

prior (prediction)

$$\begin{aligned}
 p(x_k | Z_{k-1}) &= \frac{p(x_k, Z_{k-1})}{p(Z_{k-1})} \\
 &= \frac{\int p(x_k, x_{k-1}, Z_{k-1}) dx_{k-1}}{p(Z_{k-1})} \quad \text{nuisance variable} \\
 &= \frac{\int p(x_k | x_{k-1}, Z_{k-1}) p(x_{k-1}, Z_{k-1}) dx_{k-1}}{p(Z_{k-1})} \\
 &= \frac{\int p(x_k | x_{k-1}) p(x_{k-1} | Z_{k-1}) p(Z_{k-1}) dx_{k-1}}{p(Z_{k-1})} \\
 &= \int p(x_k | x_{k-1}) p(x_{k-1} | Z_{k-1}) dx_{k-1} \quad \begin{array}{l} \text{model} \\ \text{posterior from time k-1} \end{array}
 \end{aligned}$$