$$\overset{\text{posterior}}{\boxed{p(x_k|Z_k)}} = \frac{p(x_k, Z_k)}{p(Z_k)}$$

$$\overset{p(Z_k) = p(z_k, Z_{k-1})}{=} \frac{p(x_k, z_k, Z_{k-1})}{p(z_k, Z_{k-1})}$$

$$\overset{\text{Bayes' rule}}{=} \frac{p(z_k|x_k, Z_{k-1})\,p(x_k, Z_{k-1})}{p(z_k|Z_{k-1})\,p(Z_{k-1})}$$

$$\overset{\text{independence, Bayes' rule}}{=} \frac{p(z_k|x_k)\,p(x_k|Z_{k-1})\,\cancel{p(Z_{k-1})}}{p(z_k|Z_{k-1})\,\cancel{p(Z_{k-1})}}$$

$$\overset{\text{nuisance variable}}{=} \frac{p(z_k|x_k)\,p(x_k|Z_{k-1})}{\int p(z_k|x_k)\,p(x_k|Z_{k-1})\,dx_k}$$

$$\propto \underbrace{p(z_k|x_k)}_{\text{likelihood}}\,\underbrace{p(x_k|Z_{k-1})}_{\text{prior (prediction)}}$$