COMS 6998: Deep Learning for Robotic Manipulation (Spring 2026)

Homework 0

Release: Jan. 23, 2026. *Due:* Jan. 29, 11:59 pm (ET). **No late submissions will be accepted.**

Instructions. Show your work and state assumptions. You may discuss high-level ideas with classmates, but your write-up (and any code/figures) must be your own. Cite all external sources. List collaborators (names/UNIs) at the end of your PDF.

Formatting & Figures. Typeset in LaTeX and label subparts to match problem numbering. Use SI units (meters) and radians; positive rotations are CCW. Round numerical values to 3 decimals unless stated otherwise. Embed figures in the PDF (no external links), label axes/frames clearly, and use vector graphics (PDF) or high-resolution PNG. Sketches that illuminate your thought process (e.g., the robot and its environment) are encouraged.

Submission. Submit a *single PDF* on Canvas (recommended filename: `hw0_Last_First_UNI.pdf`). Verify that text, math, and figures render correctly.

Tools & AI. Calculators and Python/MATLAB are permitted for arithmetic/plots, but *show intermediate analytic steps*; if you use code, include minimal snippets in an appendix and cite libraries. AI tools are permitted for formatting/proofreading only (e.g., LaTeX help); do not use AI to derive or check solutions.

Regrades. Request within 7 days of grade posting on Canvas with a 1–3 sentence justification. The entire problem may be re-evaluated.

Accessibility/Extensions. For documented accommodations or emergencies, contact the instructor/TA prior to the deadline when possible.

Problem 1: Homogeneous Transformations (10 points)

In this problem, you will practice converting points to homogeneous coordinates, constructing homogeneous transformation matrices, and transforming points between coordinate frames.

Setup. A point p is given in frame $\{A\}$ with Cartesian coordinates

$$p_A^{\text{cart}} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix},$$

where $p_A^{\text{cart}} = [x \quad y \quad z]^T$ denotes the position of the point in 3D *Cartesian coordinates* (i.e., ordinary coordinates without a homogeneous component).

Frame $\{B\}$ is defined relative to frame $\{A\}$ as follows:

- $\{B\}$ is rotated by $90°$ about the z-axis (counter-clockwise) relative to $\{A\}$.
- After the rotation, $\{B\}$ is translated by $t = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ meters relative to $\{A\}$.

Tasks.

1. **Convert to homogeneous coordinates (1 point).** Convert p_A^{cart} to homogeneous coordinates by appending a 1 as the fourth element, i.e., $p_A = \begin{bmatrix} p_A^{\text{cart}} \\ 1 \end{bmatrix}$. Write p_A explicitly as a 4×1 vector.

2. **Construct ${}^{A}T_{B}$ (4 points).** Construct the 4×4 homogeneous transformation matrix
$$ {}^{A}T_{B} = \begin{bmatrix} & R & & t \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$
where R is the rotation matrix corresponding to a 90° rotation about the z-axis. Write ${}^{A}T_{B}$ explicitly.

3. **Transform the point (3 points).** Compute the coordinates of p expressed in frame $\{B\}$ using
$$ p_B = {}^{B}T_{A}\, p_A, \qquad {}^{B}T_{A} = \left({}^{A}T_{B}\right)^{-1}. $$
Show your work step by step, including the computation of ${}^{B}T_{A}$.

4. **Interpret the result (2 points).** Describe qualitatively what happened to the point p: How did its x and y components change under a 90° rotation? Does the translation shift the point in the direction you expect? Optionally sketch both frames and the point before and after the transformation to verify that your result is reasonable.

Hint. Points in homogeneous coordinates (last entry = 1) are affected by both rotation and translation. Direction vectors (last entry = 0) are affected only by rotation.

Problem 2: Forward Kinematics (2R Planar Arm) (10 points)

Derive forward kinematics for the two–link planar arm in Fig. 1 using geometry and composition of planar rigid transforms.

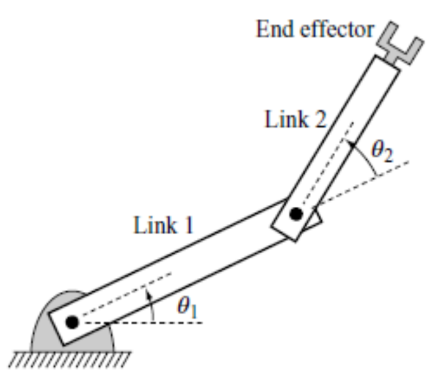

Figure 1: Two-link planar arm with joint angles θ_1 (shoulder) and *relative* elbow angle θ_2.

Robot Model. Revolute joints about $+z$; link lengths $L_1 = 1.0$ m and $L_2 = 0.8$ m. Base frame $\{0\}$ at the shoulder; end-effector frame $\{E\}$ at the tip of Link 2 with x_E along Link 2. Angles are CCW; θ_2 is the angle of Link 2 *relative to Link 1*.

Conventions. Use radians; round all numerics to 3 decimals.

Tasks.

1. **Geometric FK for position & orientation (4 points).**

   (a) Write a *vector* expression for the end–effector position $p_E = [x\ y]^T$ in the base frame by composing two planar rotations and translations (first along Link 1, then along Link 2). Do *not* skip intermediate steps. (2 pts)

   (b) From your vector expression, derive explicit *scalar* formulas for $x(\theta_1, \theta_2)$ and $y(\theta_1, \theta_2)$. (1 pt)

   (c) Determine the end–effector orientation $\phi(\theta_1, \theta_2)$ in the plane and justify your answer in one sentence. (1 pt)

2. **Pose in $SE(2)$ (3 points).**

   (a) Assemble the homogeneous transform ${}^0T_E$ using your $x(\cdot)$, $y(\cdot)$ and $\phi(\cdot)$: a 3×3 matrix with the 2×2 rotation (by ϕ) and the (x, y) translation. (2 pts)

   (b) Express ${}^0T_E$ as a product of two elementary planar rigid transforms: a base–to–link1 transform followed by a link1–to–EE transform. Write each transform explicitly (rotation about z then translation along the local x–axis), but you do *not* need to multiply them out. (1 pt)

3. **Numeric evaluation (2 points).** For $\theta_1 = 30°$ and $\theta_2 = 60°$ (convert to radians), compute numerical values of:

   $$\phi, \quad x, \quad y, \quad \text{and the full matrix } {}^0T_E.$$

   Show the numeric substitutions you used.

4. **Tool offset (gripper) (1 point).** The gripper tip frame $\{G\}$ is translated from $\{E\}$ by $d_g = 0.10$ m along x_E (no extra rotation). Define ${}^ET_G$ accordingly and write ${}^0T_G = {}^0T_E\,{}^ET_G$. Compute the numeric gripper tip position (x_G, y_G) for the angles in part (3).