

$$\begin{aligned}
 e &= \sum_{k=1}^K \int_{b_k}^{b_{k+1}} (x - y_k)^2 f_X(x) \\
 &= \int_{b_1}^{b_2} (x - y_1)^2 f_X(x) dx + \int_{b_2}^{b_3} (x - y_2)^2 f_X(x) dx + \dots + \int_{b_K}^{b_{K+1}} (x - y_K)^2 f_X(x) dx
 \end{aligned}$$

$$\frac{\partial e}{\partial y_1} = -2 \int_{b_1}^{b_2} (x - y_1) f_X(x) dx \qquad \frac{\partial e}{\partial y_1} = 0 \Rightarrow \qquad y_1 = \frac{\int_{b_1}^{b_2} x f_X(x) dx}{\int_{b_1}^{b_2} f_X(x) dx}$$

$$\frac{\partial e}{\partial y_2} = -2 \int_{b_2}^{b_3} (x - y_2) f_X(x) dx \qquad \frac{\partial e}{\partial y_2} = 0 \Rightarrow \qquad y_2 = \frac{\int_{b_2}^{b_3} x f_X(x) dx}{\int_{b_2}^{b_3} f_X(x) dx}$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\frac{\partial e}{\partial y_K} = -2 \int_{b_K}^{b_{K+1}} (x - y_K) f_X(x) dx \qquad \frac{\partial e}{\partial y_K} = 0 \Rightarrow \qquad y_K = \frac{\int_{b_K}^{b_{K+1}} x f_X(x) dx}{\int_{b_K}^{b_{K+1}} f_X(x) dx}$$