

Kinematics of miniBot-7R

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Abstract—This technical report contains the kinematics of the minibot-7R robot.

Index Terms—kinematics

I. INTRODUCTION

We want to derive the kinematics of the 7-degree-of-freedom robotic manipulator, minibot-7R. A kinematic diagram of the robot is provided in Figure 1.

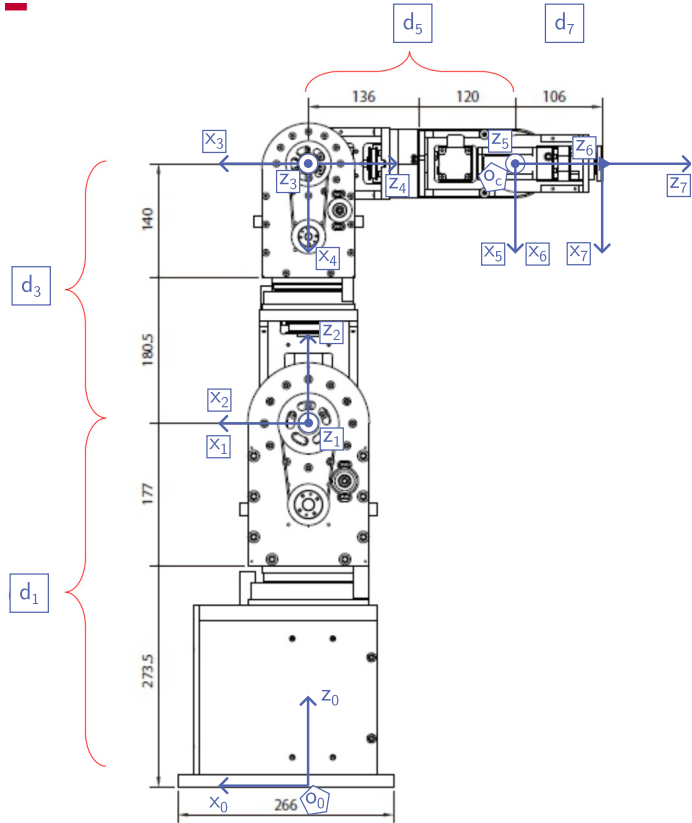


Fig. 1: Schematic of minibot-7R.

We will follow the classic Denavit-Hartenberg convention as presented in [1].

II. DENAVIT-HARTENBERG FORMULATION

The transformation matrices between consecutive frames are given as follows.

TABLE I: DH Table for minibot-7R

Link	α_i [°]	a_i	d_i [mm]	θ_i [°]
1	-90	0	450.5	θ_1^*
2	90	0	0	θ_2^*
3	-90	0	320.5	θ_3^*
4	-90	0	0	θ_4^*
5	90	0	256	θ_5^*
6	-90	0	0	θ_6^*
7	0	0	106	θ_7^*
Home: $\theta_i = 0^\circ$, $i \neq 4$;				$\theta_4 = 90^\circ$

$$\begin{aligned}
 A_1 &= \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 A_3 &= \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 A_5 &= \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c_6 & 0 & -s_6 & 0 \\ s_6 & 0 & c_6 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 A_7 &= \begin{bmatrix} c_7 & -s_7 & 0 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
 \end{aligned} \tag{1}$$

The forward kinematics map is then given by

$$f : \mathcal{Q} \triangleq \prod_{i=1}^7 \mathbb{S}^1 \rightarrow \text{SE}(3), \quad f(q) = \prod_{i=1}^7 A_i(q_i),$$

where $q_i = \theta_i$ are the joint variables.

III. VELOCITY KINEMATICS

We derive the Jacobian $J : \mathbb{R}^7 \rightarrow \mathfrak{se}(3)$, that maps the rate of change of the joint variables to the end effector twist. This map can be represented in its matrix form by the concatenation of two submatrices, J_v and J_ω , as

$$J = \begin{pmatrix} J_v \\ J_\omega \end{pmatrix}, \tag{2}$$

where J_v and J_ω are both elements of $\mathbb{R}^{3 \times 4}$. The construction of these matrices are subsequently shown, starting with J_v .

$$\begin{aligned} J_v &= [J_{v_1} \quad J_{v_2} \quad \cdots \quad J_{v_7}], \\ J_{v_i} &= z_{i-1} \times (o_c - o_{i-1}). \end{aligned} \quad (3)$$

The second part of the Jacobian matrix is constructed as

$$J_\omega = [z_0 \quad z_1 \quad \cdots \quad z_6]. \quad (4)$$

With the Jacobian constructed as in (2), the end-effector twist $\xi \in \mathfrak{se}(3)$ is related to the joint rates $\dot{q} \in \mathbb{R}^7$ as

$$J\dot{q} = \xi.$$

IV. INVERSE KINEMATICS

Suppose that we are given a point $H \in \text{SE}(3)$:

$$H = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix}.$$

We want to find a $q \in \mathcal{Q}$ such that $f(q) = H$.

We first consider the inverse position kinematics problem, which assumes that the inverse orientation kinematics may be solved by using the final three joints, i.e., whatever the rotation matrix 0R_4 is, there exist q_5 , q_6 and q_7 , such that ${}^0R_4 R_7 = R$. The inverse position kinematics problem is then to find q_1 through q_4 such that the product $\prod_1^4 A_i$ has as its translation vector the wrist center location, $o_c \in \mathbb{R}^3$.

In order to solve the inverse position problem, we first find the location of the wrist center o_c in the base coordinate system, Σ_0 :

$$o_c = o - d_7 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

We denote the current guess to the solution q^d to the inverse position kinematics problem by $q^{(k)}$ and perform the following iteration.

$$q^{(k+1)} = q^{(k)} - \alpha_k M(q^{(k)}) \left(f(q^{(k)}) - o_c \right), \quad (5)$$

where the matrix M could be taken as \bar{J}_v^\dagger or \bar{J}_v^\top . \bar{J}_v is the translation part of the Jacobian matrix that relates \dot{q}_1 through \dot{q}_4 to the rate of change of the wrist center, i.e., $\bar{J}_v \dot{q}_{1:4} = \dot{o}_c$, where

$$\bar{J}_v = [\bar{J}_{v_1} \quad \cdots \quad \bar{J}_{v_4}],$$

where $\bar{J}_{v_{i+1}} = z_i \times (o_c - o_i)$, for $i = 0, \dots, 3$.

REFERENCES

- [1] M. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. Wiley, 2020.