Before we start, we can first note that the measurement directly from our test measures the ammount of elecricity consumed, which does not necessarily all turn into physical energy being pushed into the water. We can convert this reading into the ammount of energy in the water with the efficiency of our thrusters. A standard efficienct might be somewhere between 0.6 and 0.8, however we can estimate ours as 0.5 as a starting point, then modify our equation as needed. To reflect this in our calculations, we can add an additional factor of E for efficency to the power formula, which does not appear naturaly, we are adding to account for our experimental reading. This can then be set to whatever value we wish once we finish the formula. We can now use the measurement in Watts from our static tests, which we can convert the units to metric standard units.

$$EW = \frac{J}{s} = \frac{Nm}{s} = \frac{kgm^2}{s^3}$$

$$\text{Watts} = \frac{\text{Joule}}{\text{Second}} = \frac{\text{Newton} \cdot \text{Meter}}{\text{Second}} = \frac{\text{Kilogram} \cdot \text{Meter}^2}{\text{Second}^3}.$$

We can re-condense these units with a thrust formula to get

$$\text{Power} = \text{Mass Flow Rate} \cdot \text{Accleration} \cdot \text{Distance}$$

$$EW = \frac{kg}{s}\frac{m}{s^2}m.$$

We can further breakdown the Mass flow rate into the following equation

$$\text{Mass Flow Rate} = \text{Density} \cdot \text{Volume Flow Rate}$$

$$\frac{kg}{s} = \frac{kg}{m^3}\frac{m^3}{s},$$

and then the Volume Flow Rate as the following equation

$$\text{Volume Flow Rate} = \text{Velocity} \cdot \text{Area}$$

$$\frac{m^3}{s} = \frac{m}{s}m^2.$$

Combining what we have so far we get

$$\text{Power} = \text{Density} \cdot \text{Velocity} \cdot \text{Area} \cdot \text{Acceleration} \cdot \text{Distance}.$$

$$EW = \frac{kg}{m^3}\frac{m}{s}m^2\frac{m}{s^2}m.$$

Breaking this down, we have that the power of a thruster is equal to the density of the fluid times the exit velocity of the fluid, times the surface area of the intake, times the acceleration acted on by the fluid, times the "draw distance" or the length from the input to the ouptut of the thruster. Rewriting our previous equation to include these labels gives us a formula for Thruster power

$$EP_t = \rho_f v_o A_i a_o l,$$

which written in plain text reads as

Power of the Thruster = Density of the Fluid·Velocity at the Output·Area of the Intake·Acceleration at the Output·Length of th

Going back to our experiment and comparing it with our formula for thruster power, we can notice that we measured the power, we know the density of water, and we can measure the Area of the intake and the lenght of the motor, however we still have two unkowns (Velocity and Acceleration) and only one equation. To bring this equation to one unkown, we can turn to Kinematics. Kinematics are equations that govern constant acceleration. We have the two equations

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at,$$

Where Δx is the change in position, v_0 is the initial velocity, v is the final velocity, a is the acceleration, and t is the time in seconds. In our thruster, we can use these kinematics to model the flow of water through our thruster in the static tests. We know that $v = v_o$ $a = a_o$ $\Delta x = l$ and $v_0 = 0$ since the distance the water changes speed over is the length of the thruster, and the water is still at the intake. Therefore we have the equations for our thruster

$$l = \frac{a_o t^2}{2}$$

and

$$v_o = a_o t.$$

Combining this with our earlier equation for thruster power means we have three equations and three unkowns (a_o, v_o, and t). Even without numbers, we can write both a_o and v_o in terms of t, then use these values to write our thruster power formula with one unkown.

$$a_o = \frac{2l}{t^2}$$

and

$$v_o = a_o t = \frac{2l}{t^2} t = \frac{2l}{t}.$$

We can put these back into our power formula to get

$$EP_t = \rho_f \frac{2l}{t} A_i \frac{2l}{t^2} l = 4\rho_f A_{\text{intake}} l^3 \frac{1}{t^3}.$$

This is our most reduced form of our power formula for a thruster. However, we can rearange this formula as we have measured the power, and want to know the ammount of time as it is our only unkown. Thefore we have

$$t^3 = 4\rho_f A_i l^3 \frac{1}{EP_t}$$

$$t = \sqrt[3]{4\rho_f A_i l^3 \frac{1}{EP_t}}.$$

Now that we have a formula for the ammount of time it takes water to move through the thruster, let's turn to a different formula, one for the Thrust Force generated by the thruster.

$$F_t = \rho A v_o^2$$

$$\text{Force of Thrust} = \text{Density of Fluid} \cdot \text{Area of the intake} \cdot \text{Velocity of the fluid Ouput}^2.$$

From our kinematic equations we know that $v_o = \frac{l}{t}$, therefore we have that

$$F_t = \rho_f A_i \frac{l^2}{t^2}.$$

To relate the thrust force to exclusively variables we know, we can replace the t value with our formula from the power equation giving us

$$F_t = \rho_f A_i \frac{l^2}{\left(4\rho_f A_i l^3 \frac{1}{EP_t}\right)^{\frac{2}{3}}}.$$

Now that we have the force of thrust, we can look at what we are hoping to find. We want to know the max speed of the ROV, which would nessecarily be a constant velocity. Thefore, acceleration is equal to zero. In order for the acceleration to be zero the net Force must also equal zero, and the only two forces that are acting on our ROV are the Thrust Force and the Drag Force. Therfore we have

$$F_d = F_t.$$

Looking to external resources, we can find an formula

$$c_d = \frac{2F_d}{\rho u^2 A},$$

which relates the drag force to a drag coefficient of an object moving through a fluid. This equation tells us that

$$\text{Drag Coefficient} = \frac{\text{2Force of Drag}}{\text{Density of the FluidFlow Speed of the Fluid}^2\text{Area of the object}}.$$

First, let us rewrite this equation to isolate F_d, giving us

$$F_d = \frac{c_d \rho_f u^2 A}{2}.$$

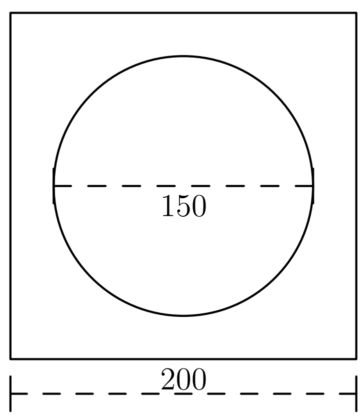
Looking at this equation we can see that we have two unkowns, the drag coefficient and the flow speed of the fluid, as we know that the drag force is equal to our thrust force, the density of water, and the surface area of our various parts of the ROV. One note here is that the drag force is not nessecarily a single force. Every face of the ROV can be modeled seperatley, and we have that

$$F_d = \sum \frac{c_d \rho_f u^2 A_n}{2},$$

with the correct drag coefficient and Area for each part. An important thing to note is that the density of water and velocity of the ROV are not related to the different objects, so we can factor them out to get

$$F_d = \frac{\rho_f u^2}{2} \sum c_d A_n.$$

Looking at the front of the ROV we can see there are 3 distinct surface shapes which face into forward movement, the slim edges of the wings, the square portion of the camera plate, and the dome of the camera. We can safely disregard the edges of the wings as the surface area in meters will be so small as to be insignificant, meaning we only need to consider the faceplate and dome. The forward face will look something like this:



We can also note that the drag coeficient for a plate is 1.05, while the coefficient for a dome is 0.42. In order to maintain the versatility of the formula we are creating, we can use w for the width of the plate, and r for the radius of the dome. Therfore, we have that the area of the dome is

$$\pi r^2,$$

and the area of the plate is

$$w^2 - \pi r^2$$

. Therfore our formula for drag force is

$$F_d = \frac{\rho_f u^2}{2} \left(0.42\pi r^2 + 1.05\left(w^2 - \pi r^2\right)\right).$$

Simplifying yeilds

$$F_d = \frac{\rho_f u^2}{2} \left(1.05 w^2 - 0.63\pi r^2\right).$$

We can now set this back equal to our thrust force to get

$$\frac{\rho_f u^2 \left(1.05 w^2 - 0.63\pi r^2\right)}{2} = \rho_f A_i \frac{l^2}{\left(4\rho_f A_i l^3 \frac{1}{EP_t}\right)^{\frac{2}{3}}}.$$

We hope to solve for the velocity u in terms of the other inputs, so we can isolate it on the left side to get

$$u^2 = \frac{2\rho_f A_i l^2}{\left(4\rho_f A_i l^3 \frac{1}{EP_t}\right)^{\frac{2}{3}} \left(1.05w^2 - 0.63\pi r^2\right) \rho_f}.$$

Finally we can cancel out the density in both the numerator and denominator and square root both sides to get

$$\text{ROV Velocity} = u = \boxed{\sqrt{\frac{2A_i l^2}{\left(4\rho_f A_i l^3 \frac{1}{EP_t}\right)^{\frac{2}{3}} \left(1.05w^2 - 0.63\pi r^2\right)}}}$$