

# Kinematics of miniBot-7R

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**Abstract**—This technical report contains the kinematics of the minibot-7R robot.

**Index Terms**—kinematics

## I. INTRODUCTION

We want to derive the kinematics of the 7-degree-of-freedom robotic manipulator, minibot-7R. A kinematic diagram of the robot is provided in Figure 1.

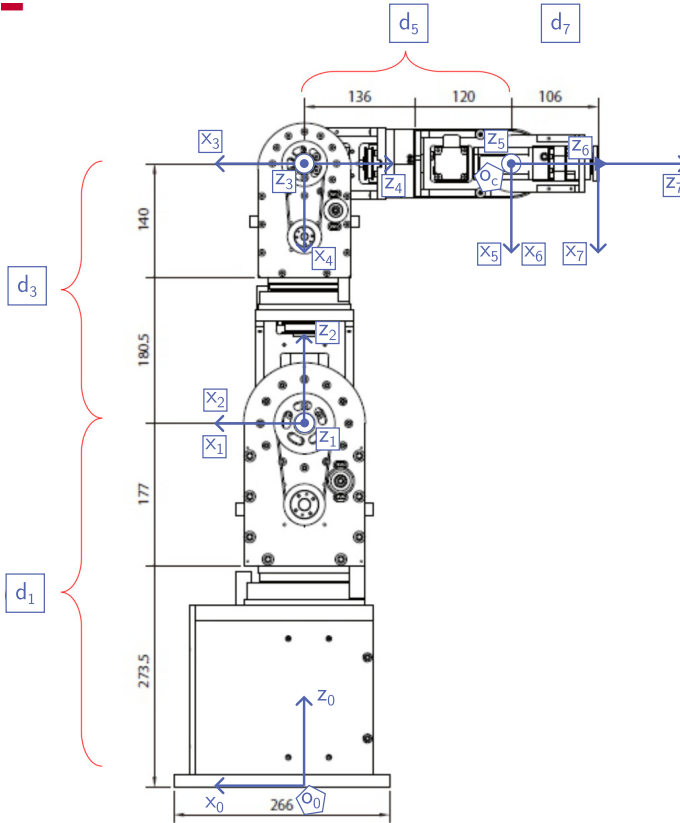


Fig. 1: Schematic of minibot-7R.

We will follow the classic Denavit-Hartenberg convention as presented in [1].

## II. DENAVIT-HARTENBERG FORMULATION

The transformation matrices between consecutive frames are given as follows.

TABLE I: DH Table for minibot-7R

Link	$\alpha_i$ [°]	$a_i$	$d_i$ [mm]	$\theta_i$ [°]
1	-90	0	450.5	$\theta_1^*$
2	90	0	0	$\theta_2^*$
3	-90	0	320.5	$\theta_3^*$
4	-90	0	0	$\theta_4^*$
5	90	0	256	$\theta_5^*$
6	-90	0	0	$\theta_6^*$
7	0	0	106	$\theta_7^*$
Home: $\theta_i = 0^\circ$ , $i \neq 4$ ; $\theta_4 = 90^\circ$				

$$\begin{aligned}
 A_1 &= \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 A_3 &= \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 A_5 &= \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c_6 & 0 & -s_6 & 0 \\ s_6 & 0 & c_6 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 A_7 &= \begin{bmatrix} c_7 & -s_7 & 0 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
 \end{aligned} \tag{1}$$

The forward kinematics map is then given by

$$f: \mathcal{Q} \triangleq \prod_{i=1}^7 \mathbb{S}^1 \rightarrow \text{SE}(3), \quad f(q) = \prod_{i=1}^7 A_i(q_i),$$

where  $q_i = \theta_i$  are the joint variables.

An alternative assignment of frames is shown in Figure 2. When the frames are assigned in this way, the DH table I still remains valid, except for the fact that at home position, we now have  $\theta_4 = -90^\circ$  rather than  $90^\circ$ .

## III. VELOCITY KINEMATICS

We derive the Jacobian  $J: \mathbb{R}^7 \rightarrow \mathfrak{se}(3)$ , that maps the rate of change of the joint variables to the end effector twist. This map can be represented in its matrix form by the concatenation of two submatrices,  $J_v$  and  $J_\omega$ , as

$$J = \begin{pmatrix} J_v \\ J_\omega \end{pmatrix}, \tag{2}$$

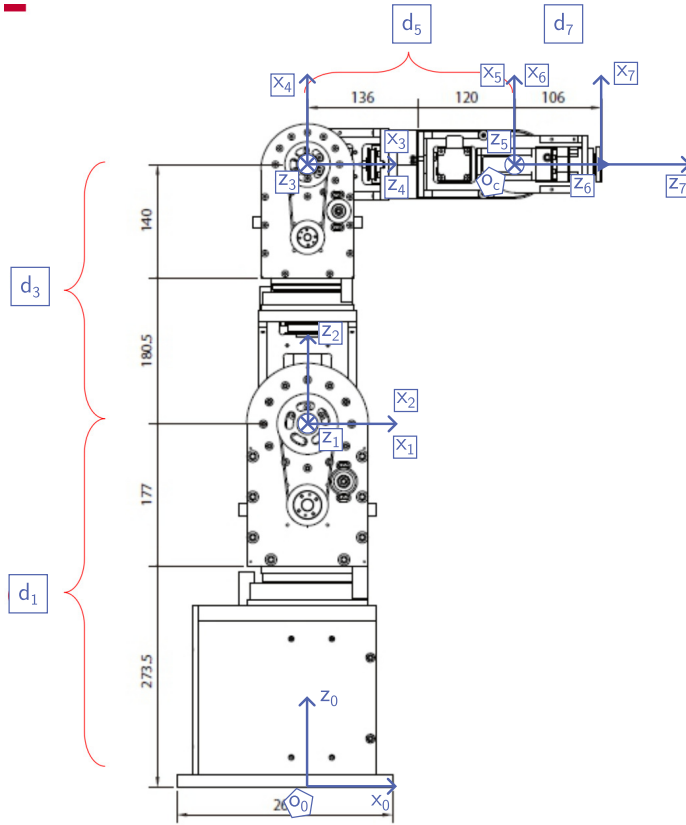


Fig. 2: Schematic of minibot-7R: Alternative DH frame assignment.

where  $J_v$  and  $J_\omega$  are both elements of  $\mathbb{R}^{3 \times 4}$ . The construction of these matrices are subsequently shown, starting with  $J_v$ .

$$J_v = [J_{v_1} \ J_{v_2} \ \cdots \ J_{v_7}], \quad (3)$$

$$J_{v_i} = z_{i-1} \times (o_c - o_{i-1}).$$

The second part of the Jacobian matrix is constructed as

$$J_\omega = [z_0 \ z_1 \ \cdots \ z_6]. \quad (4)$$

With the Jacobian constructed as in (2), the end-effector twist  $\xi \in \mathfrak{se}(3)$  is related to the joint rates  $\dot{q} \in \mathbb{R}^7$  as

$$J\dot{q} = \xi.$$

#### IV. INVERSE KINEMATICS

Suppose that we are given a point  $H \in \text{SE}(3)$ :

$$H = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix}.$$

We want to find a  $q \in \mathcal{Q}$  such that  $f(q) = H$ .

We first consider the inverse position kinematics problem, which assumes that the inverse orientation kinematics may be solved by using the final three joints, i.e., whatever the rotation matrix  ${}^0R_4$  is, there exist  $q_5$ ,  $q_6$  and  $q_7$ , such that  ${}^0R_4 R_7 = R$ . The inverse position kinematics problem is then to find  $q_1$  through  $q_4$  such that the product  $\prod_1^4 A_i$  has as its translation vector the wrist center location,  $o_c \in \mathbb{R}^3$ .

In order to solve the inverse position problem, we first find the location of the wrist center  $o_c$  in the base coordinate system,  $\Sigma_0$ :

$$o_c = o - d_7 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

We denote the current guess to the solution  $q^d$  to the inverse position kinematics problem by  $q^{(k)}$  and perform the following iteration.

$$q^{(k+1)} = q^{(k)} - \alpha_k M(q^{(k)}) (f(q^{(k)}) - o_c), \quad (5)$$

where the matrix  $M$  could be taken as  $\bar{J}_v^\dagger$  or  $\bar{J}_v^\top$ .  $\bar{J}_v$  is the translation part of the Jacobian matrix that relates  $\dot{q}_1$  through  $\dot{q}_4$  to the rate of change of the wrist center, i.e.,  $\bar{J}_v \dot{q}_{1:4} = \dot{o}_c$ , where

$$\bar{J}_v = [\bar{J}_{v_1} \ \cdots \ \bar{J}_{v_4}],$$

where  $\bar{J}_{v_{i+1}} = z_i \times (o_c - o_i)$ , for  $i = 0, \dots, 3$ .

#### REFERENCES

- [1] M. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. Wiley, 2020.