# Kinematics of miniBot-7R

Aykut C. Satici *Member, IEEE*

***Abstract*—This technical report contains the kinematics of the minibot-7R robot.**

***Index Terms*—kinematics**

## I. Introduction

We want to derive the kinematics of the 7-degree-of-freedom robotic manipulator, minibot-7R. A kinematic diagram of the robot is provided in Figure 1.

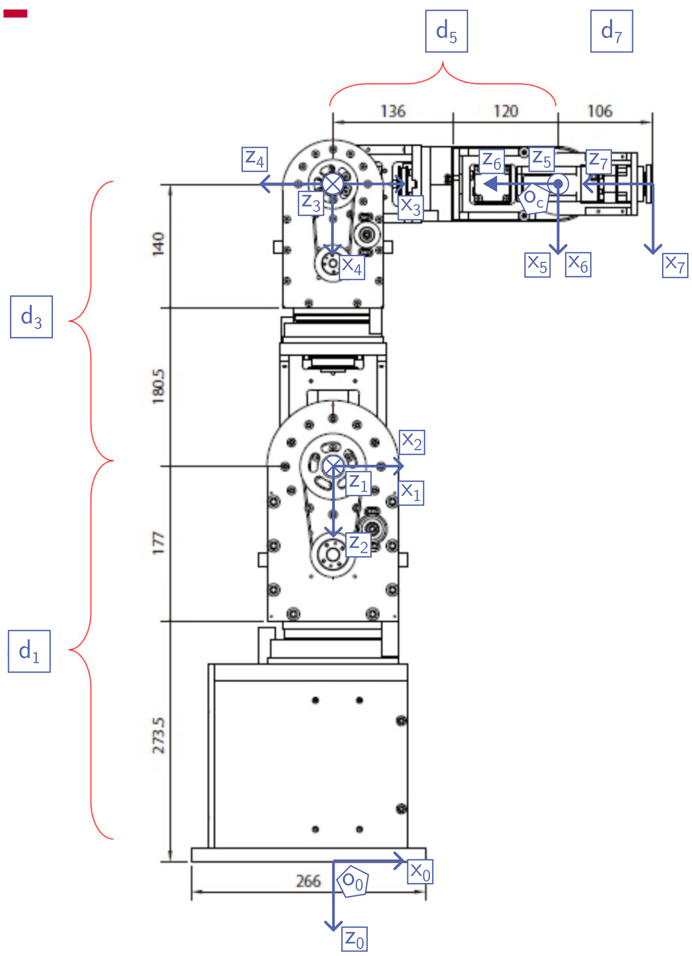


Fig. 1: Schematic of minibot-7R.

We will follow the classic Denavit-Hartenberg convention as presented in [1].

## II. Denavit-Hartenberg Formulation

The transformation matrices between consecutive frames are given as follows.

TABLE I: DH Table for minibot-7R

| *Link* | $\alpha_i$ [°] | $a_i$ | $d_i$ [mm] | $\theta_i$ [°] |
|---|---|---|---|---|
| 1 | $-90$ | 0 | 450.5 | $\theta_1^*$ |
| 2 | 90 | 0 | 0 | $\theta_2^*$ |
| 3 | $-90$ | 0 | 320.5 | $\theta_3^*$ |
| 4 | $-90$ | 0 | 0 | $\theta_4^*$ |
| 5 | 90 | 0 | 256 | $\theta_5^*$ |
| 6 | $-90$ | 0 | 0 | $\theta_6^*$ |
| 7 | 0 | 0 | 106 | $\theta_7^*$ |
| Home: | $\theta_i = 0°$, | $i \neq 4$; | | $\theta_4 = 90°$ |

$$
A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$
$$
A_3 = \begin{bmatrix} c_3 & 0 & -s_3 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$
$$
A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_6 = \begin{bmatrix} c_6 & 0 & -s_6 & 0 \\ s_6 & 0 & c_6 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$
$$
A_7 = \begin{bmatrix} c_7 & -s_7 & 0 & 0 \\ s_7 & c_7 & 0 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{1}
$$

The forward kinematics map is then given by

$$
f : \mathcal{Q} \triangleq \prod_1^7 \mathbb{S}^1 \to \mathrm{SE}(3), \quad f(q) = \prod_1^7 A_i(q_i),
$$

where $q_i = \theta_i$ are the joint variables.

## III. Velocity Kinematics

We derive the Jacobian $J : \mathbb{R}^7 \to \mathfrak{se}(3)$, that maps the rate of change of the joint variables to the end effector twist. This map can be represented in its matrix form by the concatenation of two submatrices, $J_v$ and $J_\omega$, as

$$
J = \begin{pmatrix} J_v \\ J_\omega \end{pmatrix}, \tag{2}
$$

where $J_v$ and $J_\omega$ are both elements of $\mathbb{R}^{3\times 4}$. The construction of these matrices are subsequently shown, starting with $J_v$.

$$\begin{aligned} J_v &= \begin{bmatrix} J_{v_1} & J_{v_2} & \cdots & J_{v_7} \end{bmatrix}, \\ J_{v_i} &= z_{i-1} \times (o_c - o_{i-1}). \end{aligned} \tag{3}$$

The second part of the Jacobian matrix is constructed as

$$J_\omega = \begin{bmatrix} z_0 & z_1 & \cdots & z_6 \end{bmatrix}. \tag{4}$$

With the Jacobian constructed as in (2), the end-effector twist $\xi \in \mathfrak{se}(3)$ is related to the joint rates $\dot{q} \in \mathbb{R}^7$ as

$$J\dot{q} = \xi.$$

## IV. Inverse Kinematics

Suppose that we are given a point $H \in \mathrm{SE}(3)$:

$$H = \begin{bmatrix} R & o \\ 0 & 1 \end{bmatrix}.$$

We want to find a $q \in \mathcal{Q}$ such that $f(q) = H$.

We first consider the inverse position kinematics problem, which assumes that the inverse orientation kinematics may be solved by using the final three joints, i.e., whatever the rotation matrix ${}^0R_4$ is, there exist $q_5$, $q_6$ and $q_7$, such that ${}^0R_4{}^4R_7 = R$. The inverse position kinematics problem is then to find $q_1$ through $q_4$ such that the product $\prod_1^4 A_i$ has as its translation vector the wrist center location, $o_c \in \mathbb{R}^3$.

In order to solve the inverse position problem, we first find the location of the wrist center $o_c$ in the base coordinate system, $\Sigma_0$:

$$o_c = o - d_7 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

We denote the current guess to the solution $q^d$ to the inverse position kinematics problem by $q^{(k)}$ and perform the following iteration.

$$q^{(k+1)} = q^{(k)} - \alpha_k M\left(q^{(k)}\right)\left(f\left(q^{(k)}\right) - o_c\right), \tag{5}$$

where the matrix $M$ could be taken as $\bar{J}_v^\dagger$ or $\bar{J}_v^\top$. $\bar{J}_v$ is the translation part of the Jacobian matrix that relates $\dot{q}_1$ through $\dot{q}_4$ to the rate of change of the wrist center, i.e., $\bar{J}_v \dot{q}_{1:4} = \dot{o}_c$, where

$$\bar{J}_v = \begin{bmatrix} \bar{J}_{v_1} & \cdots & \bar{J}_{v_4} \end{bmatrix},$$

where $\bar{J}_{v_{i+1}} = z_i \times (o_c - o_i)$, for $i = 0, \ldots 3$.

## References

[1] M. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. Wiley, 2020.