

Rotational Derivative

Matthew Hogencamp

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Rotational Derivative: A Prototype Idea

This document describes a non-rigorous, experimental idea for a rotational analog of the derivative applied to sinusoidal functions like $\sin(cx)$.

The hypothesis is that repeated differentiation corresponds to rotation in the complex plane, and that a general form of the n^{th} derivative of $\sin(cx)$ may follow the form:

$$D^n[\sin(cx)] = c^n \cdot \sin\left(cx + n \cdot \frac{\pi}{2}\right)$$

Motivation: This form encodes both amplitude scaling and phase shifting as functions of n . I am exploring whether this rotational framework can be extended to define fractional derivatives of sinusoidal functions, and possibly more general functions.

Disclaimer: This is a speculative, non-rigorous exploration. I am not yet sure if this idea aligns with existing fractional calculus as I have only taken up to differential equations (ODEs), intro to prob and stats, and multivar and vector calc.

Implementation: A simple MATLAB script is included to test the formula numerically at specific input values of x , c , and θ .

```
>> rotationalDerivative
Please enter theta (derivative value): 0
Please enter c value: 1
Please enter a specific x value: 0

x =
     0

ans =

    "The theta Rotational Derivative is Equal to..."

result =
     0

>>
```

Figure 1: Software properly returning the 0th derivative