Kinematics of miniBot-7R

Aykut C. Satici *Member, IEEE*

***Abstract*—This technical report contains the kinematics of the minibot-7R robot.**

***Index Terms*—kinematics**

I. Introduction

We want to derive the kinematics of the 7-degree-of-freedom robotic manipulator, minibot-7R. A kinematic diagram of the robot is provided in Figure 1.

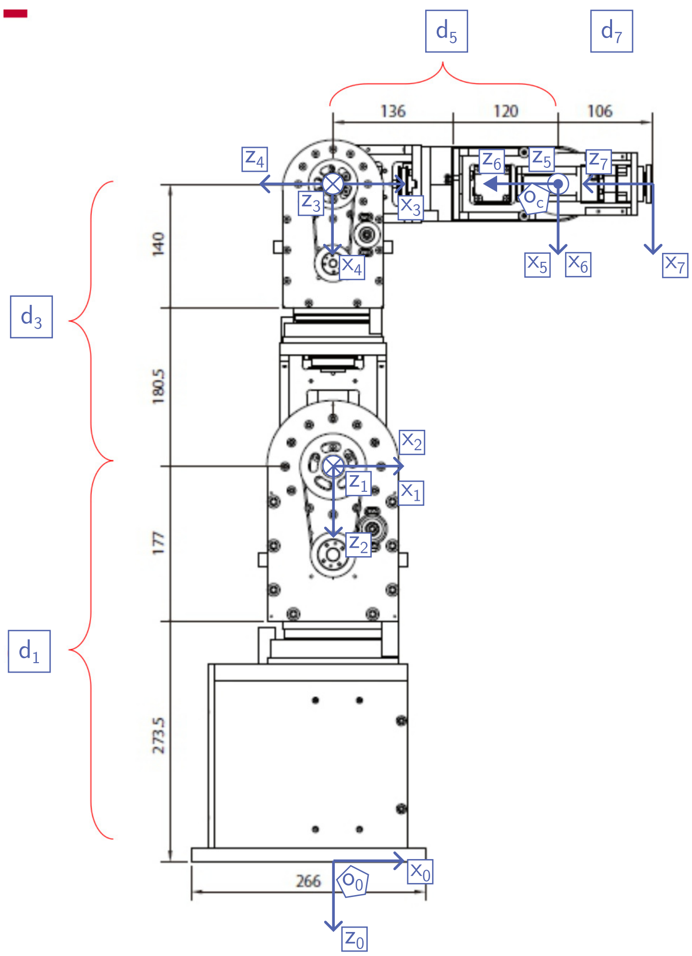

Fig. 1: Schematic of minibot-7R.

We will follow the classic Denavit-Hartenberg convention as presented in [1].

II. Denavit-Hartenberg Formulation

The transformation matrices between consecutive frames are given as follows.

TABLE I: DH Table for minibot-7R

Link	α_i [°]	a_i	d_i [mm]	θ_i [°]
1	90	0	−450.5	θ_1^*
2	−90	0	0	θ_2^*
3	90	0	−320.5	θ_3^*
4	−90	0	0	θ_4^*
5	−90	0	−256	θ_5^*
6	90	0	0	θ_6^*
7	0	0	−106	θ_7^*
Home:	$\theta_i = 0°$,	$i \neq 4$;		$\theta_4 = 90°$

$$
\begin{aligned}
A_1 &= \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_5 &= \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c_6 & 0 & s_6 & 0 \\ s_6 & 0 & -c_6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_7 &= \begin{bmatrix} c_7 & -s_7 & 0 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned} \tag{1}
$$

The forward kinematics map is then given by

$$
f : \mathcal{Q} \triangleq \prod_1^7 \mathbb{S}^1 \to \mathrm{SE}(3), \quad f(\boldsymbol{q}) = \prod_1^7 A_i(q_i),
$$

where $q_i = \theta_i$ are the joint variables.

III. Velocity Kinematics

We derive the Jacobian $\boldsymbol{J} : \mathbb{R}^7 \to \mathfrak{se}(3)$, that maps the rate of change of the joint variables to the end effector twist. This map can be represented in its matrix form by the concatenation of two submatrices, $\boldsymbol{J}_v$ and $\boldsymbol{J}_\omega$, as

$$
\boldsymbol{J} = \begin{pmatrix} \boldsymbol{J}_v \\ \boldsymbol{J}_\omega \end{pmatrix}, \tag{2}
$$

where $\boldsymbol{J}_v$ and $\boldsymbol{J}_\omega$ are both elements of $\mathbb{R}^{3\times 7}$. The construction of these matrices are subsequently shown, starting with $\boldsymbol{J}_v$.

$$\begin{aligned} \boldsymbol{J}_v &= \begin{bmatrix} \boldsymbol{J}_{v_1} & \boldsymbol{J}_{v_2} & \cdots & \boldsymbol{J}_{v_7} \end{bmatrix}, \\ \boldsymbol{J}_{v_i} &= \boldsymbol{z}_{i-1} \times (\boldsymbol{o}_c - \boldsymbol{o}_{i-1}). \end{aligned} \tag{3}$$

The second part of the Jacobian matrix is constructed as

$$\boldsymbol{J}_\omega = \begin{bmatrix} \boldsymbol{z}_0 & \boldsymbol{z}_1 & \cdots & \boldsymbol{z}_6 \end{bmatrix}. \tag{4}$$

With the Jacobian constructed as in (2), the end-effector twist $\boldsymbol{\xi} \in \mathfrak{se}(3)$ is related to the joint rates $\dot{\boldsymbol{q}} \in \mathbb{R}^7$ as

$$\boldsymbol{J}\dot{\boldsymbol{q}} = \boldsymbol{\xi}.$$

IV. Inverse Kinematics (IK)

Suppose that we are given a point $H \in \mathrm{SE}(3)$:

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{o} \\ 0 & 1 \end{bmatrix}.$$

We want to find a $\boldsymbol{q} \in \mathcal{Q}$ such that $f(\boldsymbol{q}) = \boldsymbol{H}$. We describe two methods to achieve this, which would yield equivalent solutions for a fully-actuated robot, but the differences get amplified when the robot is redundant such as miniBot-7R.

We will be using gradient-based optimization to solve for the joint angles. This is a recursive algorithm that improves on the current guess $\boldsymbol{q}^{(k)}$ to a solution $\boldsymbol{q}^d$ by performing the following iteration.

$$\begin{aligned} \bar{\boldsymbol{q}}^{(k+1)} &= \boldsymbol{q}^{(k)} - \alpha_k \boldsymbol{M}\left(\boldsymbol{q}^{(k)}\right)\delta\boldsymbol{x}, \\ \boldsymbol{q}^{(k+1)} &= \mathrm{clamp}\left(\bar{\boldsymbol{q}}^{(k+1)}, \boldsymbol{q}_{\mathrm{lb}}, \boldsymbol{q}_{\mathrm{ub}}\right). \end{aligned} \tag{5}$$

The matrix $\boldsymbol{M}$ is some sort of Jacobian, $\delta\boldsymbol{x}$ is the pertinent error that the current guess produces, and $(\boldsymbol{q}_{\mathrm{lb}}, \boldsymbol{q}_{\mathrm{ub}})$ is the lower and upper bounds on the joint angles.

A. IK by Decoupling Position and Orientation

We first consider the inverse position kinematics problem, which relies on the fact that the inverse orientation kinematics may be solved by using the final three joints due to the presence of a spherical wrist. Indeed, whatever the rotation matrix ${}^0\boldsymbol{R}_4$ is, there exist q_5, q_6 and q_7, such that ${}^0\boldsymbol{R}_4{}^4\boldsymbol{R}_7 = \boldsymbol{R}$. Since ${}^0\boldsymbol{R}_4$ will be determined once the inverse position kinematics problem is solved, one can then use q_5 through q_7 to yield ${}^4\boldsymbol{R}_7 = {}^0\boldsymbol{R}_4^\top \boldsymbol{R}$ following the steps outlined in Chapter 5.4 of [1] (pg. 151).

The inverse position kinematics problem is to find q_1 through q_4 such that the product $\prod_1^4 \boldsymbol{A}_i$ has as its translation vector the wrist center location, $\boldsymbol{o}_c \in \mathbb{R}^3$.

In order to solve the inverse position problem, we first find the location of the wrist center $\boldsymbol{o}_c$ in the base coordinate system, Σ_0:

$$\boldsymbol{o}_c = \boldsymbol{o} - d_7 \boldsymbol{R} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top.$$

The gradient descent optimization in equation (5) is then performed with $\delta\boldsymbol{x} = \left(f\left(\boldsymbol{q}^{(k)}\right) - \boldsymbol{o}_c\right)$. We can take the matrix $\boldsymbol{M}$ as the pseudo-inverse or the transpose of the Jacobian, $\boldsymbol{J}_w$, that maps the rates of changes $\dot{q}_1$ through $\dot{q}_4$ to the rate of change of the wrist center, i.e., $\boldsymbol{J}_w \dot{\boldsymbol{q}}_{1:4} = \dot{\boldsymbol{o}}_c$. This matrix $\boldsymbol{J}_w$ corresponds to the first three-by-four block of the matrix $\boldsymbol{J}_v$ from equation (3). Notice that the step size α_k may be orders of magnitude different depending on whether $\boldsymbol{M} = \boldsymbol{J}_w^\top$ or $\boldsymbol{M} = \boldsymbol{J}_w^\dagger$.

B. Fully Coupled IK

References

[1] M. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. Wiley, 2020.