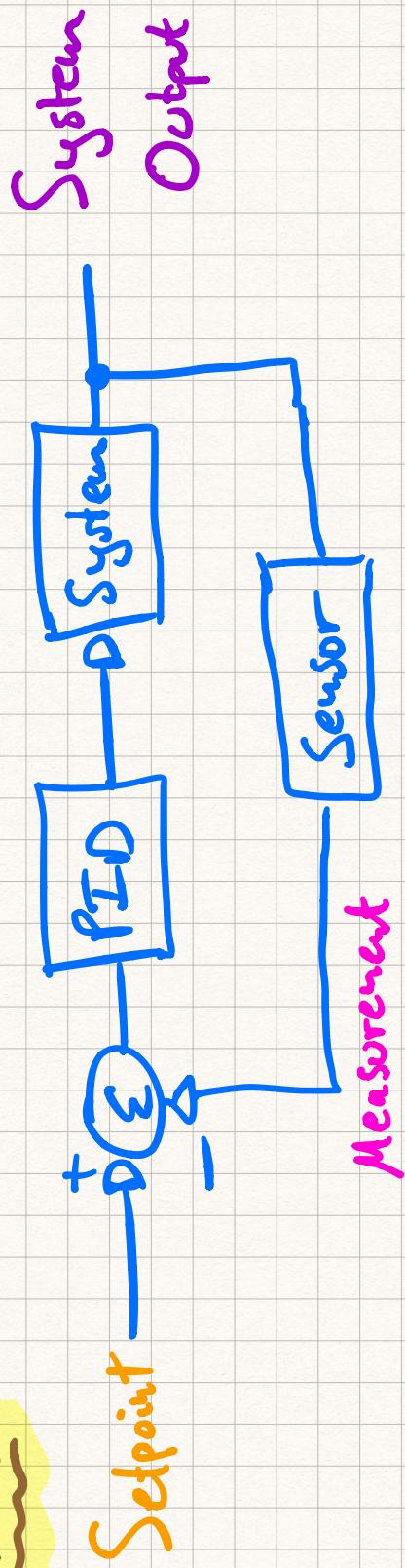


# PID Controller Implementation in Software

- Overview.
- Converting from continuous to discrete time.
- Practical considerations.
- PID code in C (for embedded systems).
- Example application.

## Overview



→ Want to make system output track desired Setpoint via feedback!

Setpoint

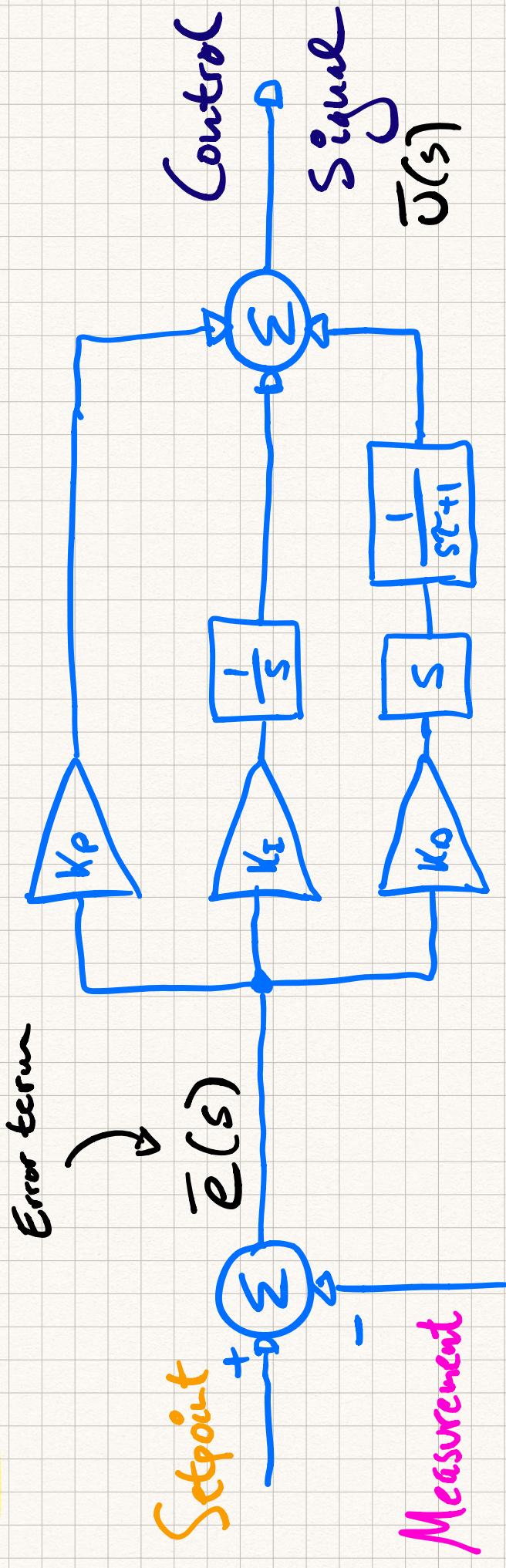
→ PID controller takes two inputs and produces a control signal.

↳ measurement

→ System usually in continuous time, PID Controller usually implemented digitally.

⇒ How can we write the controller in C?

PID in Continuous Domain



$$G(s) = \frac{\bar{U}(s)}{\bar{e}(s)} = K_p + K_I \cdot \frac{1}{s} + K_D \cdot s$$

Proportional  
Integral (Filter)  
Derivative

$$\bar{U}(s) = \frac{s}{s^2 + 1}$$

## Continuous to Discrete

Can implement this  
in code!

To difference equation!

→ T.e. S-Domain to Z-Domain (best frequency domain match)

→ Use Tustin transform (best frequency)

$$\textcircled{1} \quad \text{Substitute } S \rightarrow D \frac{z-1}{z+1}$$

$$\textcircled{2} \quad \text{Recall: } \bar{Y}(z) = \bar{X}(z) \cdot \bar{e}^{-\frac{1}{T}z} \Rightarrow y[n] = x[n-1]$$

( $\frac{1}{T}$  = Sampling time of discrete controller in seconds)

After a lot of substituting and rearranging...

$$p[n] = k_p \cdot e[n]$$

$$i[n] = \frac{k_I T}{2} (e[n] + e[n-1]) + i[n-1]$$

$$d[n] = \frac{2k_D}{2\zeta + T} (e[n] - e[n-1]) + \frac{2\zeta - T}{2\zeta + T} d[n-1]$$
$$\Rightarrow u[n] = p[n] + i[n] + d[n]$$

Controller output



## Practical Considerations

$$H(s) = \frac{1}{1 + sT}$$

→ Derivative amplifies HF noise.

Derivative filter!

→ Derivative "kick" during setpoint change.

Derivative - on - measurement!

→ Integrator can saturate output. Integrator anti-windup!



→ Limits on system input amplitude.

Clamp Controller output!

→ Choosing a sample time T.

$T_{controller} < \frac{T_{BW, system}}{10}$

## Code Structure

- header-file "library" (`#include "pid.h"`)
  - PID controller struct. (Contains gains, storage variables, ...)
  - Initialization function. (`set gains, sample time, ...`)
  - Update function.
    - Updation function (provide setpoint and measurement, returns controller output)