Rotational Derivative

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June 15th, 2025

Rotational Derivative: A Prototype Idea

This document describes a non-rigorous, experimental idea for a rotational analog of the derivative applied to sinusoidal functions like $\sin(cx)$.

The hypothesis is that repeated differentiation corresponds to rotation in the complex plane, and that a general form of the n^{th} derivative of $\sin(cx)$ may follow the form:

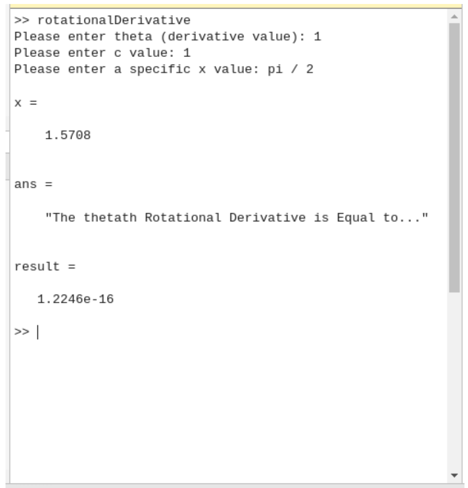
$$D^{\theta}[\sin(cx)] = c^{\theta} \cdot \sin\left(cx + \theta \cdot \frac{\pi}{2}\right)$$

Motivation: This form encodes both amplitude scaling and phase shifting as functions of n. I am exploring whether this rotational framework can be extended to define fractional derivatives of sinusoidal functions, and possibly more general functions.

Disclaimer: This is a speculative, non-rigorous exploration. I am not yet sure if this idea aligns with existing fractional calculus as I have only taken up to differential equations (ODEs), intro to prob and stats, and multivariable and vector calculus. I am an Engineering student at RIT, so, I am not the most confident in my mathematical abilities. Therefore, if anybody has any issues with this definition, or knows a bit about math and wants to reach out to me, please feel free to do so.

Implementation: A simple MATLAB script is included to test the formula numerically at specific input values of x, c, and θ.

Figure 1: Software properly returning the 0th derivative



```
>> rotationalDerivative
Please enter theta (derivative value): 1
Please enter c value: 1
Please enter a specific x value: pi / 2

x =

    1.5708


ans =

    "The thetath Rotational Derivative is Equal to..."


result =

   1.2246e-16

>>
```

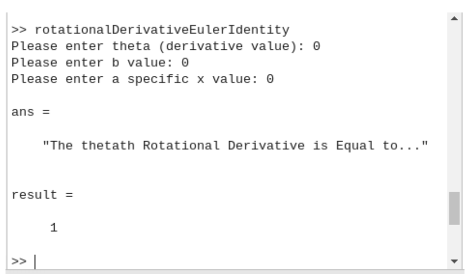
Figure 2: Software returning approximation of 1st derivative

Notably, the exponential function can be rewritten in terms of sinusoidal functions via **Euler's Identity**, letting D^{θ} be a **linear operator**: just like the integer-defined derivative.

$$D^{\theta}[e^{ibx}] = D^{\theta}[cos(bx)] + D^{\theta}[isin(bx)]$$

This allows the following formula after some derivations using the rotational definition of the derivative.

$$D^{\theta}[e^{ibx}] = (ib)^{\theta} \cdot e^{ibx}$$



```
>> rotationalDerivativeEulerIdentity
Please enter theta (derivative value): 0
Please enter b value: 0
Please enter a specific x value: 0

ans =

    "The thetath Rotational Derivative is Equal to..."


result =

     1

>>
```

Figure 3: Software returning proper input for all 0's

This could allow for the real exponential to be defined too, using complex exponentiation.

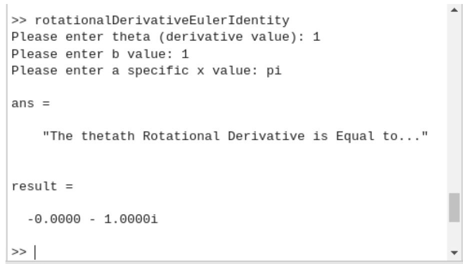


Figure 4: Software returning approx value for $1 = \theta, 1 = b, \pi = x$

Pathway for Generalizing The Derivative?

So, what has been proposed is a generalization of the derivative that is based on the assumptions that...

- The Rotational Derivative is a linear operator $\forall \theta \notin \mathbf{Z}$
- $D^{\theta}[sin(cx)] = c^{\theta} \cdot sin(cx + \theta \cdot \frac{\pi}{2})$

From these assumptions, I see this as the path of development for this derivative generalization...

- Define $D^{\theta}[x^n]$ using **Fourier Series**
- Define $D^{\theta}[e^{ax}]$ using complex exponentiation with the proposed definition obtained via Euler's Identity, or via Taylor Series after defining rotational differentiation for x^n.
- Define all other functions using Taylor Series or Fourier Series going from here (ex. Rational Functions, Logarithms, Factorials, etc.)

Here are some flaws that there might be in my conceptual roadmap...

- There might be a class of functions that **cannot be defined in relation to anything that could be constructed from the set of functions for which Rotational Differentiation is defined for.**
- There may be a class of functions whose integer derivatives disagree with their rotational derivatives $\forall \theta \in \mathbf{Z}$
- As an engineering student, I may be missing something central to the field of Mathematics.

Regardless of these concerns, I hope that this idea left you intrigued!