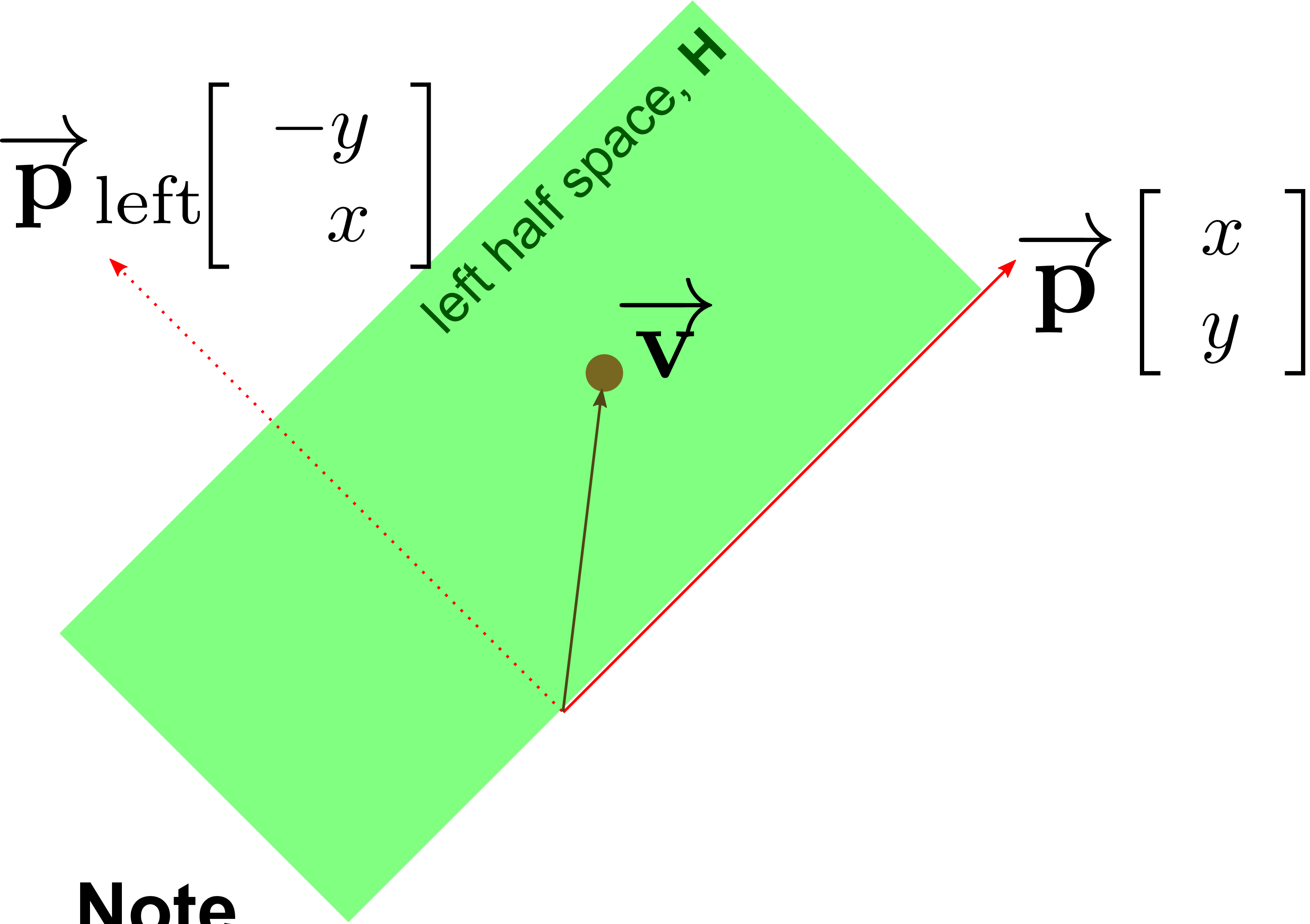

Step 1

Given $\overrightarrow{\mathbf{p}}$, compute $\overrightarrow{\mathbf{p}}_{\text{left}}$
In $\mathbb{R}^2$, simply swap components and negate y's sign. Then,

$$\overrightarrow{\mathbf{p}}_{\text{left}} \cdot \overrightarrow{\mathbf{p}} = \begin{bmatrix} -y & x \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

(orthogonal)

Step 2

$\overrightarrow{\mathbf{v}}$ is to the left of $\overrightarrow{\mathbf{p}}$ if

$$\overrightarrow{\mathbf{p}}_{\text{left}} \cdot \overrightarrow{\mathbf{v}} > 0, \quad \overrightarrow{\mathbf{v}} \in \mathbf{H}$$

$\overrightarrow{\mathbf{v}}$ is to the right of $\overrightarrow{\mathbf{p}}$ if

$$\overrightarrow{\mathbf{p}}_{\text{left}} \cdot \overrightarrow{\mathbf{v}} < 0, \quad \overrightarrow{\mathbf{v}} \notin \mathbf{H}$$

$\overrightarrow{\mathbf{v}}$ is on $\overrightarrow{\mathbf{p}}$ if

$$\overrightarrow{\mathbf{p}}_{\text{left}} \cdot \overrightarrow{\mathbf{v}} = 0, \quad \overrightarrow{\mathbf{v}} = k\overrightarrow{\mathbf{p}}, \quad k \in \mathbb{R}$$

Note

$$\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}} = \|\overrightarrow{\mathbf{a}}\| \|\overrightarrow{\mathbf{b}}\| \cos(\theta)$$

- the sign of $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}$ can be used to find left/right position, as in Step 2
- the magnitude of $\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}$ can be used to find if the vectors involved have large magnitude