

$$p_k(x) = \frac{c_k}{N h^D} \sum_{i=1}^N k\left(\left\|\frac{x-x_i}{h}\right\|^2\right)$$

c_k is a normalization constant
 $k\left(\left\|\frac{x-x_i}{h}\right\|^2\right)$ is a kernel

computing the gradient,

$$\begin{aligned} \nabla p_k(x) &= \frac{2c_k}{N h^{D+2}} \sum_{i=1}^N (x - x_i) k'\left(\left\|\frac{x-x_i}{h}\right\|^2\right) \\ &= \frac{-2c_k}{N h^{D+2}} \left[\sum_{i=1}^N x_i k'\left(\left\|\frac{x-x_i}{h}\right\|^2\right) - x \sum_{i=1}^N k'\left(\left\|\frac{x-x_i}{h}\right\|^2\right) \right] \end{aligned}$$

$$= \frac{-2c_k}{N h^{D+2}} \sum_{i=1}^N k'\left(\left\|\frac{x-x_i}{h}\right\|^2\right) \left[\frac{\sum_{i=1}^N x_i k'\left(\left\|\frac{x-x_i}{h}\right\|^2\right)}{\sum_{i=1}^N k'\left(\left\|\frac{x-x_i}{h}\right\|^2\right)} - x \right]$$

mean shift vector $m_{k'}(x)$

$$\begin{aligned} &= \frac{-2c_k c_{k'}}{c_{k'} h^2 N h^D} \sum_{i=1}^N k'\left(\left\|\frac{x-x_i}{h}\right\|^2\right) m_{k'}(x) \\ &= \frac{2c}{h^2} p_{k'}(x) m_{k'}(x) \end{aligned}$$

$$m_{k'}(x) = \frac{h^2}{2c} \frac{\nabla p_k}{p_{k'}(x)}$$

mean shift vector $m_{k'}(x)$ always points in direction of maximum increase in density