$$\hat{x}_{MMSE} = E\left[x|Z\right]$$

$$= \sum_x x p(x|Z) \xrightarrow{p(x|Z)} = \sum_{A_i} p(x, A_i|Z)$$

$$= \sum_x x \sum_{A_i} p(x|A_i) p(A_i|Z) \qquad = \sum_{A_i} \frac{p(x, A_i, Z)}{p(Z)}$$

$$= \sum_{A_i} E[x|A_i] p(A_i|Z) \qquad = \sum_{A_i} \frac{p(x|A_i, Z) p(A_i, Z)}{p(Z)}$$

$$= E\left[E[x|A_i]|Z\right] \qquad = \sum_{A_i} \frac{p(x|A_i, Z) p(A_i|Z) p(Z)}{p(Z)}$$

$$= \sum_{A_i} p(x|A_i) p(A_i|Z)$$