HW3

**Exercise 1: Bayes Filter and EKF**

(a) Describe briefly the two main steps of the Bayes filter in your own words.

(b) Describe briefly the meaning of the following probability density functions: $p(x_t \mid u_t, x_{t-1})$, $p(z_t \mid x_t)$, and $\mathrm{bel}(x_t)$, which are processed by the Bayes filter.

(c) Specify the (normal) distributions that correspond to the above mentioned three terms in EKF SLAM.

(d) Explain in a few sentences all of the components of the EKF SLAM algorithm, i. e., $\mu_t$, $\Sigma_t$, $g$, $G_t^x$, $G_t$, $R_t^x$, $R_t$, $h$, $H_t$, $Q_t$, $K_t$ and why they are needed. Specify the dimensionality of these components.

(a) two main step

- The first step is prediction step.
  - (expect pdf of $x$ at $t$ from state $x$ at $t-1$, control input $u$ at $t$)
    Considering all the possibility of state $x$ at $t-1$
- the second step is correction step
  - pdf of $x$ at $t$
    from measured value / expected pdf of $x$ at $t$.

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(b) $p(x_t \mid u_t, x_{t-1})$ $p(z_t \mid x_t)$ $bel(x_t)$

$\cdot\cdot\ p(x_t|u_t, x_{t-1}),\ p(z_t|x_t),\ bel(x_t)$

$p(x_t|u_t, x_{t-1})$

→ dynamics propagation from t-1 state with control command $u_t$

$p(z_t|x_t)$

→ probability of measured data from t state($x_t$)

$bel(x_t)$

→ pdf of $x_t$ from 0~t (control inputs / measured datas)

(c)

(c) Specify the (normal) distributions that correspond to the above mentioned three terms in EKF SLAM.

Specify the distribution)

$p(x_t|u_t, x_{t-1}) \Rightarrow \bar{\mu}_t = g(u_t, \mu_{t-1}),\ \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

(d) Explain in a few sentences all of the components of the EKF SLAM algorithm, i. e., $\mu_t$, $\Sigma_t$, $g$, $G_t^x$, $G_t$, $R_t^x$, $R_t$, $h$, $H_t$, $Q_t$, $K_t$ and why they are needed. Specify the dimensionality of these components.

- $\mu_t \in R^{2N+3}$ → means of position of robot $(x, y, \theta)$ + means of position of Landmark
- $\Sigma_t \in R^{2N+3 \times 2N+3}$ → covariance of robot position

& Landmark position'

- $g \to$ function of state & Landmark position', out $\Rightarrow 2N+3$, in $\Rightarrow (2N+3)$, $(t)$

$G_t^x \to$ Jacobian of robot position

$G_t \to$ Jacobian of robot & Landmark position

$R_t^x \to$ noise of robot position

$R_t \to$ noise of robot & Landmark position

$(G_t^x, R_t^x) \in R^{3\times3}$, $(G_t, R_t) \in R^{2n+3 \times 2n+3}$

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$h \to$ function from state to Measurement ( in $= R^{2n+3}$, out $= R^{2n}$)

$H_t \in R^{2n\times 2n+3}$, $H_t$ = Jacobian of $h$

$Q_t \in R^{2n\times2n}$, $Q_t$ = noise of measurement, $K_t \in R^{2n+3\times 2n}$ kalman gain

### Exercise 2: Jacobians

(a) Derive the Jacobian matrix $G_t^x$ of the noise-free motion function $g$ with respect to the pose of the robot. Use the odometry motion model as in exercise sheet 1:

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans}\cos(\theta_{t-1}+\delta_{rot1}) \\ \delta_{trans}\sin(\theta_{t-1}+\delta_{rot1}) \\ \delta_{rot1}+\delta_{rot2} \end{pmatrix}.$$

Do not use Octave for this part of the exercise.

$$G_t^x = \frac{\partial P}{\partial P_{t-1}} = \begin{pmatrix} \frac{\partial x_t}{\partial x_{t-1}} & \frac{\partial x_t}{\partial y_{t-1}} & \frac{\partial x_t}{\partial \theta_{t-1}} \\ \cdot & \cdot & \end{pmatrix}$$

$$= I + \begin{pmatrix} 0 & 0 & -\delta_{trans}\sin(\theta_{t-1}+\delta_{rot1}) \end{pmatrix}$$

$$1 \quad 1 \quad \Big| \quad 0 \quad 0 \quad +\delta_{trans}\cos(\theta_{t-1}+\delta_{rot1}) \Big) \\ 0 \quad 0 \quad 0$$

(b) Derive the Jacobian matrix ${}^{\text{low}}H_t^i$ of the noise-free sensor function $h$ corresponding to the $i^{\text{th}}$ measurement:

$$h(\bar{\mu}_t, j) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}, \bar{\mu}_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix},$$

where $(\bar{\mu}_{j,x}, \bar{\mu}_{j,y})^T$ is the pose of the $j^{\text{th}}$ landmark, $(\bar{\mu}_{t,x}, \bar{\mu}_{t,y}, \bar{\mu}_{t,\theta})^T$ is the pose of the robot at time $t$, and $r_t^i$ and $\phi_t^i$ are respectively the observed range and bearing of the landmark. Do not use Octave for this part of the exercise.

*Hint:* use $\frac{\partial}{\partial x}\operatorname{atan2}(y,x) = \frac{-y}{x^2+y^2}$, and $\frac{\partial}{\partial y}\operatorname{atan2}(y,x) = \frac{x}{x^2+y^2}$.

$$\text{low}(H_t^i) = \frac{h(\bar{\mu}_t, j)}{\partial \bar{\mu}_t} = \begin{pmatrix} \frac{\partial r_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t^i}{\partial \mu_{t,y}} & \frac{\partial r_t^i}{\partial \mu_{t,\theta}} \\ \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\cancel{2}\sqrt{\alpha}}(-\cancel{2}(\bar{\mu}_{j,x} - \bar{\mu}_{t,x})) & \frac{-1}{\sqrt{\alpha}}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}) & 0 \\ \frac{1}{\alpha}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}) & \frac{-1}{\alpha}(\bar{\mu}_{j,x} - \bar{\mu}_{t,x}) & -1 \end{pmatrix}$$

$$\alpha = (\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2$$