elliptical fourier

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{K} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos(kt) \\ \sin(kt) \end{bmatrix}$$

↓ differentiate

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{d^2x}{dt^2} \\ \frac{d^2y}{dt^2} \end{bmatrix} = \sum_{k=1}^{K} k \begin{bmatrix} -a_k \sin(kt) + b_k \cos(kt) \\ -c_k \sin(kt) + d_k \cos(kt) \\ k(-a_k \cos(kt) - b_k \sin(kt)) \\ k(-c_k \cos(kt) - d_k \sin(kt)) \end{bmatrix}$$

↓ square and integrate

$$\int_0^{2\pi} \begin{bmatrix} \frac{dx}{dt}^2 \\ \frac{dy}{dt}^2 \\ \frac{d^2x}{dt^2}^2 \\ \frac{d^2y}{dt^2}^2 \end{bmatrix} = \pi \sum_{k=1}^{K} k^2 \begin{bmatrix} a_k^2 + b_k^2 \\ c_k^2 + d_k^2 \\ k^2(a_k^2 + b_k^2) \\ k^2(c_k^2 + d_k^2) \end{bmatrix}$$

$$\int_0^{2\pi} \left(\frac{dx}{dt}^2 + \frac{dy}{dt}^2 \right) = \pi \sum_{k=1}^{K} k^2 (a_k^2 + b_k^2 + c_k^2 + d_k^2)$$

$$\int_0^{2\pi} \left(\frac{d^2x}{dt^2}^2 + \frac{d^2y}{dt^2}^2 \right) = \pi \sum_{k=1}^{K} k^4 (a_k^2 + b_k^2 + c_k^2 + d_k^2)$$