Adaptive Formation Control for Route-following Ground Vehicles

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ABSTRACT

A novel cooperative path planning framework is presented for maintaining formations along a desired, unknown route where spatial and temporal objectives must be considered. It uses a reference frame based on longitudinal spacing along the route and lateral orthogonal offsets to plan for route clearance and traversal around previously unknown obstacles. By defining desired positions in terms of offsets from the route, the spatial and temporal components can be decoupled. The spatial components are planned using a two-step planner for fast real-time planning. The spatially defined paths are passed to a speed adaptation algorithm for temporal considerations including inter-vehicle collision avoidance. The approach balances real-time obstacle avoidance, spatial structure of the formation, and inter-vehicle safety considerations.

Keywords: Formation control, coordinated control, platooning, distributed receding horizon control

1. INTRODUCTION

Recent years have proven formation control to be a canonical problem for multi-vehicle systems.[1–3] It allows designers of multi-agent systems to prove out collaborative architectures in terms of both available communication and control methodologies with both visual and provable results.[1, 4] Moreover, it is highly applicable in different scenarios including aligning sensor arrays,[5] fuel efficient movement,[6] as well as organized road or highway traversal.[6, 7]

Consider the problems of snow removal and route inspection. Vehicles must traverse in a formation to ensure clearance of a route for the safety of vehicles that follow. However, due to environmental conditions in snow removal or the possibilities of unknown areas for route inspection, the vehicles may not be able to depend on previously identified lanes for coordination. Moreover, they may need to adapt the formation to move around previously unknown obstacles. Finally, the vehicles may be quite large and their sluggish accelerations must be taken into account for safety considerations. In this paper, methods are developed and combined to produce an adaptive formation control framework well-suited to such problems.

Formation control in the typical sense is concerned with the temporal structure of the formation with the goal of maintaining some geometric configuration.[1, 5, 8] While such formulations have many applicable realms, they may not be fitting for large vehicles operating on highly constrained routes. Obstacle and inter-vehicle collisions necessitate the need to adapt the configuration of the vehicles for successful traversal. Moreover, purely geometric structures may prove difficult for large vehicles as route curvature could require vehicles to frequently change speed to maintain formation integrity.

Instead of a geometric approach to formation control, the proposed adaptive formation control approach is based on the idea of vehicle platooning.[7] Platooning takes into account the structure of the route being followed as the vehicles are configured with respect to the route instead of with respect to a given geometric configuration. While snapshots at different points in time may appear to have little temporal structure, the vehicles' paths are spatially structured, as shown in Figure 1. This allows vehicles to execute line-haul or create packs for fuel efficient driving.[6, 7, 9] Similar techniques were even used in Ref. 10 with the addition of timing constraints along the path to create an identifiable geometric configuration.

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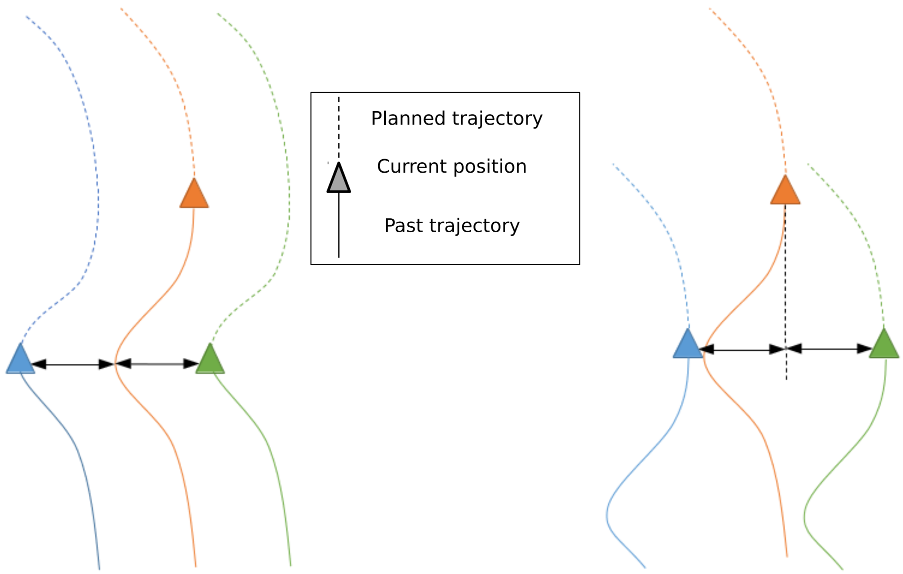


(a) Spatially-coordinated formation (b) Temporally-coordinated formation

Figure 1: In a spatially coordinated formation (a), coordination is seen in the traversed paths. In a temporally coordinated formation (b), coordination can be seen at any given snapshot in time, but the paths may not appear coordinated.

However, many platooning approaches are not well-suited for adapting to unknown or partially known environments as they are highly dependent upon the road structure for operation, (see Ref. 7 and the references therein). They depend upon lane definitions to ensure lateral inter-vehicle collision avoidance and they do not consider unknown environments.[6, 9, 11] They also do not take advantage of the route definition for longitudinal spacing, depending rather on Euclidean distance,[6] which leads to difficulty around turns.[12]

In this paper, we combine receding horizon planning with a modified platooning approach to cooperatively adapt to the environment while avoiding inter-vehicle collisions. Given a route to traverse, Euclidean planar geometry is replaced with longitudinal and lateral spacing defined with respect to the route. The lateral spacing is planned with a receding horizon approach to avoid obstacles while considering coordinated objectives. The longitudinal spacing is done using virtual spring speed adaptation designed to account for the sluggish accelerations of the vehicles. The output of the lateral plan and speed adaptation is passed into a low-level receding horizon controller which uses high-fidelity vehicle motion models over a smaller time horizon to actually control the vehicle.

The focus of this paper will be on the route based lateral planning and longitudinal speed adaptation with a brief overview of the other components. It will proceed as follows: Section 2 will give an overview of the adaptive formation control approach. Sections 3 and 4 will then detail the lateral and longitudinal spacing respectively. The paper will end with experimental results in Section 5 and concluding remarks in Section 6.

2. BACKGROUND

For safe and successful operation of multiple vehicles in an unknown or partially known environment, an autonomy paradigm based on 4D/RCS[13] is employed. Along with development of sensor processing and world modeling capabilities, the paradigm consists of a hierarchy of motion planners. This section briefly describes the adaptive formation control formulation, including how it fits within the hierarchy. A description of the underlying route-based coordinate system is given, and background is discussed for its two central components: lateral path planning and longitudinal speed control.

2.1 Adaptive Formation Control Overview

Adaptive formation control constitutes the middle of a three tiered hierarchy of path planners being developed: vehicle-, maneuver-, and primitive-level. The vehicle-level planner uses a planning horizon on the order of kilometers and is updated upon request. It also determines how the vehicles should traverse the route. The maneuver-level plans over the sensor horizon of the vehicle to produce kinematically feasible paths. Finally, the primitive-level planner takes in kinematically feasible paths and produces dynamically feasible control inputs.

While a general form for spatial and temporal constraints/objectives can be formulated, as done in Ref. 14, we identify several constraints and objectives that are vital to the successful clearance of a route, as described in Section 1, which are explicitly considered in this paper. The vehicles must satisfy the following constraints:

1. Obstacle avoidance: Vehicles must be able to plan around obstacles that may be previously unknown to the route planner.
2. Lateral path constraints: Vehicles must be capable of planning coordinated paths which are constrained as to their overlap*
3. Kinematic feasibility: The produced path must plan from the current pose of the vehicle, have a constrained curvature, and the execution must consider the acceleration capabilities of the vehicle.
4. Inter-vehicle collision avoidance: The vehicles must not come into contact with each other.

Within the above constraints, the vehicles can balance various objectives to improve the route traversal:

1. Planning-stack solution consistency: Plan the path and execution such that they stay well within the bounds required by different elements in the planning stack instead of upon the fringes of the constraints.
2. Lateral path objectives: Encourage desired lateral spacing of the vehicles along the route.
3. Longitudinal spacing: Maintain spacing between vehicles operating within the same lane.

To account for all of these constraints and objectives in real-time execution, a reference frame is defined which allows for the decoupling of the spatial and temporal considerations. As traversal is known to be with respect to a route, the spatial planning can be further simplified by planning in a single dimension consisting of the offset of the route. This allows for fast consideration of vehicle kinematics when planning around previously unknown obstacles. Furthermore, as the temporal considerations are decoupled from the spatial considerations, a speed adaptation method can be employed at an even faster rate to account for sudden changes in the motion of other vehicles. Thus, a three-stage process is designed to create and execute the coordinated paths:

1. A virtual leader path is derived by interpolating and smoothing the input route and extracting a spatial index.
2. Spatially coordinated paths are planned by each vehicle with respect to lateral offsets from the virtual leader path. The coordinated paths satisfy the kinematic requirements of pose, curvature, and obstacle avoidance as well as the path overlap objectives and constraints.
3. A speed adaptation technique is used based on longitudinal distances between the vehicles to maintain vehicle spacing. The speed adaptation ensures satisfaction of both the kinematic acceleration constraint as well as the inter-vehicle collision avoidance constraint.

Note that any number of smoothing techniques could be used to generate a sufficiently smooth virtual leader path from the given route. Thus, for sake of brevity, the particulars for the virtual leader path generation are not given.

*Lanes of a road are an example of a non-overlapping constraint while clearance of a road as done in route sweeping or snow removal requires an overlapping constraint.

2.2 Route Based Coordinate System

A key aspect of the adaptive formation control is to take advantage of the structure of the route following problem through defining a coordinate system based upon the route. This allows inherent coordination with respect to the route as planning is done in terms of lateral offsets from the route and longitudinal spacing along the route. This coordinate definition greatly simplifies the representation of constraints and objectives for the coordinated route traversal.

Intuition of the benefit of such an approach is gained through the examination of a typical road. While the road will curve along the lay of the land or the desired route, the center of each lane corresponds to a constant offset from center of the road. Nominal safe driving constitutes driving within the lane, bounded by upper and lower offset constraints. To avoid inter-vehicle collisions in the nominal traversal of the road, safe drivers do not maintain a specified euclidean distance (often travelling only a few feet apart from the vehicle in the next lane), rather they maintain a longitudinal spacing with respect to preceding vehicles in the same lane. Further temporal constraints on the paths executed by separate vehicles must then be considered by drivers in the special cases of lane merging or changing.

As mentioned, the route can be taken and smoothed to form the virtual leader path. The path is spatially indexed by s, which is the longitudinal coordinate. To define the position of the vehicle, a lateral component $r(s)$ is defined. Along with the virtual leader path, $q_l(s)$, these two components can be used to express the desired position of the vehicle in Cartesian coordinates as:

$$q_d(s) = q_l(s) + r(s)h_o(s) \tag{1}$$

where q_d denotes the desired position and $h_o(s)$ is a unit vector orthogonal to the virtual leader motion at s. While the mapping between Cartesian coordinates and the sr coordinate frame is not bijectional in general, assuming route bounds (i.e. edges of the road) and a relatively small curvature, a significant region (if not all) of the desired traversable portion of the route has a bijectional mapping between the two coordinate systems.

2.3 Lateral Path Planning

The first element of the adaptive formation control consists of planning coordinated paths which are offset from a virtual leader trajectory. It is based upon the recent work in Ref. 10 and Ref. 14 where formations are defined with respect to a defined route. While the goal of Ref. 10 was largely to use additional timing constraints to create a temporally structured formation, the work in Ref. 14 fostered the idea of spatial formations, as shown in Figure 1. In many respects, the coordinated path planning in this paper is a simplified version of the planning presented in Ref. 14.

To understand the difference between Ref. 14 and the method taken in this paper it is helpful to examine the equation for the desired position. In Ref. 14, the position included a longitudinal spacing parameter, δ:

$$q_d(s) = q_l(s + \delta(s)) + r(s)h_o(s + \delta(s)), \tag{2}$$

where $\delta(s)$ and $r(s)$ can be planned together to account for both temporal and spacial considerations. Thus, the simplification from[14] comes from the removal of the longitudinal spacing parameter, as can be seen by comparing equations (1) and (2).

By so formulating the problem, spatial planning and temporal planning can be separated, resulting in two main benefits. The first being that temporal planning can be run at a high rate for inter-vehicle collision avoidance, while the spatial planning problem (complicated due to obstacles in the environment) can be solved at a slower rate. The second main benefit comes in the form of the simplification of objectives and constraints, detailed in Section 3.

2.4 Longitudinal Speed Control

The reformulation of the desired position from (2) to (1) allows the planning of the lateral offsets to be decoupled from maintaining longitudinal spacing. Thus, the longitudinal spacing can be maintained at a very high rate to

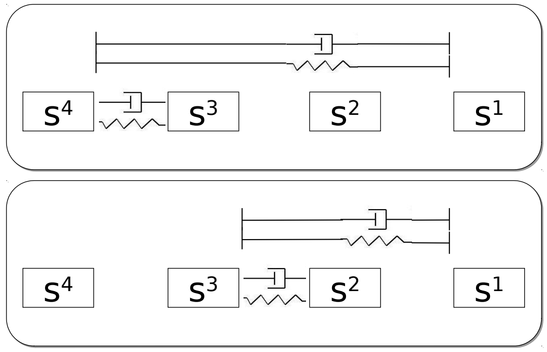


Figure 2: A virtual single directional spring damper system is shown for both vehicles 3 and 4. Note that although vehicle 4 is influenced by vehicle 3, vehicle 3 is not influenced by vehicle 4.

account for sudden or unexpected movement of other vehicles. Reactive speed adaptation methods are employed to ensure vehicle safety.

While many different methods exist for the speed adaptation of vehicle platoons,[6, 7, 9, 11, 12] we utilize a virtual spring method due to its intuitive nature and its ability to fit well into the virtual leader path framework. The methods consist of adding a virtual spring damper system between different vehicles to allow the vehicles to maintain longitudinal spacing with respect to the surrounding vehicles. This allows for an approach where the spring and damper exert a virtual force upon the vehicles when they are displaced in terms of distance or speed.

Although intuitive in nature, care must be taken to ensure that disturbances causing vehicles to change speed unexpectedly do not cause undesired oscillation in the movement of other vehicles. In Ref. 15, different interaction topologies were evaluated using a system of springs with the lead vehicle's velocity as the input and disturbances allowed in the spacings of the vehicles. Evaluation of the interaction topologies found that a virtual system of two single-directional springs[†] (one connected to the predecessor and one connected to the leader) will guarantee that no harmonic frequencies exist with respect to the leader velocity and that the gain from disturbances to the positions is bounded. The analysis determined that the connection to the leader vehicle acted much like a feed forward reference signal to attenuate all frequencies of oscillation.

To account for the undesired "accordion effect" of a virtual spring system, both Ref. 12 and Ref. 6 have proposed the introduction of two additional physics based components to the speed adaptation. The first is a virtual damper placed alongside the virtual spring to critically dampen the system, directly removing much of the oscillation. The second component is a torsional spring to reduce disturbances caused by the curvature of the road. This is meant to help with the fact that the Euclidean distance measured does not account for the actual longitudinal distance with respect to the route.

While the aforementioned work is largely concerned with the steady state maintaining of distances and the rejection of disturbances in the transients, the work in Ref. 16 focuses on establishing safe operating conditions. The central concept developed for safe operation being to avoid what are termed "reachability sets." If a vehicle is inside a reachability set and the preceding vehicle performs the worst possible motion (i.e., slamming on the brakes), the vehicle is guaranteed to collide with the preceding vehicle. For vehicle platoons, it was found that the reachability sets are a function of the deceleration ability of the vehicles. Thus, if a virtual spring-damper system is used, a virtual spring connected to the preceding vehicle must be sufficiently stiff to avoid the reachability sets.

3. PLANNING FOR LATERAL SPACING

Given a spatially indexed, sufficiently smooth virtual leader path, the formation control problem can be developed in terms of the spatial and temporal components. This section will develop the spatial planning component of

[†]Single-directional meaning that while the virtual spring exerts a force upon the follower vehicle, it exerts no such force on the preceding vehicle, as depicted in Figure 2.

the adaptive formation control while Section 4 will develop the temporal planning using adaptive speed control.

The spatial planning consists of a two step, receding horizon process where a discrete search over the lateral offsets is performed and fed into the inputs of a continuous spline which is then optimized locally. The path is fed into the speed control and the process is repeated. The discrete portion is largely responsible for a quick search over the potentially non-convex obstacle topology, the continuous portion is largely responsible for path feasibility, and the speed adaptation responsible for temporal constraints.

3.1 Discrete Search Through Obstacle Topology

The first component of the lateral offset planning consists of a discrete search over the obstacle topology to find the best path in terms of space for the vehicles. The desired trajectory defined in (1) is discretized in terms of the spatial index s into a vector of N equally spaced spatial indices

$$s = \begin{bmatrix} s_1 & \dots & s_N \end{bmatrix}^T .$$

Given that s_0 is the spatial index of the vehicle at the beginning of the optimization and Δ is the spacing between adjacent spatial indices, then $s_i = s_0 + \Delta i$. The discrete planner then searches through the topology of obstacles to find an appropriate offset for each spatial index. A slight abuse of notation is used for simplicity. Where $r(s)$ denotes a continuous mapping, we define the vector $r \in \mathbb{R}^N$ as:

$$r = \begin{bmatrix} r_1 & \dots & r_N \end{bmatrix}^T ,$$

where r_i corresponds to the offset at s_i.

The planning consists of nodes on multiple "slices", where each slice consists of the possible lateral offsets orthogonal to the corresponding spatial index of the virtual leader. Nodes on slices of adjacent spacial indices are connected by directional edges pointing from the previous slice to the next slice, while nodes on the same slice are not connected. A heuristically informed search over the possible offsets is performed as described in Ref. 14.

While the removal of an additional time component in the redefinition of the desired position in (1) removes the ability for the discrete planner to simultaneously perform temporal and spatial planning, it simplifies the design of costs and constraints in two significant ways:

1. Kinematic feasibility: Constrained curvature can be approximated by limits on the possible change of offset between two adjacent spatial indices. In Ref. 14, a constraint involving both the offset and time index would need to be formulated to ensure that the discrete planner did not plan paths orthogonal to the direction of motion.

2. Consistency with the route plan: This can be implemented through upper and lower limits for the terminal offset value. In Ref. 14, a temporal constraint would also be required to ensure that progress is being made along the route[‡].

Apart from being concerned with obstacle avoidance, the discrete planner takes into consideration the lateral spatial constraints and objectives. Leading vehicles utilize costs to encourage traversal through larger areas to consider subsequent vehicle motion. Following vehicles consider paths through the obstacles which allow for the satisfaction of spatial constraints with respect to preceding vehicles.

3.2 Continuous Splines for Path Refinement

The output of a discrete trajectory of offset values is passed as initial conditions into a control theoretic spline,[17] which allows for refined kinematic and constraint considerations due to its continuous representation of the trajectory. The reformulation in (1) has an even greater impact upon the simplicity of the cost and constraint representation for the continuous planner than the discrete planner. It allows the optimization to become a convex problem, greatly increasing the speed of optimization.

[‡]Note that a temporal constraint to require progression along the route could conflict with inter-vehicle collision avoidance if a preceding vehicle stopped moving

3.2.1 Spline Formulation

The spline provides a continuous representation of the offset which can in turn be refined to better meet the constraints and objectives of the problem being solved. Allowing $x_r(s) \in \mathbb{R}^2$ with the dynamic equation:

$$\frac{dx}{ds}(s) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(s) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(s), \tag{3}$$

the input $u(s) \in \mathbb{R}$ can be defined[§] in terms of a linear equation including the vector r such that

$$x(s_i) = \begin{bmatrix} r_i & \frac{dr_i}{ds} \end{bmatrix}^T . \tag{4}$$

This allows the result of the discrete planner to be used directly within a continuous representation, which can then consider orientation of the path and its derivatives for kinematic considerations. The formulation allows for the N values of r to be optimized while taking into consideration the effect that the change of r has on the continuous spline.

3.2.2 Objective and Constraint Evaluation

The problem constraints and objectives discussed in Section 2.1 are now considered in terms of the spline optimization:

1. Obstacle avoidance and spatial path constraints: Represented in terms of upper and lower bounds on each offset as shown in Figure 3 and expressed as $a \leq r \leq b$, where $\leq$ is in terms of each element[¶].
2. Kinematic feasibility (initial condition): The value for $x(s_0)$ and r_i can be adjusted as described in the Appendix to ensure that the initial position and orientation of the path match that of the vehicle.
3. Kinematic feasibility (curvature cost and constraint): Curvature of the path can be approximated by $u(s)$, which is the second derivative of the offset and a linear function of r. By penalizing $u(s)$, the curvature with respect to the route will be penalized. Thus, given a bounded curvature of the route and a soft constraint implemented in terms of a cost, the curvature of the path will be limited and encouraged to be that of the route.
4. Planning stack: In addition to the curvature cost, the constraints $a_N \leq r_N \leq b_N$ can be defined to not allow more deviation from the discrete planner terminal point than desired.
5. Spatial path constraints: objectives can be designed on r_i to balance traversal of the middle of the road (i.e., middle of a_i and b_i) with an offset from the middle to achieve the desired lateral spacing with respect to other vehicles.

3.2.3 Optimization Problem

The optimization problem to be solved can then take the form:

$$\min_r J(r) = \sum_{k=1}^{M} L(x_r(s_k), u_r(s_k)) + \Phi(x(s_M)) \tag{5}$$

subject to $a \leq r \leq b$ and the analytic solution for x_r and u_r in equations (17) and (12) of Appendix A. The function L is defined to balance curvature, middle of the road, and lateral spacing objectives while Φ can be used to penalize deviation from the discrete planner terminal point. Note that the final index M was used to denote a difference from N, the number of elements in r. Despite only the N parameters being optimized, a much finer evaluation of the offset trajectory can be used in evaluating the cost than that passed in from the discrete planner.

[§]Details are presented in the Appendix, including an analytic solution for $x(s)$.

[¶]Note that the values for a and b must be solved for as a pre-optimization step

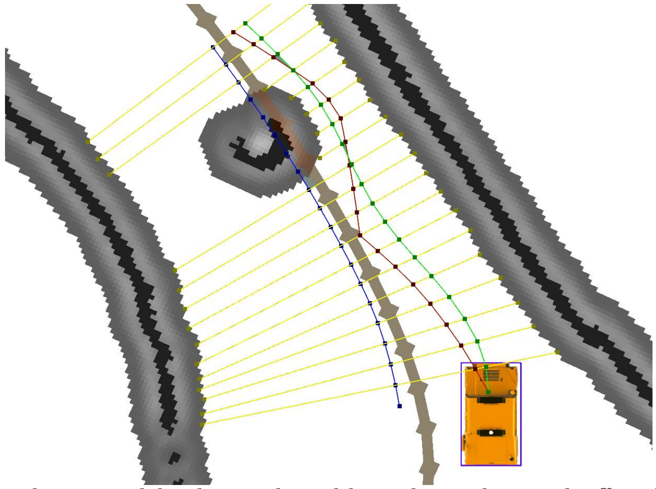


Figure 3: This image shows the virtual leader path in blue, the orthogonal offset lines in yellow, the discrete plan in red, and the continuous plan in green.

4. SPEED ADAPTATION FOR LONGITUDINAL SPACING

The final component of adaptive formation control consists of the execution of the planned path. The execution must satisfy temporal constraints mainly concerned with inter-vehicle collision avoidance and acceleration constraints. For safety considerations, it is important that the vehicle spacing be monitored and updated at a high rate. Moreover, the longitudinal speed adaptation must seek to satisfy desired vehicle spacings in a smooth, non-oscillatory fashion. To accomplish these tasks, this Section presents various modifications to the physics based interaction models presented in Section 2.4.

Based on the interaction topology analysis in Ref. 15, a double spring system is used along with dampers as done in Ref. 6, 12 with some modifications to achieve the desired behavior:

1. Distance is measured according to the spatial index of the virtual leader path to avoid the need for a torsional spring to account for route curvature.
2. An additional stiff spring is added, acting much like a door-stop where a force is exerted only if the spring is compressed, to avoid reachability sets[‖].

The resulting equation for the desired velocity for vehicle i is given by:

$$\begin{aligned} F^i = & \underbrace{-k_{sp}(s^i - s^{i-1} + d^{i,i-1}) - k_{dp}(v_d^i - v_d^{i-1})}_{\text{Spring-damper wrt preceding vehicle}} \\ & \underbrace{-k_{sl}(s^i - s^1 - d^{i,1}) - k_{dl}(v_d^i - v^l)}_{\text{Spring-damper wrt to lead vehicle}} \\ & \underbrace{-k_{sd}(s^i - s^{i-1} + d_d^i)}_{\text{``Door-stop'' spring}} \end{aligned} \tag{6}$$

$$\dot{v}_d^i = m^i F^i \tag{7}$$

where the superscript denotes the vehicle index, F^i is the virtual force, m^i is the virtual mass, s^i is the spacial index of vehicle i as projected onto the virtual leader path, $d^{i,j} \geq 0$ is the desired lateral spacing between vehicles

[‖]To ensure that reachability sets are not entered, the spring between a vehicle and its predecessor could be designed to be sufficiently stiff. However, this would cause the unnatural duplication of mild movements of a predecessor by its follower

i and j, k_{sp} and k_{dp} are the spring and damper coefficients related to the preceding vehicle, k_{sl} and k_{dl} are the spring and damper coefficients related to the leader (with v^l replaced with v^{vl} for the first vehicle in formation), k_{sd} is the coefficient for the safety or "door-stop" spring, and d_d^i is the distance at which it is activated (i.e. $k_{sd} = 0$ for $s^i - s^{i-1} + d_d^i \leq 0$).

A virtual leader speed is introduced to the lead vehicle in order to slow it down when other vehicles become displaced. When using a single-directional virtual spring-damper system, vehicles ignore all other vehicles behind them. By introducing a virtual leader speed, the lead vehicle will slow down to allow vehicles to regain their relative positioning. Originally inspired from the speed adaptation work presented in Ref. 5 to ensure formation integrity, a ramp function was found sufficient to adapt the speed. Allow the aggregate distance between the n vehicles to be defined as

$$\delta = \sum_{i=1}^{n-1} \min(d^{i,i+1}, s^i - s^{i+1}) - d^{i,i+1}, \tag{8}$$

the nominal velocity of the formation as v^{nom}, the threshold for starting the ramp down of the velocity as δ_1 , and the zero velocity threshold as δ_2. The virtual leader velocity can be defined as:

$$v^{vl} = \begin{cases} v^{nom} & \delta < \delta_1 \\ v^{nom}(1 - \frac{\delta - \delta_1}{\delta_2 - \delta_1}) & \delta_1 \leq \delta \leq \delta_2 \\ 0 & otherwise \end{cases} \tag{9}$$

The gains for the spring-damper systems can be designed to be critically damped as in Ref. 6, 12 to avoid oscillations. Furthermore, the vehicle acceleration abilities can be taken into account by saturating the desired acceleration by the minimum and maximum vehicle accelerations[**]. The desired velocity can also be saturated to avoid negative velocities or to exceed the maximum desired velocity of the vehicle.

While the "door-stop" spring is designed to avoid the "reachability sets" for safety, the remaining spring-dampers can be evaluated for stability in terms of converging to the desired steady state as well as bounded disturbance propagation throughout the system. The proofs for the stability and disturbance rejection properties discussed in Ref. 15 and Ref. 12 can be followed almost exactly The subtle differences include changing the input of the system to be the virtual leader velocity, expressing the differences between the integral of v_d^i and s^i as disturbances, and ignoring the "door-stop" spring in the analysis (with the assumption that d_d is sufficiently smaller than d_p to ignore its effect on the steady state stability).

Note that the approach taken in this section will allow vehicles in a single lane to operate safely. It will be the subject of future work to consider the cases of merging and/or changing lanes.

5. EXPERIMENTATION RESULTS

5.1 Simulation

Implementation

The planning and speed adaptation algorithms were implemented in C++, utilizing the Robot Operating System (ROS) framework.[18] Each planner was implemented in a separate process (ROS node). A distributed simulator was used with a separate computer for each vehicle and one for a ROS Stage simulator to simulate vehicle kinematics and lidar information In addition to the planners, each vehicle maintained its own world model. Each vehicle computer consisted of a 2.066 GHz Intel i7-4790T processor with 16 GB of 1600 MHz DDR3 RAM.

[**]Most likely conservative estimates to avoid the need to account for friction, drag, etc.

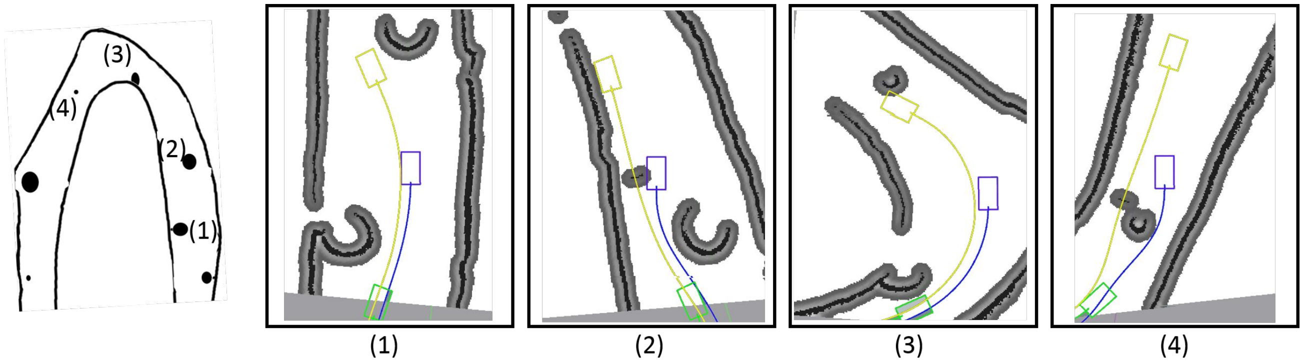


Figure 4: The far left image shows the simulated environment with numbers showing the approximate position of the snapshots in the right four images. The vehicles are shown as rectangles with the traversed paths extending from the rear of the rectangle. Note the noise in the center and right images from imperfect removal of the lead vehicle from the second vehicle's costmap.

Scenario

The simulation consisted of a route clearance scenario. The leading vehicles were constrained to have overlapping paths, with a maximum of 2 meters apart. The third vehicle was then constrained to stay within the path cleared by the previous two vehicles. The second vehicle had a desired spacing of 15 meters from the lead vehicle and the third had a desired spacing of 20 meters from the second.

The discrete planner used a 20 meter receding planning horizon with a 1 meter longitudinal resolution. The discrete planner was constrained to plan within 20 meters laterally from the commanded path. It used a search resolution of 0.2 meters with a maximum of 0.5 meter change in offset between points. A hard constraint on obstacle avoidance was implemented through the removal of edges and nodes. A soft constraint was implemented as a large cost for offsets which violated the vehicle overlap constraints. The soft constraint allowed for following vehicles to split around obstacles when necessary, as shown in Figure 4.

The continuous planner used the same 20 meter receding horizon. A 10 meter bound on the offset from the discrete planner was given, although it rarely used those bounds due to the obstacle limits, depicted in Figure 3, and the vehicle overlap constraints. The upper and lower bounds for vehicle overlap constraints were calculated from the preceding vehicle(s) executed paths, expanded to include the discrete planner when necessary.

Results

Three metrics are considered to measure the effectiveness of the adaptive formation control: lateral spacing between the lead vehicles, percent overlap time, and longitudinal vehicle spacing. The lateral spacing between the first and second vehicles averaged 1.64 meters, overlapping 85.49% of the time, with an average longitudinal spacing of 16.26 meters. The second and third vehicles had an average longitudinal spacing of 20.25 meters. The third vehicle stayed in the cleared path 74.96% of the time[††]. It is also worth noting that the discrete planner required an average of 0.045 seconds and the continuous planner required an average of 0.16 seconds to compute desired trajectories.

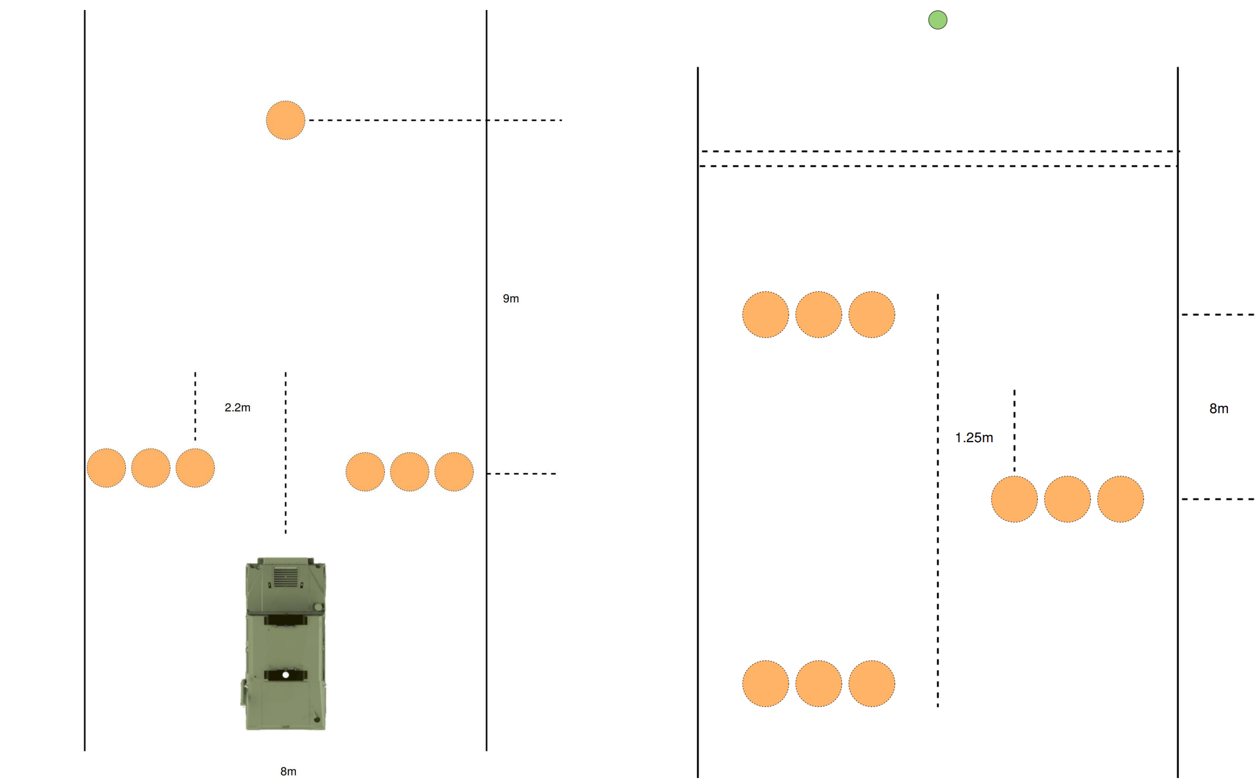
5.2 On Vehicle

Implementation

The formation control was also implemented and tested on the vehicles depicted in Figure 5. Implementation details are very similar to the simulation with the exception of sensory data being a combination of a small lidar and stereo vision. The sensor processing, world modeling, and planning were performed on individual computers within each vehicle. Communication between vehicles was performed using commercial 2.4 GHz wifi.

[††]Note that the percent overlap time is less than might be expected due to the noise from the inaccurate removal of preceding vehicle from the world model.

Figure 5: Vehicles that support the software used in testing. The two tan HMMWVs on the right were used in the test scenarios described in Section 5.2.



(a) Part 1: Gate and Dodge (b) Part 2: Slalom

Figure 6: Maneuverability Course

Test Scenarios

Three scenarios were tested with each scenario using the two tan High Mobility Multipurpose Wheeled Vehicles (HMMWVs) in Figure 5. Each scenario was run 5 times and the results are averaged over all runs for that scenario. The leading vehicles were constrained to have overlapping paths, with a maximum of 2 meters apart. The second vehicle had a desired spacing of 15 meters from the lead vehicle.

The first scenario involved traveling down an open two-lane road.The second scenario used the same two-lane road but added a maneuver course (Figure 6) to the beginning of the traveled section of road. Figure 7 shows an example of two other vehicles (used in previous testing) completing a maneuver course similar to the one specified in Figure 6. The third scenario involved traveling down a congested single lane road.

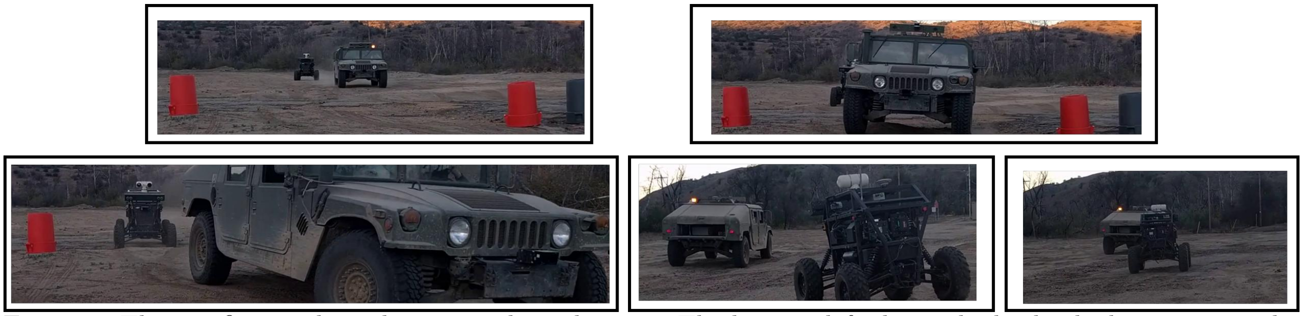


Figure 7: The top figures show the approach to the gate. The bottom left shows the lead vehicle moving to the right of the barrel. The bottom middle shows the second vehicle returning to the overlap path. The bottom right shows the vehicles again in overlapping paths.

Results

The same three metrics from the simulation testing are considered to measure the effectiveness of the adaptive formation control: lateral spacing between the lead vehicles, percent overlap time, and longitudinal vehicle spacing. For scenario one, the lateral spacing between the first and second vehicles averaged 1.14 meters and overlapped 100% of the time. The longitudinal spacing averaged 17.37 meters over the 5 runs. In scenario two, the first and second vehicles had an average lateral spacing of 1.11 meters, overlapped 98.5% of the time and maintained an average longitudinal spacing of 16.66 meters. In the third scenario, the two vehicles averaged 0.93 meters apart laterally, overlapped 100% of the time, with an average longitudinal spacing of 17.49 meters.

Table 1: Summary of results from the three scenarios.

	Scenario 1	Scenario 2	Scenario 3
Lateral Spacing (m)	1.14	1.11	0.93
Overlapping %	100	98.5	100
Longitudinal Spacing (m)	17.37	16.66	17.49

6. CONCLUDING REMARKS

A novel adaptive formation control framework was presented for maintaining formations along a desired, unknown route where spatial and temporal objectives must be considered. By defining the reference frame in terms of longitudinal spacing along the route and lateral offsets from the route, temporal and spatial objectives could be decoupled. The spatial considerations are taken into account through a two-step planning approach. A discrete component first evaluates the obstacle topology and a continuous component considers kinematic constraints of the vehicles. The output of the spatial planner is then given to a speed adaptation algorithm to take into account temporal considerations such as inter-vehicle collision avoidance and accelerations constraints.

The adaptive formation control was demonstrated in both a distributed simulator and actual vehicles. It was able to adapt the formation to the environment surrounding the route to avoid obstacles while balancing the objectives of the problem. Both spatial constraints defined in terms of path overlap and temporal constraints defined in terms of inter-vehicle collision avoidance were satisfied.

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APPENDIX A. CONTROL THEORETIC SPLINES

A.1 Control Theoretic Spline Overview

This appendix gives a very brief overview of control theoretic splines. For a detailed exposition, the reader is encouraged to refer to Ref. 17.

Allow $x(s) \in \mathbb{R}^n$ to be a state vector defined in terms of the linear dynamic equation

$$\frac{dx}{ds} = Ax(s) + Bu(s); x(0) = \mathit{0} \tag{10}$$

and output $y = Cx(s)$ with input $u(s) \in \mathbb{R}$. A vector, $l(s) \in \mathbb{R}^N$ can be defined in terms of the spatial indices such that the i^{th} component can be written as

$$l_i(s) = \begin{cases} Ce^{As_i - s}B & s_i > s \\ 0 & otherwise \end{cases} \tag{11}$$

with a Grammian $G = \int_{s_0}^{s_N} l(s)l^T(s)ds$ where $s_i > s_j \; \forall i > j$. It was shown in Ref. 17 that if the input is defined as

$$u(s) = \alpha^T G^{-1} l(t), \tag{12}$$

then $y(s_i) = \alpha_i$.

A.2 Adjusting Initial Conditions

Note that in (10) and (12) it is assumed that $x(0) = 0$. However, a non-zero initial condition is almost always desired. Since $x_1(0)$ corresponds to the initial offset, it must be non-zero for the plan to start at the initial position of the vehicle, as defined by (1). Moreover, using the matrices in (16), $x_2(0)$ corresponds to $\frac{dr}{ds}(0)$, which can be set to give the appropriate starting orientation of the vehicle. This is done by taking the derivative of (1) with respect to s, using the offset required for initial position and solving for $x_2(0)$ to ensure that $\frac{dq_d}{ds}(0)$ points in the direction of the initial vehicle orientation. Thus it is imperative to be able to start with arbitrary initial conditions.

Note that the solution for a linear system is known to be:[19]

$$y(s) = Ce^{A(s-s_0)}x(s_0) + \int_{s_0}^{s} Ce^{A(s-s_0-t)}Bu(t)dt \tag{13}$$

where the first component is zero for $x(s_0) = 0$. Given that $y(s_i) = \alpha_i$ when $x(s_0) = 0$, the solution for the non-zero initial condition can then be written as:

$$y(s_i) = Ce^{A(s-s_0)}x(s_0) + \alpha_i. \tag{14}$$

Thus, if the desired value for $y(s_i)$ is r_i, then

$$\alpha_i = r_i - Ce^{A(s-s_0)}x(s_0) \tag{15}$$

will produce the desired result.

A.3 Analytic Solution

Consider the simple two dimensional case with matrices:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{16}$$

The trajectory $y(s) = Cx(s)$ then interpolates the r_i values and the second dimension of $x(s)$ being the derivative to r_i, $\frac{dr}{ds}(s)$, with the input $u(s) = \frac{d^s r}{ds^2}$. In this scenario,

$$e^{At} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}.$$

Using algebra to simplify the equation into simple integrands, the solution for $x(s)$ can be expressed as

$$x(s) = \begin{bmatrix} x_{01} + s x_{02} \\ x_{02} \end{bmatrix} + \sum_{i=1}^{k-1} \tau_i s_i^2 \begin{bmatrix} \frac{s}{2} - \frac{s_i}{6} \\ \frac{1}{2} \end{bmatrix} + \sum_{k}^{N} \tau_i s \begin{bmatrix} \frac{s s_i}{2} - \frac{s^2}{6} \\ s_i - \frac{s}{2} \end{bmatrix} \tag{17}$$

where $\tau = r^T G^{-1}$ and $s_{k-1} \leq s \leq s_k$.

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