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Safe robust adaptive control under both parametric and nonparametric uncertainty

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ABSTRACT
This article presents a method for guaranteeing the safety of a system with both parametric and nonparametric uncertainties, while at the same time decreasing the conservatism compared to existing approaches. This is obtained by combining robust adaptive control barrier functions (RaCBF) and Gaussian process control barrier functions (GPCBF). We provide a condition under which the considered system is safe with a given probability, and show that the proposed method is less conservative than GPCBF. We evaluate the method through a simulation study, where we consider a force controlled robot manipulator in contact with a partially unknown environment. The results show that our proposed GPRaCBF can guarantee bounds on the contact forces despite parametric and nonparametric uncertainties in the contact dynamics and outperforms GPCBF in terms of the conservatism.

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Safety guarantees; system uncertainty; robust control; force control

1. Introduction

As the demand for flexibility and quick reconfigurability in the manufacturing industry increases [1], more and more robots are being used in tasks that involve contact with the environment and the workpiece. Here, the need for robot force control may arise either as a means of adapting to uncertainty in the environment, or directly from using the robot in assembly applications. Such applications are often safety critical, as too high interaction forces can lead to damage to the environment and compromise the safety of the robotic setup, e.g. in cases where collaborative robots work side-by-side with humans [2–4].

In safety-critical robotics applications, control barrier functions (CBFs) have been widely studied in recent years and demonstrated their effectiveness to ensure safety guarantees in, for instance, bipedal walking [5], multi-robot planning [6], and quadrotor control [7]. Most approaches have considered limits in terms of position, e.g.for obstacle avoidance [8] or separation monitoring [9]. However, CBFs have not been widely studied in the context of robot force control, where the constraints place bounds on the maximum forces and torques the manipulator may apply, and not on its position. The approaches that have considered such cases, e.g. [10], have adopted simple and fully certain environment models.

However, the lack of certainty in the contact dynamics is an acknowledged problem that has been studied in the context of safety for force control and manipulation [11,12]. This uncertainty makes using CBFs for safe contact force control a challenge, as their safety guarantees depend entirely on an accurate model of the process dynamics. Acknowledging the limitations of CBFs in terms of their dependence on an accurate dynamic model, recent research has considered how to preserve the safety guarantees in the presence of uncertainty. Different types of uncertainty have been studied. Parametric uncertainty was considered in [13,14], which introduce adaptive control barrier functions (aCBF) and RaCBF, respectively. The RaCBF framework ensures that the conservatism of the safety-ensuring control can be minimized by adapting to the unknown parameters. Many approaches to including nonparametric uncertainty have also been proposed, including [15], which incorporates uncertainty in the model given by a Gaussian process (GP) per state in the system, and [16], which includes a single GP to model the uncertainty of the system in the control barrier function directly; hence, only one GP is needed, which significantly simplifies the online learning of the GP [17].

In this article, we combine RaCBF [14] and Gaussian process control barrier functions (GPCBF) [16] to guarantee the safety of a system in the presence of

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both parametric and nonparametric uncertainties in the model. By exploiting the structure of the uncertainty, we show that our approach is less conservative than GPCBF under equal conditions. We present simulation results that showcase the advantages of our method in a robot force control application despite modeling errors in both the parameters and the structure of the contact dynamics model.

The remainder of the article is organized as follows: Section 2 presents the problem formulation. Section 3 reviews preliminary theory on RaCBF and GPCBF. Section 4 introduces our proposed methodology, provides a proof that the resulting system is safe with a given probability, and analyzes the conservatism of our approach compared to GPCBF. We subsequently verify our proposed method in a simulation of a robot force control application in Section 5, and provide conclusions in Section 6.

2. Problem formulation

The problem addressed in this article is the design of safety-ensuring controllers for systems with both parametric and nonparametric uncertainties. Thus, we consider the following control affine system:

$$\dot{x} = f(x) + F(x)\theta^* + g(x)u, \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ is the state, and f, g are locally Lipschitz continuous functions, $F : \mathcal{X} \to \mathbb{R}^{n\times k}$ is smooth on $\mathcal{X}$ and models parametric uncertainty about f, $F(0) = 0$, $\theta^* \in \Theta \subset \mathbb{R}^k$ is an unknown parameter, and $u \in \mathcal{U} \subset \mathbb{R}^m$ is a control input. According to [5], the safe set

$$\mathcal{S} = \{x \in \mathcal{X} \mid h(x) \geq 0\} \tag{2}$$

is defined as the superlevel set of a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$. If the set $\mathcal{S}$ is forward invariant, then the control system (1) is said to be *safe* for the set $\mathcal{S}$. To render $\mathcal{S}$ safe, we define a CBF. The function $h : \mathcal{X} \to \mathbb{R}$ is a CBF for the control system (1) if there exists an extended class $\mathcal{K}_\infty$ function α such that

$$\sup_{u\in\mathcal{U}}[L_f h(x) + L_{F(x)\theta^*} h(x) + L_g h(x)u + \alpha\,(h(x))] \geq 0. \tag{3}$$

There exist several methods that address the safety of the system (1) via CBFs given by (3); however, when the system dynamics is uncertain the analysis becomes more complicated. Methods that consider parametric uncertainties assume that θ^* is unknown, but that f, F, and g are known, see, e.g. [14], whereas methods that consider nonparametric uncertainties do not exploit the structure of the uncertainty, but assume that f and g are unknown, see, e.g. [16].

In this work, we combine parametric and nonparametric methods to guarantee the safety of an uncertain system, while at the same time decreasing the conservatism compared to [16]; i.e. we solve the following problem:

Problem 2.1: *Consider a control system* (1) *where f, g, and θ^* are not known exactly; however, bounds on the uncertainty are given as follows:*

$$-\tilde{\vartheta} \leq \tilde{\theta} \leq \tilde{\vartheta} \tag{4}$$

$$||\Delta||_k \leq B, \tag{5}$$

where $\tilde{\theta} = \hat{\theta} - \theta^*$, $\hat{\theta}, \hat{g}, \hat{f}$ are estimates of θ^*, g, f, $\Delta(x, u) = L_f h(x) + L_g h(x)u - L_{\hat{f}} h(x) - L_{\hat{g}} h(x)u$, $\tilde{\vartheta} \in \mathbb{R}^k$ (see (9)), $B \in \mathbb{R}$ is an upper bound on the RKHS norm of Δ (see (19)) and assume that $\Delta(x, u) + \epsilon$ is measured, where ϵ is a zero mean Gaussian noise process with variance σ. Design a control signal u that makes the system (1) *safe with probability at least $1 - \delta$.*

A solution to Problem 2.1 that combines RaCBF with GPCBF is provided in Section 4; the individual methods RaCBF and GPCBF are explained in the following section.

3. Preliminaries

This section presents background on control barrier functions that consider parametric uncertainties and control barrier functions that consider nonparametric uncertainty.

3.1. Parametric uncertainty

To ensure safety despite parametric uncertainty, we leverage adaptive control methods for estimating the unknown parameter θ^* in the model (1). In addition, the safe set $\mathcal{S}$ in (2) is modified to include parameter dependence in the CBF and the safe set is tightened according to the maximum possible parameter estimation error.

The following description follows [13], which defines adaptive Control Barrier Functions (aCBF), and [14], which defines Robust adaptive Control Barrier Functions (RaCBF). To study a system with parametric uncertainty, the system dynamics (1) is extended with the dynamics of an adaptive parameter estimator as

$$\begin{bmatrix}\dot{x}\\ \dot{\hat{\theta}}\end{bmatrix} = \begin{bmatrix}f(x) + F(x)\theta^* + g(x)u\\ \Gamma\tau(x, \hat{\theta})\end{bmatrix} \tag{6}$$

$$\text{with} \quad \tau(x, \hat{\theta}) = -\left(\frac{\partial h_r}{\partial x}(x, \hat{\theta})F(x)\right)^\top, \tag{7}$$

where $\hat{\theta} \in \Theta \subset \mathbb{R}^k$ is the estimated parameter, and τ is an update law that is locally Lipschitz continuous on $\mathcal{X} \times \Theta$, and Γ is an adaptive gain. The safe set $\mathcal{S}$ is reformulated to the parameterized safe set

$$\mathcal{S}_\theta = \{x \in \mathcal{X} \mid h_r(x, \theta) \geq 0\}, \tag{8}$$

where $h_r : \mathcal{X} \times \Theta \to \mathbb{R}$ is a continuously differentiable function.

In particular, the robust adaptive control barrier function (RaCBF) introduced in [14] makes the safety subsets tight in $\mathcal{S}_\theta$ by introducing a maximum possible parameter estimation error in the following definitions of superlevel sets:

$$\mathcal{S}_\theta^r = \left\{x \in \mathcal{X} : h_r(x, \theta) \geq \frac{1}{2}\tilde{\vartheta}^\top \Gamma^{-1} \tilde{\vartheta}\right\}, \tag{9}$$

where $\mathcal{S}_\theta^r \subset \mathcal{S}_\theta$ is the tightened set, and $\tilde{\vartheta} \in \mathbb{R}^k$ is the maximum possible error between θ^* and $\hat{\theta}$, and $\Gamma \in \mathbb{R}^{n \times n}$ is an adaptive gain satisfying $\lambda_{\min}(\Gamma) \geq \frac{||\tilde{\vartheta}||^2}{2h_r(x,\theta)}$. We say that $h_r(x, \theta)$ is a RaCBF if there exists an extended class $\mathcal{K}_\infty$ function α such that for the system (6) and any $\theta \in \Theta$:

$$\sup_{u \in \mathcal{U}} \left[\frac{\partial h_r}{\partial x}(x, \theta) \left(\mathcal{F}(x, \theta) + g(x)u\right)\right] \geq -\alpha \left(h_r(x, \theta) - \frac{1}{2}\tilde{\vartheta}^\top \Gamma^{-1} \tilde{\vartheta}\right) \tag{10}$$

with

$$\mathcal{F}(x, \theta) = f(x) + F(x)\lambda(x, \theta), \tag{11}$$

$$\lambda(x, \theta) \triangleq \theta - \Gamma \left(\frac{\partial h_r}{\partial \theta}(x, \theta)\right)^\top. \tag{12}$$

Consequently, a min-norm quadratic programming (QP) problem with the safety constraint (10) can be constructed as follows [14]:

$$u_{\mathsf{safe}} = \arg\min_{u \in \mathcal{U}} \frac{1}{2}||u - u_{\mathsf{nominal}}||^2 \quad (\mathbf{RaCBF-QP})$$
$$\text{s.t. } \frac{\partial h_r}{\partial x}(x, \hat{\theta}) \left(\mathcal{F}(x, \hat{\theta}) + g(x)u\right) + \alpha \left(h_r(x, \hat{\theta}) - \frac{1}{2}\tilde{\vartheta}^\top \Gamma^{-1} \tilde{\vartheta}\right) \geq 0.$$

The possible conservatism of RaCBFs originates from the introduction of $\tilde{\vartheta}$; however, the gain of the parameter estimator Γ can be increased to reduce the magnitude of $\tilde{\vartheta}^\top \Gamma^{-1} \tilde{\vartheta}$. In addition, set membership identification (SMID) can be used for decreasing $\tilde{\vartheta}$ over time [14]. Therefore, one can make the conservatism arbitrarily small.

3.2. Nonparametric uncertainty

To consider nonparametric uncertainty, we assume that only an approximate model of the system dynamics (1) of the form:

$$\dot{x} = \hat{f}(x) + \hat{g}(x)u \tag{13}$$

is available, where $\hat{f}$ and $\hat{g}$ are approximations of f and g. Since the model is not perfectly known, condition (3) cannot be evaluated. However, according to [16], it can be reformulated based on the model error by introducing a nonparametric uncertainty $\Delta_{np}(x, u)$ into condition (3) as follows:

$$\sup_{u \in \mathcal{U}} [L_{\hat{f}} h(x) + L_{\hat{g}} h(x)u + \Delta_{np}(x, u) + \alpha(h(x))] \geq 0, \tag{14}$$

where $\Delta_{np}(x, u) = (L_f h(x) - L_{\hat{f}} h(x)) + (L_g h(x) - L_{\hat{g}} h(x))u$ is the error between the actual Lie derivative and the approximated one. We assume that $\Delta_{np}(x, u)$ is i.i.d Gaussian noise, which makes it suitable for being approximated by a GP.

A GP is an extension of multivariate Gaussian distributions to an infinite dimensional stochastic process in which any finite combination of dimensions has a joint Gaussian distribution [18]. A GP with the input domain $\mathcal{X}$ can be specified by the mean function, $\mu : \mathcal{X} \to \mathbb{R}$ and the covariance function $k : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$:

$$z(x) \sim \mathcal{GP}\left(\mu(x), k(x, x_*)\right), \tag{15}$$

where $x, x_* \in \mathcal{X}$, and the unknown function $z(\cdot)$ is sampled from a GP at a new query point x_*. To predict $\Delta_{np}(x, u)$, which depends on both x and u, Castañeda et al. [19] proposed to use the affine dot product (ADP) compound kernel.

Definition 3.1 (ADP compound kernel): The affine dot product compound kernel $k_c : \bar{\mathcal{X}} \times \bar{\mathcal{X}} \to \mathbb{R}$ is defined as

$$k_c\left(\begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x_* \\ y_* \end{bmatrix}\right) := y^\top \operatorname{Diag}([k_1(x, x_*), \ldots, k_{m+1}(x, x_*)])y_*, \tag{16}$$

where $\bar{\mathcal{X}} := \mathcal{X} \times \mathbb{R}^{m+1}$ and $y = [1 \;\; u^\top]^\top$.

From N data points $\{\Delta_{np}(x_i, u_i)\}_{i=1}^N$, the mean and variance of the GP estimate of $\Delta_{np}(x, u)$ can be derived at a new query point (x_*, y_*) as follows [19]:

$$\mu_* = \mathbf{z}^\top (K_c + \sigma_n^2 I)^{-1} \mathbf{k}_{*Y}^\top y_*, \tag{17}$$

$$\sigma_*^2 = y_*^\top \left(\operatorname{Diag}(G) - \mathbf{k}_{*Y}(K_c + \sigma_n^2 I)^{-1} \mathbf{k}_{*Y}^\top\right) y_*,$$

$$G = \begin{bmatrix} k_1(x_*, x_*) \\ \vdots \\ k_{m+1}(x_*, x_*) \end{bmatrix}, \tag{18}$$

where $K_c \in \mathbb{R}^{N\times N}$ is the Gram matrix of k_c with the input data $X \in \mathbb{R}^{n\times N}$ and $Y \in \mathbb{R}^{(m+1)\times N}$, and $\mathbf{k}_{*Y} \in \mathbb{R}^{(m+1)\times N}$ is provided by

$$\mathbf{k}_{*Y} = \begin{bmatrix} K_{1*} \\ \vdots \\ K_{m+1*} \end{bmatrix} \circ Y, \quad K_i = [k_i(x_*, x_1), \dots, k_i(x_*, x_N)].$$

To ensure the probabilistic bounds of the posterior of $\Delta_{np}(x,u)$, the target function should be in a reproducing kernel Hilbert space (RKHS, Wendland [20]), $\mathcal{H}_k(\bar{\mathcal{X}})$ of the selected kernel. The RKHS norm of function h, denoted by $||h||_k$, is a measure of smoothness as given by

$$||h(x) - h(x')||_2 \le ||h||_k ||k(x,\cdot) - k(x',\cdot)||_k \quad \forall\, x, x' \in \mathcal{X}. \tag{19}$$

We leverage the theorem from [19] directly to ensure the probabilistic boundedness as follows:

Theorem 3.2 ([19]): *Let k_i be the $m+1$ bounded kernel for $i = 1, \dots, m+1$. We assume that $\Delta(x,u) = \Phi\left[\begin{smallmatrix}1\\u\end{smallmatrix}\right]$, and each element of $\Phi \in \mathcal{H}_{k_i}$ with bounded RKHS norm. Moreover, it is assumed to measure $z_i := \Delta(x_i,u_i) + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0,\sigma_n^2)$. Subsequently, let μ_* and σ_*^2 be the mean and variance of the posterior for Δ using the ADP compound kernel at a query point (x_*, u_*). Then, with a probability of at least $1-\delta$, the following holds:*

$$|\mu_* - \Delta(x_*, u_*)| \le \beta\sigma_*,$$

where $\beta = (2B^2 + 300\gamma_{N+1}\ln^3(\frac{N+1}{\delta}))^{0.5}$, N is the number of data points, γ_{N+1} is the maximum information gain, and B is an upper bound on $||\Delta||_k$.

From Theorem 3.2, it can be concluded that the safety constraint at a query point (x,u):

$$L_{\hat{f}}h(x) + L_{\hat{g}}h(x)u + \Delta_{np}(x,u) + \alpha\,(h(x)) \ge 0 \tag{20}$$

is satisfied with probability at least $1-\delta$ if

$$L_{\hat{f}}h(x) + L_{\hat{g}}h(x)u + \mu(x,u) - \beta\sigma(x,u) + \alpha\,(h(x)) \ge 0. \tag{21}$$

Due to the structure of the ADP compound kernel, the following second-order cone program (SOCP) can be used for finding a safe control input with probability at least $1-\delta$ [16]:

$$u_{\mathsf{safe}} = \arg\min_{u\in\mathcal{U}} \frac{1}{2}||u - u_{\mathsf{nominal}}||^2 \quad \textbf{(GPCBF} - \textbf{SOCP)}$$
$$\text{s.t. } L_{\hat{f}}h(x) + L_{\hat{g}}h(x)u + \mu(x,u) - \beta\sigma(x,u) + \alpha(h(x)) \ge 0.$$

The possible conservatism of the GPCBF framework originates from the term $\beta\sigma(x,u)$, which depends on B_{GP}:

$$||\Delta_{np}||_k \le B_{GP}, \tag{22}$$

where Δ_{np} is given at (14).

This implies that the smaller the RKHS-norm of the uncertainty is, the less conservative the constraint is. The method can be used in an episodic fashion, where μ and σ are gradually updated.

The choice of upper bound on the RKHS norm, B_{GP}, is application specific. In practice, it is hard to determine the bound. More details can be found in [21].

4. Proposed method

This section presents the proposed method for solving *Problem 2.1*, which we denote GPRaCBF and which combines RaCBF and GPCBF. We provide a SOCP for ensuring safety with probability $1-\delta$ and show that GPRaCBF is less conservative than GPCBF.

We consider a system governed by the dynamics given in (1), where f, g, and θ^* are not known exactly, i.e.the utilized model is

$$\dot{x} = \hat{f}(x) + F(x)\hat{\theta} + \hat{g}(x)u. \tag{23}$$

We say that $h_r(x,\theta)$ is a Gaussian Process Robust adaptive Control Barrier Function (GPRaCBF) if there exists an extended class $\mathcal{K}_\infty$ function α such that for the system (6) and any $\theta \in \Theta$, with probability $1-\delta$:

$$\sup_{u\in\mathcal{U}} \left[\frac{\partial h_r}{\partial x}(x,\theta)\left(\hat{\mathcal{F}}(x,\theta) + \hat{g}(x)u\right) + \mu(x,u) - \beta\sigma(x,u) \right] \ge -\alpha\left(h_r(x,\theta) - \frac{1}{2}\tilde{\vartheta}^\top\Gamma^{-1}\tilde{\vartheta}\right), \tag{24}$$

where $\hat{\mathcal{F}}(x,\theta) = \hat{f}(x) + F(x)\lambda(x,\theta)$, $\tilde{\vartheta}$ is given in (4) and β, μ, and σ are given by Theorem 3.2 with $\Delta(x,u) = L_f h_r(x) + L_g h_r(x)u - L_{\hat{f}}h_r(x) - L_{\hat{g}}h_r(x)u$.

Remark 4.1: $\Delta(x,u)$ may be approximated in an episodic manner similar to GPCBF [16], where the parameter estimate is also updated. Then, the standard deviation of the estimate of Δ is gradually decreased as several episodes run by. Since we assume a high probabilistic bound, episodic data collection always incurs infeasibility of optimization if the system becomes unsafe. Therefore, according to [16], the episodes stop before the system becomes unsafe, and then we collect the data. Afterwards, the episodes iterate until the optimization is always feasible in the whole period.

The following theorem provides a probabilistic safety condition for system (1) with both parametric and non-parametric uncertainty, i.e., the theorem combines the notions of RaCBF and GPCBF, see [14,16].

Theorem 4.1: *Let $\mathcal{S}^r_{\hat{\theta}} \subset \mathbb{R}^n$ be a superlevel set of a continuously differentiable function $h_r : \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}$. If h_r is a GPRaCBF on $\mathcal{S}^r_{\hat{\theta}}$, then any locally Lipschitz continuous controller satisfying the following:*

$$\frac{\partial h_r}{\partial x}(x,\theta)[\hat{f}(x) + F(x)\lambda(x,\theta) + \hat{g}(x)u] + \mu(x,u) - \beta\sigma(x,u) \geq -\alpha\left(h_r(x,\theta) - \tfrac{1}{2}\tilde{\vartheta}^\top\Gamma^{-1}\tilde{\vartheta}\right) \tag{25}$$

renders the set $\mathcal{S}^r_{\hat{\theta}}$ safe with probability at least $1-\delta$ for the uncertain system with adaptation law and adaptation gain as [14]:

$$\dot{\hat{\theta}} = -\Gamma\left(\frac{\partial h_r}{\partial x}(x,\hat{\theta})F(x)\right)^\top, \tag{26}$$

$$\lambda_{\min}(\Gamma) \geq \frac{||\tilde{\vartheta}||^2}{2h_r(x,\theta)}, \tag{27}$$

where $\tilde{\vartheta} \in \mathbb{R}^k$ is the maximum possible parameter error, $h_r(x,\theta) > 0$ can be decided freely based on the desired conservatism, and $\Gamma = \Gamma^\top$ is an admissible symmetric positive definite matrix. Furthermore, the original set $\mathcal{S}_{\hat{\theta}}$ is also safe for the uncertain system.

Proof: This proof is inspired by Lopez et al. [14] and Krstić and Kokotović [22]. Consider the candidate control barrier function as follows:

$$h(x,\hat{\theta}) = h_r(x,\hat{\theta}) - \tfrac{1}{2}\tilde{\theta}^\top\Gamma^{-1}\tilde{\theta}, \tag{28}$$

where the minimum eigenvalue of Γ must satisfy (27) for any $h_r(x_r,\theta_r) > 0$. We compute the Lie derivative of h with respect to (1):

$$\dot{h} = \dot{h}_r(x,\hat{\theta}) - \tilde{\theta}^\top\Gamma^{-1}\dot{\tilde{\theta}} \tag{29}$$

$$= \frac{\partial h_r}{\partial x}[f(x) + F(x)\theta^* + g(x)u] + \frac{\partial h_r}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tilde{\theta}^\top\Gamma^{-1}\dot{\hat{\theta}}. \tag{30}$$

The Lie derivative can be reformulated in terms of the approximate model (23) and an error $\Delta(x,u)$ as follows:

$$\dot{h} = \frac{\partial h_r}{\partial x}[\hat{f}(x) + F(x)\theta^* + \hat{g}(x)u] + \Delta(x,u) + \frac{\partial h_r}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tilde{\theta}^\top\Gamma^{-1}\dot{\hat{\theta}}, \tag{31}$$

where $\Delta(x,u) = L_f h_r(x) + L_g h_r(x)u - L_{\hat{f}} h_r(x) - L_{\hat{g}} h_r(x)u$. We add and subtract $\frac{\partial h_r}{\partial x}F(x)\left(\hat{\theta} - \Gamma\left(\frac{\partial h_r}{\partial \hat{\theta}}\right)^\top\right)$:

$$\dot{h} = \frac{\partial h_r}{\partial x}[\hat{f}(x) + F(x)\lambda(x,\hat{\theta}) + \hat{g}(x)u] + \Delta(x,u) + \frac{\partial h_r}{\partial \hat{\theta}}\dot{\hat{\theta}} - \tilde{\theta}^\top\Gamma^{-1}\dot{\hat{\theta}} - \frac{\partial h_r}{\partial x}F(x)\left(\tilde{\theta} - \Gamma\left(\frac{\partial h_r}{\partial \hat{\theta}}\right)^\top\right), \tag{32}$$

where $\lambda(x,\theta) = \theta - \Gamma(\frac{\partial h_r}{\partial \theta}(x,\theta))^\top$. We use (26) and obtain the following:

$$\dot{h} = \frac{\partial h_r}{\partial x}[\hat{f}(x) + F(x)\lambda(x,\hat{\theta}) + \hat{g}(x)u] + \Delta(x,u). \tag{33}$$

By (25) and Theorem 3.2, we conclude that with probability $1-\delta$:

$$\begin{aligned}\dot{h} &= \frac{\partial h_r}{\partial x}[\hat{\mathcal{F}}(x,\hat{\theta}) + \hat{g}(x)u] + \mu(x,u) - \beta\sigma(x,u)\\ &\geq -\alpha\left(h_r(x,\theta) - \tfrac{1}{2}\tilde{\vartheta}^\top\Gamma^{-1}\tilde{\vartheta}\right)\\ &\geq -\alpha\left(h(x,\theta)\right).\end{aligned} \tag{34}$$

The second inequality is obtained from the bound on the maximal parameter estimation error. From (27), $h \geq 0$ and $h_r \geq h$; hence, $h_r \geq \frac{1}{2}\tilde{\vartheta}^\top\Gamma^{-1}\tilde{\vartheta} \geq 0$ and $\mathcal{S}^r_\theta$ is safe. ■

The following SOCP can be used for finding a safe control input with probability at least $1-\delta$:

$$\begin{aligned} u_{\mathrm{safe}} = \arg\min_{u\in\mathcal{U}} \ & \frac{1}{2}||u - u_{\mathrm{nominal}}||^2 \quad (\textbf{GPRaCBF} - \textbf{SOCP})\\ \text{s.t.}\ & \frac{\partial h_r}{\partial x}(x,\hat{\theta})\left(\hat{\mathcal{F}}(x,\hat{\theta}) + \hat{g}(x)u\right) + \mu(x,u) - \beta\sigma(x,u)\\ & + \alpha\left(h_r(x,\hat{\theta}) - \frac{1}{2}\tilde{\vartheta}^\top\Gamma^{-1}\tilde{\vartheta}\right) \geq 0.\end{aligned}$$

Note that the feasibility of the above GPRaCBF-SOCP is not always guaranteed. This intuitively arises from the exploration-exploitation dilemma, as the SOCP may not be able to find a safe control input in regions where the uncertainty of the GP, $\sigma(x,u)$, is too high. To avoid repetition, we direct the reader to Lemmas 2 and 3 in [16] for necessary and sufficient conditions for the feasibility of the GP-CBF-CLF-SOCP, which are directly applicable to the GPRaCBF. For application in a practical control system, Algorithm 1 in [17] provides an implementation of a safe learning framework based on the sufficient condition for the feasibility of the SOCP. We adopt this implementation verbatim in our approach, albeit considering the GPRaCBF-SOCP instead.

4.1. Analysis of conservatism

It is relevant to compare the conservatism of the proposed GPRaCBF with the GPCBF proposed in [16] to determine the difference between the methods. Both methods rely on Theorem 3.2 to determine the uncertainty, which is given by β and $\sigma(x,u)$. Since σ is related to the measurement noise, only β will be different for the two methods. In the computation of β, only the RKHS norm bound B on the uncertainty will be different; hence, the RKHS norm of the uncertainties will determine the difference between GPRaCBF and GPCBF. We define a trend-capturing approximation; this is necessary as an erroneous parametric model will increase the conservatism of GPRaCBF compared to GPCBF.

Definition 4.2 (Trend-capturing approximation): Let k be a bounded kernel, and h be a function with RKHS norm $||h||_k$. The function $\hat{h}$ is said to be a *trend-capturing approximation* of h if

$$||e||_k \leq ||h||_k \tag{35}$$

where $e = h - \hat{h}$ is the approximation error.

For simplicity, the following analysis assumes that $\hat{g} = g$ and that h_r does not depend on θ. Furthermore, for fairness of the comparison, we define the relation between estimates as $\hat{f}_{np}(x) = \hat{f}(x) + F(x)\hat{\theta}(0)$, and without loss of generality, we define $\hat{f}$ such that $\hat{\theta}(0) = 0$.

Proposition 4.3: *Let $\hat{g} = g$ and let $L_{F(x)\theta^*}h_r$ be a trend-capturing approximation of $L_{\hat{f}(x)-f(x)}h_r$, i.e.*

$$||L_{\hat{f}}h_r - L_f h_r - L_{F(x)\theta^*}h_r||_k \leq ||L_{\hat{f}}h_r - L_f h_r||_k. \tag{36}$$

Then

$$||\Delta||_k \leq ||\Delta_{np}||_k, \tag{37}$$

where

$$\Delta(x) = L_f h_r(x) - L_{\hat{f}} h_r(x) \tag{38}$$

$$\Delta_{np}(x) = L_{f_{np}} h_r(x) - L_{\hat{f}_{np}} h_r(x). \tag{39}$$

From the previous proposition, we conclude that $B_{GP} \geq B_{GPRa}$, i.e., the proposed GPRaCBF is less conservative than GPCBF when $L_{F(x)\theta^*}h_r$ is a trend-capturing approximation of $L_{\hat{f}-f}h_r$.

5. Simulation case study: Robot force control

While we emphasize the general applicability of our method, we evaluate it in a simulation of a robot force-control application. A typical robotic assembly application will require that the controller maintains a constant contact force with the environment, while at the same imposing bounds on the maximum forces and torques the manipulator may apply to ensure the integrity of the workpiece.

A notable challenge when ensuring safe force control of a robot is that the dynamics of the contact force is partially unknown. In addition, modeling errors of the robot will result in deviations in the controlled robot's dynamics. For CBF-based methods that do not consider uncertainty, this will usually result in failure to comply with the constraint, while – as will be shown below – methods that do consider uncertainty can be overly conservative. This can, in both cases, have a detrimental effect on product quality (e.g., in material removing applications).

We consider a robot manipulator in contact with an environment, where the robot is controlled by a nominal controller with an inverse dynamic control in its inner loop as shown in Figure 1. In particular, we consider a force controlled robot where the magnitude of the contact force must be bounded with a particular value f_{limit}. The dynamics of the robot controlled by an inverse dynamics controller is given by

$$\ddot{x} = y + w_r, \tag{40}$$

where w_r could be added to the robot dynamics to encapsulate modeling errors in the controlled robot dynamics. In our simulation, we assume that the inverse dynamics controller is accurate.

To model the contact dynamics we use the Kelvin-Voigt model [23], as this is a simple model that gives a linear relation between force and motion. In particular, the force exerted by the material on the object is

$$\hat{f}_c = \begin{cases} \hat{k}x + \hat{b}\dot{x} + w_c, & x \geq 0, \\ 0, & x < 0, \end{cases} \tag{41}$$

where x is the penetration of the object [m], $\hat{k}$ is an estimate of the elastic parameter of the contact [N/m], and $\hat{b}$ is an estimate the viscous parameter of the contact [N/(m/s)]. Note that $\hat{k}$ and $\hat{b}$ may deviate from the true values of the contact dynamics. In addition to this, as our model structure may not be correct, we add a nonparametric uncertainty w_c to encapsulate differences caused by an erroneous model structure.

In the following sections, we assume that our knowledge of the contact dynamics is always given by (41). We then vary how well these assumptions fit the real dynamics of the system. In the first simulation, we assume that we know partial information about the real contact model, i.e., the structure of the model is a good fit, but we do not know the inherent parameters. In the second

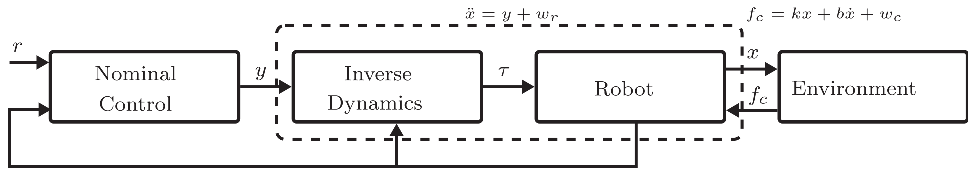


Figure 1. Block diagram of the considered system with uncertainty in the robot dynamics given by w_r and uncertainty in the contact dynamics given by w_c.

simulation, we assume that we do not have any prior knowledge about the real contact force model, i.e., neither the structure nor the parameter values are a good fit of the real dynamics. In both cases, we show that the proposed method ensures the safety guarantees with probability at least $1 - \delta = 0.95$ provided by Castañeda et al. [16], and we compare our proposed method, GPRaCBF, against multiple existing baselines of safe controllers (aCBF, RaCBF and GPCBF) and show that GPRaCBF is able to enforce safety constraints in cases where other methods fail, while being less conservative than GPCBF. In both simulations, we use squared exponential covariance function k [18] in GP-based methods and assume that a measurement noise, $\epsilon \sim \mathcal{N}(0, \sigma^2)$, exists.

5.1. Case 1: Parametric uncertainty

Firstly, we assume that the structure of the real contact force model is known, i.e., $w_c = 0$ in (41). The inherent parameters in the real model, k, b are unknown, but we instead know an estimate of them, $\hat{k}, \hat{b}$. Thus, our model of the contact dynamics can be modeled as follows:

$$\hat{f}_c = -\hat{k}x - \hat{b}\dot{x}, \tag{42}$$

where $\hat{f}_c$ is the modeled contact force, and $\hat{k}$ and $\hat{b}$ are the estimated parameters. In this case, we simulate the actual contact dynamics to follow:

$$f_c = -kx - b\dot{x}, \tag{43}$$

where f_c, k and b are the real contact force and parameters, respectively. In this simulation, we set $k = 1000, b = 10, \hat{k} = 1080, \hat{b} = 20$. Our system model is thus given by

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{x} \\ 0 \end{bmatrix}}_{f(x)} - \frac{1}{m} \underbrace{\begin{bmatrix} 0 & 0 \\ x & \dot{x} \end{bmatrix}}_{F(x)} \underbrace{\begin{bmatrix} k \\ b \end{bmatrix}}_{\theta^*} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{g(x)} u, \tag{44}$$

where $\mathbf{x} = [x \ \dot{x}]^\top$ is the system states representing the position and velocity of the end-effector of a robot, and u is the force control input from the robot, and $m{>}0, k{>}0, b{>}0$ are mass, stiffness, and damping, respectively.

To ensure safe contact force control, our safety constraint is defined as follows:

$$f_c \geq f_{\text{limit}}, \tag{45}$$

where f_{limit} is the minimum safety limit for the contact force and is in this case set to -8 N for the simulations. We simulate the nominal and the adaptive safe controllers when given the desired force trajectory as shown in Figure 2. Adaptive safe controllers such as aCBF and RaCBF can enforce the safety constraint, as shown in Figure 2(a). This is because the adaptive controllers adapt to parametric uncertainty. Note that RaCBF is less conservative than aCBF since we explicitly introduce knowledge on the maximum possible errors in the RaCBF-QP constraint.

5.2. Case 2: Nonparametric uncertainty

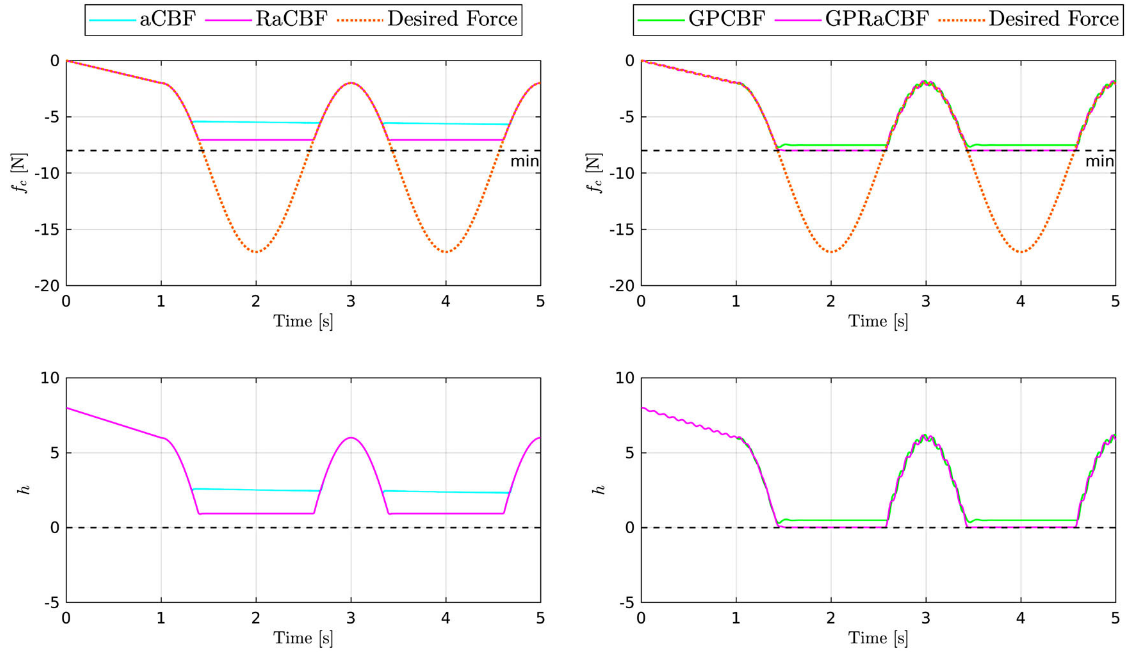
In a typical robotics contact force control application, discrepancies between our model and the real contact dynamics will not just be limited to parametric uncertainties, but will often involve discrepancies in model structure, i.e., nonparametric uncertainty. This will lead to the violation safety guarantees. To simulate this situation, we now change the simulated contact dynamics to follow the Hunt-Crossley model [24]:

$$f_c = -k_{hc}x^n - b_{hc}\dot{x}x^n, \tag{46}$$

where $k_{hc}{>}0, b_{hc}{>}0, n{>}0$ are the parameters of the model. In this simulation, we set $k_{hc} = 1080, b_{hc} = 20, n = 1.2$.

In terms of our knowledge of the system, however, our assumption is still that the contact dynamics follow a model of the same form as (41). Because our model now has structural differences with respect to the actual dynamics, nonparametric uncertainty has now been introduced into our scenario (in addition to the existing parametric uncertainty). The overall system dynamics are given by:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{x} \\ f_c - \hat{f}_c \end{bmatrix}}_{f(x)} - \frac{1}{m} \underbrace{\begin{bmatrix} 0 & 0 \\ x & \dot{x} \end{bmatrix}}_{F(x)} \underbrace{\begin{bmatrix} k \\ b \end{bmatrix}}_{\theta^*} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{g(x)} u, \tag{47}$$



(a) Parametric uncertainty only (b) Parametric and nonparametric uncertainty

Figure 2. Safe contact force control performance of each controller. The graphs in the first row represent the measured contact forces, and the graphs in the next row show the performance of safety guarantees, $h \geq 0$. (a) shows the simulation results of the existing adaptive CBF controllers when given only parametric uncertainty. (b) shows the performance of GP-based controllers in the presence of parametric and non-parametric uncertainty both and measurement noise.

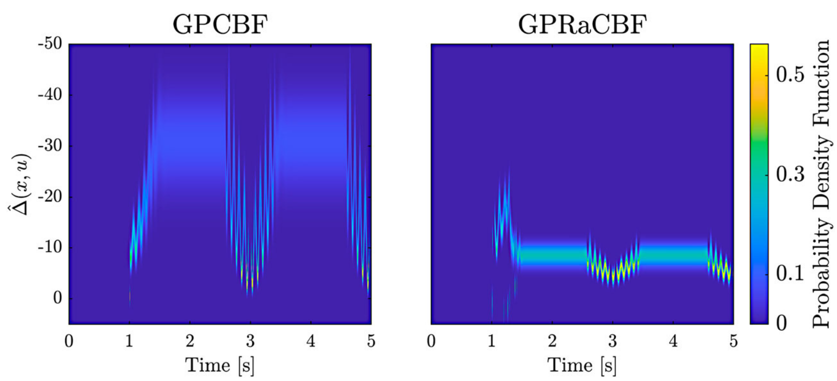


Figure 3. The posterior probability distributions of the uncertainty, $\hat{\Delta}(x, u)$ on each GP-based controller during simulations.

where f_c corresponds to (46) and $\hat{f}_c$ corresponds to (42). Note that neither the nominal P, nor the CBF, aCBF or RaCBF controllers will be able to ensure the safety of systems with nonparametric uncertainty. As shown in Figure 2(b), GPCBF and GPRaCBF are able to ensure the safety constraint despite our lack of knowledge on the real contact force model, as model discrepancies will be encapsulated in the posterior distribution of the GP based on the data collected online. To collect data and train the GP online, we leverage the framework for safe online learning proposed by Castañeda et al. [17].

Notice that a key difference between GPCBF and the proposed method, GPRaCBF, lies in the conservatism of the controllers. This can be observed in Figure 2(b), where the value of h is smaller for GPRaCBF than GPCBF. This is because we introduce robust adaptation to parametric uncertainty with the maximum possible error in the formulation of GPRaCBF, which satisfies Proposition 4.3. Consequently, we alleviate the conservatism of the subset of the safety set when compared to GPCBF. Moreover, as shown in Figure 3, the slope of the nonparametric uncertainty is lower when using GPRaCBF than GPCBF. This is because GPRaCBF estimates and decreases the inherent parametric uncertainty in the real model independently from the nonparametric uncertainty. Lastly, assuming a finite amount of data, reliability is higher when using GPRaCBF since the GP's posterior distribution has smaller standard deviations than the existing method.

6. Conclusions

In this article, we presented a method for ensuring the safety of a system with parametric and non-parametric uncertainties. We combined the ability of RaCBF to adapt to unknown parameters with the ability of GP to encode nonparametric errors and then integrated this into GPRaCBF, which can be formulated as a SOCP. We provided an optimal input signal that makes the system safe with probability $1 - \delta$ at each time step. Since our method exploited the structure of the uncertainty, its performance was less conservative than GPCBF. We provided theoretical proof of safety guarantees of the proposed method and verified the method in a simulation case study, where we also demonstrated that the method can be applied to a robot force control application with uncertainty in the contact dynamics. Additionally, robot modeling errors can also be encapsulated into nonparametric uncertainty simultaneously; therefore, our method can be used in general robotic use cases.

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