Lecture 8: Kinematics: Path and Trajectory Planning

- Concept of Configuration Space

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- For example, for a rigid object moving on a plane

$$\mathcal{Q} = \{x,\, y,\, \theta\} = \mathbb{R}^2 \times S^1$$

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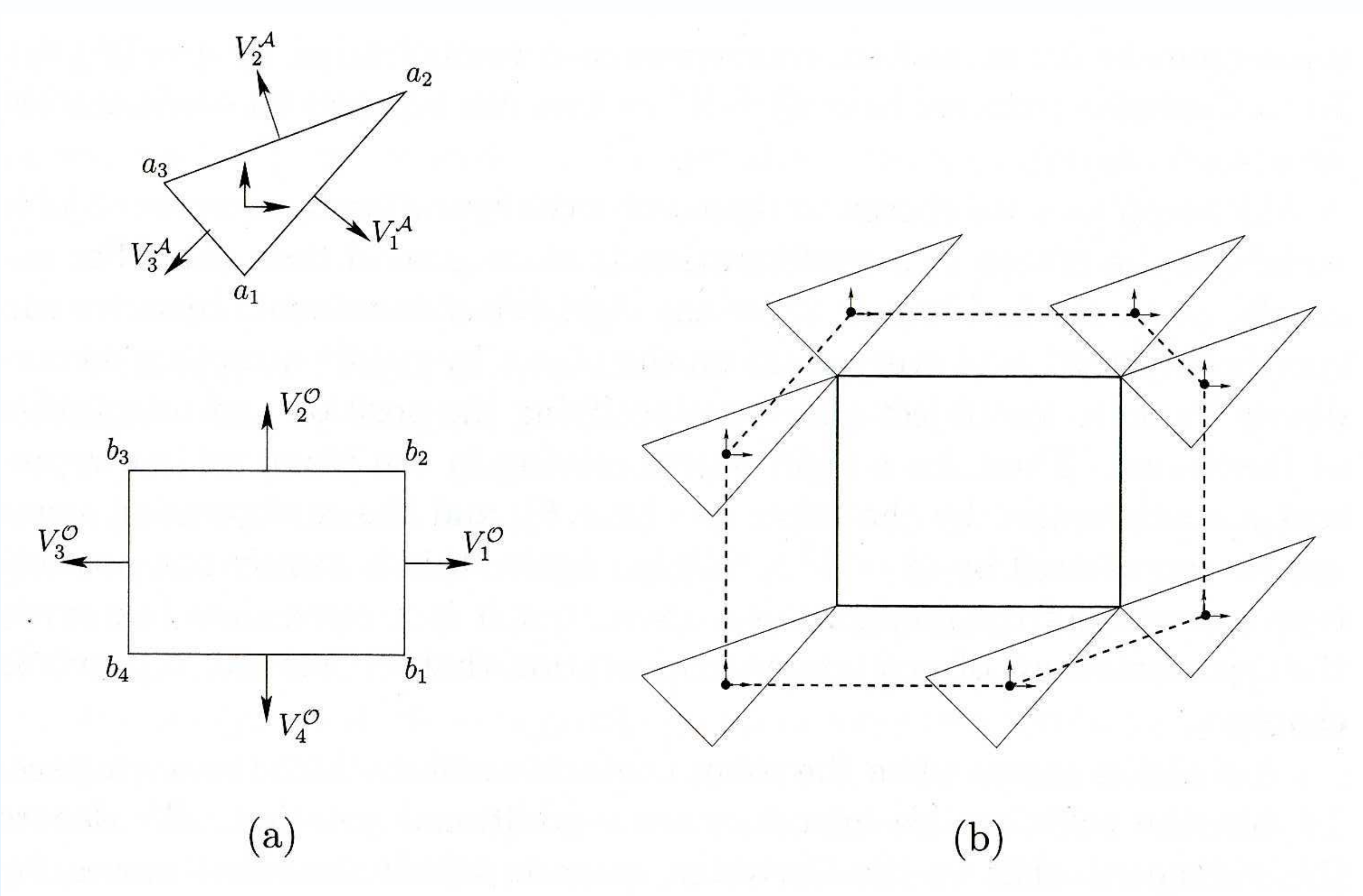
$$\mathcal{QO} := \{q \in \mathcal{Q} : \mathcal{A}(q) \cap O_i \neq \emptyset,\ \forall\, i\}$$

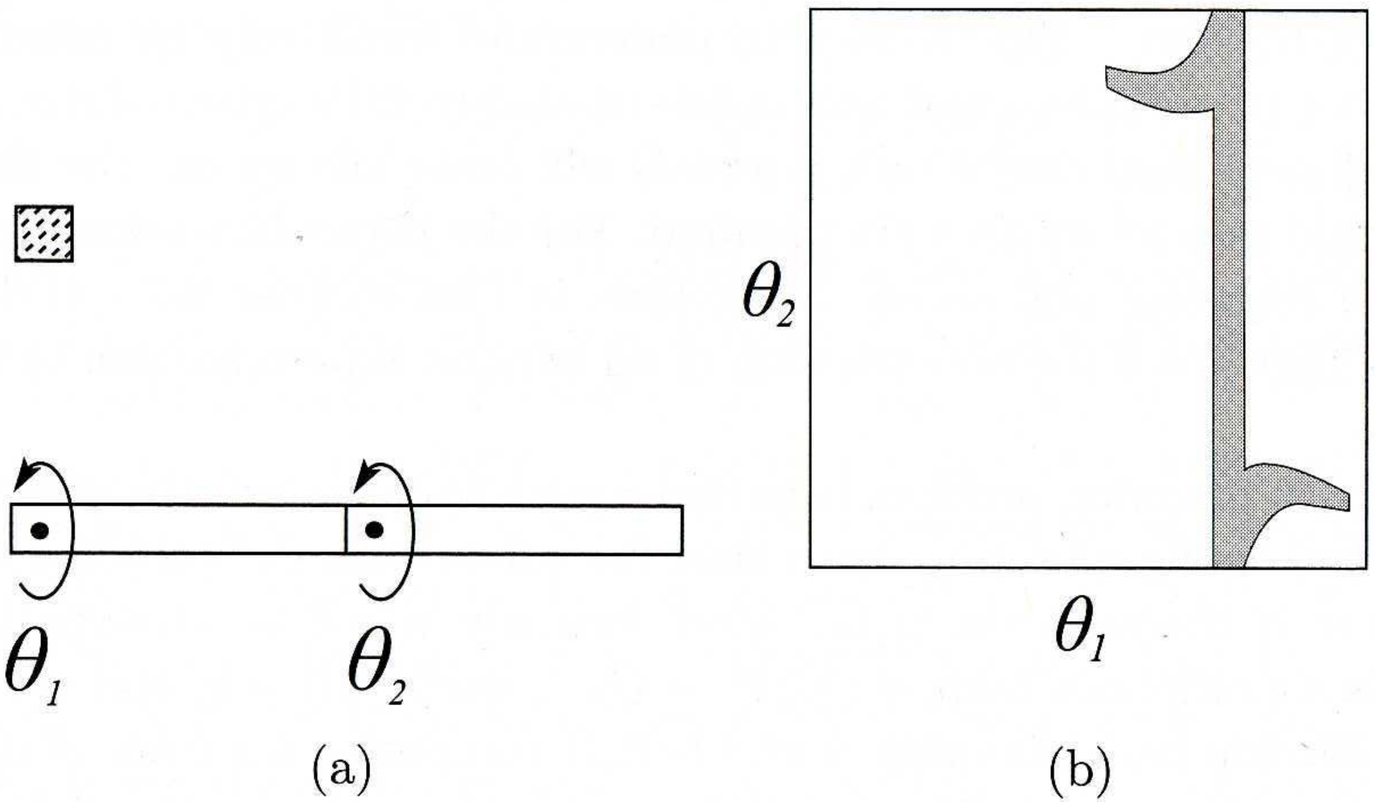
- Then collision-free configurations are defined by

$$\mathcal{Q}_{free} := \mathcal{Q} \setminus \mathcal{QO}$$



(a) The end-effector of the robot has a from of triangle. It moves in a plane. The plane contains a rectangular obstacle.

(b) $\mathcal{QO}$ is the set with the dashed boundary



(a) Two-links planar arm robot. The workspace has a single square obstacle.

(b) The configuration space and the set $\mathcal{QO}$ occupied by the obstacle is in gray.

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Formally, the task is to find a continuous function $\gamma(\cdot)$ such that

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Common additional requirements:

- Some intermediate points q_i can be given
- Smoothness of a path
- Optimality (length, curvature, etc)

Path Planning: Potential Field Approach

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  - global minimum at q_f $\Rightarrow$ this point is attractive
  - maximum or to be $+\infty$ in the points of $\mathcal{QO}$ $\Rightarrow$ these points repel the robot
- Try to find such function $U(\cdot)$ constructed in a simple from, where we can easily add or remove an obstacle and change q_f. The common form for $U(\cdot)$ is

$$U(q) = U_{att}(q) + \left(U_{rep}^{(1)}(q) + U_{rep}^{(2)}(q) + \cdots + U_{rep}^{(N)}(q)\right)$$

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  - Choose the way measure the distance $d(\cdot)$ in $\mathcal{Q}$
  - Choose $\varepsilon > 0$ and find k neighbors of distance no more than ε that can be connected to the current one

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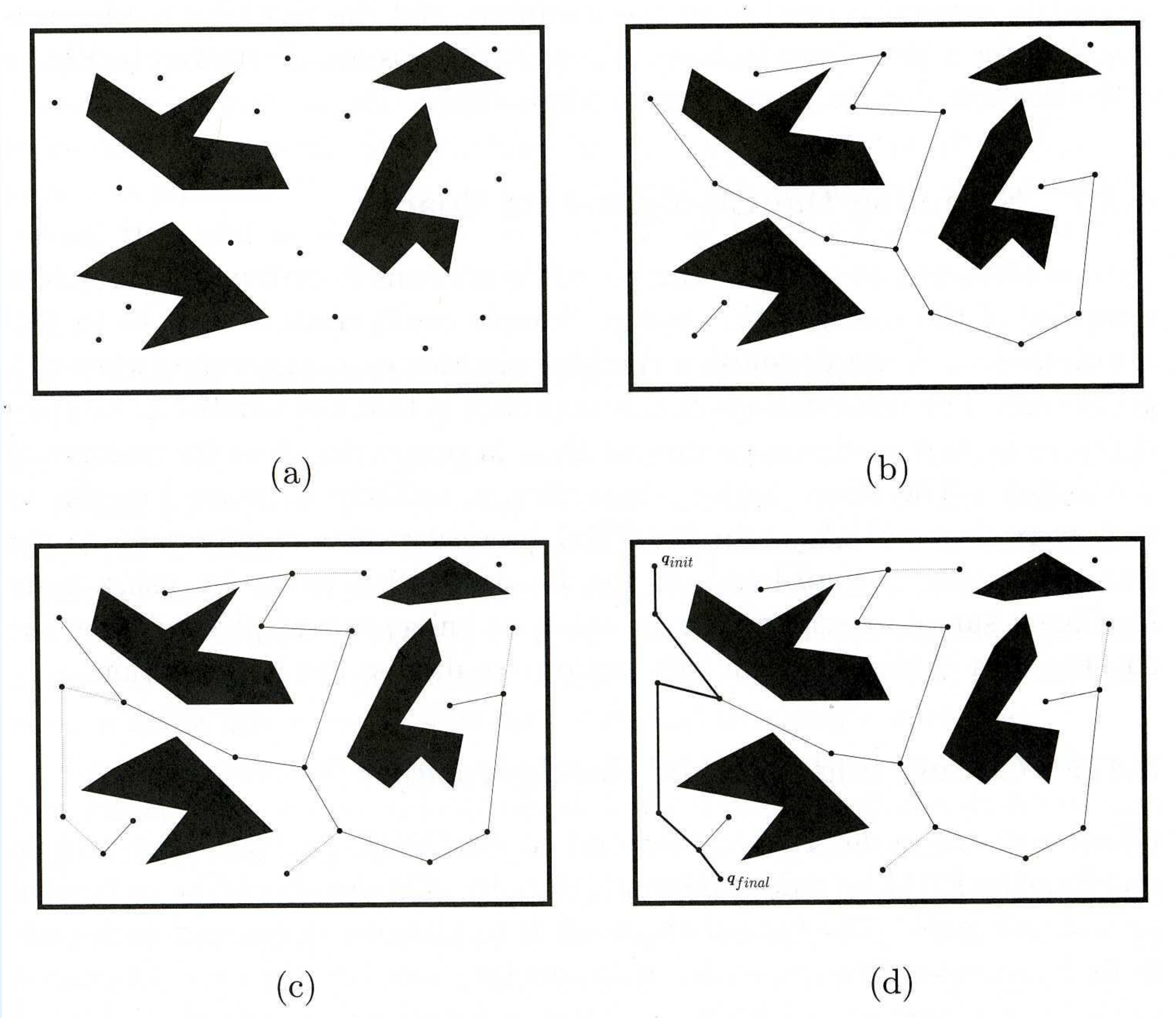
- Make enhancement, that is, try to connect disjoint components
- Try to compute a smooth path from a family of points



Steps in constructing probabilistic roadmap

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Trajectory Planning

Trajectory is a path $\gamma : [0,1] \longrightarrow \mathcal{Q}_{free}$ with explicit parametrization of time

$$[T_s, T_f] \ni t \ \rightsquigarrow \ \tau \in [0,1] : \ q(t) = \gamma(\tau) \in \mathcal{Q}_{free}$$

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This means that we make specifications on

- velocity $\frac{d}{dt} q(t)$ of a motion;
- acceleration $\frac{d^2}{dt^2} q(t)$ of a motion;
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- $\ldots$

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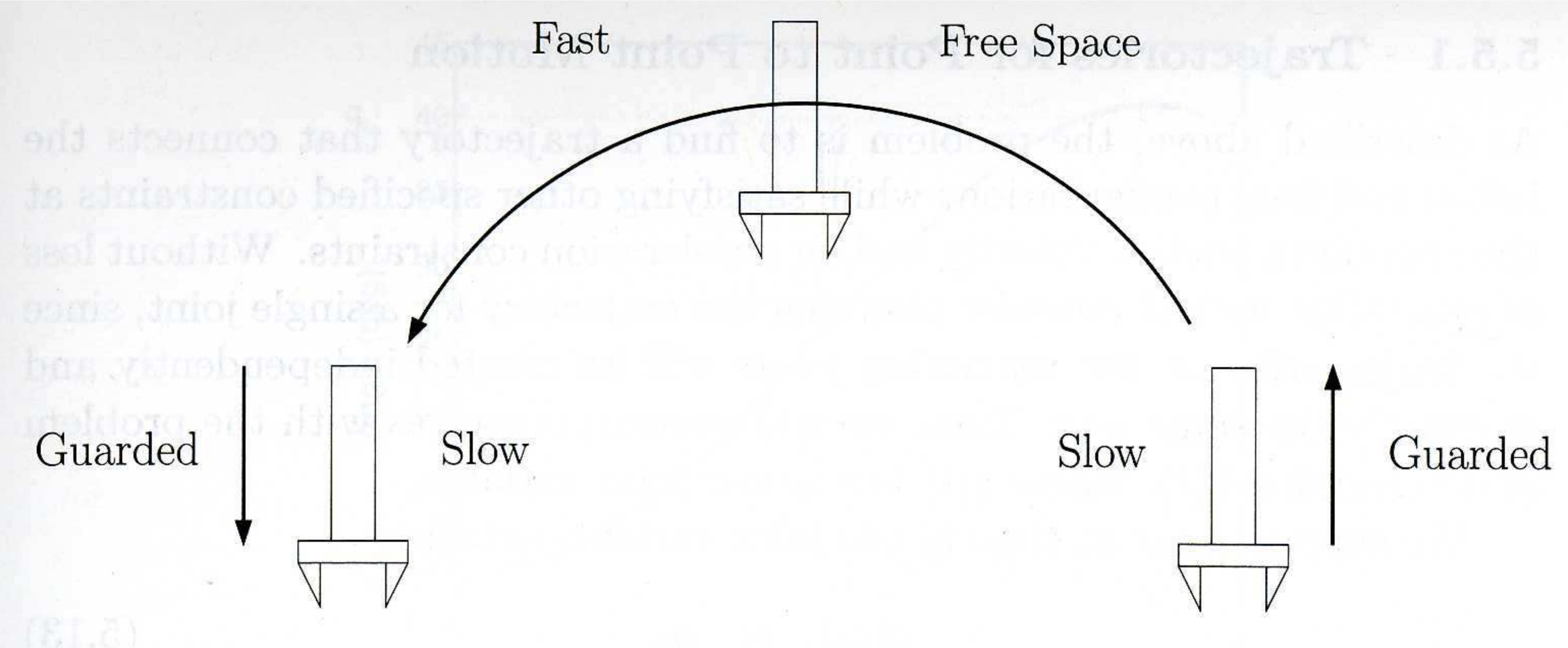
- ...

In fact, it is common that the path is not given completely, but as a family of snap-shots

$$q_s, \quad q_1, \quad q_2, \quad q_3, \quad \ldots, \quad q_f$$

So that we have substantial freedom in generating trajectories.



Decomposition of a path into segments with fast and slow velocity profiles

Trajectories for Point to Point Motion

Consider the i^{th} joint of a robot and suppose that the specification

$$\text{at time } t = t_0 \text{ is :} \qquad q_i(t_0) = q_0, \ \frac{d}{dt}q(t_0) = v_0$$

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In addition, we might be given constraints of accelerations

$$\frac{d^2}{dt^2}q(t_0) = \alpha_0 \qquad \frac{d^2}{dt^2}q(t_f) = \alpha_f$$

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In addition, we might be given constraints of accelerations

$$\frac{d^2}{dt^2}q(t_0) = \alpha_0 \qquad \frac{d^2}{dt^2}q(t_f) = \alpha_f$$

If we choose to generate a polynomial

$$q(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_m t^m$$

that will satisfy the interpolation constraints,

what degree this polynomial should be chosen?

Trajectories for Point to Point Motion

The interpolation constraints

at time $t = t_0$ is : $\quad q_i(t_0) = q_0, \; \frac{d}{dt}q(t_0) = v_0$

at time $t = t_f$ is : $\quad q_i(t_f) = q_f, \; \frac{d}{dt}q(t_f) = v_f$

for the 3^{rd}-order polynomial

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3, \quad \frac{d}{dt}q(t) = a_1 + 2a_2 t + 3a_3 t^2$$

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$$\begin{aligned} q_0 &= a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 \\ v_0 &= a_1 + 2a_2 t_0 + 3a_3 t_0^2 \\ q_f &= a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\ v_f &= a_1 + 2a_2 t_f + 3a_3 t_f^2 \end{aligned}$$

Trajectories for Point to Point Motion

The equations

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written in matrix form are

$$\begin{bmatrix} q_0 \\ v_0 \\ q_f \\ v_f \end{bmatrix} = \begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Trajectories for Point to Point Motion

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$$
\begin{aligned}
q_0 &= a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 \\
v_0 &= a_1 + 2 a_2 t_0 + 3 a_3 t_0^2 \\
q_f &= a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \\
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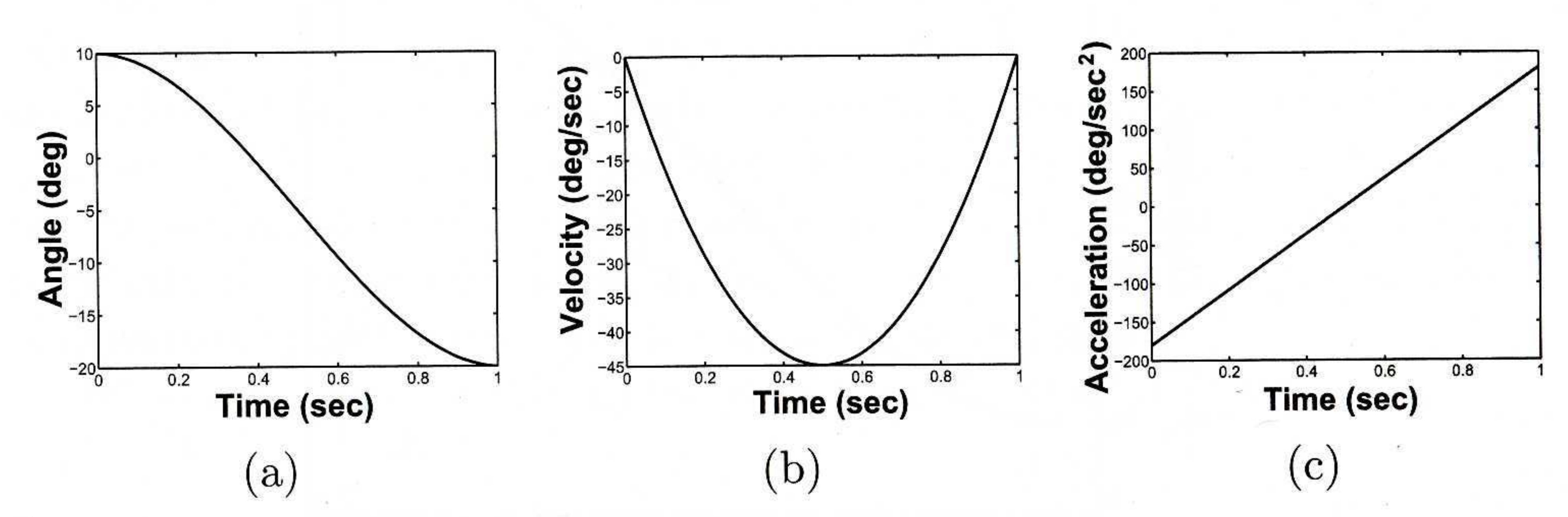
$$

What is the determinant of this matrix?



The parameters for interpolation

$$t = 0 \text{ and } t_f = 1, \quad q_0 = 10 \text{ and } q_f = -20, \quad v_0 = v_f = 0$$

what is wrong with the trajectory?

Trajectories for Point to Point Motion

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and additional constraints of accelerations

$$\frac{d^2}{dt^2} q(t_0) = \alpha_0 \qquad \frac{d^2}{dt^2} q(t_f) = \alpha_f$$

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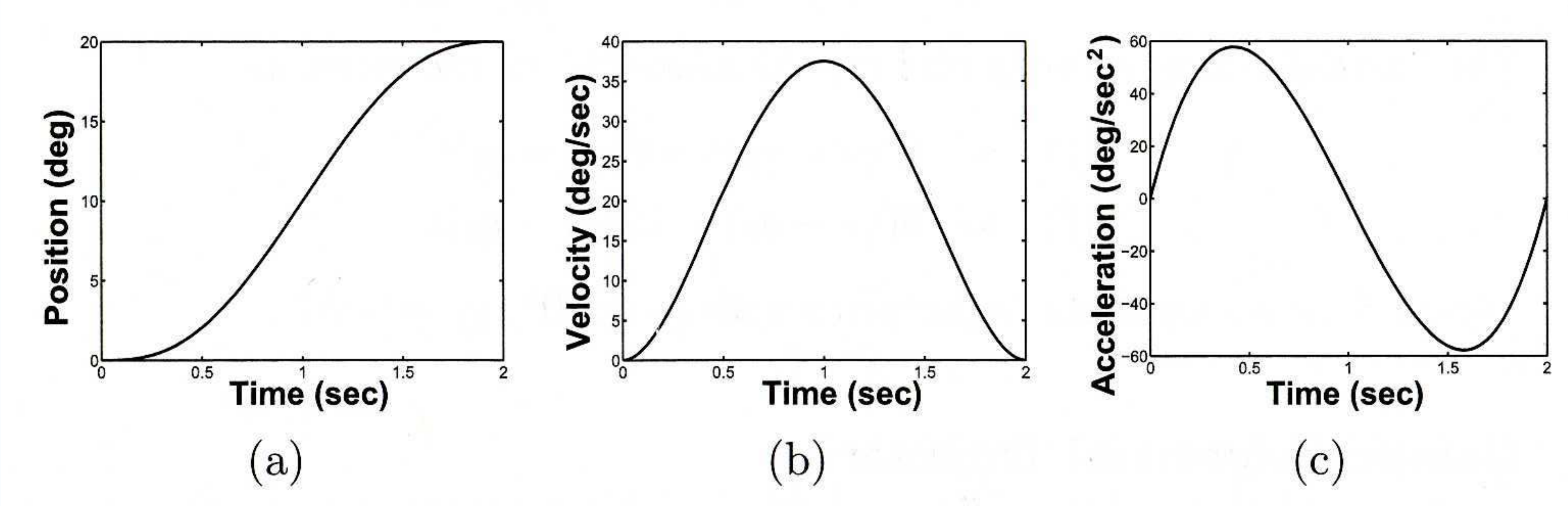
and additional constraints of accelerations

$$\frac{d^2}{dt^2}q(t_0) = \alpha_0 \qquad \frac{d^2}{dt^2}q(t_f) = \alpha_f$$

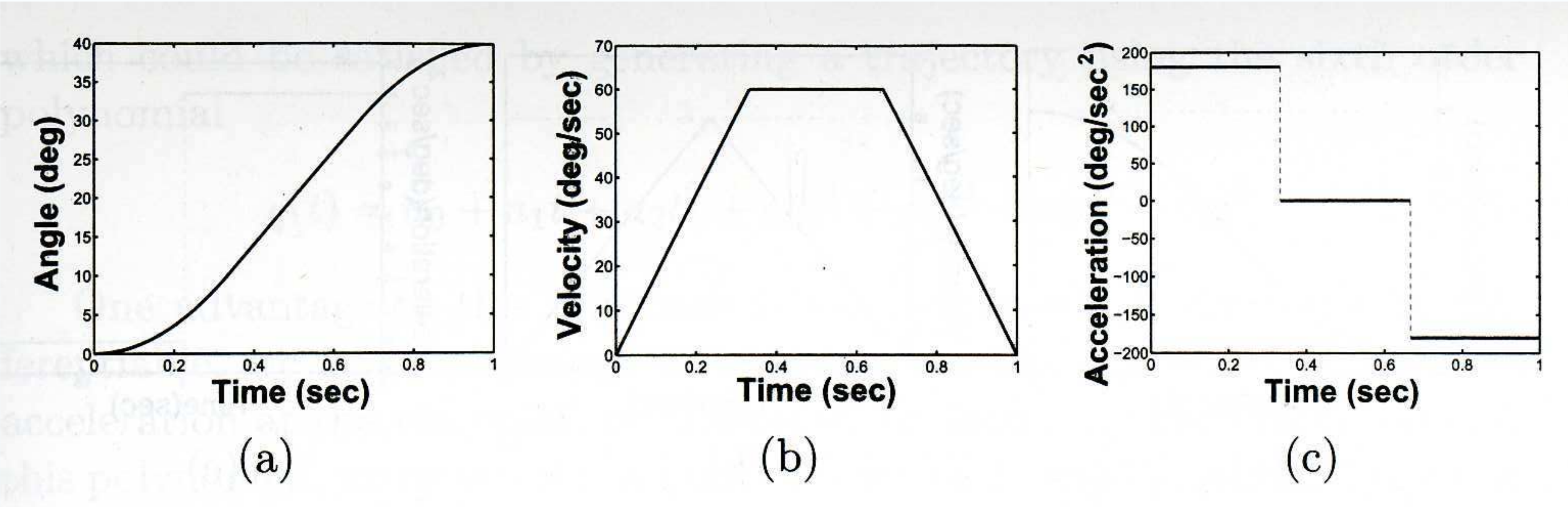
To find interpolating polynomial we need to choose a polynomial of order ≥ 5

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$



The parameters for interpolation

$$t = 0 \text{ and } t_f = 2, \quad q_0 = 0 \text{ and } q_f = 20, \quad v_0 = v_f = 0$$



Interpolation by LSPB: Linear segments with parabolic blends