

$$\{x : (y_\alpha - x)^2 < (y_\beta - x)^2\} \subset S_\alpha, \quad \forall \beta \neq \alpha$$

$$\begin{aligned} (y_\alpha - x)^2 &< (y_\beta - x)^2 \\ y_\alpha^2 + x^2 - 2y_\alpha x &< y_\beta^2 + x^2 - 2y_\beta x \\ \frac{1}{2}y_\alpha^2 - y_\alpha x &< \frac{1}{2}y_\beta^2 - y_\beta x \\ x(y_\beta - y_\alpha) - \frac{1}{2}(y_\beta^2 - y_\alpha^2) &< 0 \\ (y_\beta - y_\alpha) \left(x - \frac{1}{2}(y_\beta + y_\alpha)\right) &< 0 \end{aligned}$$

$$\{x : (y_\beta - y_\alpha) \left(x - \frac{1}{2}(y_\beta + y_\alpha)\right) < 0\} \subset S_\alpha, \quad \forall \beta \neq \alpha$$