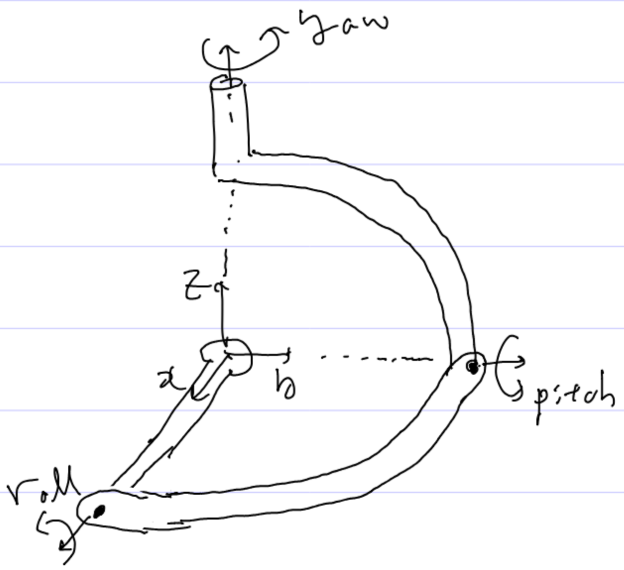
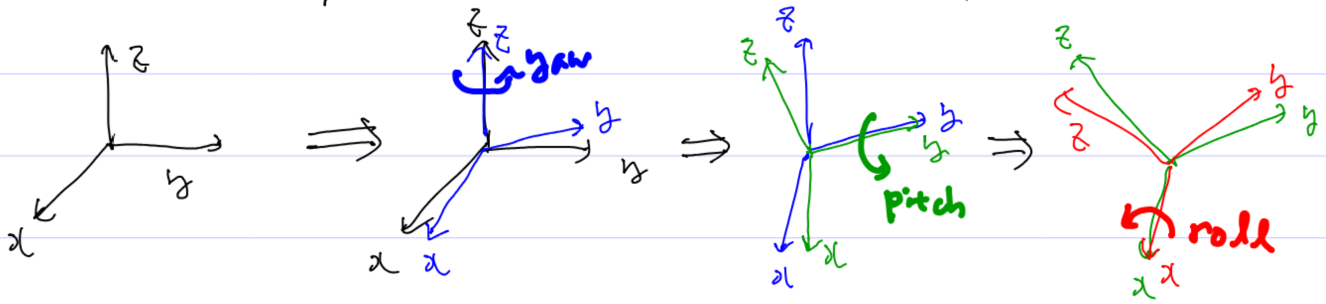
<Euler Angles (Roll, Pitch, Yaw)>



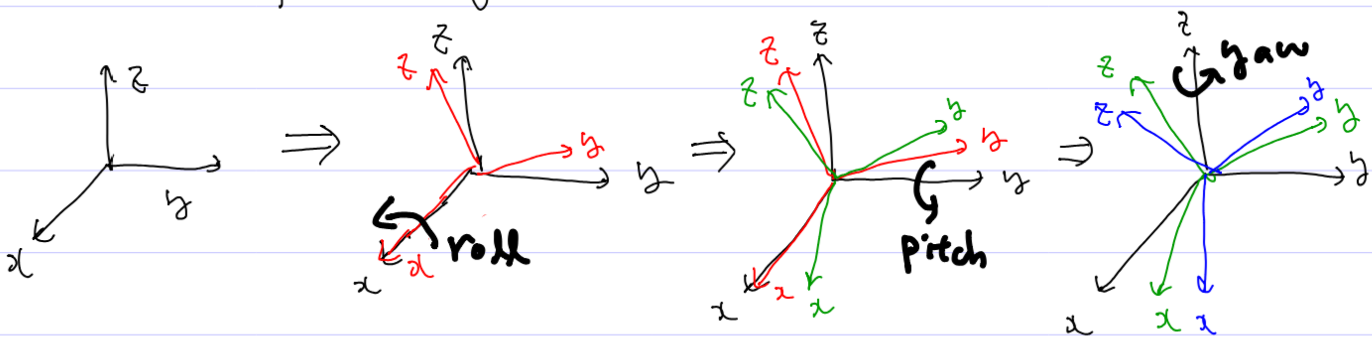
$$R = R_z(y)\,R_y(p)\,R_x(r)$$

※ 2 Interpretations

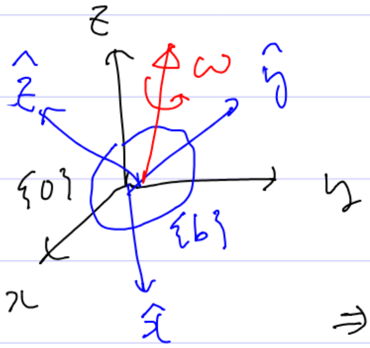
i) yaw → pitch → roll w.r.t. body frames.



ii) roll → pitch → yaw w.r.t inertial frame



⟨Angular Velocity⟩



$$\hat{x}^o = R_{ob}\,\hat{i}$$
$$\hat{y}^o = R_{ob}\,\hat{j}$$
$$\hat{z}^o = R_{ob}\,\hat{k}$$
$$\Rightarrow \underline{[\hat{x}^o \;\; \hat{y}^o \;\; \hat{z}^o] = R_{ob}[\hat{i} \;\; \hat{j} \;\; \hat{k}] = R_{ob}} \quad \cdots ①$$

$$\dot{\hat{x}}^o = \omega^o \times \hat{x}^o$$
$$\dot{\hat{y}}^o = \omega^o \times \hat{y}^o$$
$$\dot{\hat{z}}^o = \omega^o \times \hat{z}^o$$

$$\therefore\; \underline{\dot{R}_{ob}} = [\dot{\hat{x}}^o \;\; \dot{\hat{y}}^o \;\; \dot{\hat{z}}^o]$$

$$= \omega^o \times [\hat{x}^o \;\; \hat{y}^o \;\; \hat{z}^o]$$

from ①

$$= \underline{\omega^o \times R_{ob}} \quad \cdots ②$$

⟨Euler Angle and Angular Velocity⟩

- Euler Angle : $R_{ob} = R_z(y)\,R_y(p)\,R_x(r) \quad \cdots ③$
- $\dot{R}_x = \dot{r}\hat{i} \times R_x, \quad \dot{R}_y = \dot{p}\hat{j}\times R_y, \quad \dot{R}_z = \dot{y}\hat{k}\times R_z \quad \cdots ④$
- From ③, ④

using

$(a\times R_1)R_2 = a\times(R_1R_2)$

$R(a\times b) = (Ra)\times(Rb)$

$$\dot{R}_{ob} = \dot{R}_z R_y R_x + R_z \dot{R}_y R_x + R_z R_y \dot{R}_x$$
$$= (\dot{y}\hat{k}\times R_z)R_y R_x + R_z(\dot{p}\hat{j}\times R_y)R_x + R_z R_y(\dot{r}\hat{i}\times R_x)$$
$$= \dot{y}\hat{k}\times \underbrace{(R_z R_y R_x)} + (R_z\dot{p}\hat{j})\times\underbrace{(R_z R_y R_x)} + (R_z R_y \dot{r}\hat{i})\times\underbrace{(R_z R_y R_x)}_{=R_{ob}}$$

$$\therefore\; \underline{\dot{R}_{ob} = (\dot{y}\hat{k} + R_z\dot{p}\hat{j} + R_z R_y \dot{r}\hat{i})\times R_{ob}}$$

from ②, $\underline{\dot{R}_{ob} = \omega^o \times R_{ob}}$

$$\boxed{\therefore\; \omega^o = \dot{y}\hat{k} + R_z\dot{p}\hat{j} + R_z R_y\dot{r}\hat{i} = [R_z R_y\hat{i} \;\; R_z\hat{j} \;\; \hat{k}]\begin{bmatrix}\dot{r}\\ \dot{p}\\ \dot{y}\end{bmatrix}}$$

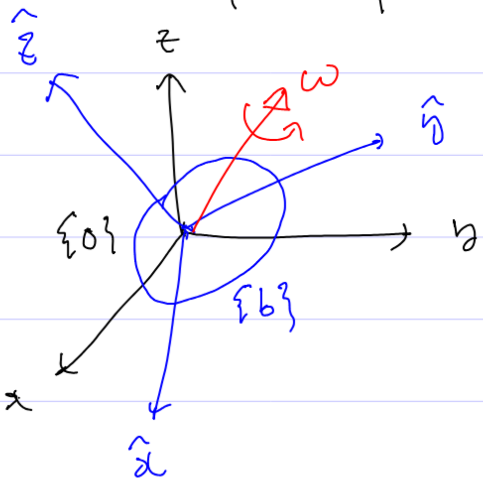
$$R_z R_y \hat{i} = \begin{bmatrix} C_y & -S_y & 0 \\ S_y & C_y & 0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} C_p & 0 & S_p \\ 0 & 1 & 0 \\ -S_p & 0 & C_p\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix} C_y C_p \\ S_y C_p \\ -S_p\end{bmatrix}$$

$$R_z\hat{j} = \begin{bmatrix} C_y & -S_y & 0 \\ S_y & C_y & 0 \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-S_y \\ C_y \\ 0\end{bmatrix}$$

$$= \underline{\begin{bmatrix} C_y C_p & -S_y & 0 \\ S_y C_p & C_y & 0 \\ -S_p & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{r}\\ \dot{p}\\ \dot{y}\end{bmatrix}}$$

$\cdots ⑤$

<Gyroscope>



☆ Gyro measures angular velocity 'ω' expressed in '$\{b\}$ frame'

which means $\omega^b = \begin{bmatrix} \omega^b_x \\ \omega^b_y \\ \omega^b_z \end{bmatrix}$

$$\omega^0 = \omega^b_x \hat{x}^0 + \omega^b_y \hat{y}^0 + \omega^b_z \hat{z}^0 = \underbrace{\begin{bmatrix} \hat{x}^0 & \hat{y}^0 & \hat{z}^0 \end{bmatrix}}_{=R_{ob}} \underbrace{\begin{bmatrix} \omega^b_x \\ \omega^b_y \\ \omega^b_z \end{bmatrix}}_{=\omega^b}$$

$$\therefore \ \omega^0 = R_{ob}\, \omega^b \quad \cdots (6)$$

<Gyroscope and Euler Angles>

from (6), $\omega^b = R_{ob}^T \omega^0$

from (5)

$$\omega^b = R_{ob}^T \begin{bmatrix} C_y C_p & -S_y & 0 \\ S_y C_p & C_y & 0 \\ -S_p & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{r} \\ \dot{p} \\ \dot{y} \end{bmatrix}$$