

$$\begin{aligned}
 MSE &= \sum_{j=0}^{N-1} \int_{b_j}^{b_{j+1}} (x - y_j)^2 f_X(x) dx \\
 &= \int_{b_0}^{b_1} (x - y_0)^2 f_X(x) dx + \int_{b_1}^{b_2} (x - y_1)^2 f_X(x) dx + \dots + \int_{b_{N-1}}^{b_N} (x - y_{N-1})^2 f_X(x) dx
 \end{aligned}$$

$\frac{\partial MSE}{\partial y_0} = -2 \int_{b_0}^{b_1} (x - y_0) f_X(x) dx$	$\frac{\partial MSE}{\partial y_0} = 0 \Rightarrow$	$y_0 = \frac{\int_{b_0}^{b_1} x f_X(x) dx}{\int_{b_0}^{b_1} f_X(x) dx}$
$\frac{\partial MSE}{\partial y_1} = -2 \int_{b_1}^{b_2} (x - y_1) f_X(x) dx$	$\frac{\partial MSE}{\partial y_1} = 0 \Rightarrow$	$y_1 = \frac{\int_{b_1}^{b_2} x f_X(x) dx}{\int_{b_1}^{b_2} f_X(x) dx}$
$\vdots$	$\vdots$	$\vdots$
$\frac{\partial MSE}{\partial y_{N-1}} = -2 \int_{b_{N-1}}^{b_N} (x - y_{N-1}) f_X(x) dx$	$\frac{\partial MSE}{\partial y_{N-1}} = 0 \Rightarrow$	$y_{N-1} = \frac{\int_{b_{N-1}}^{b_N} x f_X(x) dx}{\int_{b_{N-1}}^{b_N} f_X(x) dx}$