$$E^*_{snake} = \int_0^1 [\underbrace{E_{int}(v(s))}_{\text{internal}} + \underbrace{E_{ext}(v(s))}_{\text{external}}]\, ds$$

$$= \int_0^1 \left[\frac{\alpha(s)}{2} \underbrace{|v_s(s)|^2}_{\text{membrane}} + \frac{\beta(s)}{2} \underbrace{|v_{ss}(s)|^2}_{\text{thin plate}} - \underbrace{\nabla I(x(s), y(s))}_{\text{gradient}} \right] ds$$

$$= \int_0^1 \left[\frac{\alpha(s)}{2} \left(x_s^2 + y_s^2\right) + \frac{\beta(s)}{2} \left(x_{ss}^2 + y_{ss}^2\right) - \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \right] ds$$