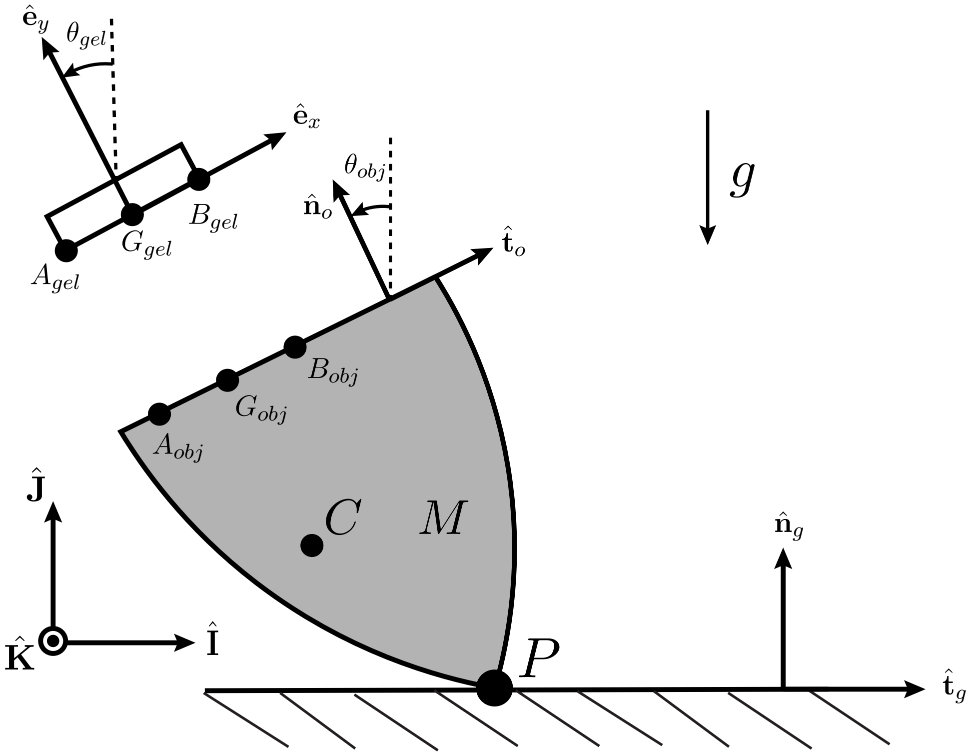

State variables

Position and orientation of the object. The reference point is the point of contact between the gel pad and the object:

$$[x_{G/obj}, y_{G/obj}, \theta_{obj}]^T$$

Position and orientation of the gel pad. The reference point is the point of contact between the gel pad and the object:

$$[x_{G/gel}, y_{G/gel}, \theta_{gel}]^T$$

Vector, Frame, and Parameter Definitions

- $\hat{\mathbf{n}}_g$ and $\hat{\mathbf{t}}_g$ are the respective normal and tangents to the ground surface
- $\hat{\mathbf{n}}_o$ and $\hat{\mathbf{t}}_o$ are the respective normal and tangents to the object at the contact patch
- $\hat{\mathbf{I}}, \hat{\mathbf{J}}, \hat{\mathbf{K}}$ are the basis vectors of the world reference frame, which is inertial.
- $\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_y$ are the basis vectors of the gel reference frame, which is body fixed to the gel pad at G.

- M is the mass of the object.
- g is the magnitude of gravity.

Point Definitions

- Point A and B define corners of the contact patch between the gel pad and the object. We will probably assume that this contact patch is convex, meaning that the corners of the contact patch will be the generators for any point inside the contact patch.
- Point G is our master reference point on the contact patch. We will probably assume that it is the centroid of the contact patch.
- Note that A B and G can be either material points of the object or the gel pad. We will use the subscript *gel* to denote a material point of the gel pad, and the subscript *obj* to denote a material point of the object.
- C is the center of mass of the object
- P is the contact point between the object and the ground
- We use $\vec{\boldsymbol{r}}_{point}$ and $\vec{\boldsymbol{v}}_{point}$ to denote the respective position and velocity of a given point in the world frame.

Possible Motion Constraints

Non-penetration constraints: The material contact point of the gel cannot fall inside the object, and visa versa:

$$\left(\vec{\boldsymbol{v}}_{A/gel} - \vec{\boldsymbol{v}}_{A/obj}\right) \cdot \hat{\mathbf{n}}_O \geq 0$$
$$\left(\vec{\boldsymbol{v}}_{B/gel} - \vec{\boldsymbol{v}}_{B/obj}\right) \cdot \hat{\mathbf{n}}_O \geq 0$$

The object cannot penetrate the ground:

$$\left(\vec{\boldsymbol{v}}_{P/obj} - \vec{\boldsymbol{v}}_{P/grnd}\right) \cdot \hat{\mathbf{n}}_g \geq 0$$

No-slip constraint: There is no-slip between the object and the gel. It is unclear whether or not we will apply this constraint:

$$\left(\vec{\boldsymbol{v}}_{A/gel} - \vec{\boldsymbol{v}}_{A/obj}\right) \cdot \hat{\mathbf{t}}_O = 0$$
$$\left(\vec{\boldsymbol{v}}_{B/gel} - \vec{\boldsymbol{v}}_{B/obj}\right) \cdot \hat{\mathbf{t}}_O = 0$$

There is no-slip between the object and the ground. It is unclear whether or not we will apply this constraint:

$$\left(\vec{\boldsymbol{v}}_{P/obj} - \vec{\boldsymbol{v}}_{P/grnd}\right) \cdot \hat{\mathbf{t}}_g = 0$$

Glue constraint: This is a combination of the no-slip constraint with a stronger version of the non-penetration constraint. We act as if the gel-pad is glued to the object. It is unclear how useful this constraint will be, though it is easy to express:

$$\left(\vec{\boldsymbol{v}}_{A/gel} - \vec{\boldsymbol{v}}_{A/obj}\right) = 0$$
$$\left(\vec{\boldsymbol{v}}_{B/gel} - \vec{\boldsymbol{v}}_{B/obj}\right) = 0$$

Similarly, we assume that the tip of the object is glued to the ground:

$$\left(\vec{\boldsymbol{v}}_{P/obj} - \vec{\boldsymbol{v}}_{P/grnd}\right) = 0$$

Motion generation constraints: The motion of the material points A and B on the gel in the world frame can be computed as a function of the velocities of the generalized coordinates, and the current state:

$$\vec{\boldsymbol{v}}_{A/gel} = \vec{\boldsymbol{v}}_{G/gel} + \dot{\theta}_{gel}\hat{\mathbf{k}} \times \left(\vec{\boldsymbol{r}}_{A/gel} - \vec{\boldsymbol{r}}_{G/gel}\right)$$
$$\vec{\boldsymbol{v}}_{B/gel} = \vec{\boldsymbol{v}}_{G/gel} + \dot{\theta}_{gel}\hat{\mathbf{k}} \times \left(\vec{\boldsymbol{r}}_{B/gel} - \vec{\boldsymbol{r}}_{G/gel}\right)$$

The motion of the material points A and B on the object in the world frame can be computed as a function of the velocities of the generalized coordinates, and the current state:

$$\vec{\boldsymbol{v}}_{A/obj} = \vec{\boldsymbol{v}}_{G/obj} + \dot{\theta}_{obj}\hat{\mathbf{k}} \times \left(\vec{\boldsymbol{r}}_{A/obj} - \vec{\boldsymbol{r}}_{G/obj}\right)$$
$$\vec{\boldsymbol{v}}_{B/obj} = \vec{\boldsymbol{v}}_{G/obj} + \dot{\theta}_{obj}\hat{\mathbf{k}} \times \left(\vec{\boldsymbol{r}}_{B/obj} - \vec{\boldsymbol{r}}_{G/obj}\right)$$

The motion of the material point P on the object in the world frame can be computed as a function of the velocities of the generalized coordinates, and the current state:

$$\vec{\boldsymbol{v}}_{P/obj} = \vec{\boldsymbol{v}}_{G/obj} + \dot{\theta}_{obj}\hat{\mathbf{k}} \times \left(\vec{\boldsymbol{r}}_{P/obj} - \vec{\boldsymbol{r}}_{G/obj}\right)$$

Force Constraints

Static Equilibrium: The sum of forces acting on the object must equal zero:

$$\vec{\boldsymbol{F}}_{A/obj} + \vec{\boldsymbol{F}}_{B/obj} + \vec{\boldsymbol{F}}_{grav} + \vec{\boldsymbol{F}}_{P/obj} = 0$$

The sum of torques acting on the object (with respect to point G) must equal zero:

$$(\vec{\boldsymbol{r}}_A - \vec{\boldsymbol{r}}_G) \times \vec{\boldsymbol{F}}_{A/obj} + (\vec{\boldsymbol{r}}_A - \vec{\boldsymbol{r}}_G) \times \vec{\boldsymbol{F}}_{B/obj} + (\vec{\boldsymbol{r}}_A - \vec{\boldsymbol{r}}_G) \times \vec{\boldsymbol{F}}_{grav} + (\vec{\boldsymbol{r}}_A - \vec{\boldsymbol{r}}_G) \times \vec{\boldsymbol{F}}_{P/obj} = 0$$

Friction Cone Constraints: The force that the gel pad can exert on the object is a positive linear combination of the two friction cone generators:

$$\vec{\boldsymbol{F}}_{A/obj} = \alpha_1 \left(-\hat{\mathbf{n}}_O + \mu_1 \hat{\mathbf{t}}_O\right) + \alpha_2 \left(-\hat{\mathbf{n}}_O - \mu_1 \hat{\mathbf{t}}_O\right), \quad \alpha_1, \alpha_2 \geq 0$$

$$\vec{\boldsymbol{F}}_{B/obj} = \alpha_3 \left(-\hat{\mathbf{n}}_O + \mu_1 \hat{\mathbf{t}}_O\right) + \alpha_4 \left(-\hat{\mathbf{n}}_O - \mu_1 \hat{\mathbf{t}}_O\right), \quad \alpha_3, \alpha_4 \geq 0$$

The force that the ground can exert on the object is a positive linear combination of the two friction cone generators:

$$\vec{\boldsymbol{F}}_{P/obj} = \alpha_5 \left(-\hat{\mathbf{n}}_g + \mu_2 \hat{\mathbf{t}}_g\right) + \alpha_6 \left(-\hat{\mathbf{n}}_g - \mu_2 \hat{\mathbf{t}}_g\right), \quad \alpha_5, \alpha_6 \geq 0$$

Gravity: The force of gravity is given by:

$$\vec{\boldsymbol{F}}_{grav} = -Mg\hat{\mathbf{J}}$$

Control Inputs: The control inputs to the system are the force and torque (with respect to G) exerted by the gel pad, which are given by:

$$\vec{\boldsymbol{F}}_{gel} = \vec{\boldsymbol{F}}_{A/obj} + \vec{\boldsymbol{F}}_{B/obj}$$

$$\vec{\boldsymbol{\tau}}_{G/gel} = (\vec{\boldsymbol{r}}_A - \vec{\boldsymbol{r}}_G) \times \vec{\boldsymbol{F}}_{A/obj} + (\vec{\boldsymbol{r}}_A - \vec{\boldsymbol{r}}_G) \times \vec{\boldsymbol{F}}_{B/obj}$$

Control Strategy

Create a policy $\vec{\boldsymbol{F}}_G(x_{G/gel}, y_{G/gel}, \theta_{gel})$, $\tau_G\left(x_{G/gel}, y_{G/gel}, \theta_{gel}\right)$

- Output values of policy $\vec{\boldsymbol{F}}_G, \tau_G$ must satisfy the force constraints for every state value.
- We use the motion constraints to determine the admissible variations of the system.
- If we can find a policy for which the virtual work function is locally convex, then it should correspond to a stabilizing controller, right?