Kinematics of miniBot-7R

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***Abstract*—This technical report contains the kinematics of the minibot-7R robot.**

***Index Terms*—kinematics**

I. Introduction

We want to derive the kinematics of the 7-degree-of-freedom robotic manipulator, minibot-7R. A kinematic diagram of the robot is provided in Figure 1.

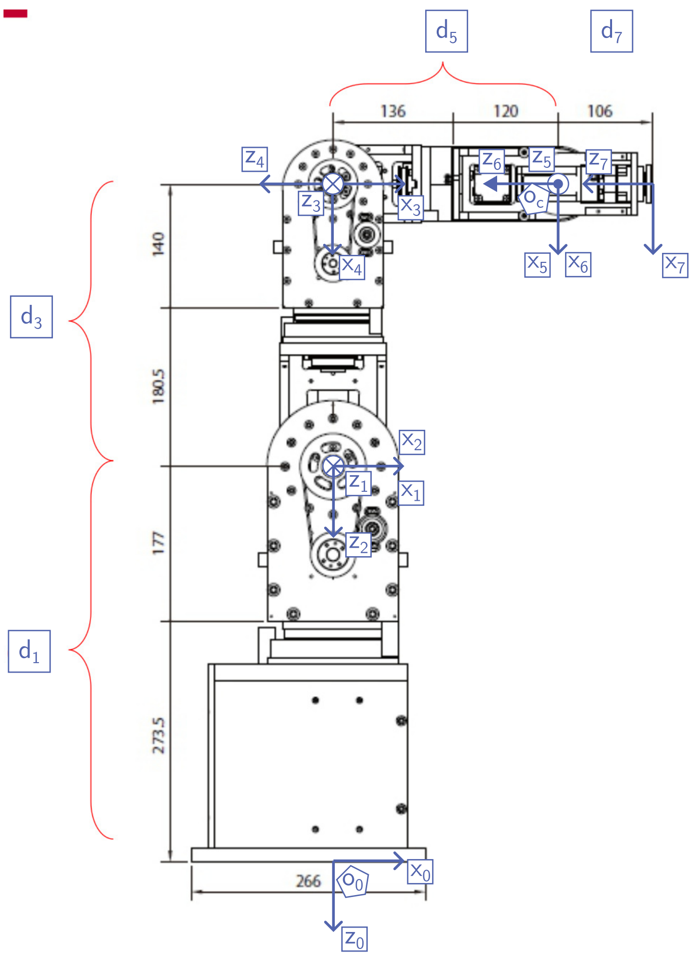

Fig. 1: Schematic of minibot-7R.

We will follow the classic Denavit-Hartenberg convention as presented in [1].

II. Denavit-Hartenberg Formulation

The transformation matrices between consecutive frames are given as follows.

TABLE I: DH Table for minibot-7R

Link	α_i [°]	a_i	d_i [mm]	θ_i [°]
1	90	0	-450.5	θ_1^*
2	-90	0	0	θ_2^*
3	90	0	-320.5	θ_3^*
4	-90	0	0	θ_4^*
5	-90	0	-256	θ_5^*
6	90	0	0	θ_6^*
7	0	0	-106	θ_7^*
Home:	$\theta_i = 0°$,	$i \neq 4$;		$\theta_4 = 90°$

$$
\begin{aligned}
A_1 &= \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_5 &= \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c_6 & 0 & s_6 & 0 \\ s_6 & 0 & -c_6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_7 &= \begin{bmatrix} c_7 & -s_7 & 0 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned} \tag{1}
$$

The forward kinematics map is then given by

$$
f : \mathcal{Q} \triangleq \prod_1^7 \mathbb{S}^1 \to \mathrm{SE}(3), \quad f(q) = \prod_1^7 A_i(q_i),
$$

where $q_i = \theta_i$ are the joint variables.

III. Velocity Kinematics

We derive the Jacobian $J : \mathbb{R}^7 \to \mathfrak{se}(3)$, that maps the rate of change of the joint variables to the end effector twist. This map can be represented in its matrix form by the concatenation of two submatrices, J_v and J_ω, as

$$
J = \begin{pmatrix} J_v \\ J_\omega \end{pmatrix}, \tag{2}
$$

where J_v and J_ω are both elements of $\mathbb{R}^{3\times 4}$. The construction of these matrices are subsequently shown, starting with J_v.

$$\begin{aligned} J_v &= \begin{bmatrix} J_{v_1} & J_{v_2} & \cdots & J_{v_7} \end{bmatrix}, \\ J_{v_i} &= z_{i-1} \times (o_c - o_{i-1}). \end{aligned} \tag{3}$$

The second part of the Jacobian matrix is constructed as

$$J_\omega = \begin{bmatrix} z_0 & z_1 & \cdots & z_6 \end{bmatrix}. \tag{4}$$

With the Jacobian constructed as in (2), the end-effector twist $\xi \in \mathfrak{se}(3)$ is related to the joint rates $\dot{q} \in \mathbb{R}^7$ as

$$J\dot{q} = \xi.$$

IV. Inverse Kinematics

Suppose that we are given a point $H \in \mathrm{SE}(3)$:

$$H = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix}.$$

We want to find a $q \in \mathcal{Q}$ such that $f(q) = H$.

We first consider the inverse position kinematics problem, which assumes that the inverse orientation kinematics may be solved by using the final three joints, i.e., whatever the rotation matrix ${}^0R_4$ is, there exist q_5, q_6 and q_7, such that ${}^0R_4{}^4R_7 = R$. The inverse position kinematics problem is then to find q_1 through q_4 such that the product $\prod_1^4 A_i$ has as its translation vector the wrist center location, $o_c \in \mathbb{R}^3$.

In order to solve the inverse position problem, we first find the location of the wrist center o_c in the base coordinate system, Σ_0:

$$o_c = o - d_7 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

We denote the current guess to the solution q^d to the inverse position kinematics problem by $q^{(k)}$ and perform the following iteration.

$$\begin{aligned} \bar{q}^{(k+1)} &= q^{(k)} - \alpha_k M\left(q^{(k)}\right)\left(f\left(q^{(k)}\right) - o_c\right), \\ q^{(k+1)} &= \mathrm{clamp}\left(\bar{q}^{(k+1)}, q_{\mathrm{lb}}, q_{\mathrm{ub}}\right). \end{aligned} \tag{5}$$

where the matrix M could be taken as the pseudo-inverse or the transpose of the Jacobian, J_w, that maps the rates of changes $\dot{q}_1$ through $\dot{q}_4$ to the rate of change of the wrist center, i.e., $J_w\dot{q}_{1:4} = \dot{o}_c$, where

$$J_w = \begin{bmatrix} J_{w,v_1} & \cdots & J_{w,v_4} \end{bmatrix},$$

where $J_{w,v_{i+1}} = z_i \times (o_c - o_i)$, for $i = 0, \ldots 3$. Notice that the step size α_k may be orders of magnitude different depending on whether $M = J_w^\top$ or $M = J_w^\dagger$.

References

[1] M. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. Wiley, 2020.