

1. PREDICTION

1. predicted estimate: state

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1} \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_{k-1} \mathbf{u}_{k-1}$$

$$\mu_1 = A\mu_{fused} + Bu$$

2. predicted estimate: state-covariance

$$\mathbf{P}_{k|k-1} = \mathbf{A}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{A}_{k-1}^T + \mathbf{Q}_k$$

$$\begin{aligned}\sigma_1^2 &= A\sigma_{fused}^2 A + Q \\ &= A^2\sigma_{fused}^2 + Q\end{aligned}$$

3. gain

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{C}_k^T \left(\underbrace{\mathbf{C}_k \mathbf{P}_{k|k-1} \mathbf{C}_k^T + \mathbf{R}_k}_{\text{innovation: covariance}} \right)^{-1}$$

$$\begin{aligned}K &= \sigma_1^2 C (C\sigma_1^2 C + \sigma_2^2)^{-1} \\ &= \frac{C\sigma_1^2}{C^2\sigma_1^2 + \sigma_2^2}\end{aligned}$$

2. UPDATE

4. updated estimate: state

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\underbrace{\mathbf{y}_k - \mathbf{C}_k \hat{\mathbf{x}}_{k|k-1}}_{\text{innovation: measurement}} \right)$$

$$\mu_{fused} = \mu_1 + K(\mu_2 - C\mu_1)$$

5. updated estimate: state-covariance

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{C}_k) \mathbf{P}_{k|k-1}$$

$$\sigma_{fused}^2 = (1 - KC)\sigma_1^2$$

system model estimate: state covariance noise covariance