$$x(t) = a_0 + a_k \cos(kt) + b_k \sin(kt)$$

$$\int_0^{2\pi} \frac{dx}{dt}^2 = \int_0^{2\pi} \left[(-a_1 \sin(t) + b_1 \cos(t)) + (-2a_2 \sin(2t) + 2b_2 \cos(2t)) + \ldots\right]^2 dt$$

$$= \int_0^{2\pi} \left[a_1^2 \sin^2(t) + b_1^2 \cos^2(t) + 4a_2^2 \sin^2(2t) + 4b_2^2 \cos^2(2t) + \ldots\right] dt$$

$$= \int_0^{2\pi} \sum_{k=1}^{K} k^2 \left[a_k^2 \sin^2(kt) + b_k^2 \cos^2(kt)\right] dt$$

$$= \pi \sum_{k=1}^{K} k^2 \left[a_k^2 + b_k^2\right]$$