

$$\hat{x}_{MMSE} = E[x|Z]$$

$$= \sum_x x \underbrace{p(x|Z)}_{\xrightarrow{p(x|Z)}} \rightarrow = \sum_{A_i} p(x, A_i|Z)$$

$$= \sum_x x \sum_{A_i} \underbrace{p(x|A_i)p(A_i|Z)}_{\text{joint distribution}}$$

$$= \sum_{A_i} \underbrace{E[x|A_i]}_{\text{conditional expectation}} \underbrace{p(A_i|Z)}_{\text{marginal distribution}}$$

$$= E[E[x|A_i]|Z]$$

$$= \sum_{A_i} \frac{p(x, A_i, Z)}{p(Z)}$$

$$= \sum_{A_i} \frac{\underbrace{p(x|A_i, Z)}_{\text{joint distribution}} \underbrace{p(A_i, Z)}_{\text{joint distribution}}}{p(Z)}$$

$$= \sum_{A_i} \frac{\underbrace{p(x|A_i, Z)}_{\text{joint distribution}} \underbrace{p(A_i|Z)p(Z)}_{\text{joint distribution}}}{p(Z)}$$

$$= \sum_{A_i} \underbrace{p(x|A_i)p(A_i|Z)}_{\text{joint distribution}}$$