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Distributed Formation Tracking Control of Multiple Mobile Robotic Systems

Chunyu Chen[1], Yifan Xing[1], Vladimir Djapic[2], and Wenjie Dong[1]
[1]Department of Electrical Engineering, The University of Texas-Pan American, Edinburg, TX
[2]SPAWAR Systems Center Pacific, San Diego, CA

Abstract— **In this paper formation tracking control of multiple wheeled mobile robots is studied. The reference trajectory is considered as a virtual leader vehicle system while the real multiple vehicle systems are considered as follower agents. Chained-form systems and theories of cascaded systems and communication graph are introduced to design control methods for kinematic systems. In addition to distributed control algorithms for kinematic multi-vehicle systems, formation control of vehicles' dynamics is addressed with the aid of backstepping method, parametrical uncertainties of vehicles' mechanics are estimated by sliding mode control. Simulation results are presented to verify the proposed control laws.**

I. INTRODUCTION

Cooperation of multiple vehicles receive more attention in recent years as group performances guarantee higher efficiency and robustness. Before developing capabilities for multivehicle systems, it is important to study the coordination problem since individual vehicle in the group is expected to move in specific manners. Coordination of multivehicle systems can be transformed into consensus problem as all vehicle systems are supposed to converge to a reference state. From analysis of vehicle models, it is known kinematic of vehicle systems is a first-order nonlinear system, by linearization from state transform or other nonlinear control methods, vehicle kinematic system can be stablized and converge to consensus values. Consensus on multiple single-integrator systems is addressed in [1][2][3][4][5][6][7]. Consensus control algorithms for multiple second-order systems are studied in [8][9][10]. In [12], The authors use decentralized discrete-time block control scheme to achieve formation and trajectory tracking, both first-order and second-order systems are discussed. Each agent is provided with discrete-time state observers and formation tracking can be achieved by applying the proposed block control with a consensus scheme. In [13], distributed consensus tracking algorithms without velocity measurements under both fixed and switching network topologies are proposed for a leader-follower communication pattern. A mild connectivity requirement is adopted for distributed consensus tracking and swarm tracking.

Formation tracking is one common problem of multiple vehicle coordination, unlike centralized formation tracking control, distributed formation tracking control utilizes information of vehicle's own states and that from its neighbors. Distributed formation control is addressed in [14][15][16][17]. In [14], a new kinematic model for leader-follower system is addressed, globally stable controller is designed with the aid of backstepping methods. In [15][16], formation stabilization for multiple nonholonomic mobile robots is addressed and distributed control methods are studied for vehicles kinematic systems with the aid of cascaded system, a leader-follower graph is applied to describe the communication topology of multivehicle systems. In [17], Distributed formation tracking methods are addressed with the aid of σ process by suitable variable transformation. Delayed communication is considered for the proposed controllers. In [18], a novel formation control technique of multiple wheeled mobile robots employing artificial potential field based navigation is addressed , the communication graph is considered as a leader-follower topology, the leader motions by artificial field and followers motion by control.

In real-life operations, control of kinematics is not practical due to the unavailability of velocities, engine-generated torques actually control the motion of vehicles. From analysis of vehicle dynamics, ut is learned parametrical uncertainties exist in the dynamic systems. Uncertainties are either estimated by adaptive control methods [22][23] or through neural networks' on-line training process [24][25]. In [22][23], parametrical uncertainties are estimated by adaptive control methods and a dynamics-based controller is designed with the aid of kinematic-based controller and backstepping methods. In [24][25], Neural networks are applied to estimate the dynamics of the robots, computed-torque controller is designed with the aid of kinematic-based formation controller and backstepping methods.

In this paper, distributed formation tracking of multiple wheeled unicycles is considered. Variable transformations are utilized to change vehicle' kinematic system into chained-form system. Exponential stability of cascaded system is introduced for controlling distributed formation tracking algorithms for transformed kinematic systems, exponential stability of the chained-form systems is proved to guarantee all vehicles track the desired reference trajectory with fixed formation. Distributed controller for vehicles' dynamics is designed with the aid of backstepping methods and kinematics-based controller. Sliding mode control method is applied to estimate the parametrical uncertainties. Compared with the distributed algorithms in [27], the control methods proposed in this paper is more practical and implementable since the derivatives in kinematic control part is removed. Besides, the variable transformation and control laws are conciser compared with that in [27], which also simplifies the control in real-life operations. Simulations have been done to prove the proposed control algorithms.

II. PROBLEM STATEMENT

With the aid of Lagrange-D'Alembert principle, m unicycle systems with three generalized states are defined as

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + J^\top(q_{*j})\lambda_j \quad (1)$$

$$J(q_{*j})\dot{q}_{*j} = 0 \quad (2)$$

Then derivatives of generalized states $\dot{q}_{*j}$ can be defined as

$$\dot{q}_{*j} = g(q_{*j})V_{*j} = \begin{bmatrix} \cos\theta_j & 0 \\ \sin\theta_j & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_j \\ \omega_j \end{bmatrix} \quad (3)$$

The communication graph between vehicles as well as the Laplacian matrix are defined the same way as in [27], due to space limitation the details are limited here.

The time-varying reference signal is defined by a sinsudal trajectory

$$\dot{x}_0 = v_0\cos\theta_0, \quad \dot{y}_0 = v_0\sin\theta_0, \quad \dot{\theta}_0 = \omega_0 \quad (4)$$

the desired formation is denoted by $\mathcal{P}$ with a group of orthogonal coordinates $p_j = (p_{jx}, p_{jy})$ for $(1 \le j \le m)$, p_j satisfy $\sum_{j=1}^{m} p_{jx} = 0$ and $\sum_{j=1}^{m} p_{jy} = 0$. $p_0 = (p_{0x}, p_{0y})$ for the virtual leader is supposed to be $(0, 0)$.

Distributed formation tracking control is defined as designing control laws $V_{*j} = [v_j, \omega_j]$ for system j by using its own state information and its neighbors' state information such that

$$\lim_{t\to\infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix} \quad (5)$$

$$\lim_{t\to\infty}(\theta_i - \theta_0) = 0 \quad (6)$$

$$\lim_{t\to\infty}\left[\sum_{i=1}^{m}\frac{x_i}{m} - x_0\right] = 0, \lim_{t\to\infty}\left[\sum_{l=1}^{m}\frac{y_i}{m} - y_0\right] = 0 \quad (7)$$

for $1 \le i \ne j \le m$.

In order to transform kinematic system into chained-form system, we utilize the state transform proposed in [27] and have

$$\dot{q}_{1j} = v_{1j} \quad (8)$$

$$\dot{q}_{2j} = v_{2j} \quad (9)$$

$$\dot{q}_{3j} = v_{1j}q_{2j} \quad (10)$$

The virtual leader in (4) is also transformed to the chained-form system in (8)-(10) as

$$\dot{q}_{10} = v_{10}, \quad \dot{q}_{20} = v_{20}, \quad \dot{q}_{30} = v_{10}q_{20}$$

Lemma 1: If $\lim_{t\to\infty}(q_{1j} - q_{10}) = 0$, $\lim_{t\to\infty}(q_{2j} - q_{20}) = 0$, and $\lim_{t\to\infty}(q_{3j} - q_{30}) = 0$ for $1 \le j \le m$, then (5)-(7) hold.

The proof is omitted here due to space limitations.

Define $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]$, by Lemma 1, the control problem can be transformed to designing control laws $v_{*j} = v_{1j}, v_{2j}$ such that

$$\lim_{t\to\infty}(q_{*j} - q_{*0}) = 0 \quad (11)$$

with the its own states information and that from its neighbors.

Notice in Eqn. (1) there exist nonholonomic constraints $J^\top(q_{*j})\lambda_j$, besides, $v_{*j} = v_{1j}, v_{2j}$ are not the real control inputs $V_{*j} = [v_j, \omega_j]$, by the same transformation proposed in [27], we have

$$\hat{M}_j(q_{*j})\dot{v}_{*j} + \hat{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \hat{G}_j(q_{*j}) = \hat{B}_j(q_{*j})\tau_j \quad (12)$$

By (12) it is known τ_j are the real control inputs, we first propose a control law of v_{*j}, then we design control laws of τ_j with the knowledge of backstepping method.

III. MAIN RESULTS

Before designing distributed formation tracking control of multi-vehicle systems, we introduce a theorem of exponential stability of cascaded system. Consider the cascaded system

$$\begin{aligned} \dot{x}_1 &= f_1(t, x_1) + g(t, x_1, x_2)x_2 \\ \dot{x}_2 &= f_2(t, x_2) \end{aligned} \quad (13)$$

where $x_1 \in \mathcal{R}^n$, $x_2 \in \mathcal{R}^n$,$f_1(t, x_1)$ is continuously differentiable in (t, x_1), $g(t, x_1, x_2)$ and $f_2(t, x_2)$ are locally Lipschitz in (x_1, x_2) and x_2.

Theorem 1: (13) is globally exponential stable if it satisfies the following three assumptions [28][29].

Assumption 1: $\dot{x}_1 = f_1(t, x_1)$ is globally exponential stable.

Assumption 2: the interconnection function $g(t, x_1, x_2)$ satisfies for all $t_0 \ge 0$

$$||g(t, x_1, x_2)|| \le \theta_1(||x_2||) + \theta_2(||x_2||)x_1$$

where θ_1 and θ_2 are continuous functions.

Assumption 3: $\dot{x}_2 = f_2(t, x_2)$ is globally exponentially stable.

Consider a linear time-varying system

$$\dot{x} = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \phi(t) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & 0 \\ 0 & \vdots & \phi(t) & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u \quad (14)$$

where $\phi(t)$ is a bounded continuously differentiable Lipschitz function.

Lemma 2: The control algorithm

$$u = -k_2x_1 - k_3\phi(t)x_2 - k_4x_3 - k_5\phi(t)x_4 - ... \quad (15)$$

ensures system (14) is globally exponentially stable if k_i are such that

$$\lambda^n + k_1\lambda^{n-1} + ... + k_n$$

is Hurwitz [28].

IV. Distributed Controller for Kinematic Systems

The following assumption is made on the leader agent.

Assumption 4: The $\frac{d^i v_{*0}}{dt^i}$ $(0 \le i \le 2)$ are bounded and $\int_t^{t+T} v_{*0}^2(\tau)d\tau > \alpha$ for some $\alpha > 0$ and $T > 0$.

Assumption 4 means signal v_{*0} is persistently excited signal (PE signal).

Theorem 2: For m systems in (8)-(10), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws

$$\begin{aligned} v_{1j} = u_{1j} = &- \sum_{i\in\mathcal{N}_j} a_{ji}(q_{1j} - q_{1i}) \\ &- a_{j,m+1}(q_{1j} - q_{10}) + \delta_{1j} \end{aligned} \tag{16}$$

$$\begin{aligned} \dot{\delta}_{1j} = &- \sum_{i\in\mathcal{N}_j} a_{ji}(\delta_{1j} - \delta_{1i}) - a_{j,m+1}(\delta_{1j} - \delta_{10}) \\ &- \rho_1 \mathrm{sign}[\sum_{i\in\mathcal{N}_j} a_{ji}(\delta_{1j} - \delta_{1i}) - a_{j,m+1}(\delta_{1j} - \delta_{10})] \end{aligned} \tag{17}$$

for $1 \le j \le m$, $\delta_{10} = u_{10}$, guarantee that q_{1j} globally exponentially converge to q_{10}.

Proof: Define $\tilde{\delta}_{1j} = \delta_{1j} - \delta_{10}$, (17) can be written as

$$\begin{aligned} \dot{\tilde{\delta}}_{1j} = &- \sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j} \\ &- \rho_1 \mathrm{sign}\left[\sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{\delta}_{1j} - \tilde{\delta}_{1i}) - a_{j,m+1}\tilde{\delta}_{1j}\right] - \dot{\delta}_{10} \end{aligned} \tag{18}$$

Define $\delta_{1*} = [\delta_{1j}, \delta_{2j}, ..., \delta_{mj}]$, with the aid of Laplacian matrix (18) can be written as

$$\dot{\tilde{\delta}}_{1*} = -(\mathcal{L} + B)\tilde{\delta}_{1*} - \rho_1 \mathrm{sign}\left[(\mathcal{L} + B)\tilde{\delta}_{1*}\right] - \dot{\delta}_{10}\mathbf{1} \tag{19}$$

where $\mathcal{L}$ is Laplacian matrix of the communication diagraph, $B = diag(a_{1,m+1}, a_{2,m+1}, ..., a_{m,m+1})$ is the diagonal matrix representing communication with the virtual leader, then δ_1 can be proved to globally exponentially converge to δ_{10}.

Substitute v_{1j} in (16) into (8) we have

$$\dot{q}_{1j} = - \sum_{i\in\mathcal{N}_j} a_{ji}(q_{1j} - q_{1i}) - a_{j,m+1}(q_{1j} - q_{10}) + \delta_{1j} \tag{20}$$

define $\tilde{q}_{1j} = q_{1j} - q_{10}$, (20) can be written as

$$\dot{\tilde{q}}_{1j} = - \sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{q}_{1j} - \tilde{q}_{1i}) - a_{j,m+1}\tilde{q}_{1j} + \tilde{\delta}_{1j} \tag{21}$$

since $\tilde{\delta}_1$ is globally exponentially stable, it can be proved q_{1j} globally exponentially converge to q_{10}. ■

Theorem 3: For m systems in (8)-(10), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws

$$v_{2j} = u_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} \tag{22}$$

$$\begin{aligned} \dot{\delta}_{2j} = &- \sum_{i\in\mathcal{N}_j} a_{ji}(\delta_{2j} - \delta_{2i}) - a_{j,m+1}(\delta_{2j} - \delta_{20}) \\ &- \rho_2 \mathrm{sign}\left[\sum_{i\in\mathcal{N}_j} a_{ji}(\delta_{2j} - \delta_{2i}) - a_{j,m+1}(\delta_{2j} - \delta_{20})\right] \end{aligned} \tag{23}$$

for $1 \le j \le m$, $\delta_{20} = u_{20} + k_2 q_{20} + k_3 u_{10} q_{30}$, guarantee that q_{2j} globally exponentially converge to q_{20} and q_{3j} globally exponentially converge to q_{30}.

Proof: substitute v_{2j} in (22) into (9), it follows that

$$\dot{q}_{2j} = -k_2 q_{2j} - k_3 u_{1j} q_{3j} + \delta_{2j} \tag{24}$$

Define $\zeta_{*j} = [\zeta_{1j}, \zeta_{2j}]$ as

$$\zeta_{1j} = \sum_{i\in\mathcal{N}_j} a_{ji}(q_{2j} - q_{2i}) - a_{j,m+1}(q_{2j} - q_{20})$$

$$\zeta_{2j} = \sum_{i\in\mathcal{N}_j} a_{ji}(q_{3j} - q_{3i}) - a_{j,m+1}(q_{3j} - q_{30})$$

differentiate ζ_{*j} along (24), (10) and define $\tilde{\delta}_{2j} = \delta_{2j} - \delta_{20}$, $\tilde{u}_{1j} = u_{1j} - u_{10}$, then we have

$$\begin{aligned} \dot{\zeta}_{1j} = &-k_2\zeta_{1j} - k_3 u_{10}\zeta_{2j} + \\ &\sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ &-k_3[\sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{u}_{1j} q_{3j} - \tilde{u}_{1j} q_{3i}) - a_{j,m+1}\tilde{u}_{1j} q_{3j}] \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{\zeta}_{2j} = u_{10}\zeta_{1j} + [&\sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{u}_{1j} q_{2j} - \tilde{u}_{1j} q_{2i}) \\ &- a_{j,m+1}\tilde{u}_{1j} q_{2j}] \end{aligned} \tag{26}$$

notice $\dot{\delta}_{2j}$ and $\dot{\delta}_{1j}$ have the same structure, by (18) we have

$$\begin{aligned} \dot{\tilde{\delta}}_{2j} = &- \sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ &- \rho_2 \mathrm{sign}\left[\sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j})\right] - \dot{\delta}_{20} \end{aligned} \tag{27}$$

Define $x_1 = [[\zeta_{11}, \zeta_{21}], [\zeta_{12}, \zeta_{22}], ..., [\zeta_{1m}, \zeta_{2m}]]$ $x_2 = [[\tilde{\delta}_{21}, \tilde{q}_{11}, \tilde{\delta}_{11}], ..., [\tilde{\delta}_{2m}, \tilde{q}_{1m}, \tilde{\delta}_{1m}]]$ by (25) and (26) we have

$$f_{1j} = \begin{bmatrix} -k_2 & -k_3 u_{10} \\ u_{10} & 0 \end{bmatrix}, f_1(t, x_1) = [f_{11}, ..., f_{1m}]$$

$$g_j = \begin{bmatrix} \sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{\delta}_{2j} - \tilde{\delta}_{2i}) - a_{j,m+1}\tilde{\delta}_{2j} \\ -k_3[\sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{u}_{1j} q_{3j} - \tilde{u}_{1j} q_{3i}) - a_{j,m+1}\tilde{u}_{1j} q_{3j}] \\ \sum_{i\in\mathcal{N}_j} a_{ji}(\tilde{u}_{1j} q_{2j} - \tilde{u}_{1j} q_{2i}) - a_{j,m+1}\tilde{u}_{1j} q_{2j} \end{bmatrix}$$

$$g(t, x_1, x_2)x_2 = [g_1, g_2, ..., g_m]$$

by (27), (21) and (18) we have

$$f_{2j}=\begin{bmatrix} -\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{2j}-\tilde{\delta}_{2i})-a_{j,m+1}\tilde{\delta}_{2j} \\ -\rho_2\text{sign}\left[\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{2j}-\tilde{\delta}_{2i})-a_{j,m+1}\tilde{\delta}_{2j})\right] \\ -\dot{\delta}_{20} \\ -\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{q}_{1j}-\tilde{q}_{1i})-a_{j,m+1}\tilde{q}_{1j}+\tilde{\delta}_{1j} \\ -\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{1j}-\tilde{\delta}_{1i})-a_{j,m+1}\tilde{\delta}_{1j} \\ -\rho_1\text{sign}\left[\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{1j}-\tilde{\delta}_{1i})-a_{j,m+1}\tilde{\delta}_{1j})\right] \\ -\dot{\delta}_{10} \end{bmatrix}$$

$$f_2(t,x_2)=[f_{21},f_{22},...,f_{2m}]$$

notice the second order matrix f_{1j} is special case of closed-loop systems of (14), since u_{10} is continuous PE signal, then by Lemma 2, $\dot{x}_1=f_1(t,x_1)$ is globally exponentially stable, then the Assumption 1 in Theorem 1 holds. g_j can be proved to satisfy Assumption 2 in Theorem 1 and from the expression of f_{2j} it can be proved $\dot{x}_2=f_2(t,x_2)$ is globally exponentially stable and Assumption 3 holds. Therefore, $x_1=[[\zeta_{11},\zeta_{21}],[\zeta_{12},\zeta_{22}],...,[\zeta_{1m},\zeta_{2m}]]$ is globally exponentially stable. From the definition of ζ_{1j} and ζ_{2j} we have

$$\zeta_{1*} \quad = \quad (\mathcal{L}+B)\tilde{q}_{2*}$$

since ζ_{1*} is globally exponentially stable, then $\tilde{q}_{2*}=(\mathcal{L}+B)^{-1}\zeta_{1*}$ is exponentially stable and q_{2j} globally exponentially converge to q_{20}. Similarly, we can prove q_{3j} globally exponentially converge to q_{30}. ■

Theorem 4: For m systems in (8)-(10), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws (16)-(17) and (22)-(23) guarantee (5)-(7) hold.

Proof: By Theorem 2 q_{1j} globally exponentially converge to q_{10}. By Theorem 3 q_{2j} globally exponentially converge to q_{20}, q_{3j} globally exponentially converge to q_{30}. By Lemma 1 (5)-(7) hold. ■

V. Distributed controller for dynamics

From the dynamics in (12) it is known the real control inputs are torques generated by vehicle engines, in this section, distributed control algorithms are proposed with the aid of backstepping methods and sliding mode control.

Define $\check{v}_{*j}=v_{*j}-u_{*j}$, where $v_{*j}=[v_{1j},v_{2j}]$, $u_{*j}=[u_{1j},u_{2j}]$, we have

Theorem 5: For m systems in (8)-(10), if a directed spanning tree exists in the directed communication graph with the virtual leader the root of the tree, then the distributed control laws

$$\tau_j= \hat{B}_j^{-1}(\tilde{M}_j\dot{u}_{*j}+\tilde{C}_ju_{*j}+\tilde{G}_j-k\check{v}_{*j}-Y_j\beta sign(Y_j^T\check{v}_{*j})) \quad (28)$$

where k satisfies $||\tilde{a}-\hat{a}||<k$, and (16)-(17),(22)-(23) guarantee (5)-(7) hold.

Proof: Denote the inertia parameter error vector $\check{a}=\tilde{a}-\hat{a}$, choose Lyapunov function $V_2=\frac{1}{2}\check{v}_{*j}^T\hat{M}_j\check{v}_{*j}$, differentiate V_2 it follows that

$$\dot{V}_2=\frac{1}{2}\check{v}_{*j}^T\dot{\hat{M}}_j\check{v}_{*j}+\check{v}_{*j}^T\hat{M}_j\dot{\check{v}}_{*j} \quad (29)$$

substitute (28) into (12) we have

$$\hat{M}_j\dot{\check{v}}_{*j}=-\hat{C}_j\check{v}_{*j}-k\check{v}_{*j}+Y_j\check{a}+Y_j\beta sign(Y_j^T\check{v}_{*j}) \quad (30)$$

substitute $\hat{M}_j\dot{\check{v}}_{*j}$ into (29) it follows that

$$\dot{V}_2<-\check{v}_{*j}^Tk\check{v}_{*j}$$

then $v_{*j}-u_{*j}$ can be proved to globally exponentially converge to zero.

(16) and (22) are rewritten as

$$v_{1j}=-\sum_{i\in\mathcal{N}_j}a_{ji}(q_{1j}-q_{1i})-a_{j,m+1}(q_{1j}-q_{10}) +\delta_{1j}+\check{v}_{1j} \quad (31)$$

$$v_{2j}=-k_2q_{2j}-k_3u_{1j}q_{3j}+\delta_{2j}+\check{v}_{2j} \quad (32)$$

$\zeta_{*j}=[\zeta_{1j},\zeta_{2j}]$ are the same as defined in Section III, it follows that

$$\begin{aligned}\dot{\zeta}_{1j}=&-k_2\zeta_{1j}-k_3u_{1,m+1}\zeta_{2j} \\ &+\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{2j}-\tilde{\delta}_{2i})-a_{j,m+1}\tilde{\delta}_{2j} \\ &+\sum_{i\in\mathcal{N}_j}a_{ji}(\check{u}_{2j}-\check{u}_{2i})-a_{j,m+1}\check{u}_{2j} \\ &-k_3[\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{u}_{1j}q_{3j}-\tilde{u}_{1j}q_{3i})-a_{j,m+1}\tilde{u}_{1j}q_{3j}] \quad (33)\end{aligned}$$

$$\dot{\tilde{q}}_{1j}=-\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{q}_{1j}-\tilde{q}_{1i})-a_{j,m+1}\tilde{q}_{1j}+\tilde{\delta}_{1j}+\check{u}_{2j} \quad (34)$$

$\dot{\zeta}_{2j}$, $\tilde{\delta}_{1j}$ and $\tilde{\delta}_{2j}$ are unchanged as expressed in (26), (18) and (27).

$$g_j=\begin{bmatrix} \sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{2j}-\tilde{\delta}_{2i})-a_{j,m+1}\tilde{\delta}_{2j} \\ \sum_{i\in\mathcal{N}_j}a_{ji}(\check{u}_{2j}-\check{u}_{2i})-a_{j,m+1}\check{u}_{2j} \\ -k_3[\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{u}_{1j}q_{3j}-\tilde{u}_{1j}q_{3i})-a_{j,m+1}\tilde{u}_{1j}q_{3j}] \\ \sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{u}_{1j}q_{2j}-\tilde{u}_{1j}q_{2i})-a_{j,m+1}\tilde{u}_{1j}q_{2j} \end{bmatrix}$$

$$g(t,x_1,x_2)x_2=[g_1,g_2,...,g_m]$$

$$f_{2j}=\begin{bmatrix} -\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{2j}-\tilde{\delta}_{2i})-a_{j,m+1}\tilde{\delta}_{2j} \\ -\rho_2\text{sign}\left[\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{2j}-\tilde{\delta}_{2i})-a_{j,m+1}\tilde{\delta}_{2j})\right] \\ -\dot{\delta}_{20} \\ -\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{q}_{1j}-\tilde{q}_{1i})-a_{j,m+1}\tilde{q}_{1j}+\tilde{\delta}_{1j}+\check{u}_{1j} \\ -\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{1j}-\tilde{\delta}_{1i})-a_{j,m+1}\tilde{\delta}_{1j} \\ -\rho_1\text{sign}\left[\sum_{i\in\mathcal{N}_j}a_{ji}(\tilde{\delta}_{1j}-\tilde{\delta}_{1i})-a_{j,m+1}\tilde{\delta}_{1j})\right] \\ -\dot{\delta}_{10} \end{bmatrix}$$

f_{1j} are the same as that in Section III, it can be proved that the modified cascaded system

$$\begin{aligned}\dot{x}_1 &= f_1(t,x_1)+g(t,x_1,x_2)x_2 \\ \dot{x}_2 &= f_2(t,x_2)\end{aligned}$$

still satisfies the Assumptions in Theorem 1 and $q_{*j}=[q_{1j},q_{2j},q_{3j}]$ exponentially converge to $q_{*0}=[q_{10},q_{20},q_{30}]$, by Lemma 1, (5)-(7) hold. ■

VI. Distributed Controller for Time-Varying Communication Topology

In real-life operations, due to disconnection or creation of links and nodes failures, the communication topology is time-varying. It has been proved through a infinite sequence of nonoverlapping, uniformly bounded time interval, if the union of the graphs across each interval has a directed spanning tree, then multi-agent systems reach consensus with the aid of theories of SIA matrices [5].

Theorem 6: For m systems in (8)-(10), if at any nonoverlapping, uniformly bounded time interval, a directed spanning tree exists for union of the graphs across the interval, with the virtual leader the root of the spanning tree, then the distributed control laws (16)-(17),(22)-(23) and (28) guarantee (5)-(7) hold.

VII. Simulation Results

To show the effectiveness of the proposed results, simulation has been done for four robots. The desired geometric pattern $\mathcal{P}$ is shown in Fig. 1. Assume the format of the robot systems is in square shape. The pattern $\mathcal{P}$ can be described by orthogonal coordinates $(p_{1x}, p_{1y}) = (0, 1)$, $(p_{2x}, p_{2y}) = (-1, 0)$, $(p_{3x}, p_{3y}) = (0, -1)$ and $(p_{4x}, p_{4y}) = (1, 0)$. For the leading agent, assume the reference trajectory is $(x_5, y_5, \theta_5) = (10\sin(t),\ -10\cos(t),\ t)$ and $(p_{5x}, p_{5y}) = (0, 0)$, $v_5 = 10$ and $\omega_5 = 1$.

Fig. 2 represents the communication graph for the multivehicle systems, ρ_1, ρ_2, k_2 and k_3 are assigned 2, k in dynamic control laws is 10. Fig. 4 shows the centroid of x_j $(1 \le j \le 4)$ (i.e., $\sum_{j=1}^{4} x_j/4$) and x_0. Fig. 5 shows the centroid of y_j $(1 \le j \le 4)$ (i.e., $\sum_{j=1}^{4} y_j/4$) and y_0. Fig. 6 shows $(\theta_j - \theta_0)$ $(1 \le j \le 4)$. Fig. 7 shows the formation tracking of the 4 followers, the blue spots represent each follower robot, the black spot represent the trajectory of centroid of the robots.

Assume the information communication graph switches according to the following logic.

$$\mathcal{G} = \begin{cases} \mathcal{G} \text{ in Fig. 2}, & \text{if } t - \text{round}(t) \ge 0 \\ \mathcal{G} \text{ in Fig. 3}, & \text{if } t - \text{round}(t) < 0 \end{cases}$$

For the switching topologies defined above, Fig. 8 shows the centroid of x_j $(1 \le i \le 4)$ and x_0. Fig. 9 shows the centroid of y_j $(1 \le j \le 4)$ and y_0. Fig. 10 shows $(\theta_j - \theta_0)$ $(1 \le j \le 4)$. Fig. 11 shows the formation tracking of the 4 followers.

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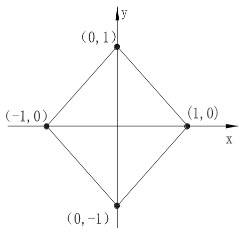

Fig. 1. Desired geometric formation.

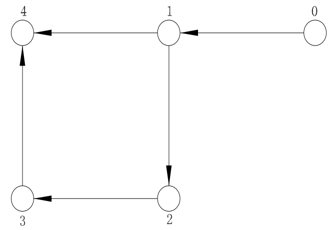

Fig. 2. Information exchange graph $\mathcal{G}$.

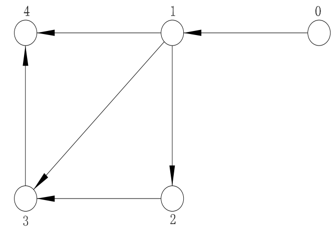

Fig. 3. Information exchange graph $\mathcal{G}$.

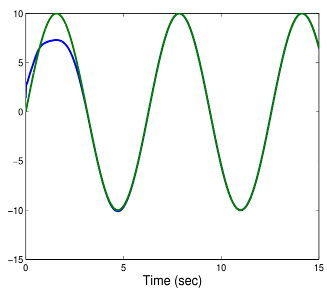

Fig. 4. Response of the centroid of x_j for $1 \leq j \leq 4$ and x_0.

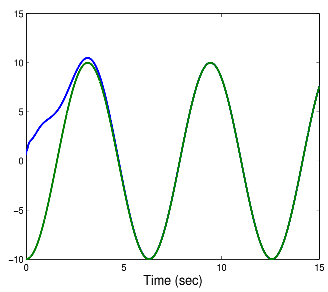

Fig. 5. Response of the centroid of y_j for $1 \leq j \leq 4$ and y_0.

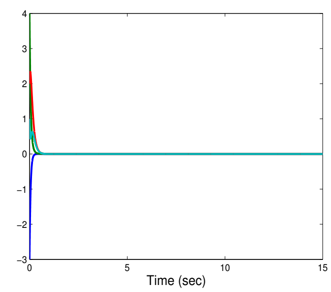

Fig. 6. Response of $(\theta_j - \theta_0)$ for $1 \leq j \leq 4$.

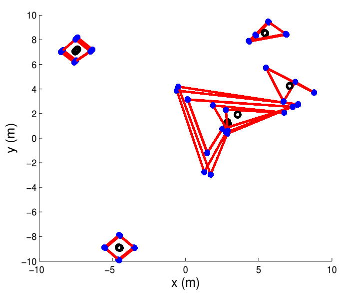

Fig. 7. formation tracking of 4 followers.

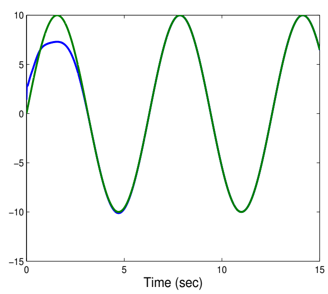

Fig. 8. Response of the centroid of x_j for $1 \leq j \leq 4$ and x_0.

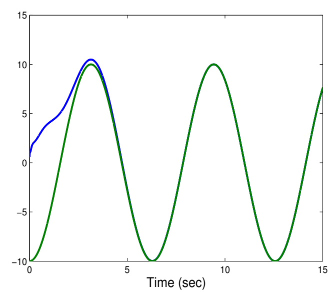

Fig. 9. Response of the centroid of y_j for $1 \leq j \leq 4$ and y_0.

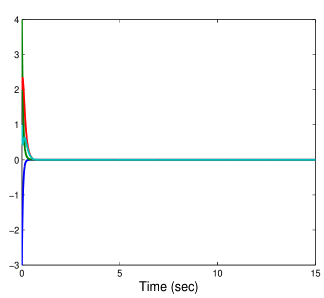

Fig. 10. Response of $(\theta_j - \theta_0)$ for $1 \leq j \leq 4$.

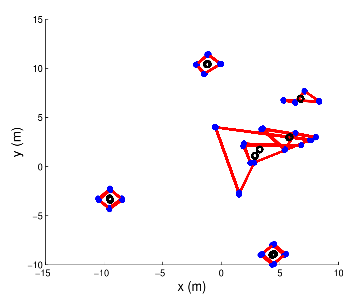

Fig. 11. formation tracking of 4 followers.