Reach Alpha MKII
Kinematic and Dynamic Properties

Version 7

August 2022

1 Kinematics

1.1 Denavit-Hartenberg Parameters

To describe the transformations between links the Standard Denavit-Hartenberg (DH) method has been used.

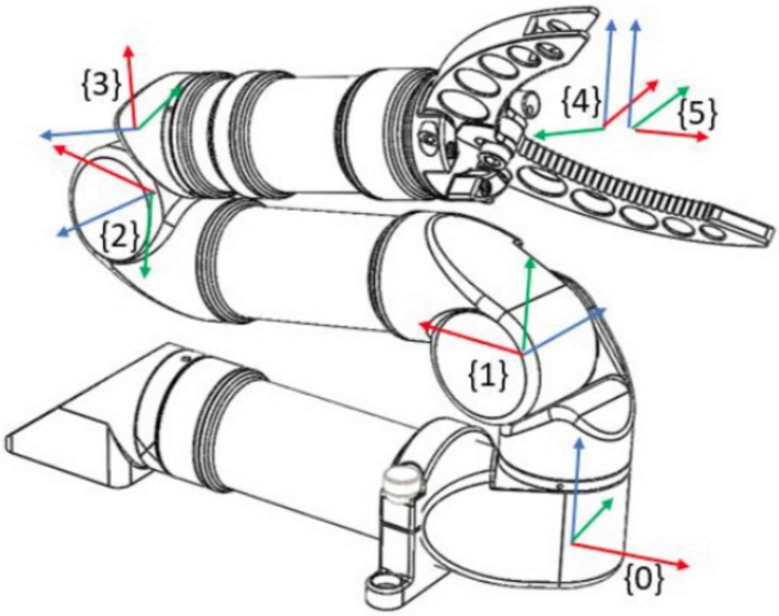

Figure 1: Reach Alpha 5 - Standard DH Frames (x,y,z)

The DH parameters are given for the Reach Alpha 5 standard configuration in Table 1. These values define the frame locations shown in Figure 1.

Link	d (mm)	θ	a (mm)	α
0	46.2	$\theta_0 + \pi$	20	$\pi/2$
1	0	$\theta_1 + \theta_c - \pi/2$	a_1	π
2	0	$\theta_2 + \theta_c - \pi/2$	20	$-\pi/2$
3	-180	$\theta_3 + \pi/2$	0	$\pi/2$
4	0	$-\pi/2$	0	0

Table 1: Reach Alpha 5 - DH Parameters, where $\theta_c = \tan^{-1}\left(\frac{40}{145.3}\right)$ and $a_1 = \sqrt{40^2 + (145.3)^2}$

where d is offset along previous z to the common normal, θ is the angle about z_{i-1}, from x_{i-1} to x_i, a is the length of the common normal, α is the angle about the common normal, from z_{i-1} axis to z_i axis. The mapping between DH frames conveyed in Table 1 and the inertial frames is described in Section 2.

1.2 Workspace

For the Reach Alpha 5, the outer reachable limits form the torus described by,

$$\left(\sqrt{x^2+y^2}-a_0\right)^2+(z-d_0)^2 \leq \left(a_1+\sqrt{d_3^2+a_2^2}\right)^2$$

The inner reachable limit is the torus,

$$\left(\sqrt{x^2+y^2}-a_0\right)^2+(z-d_0)^2 \geq \left((39.94+a_2)^2+(145.3+d_3)^2\right)$$

The inner reachable limit with the arm in the downward position is the torus,

$$\left(\sqrt{x^2+y^2}-a_0\right)^2+(z-d_0+145.3)^2 \geq (-d_3)^2$$

These are shown in Figure 2

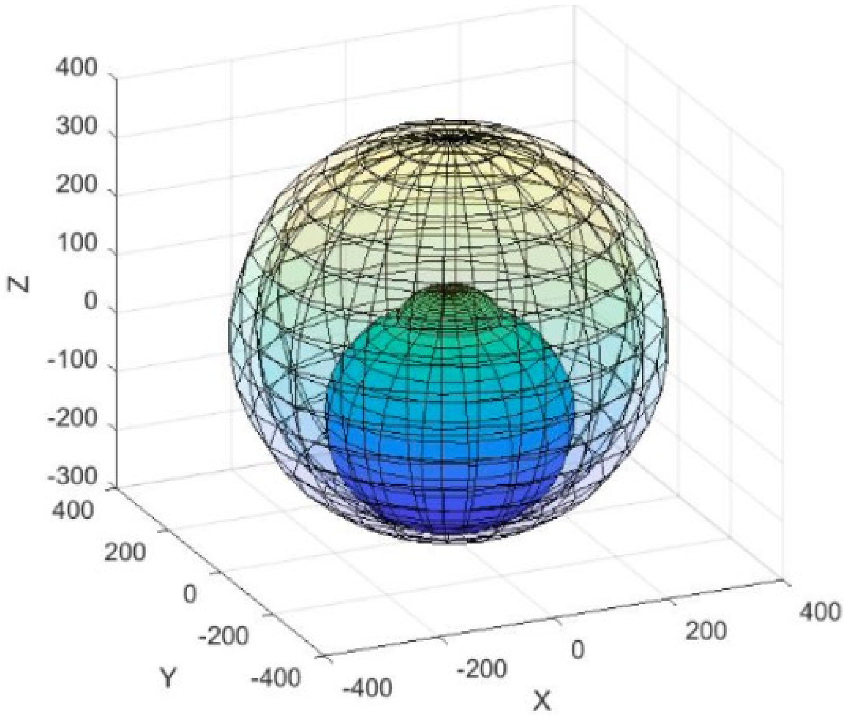

Figure 2: Reach Alpha 5 - Reachable workspace without self collision showing inner and outer limits

1.3 Inverse Kinematics - Underarm Pose

The following section describes the analytical solution for the Reach Alpha 5 underarm pose.

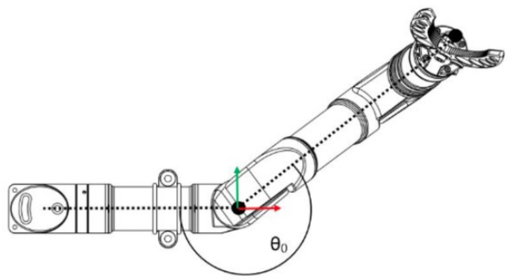

Figure 3: Reach Alpha 5 - Calculation of θ_0 underarm pose

From Figure 3, θ_0 is given by,

$$\theta_0 = \arctan 2\,(y, x) + \pi$$

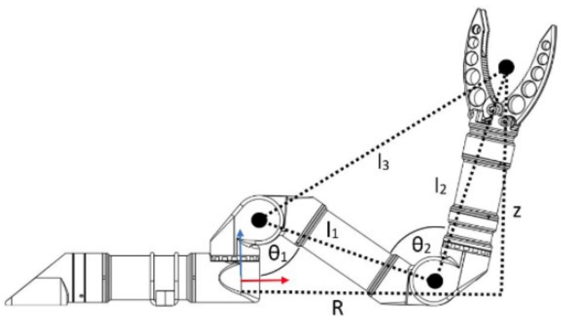

Figure 4: Reach Alpha 5 - Calculation of θ_1 and θ_2 underarm pose

From Figure 4, θ_1 and θ_2 are given by,

$$R = \sqrt{x^2 + y^2}$$
$$l_1 = a_1$$
$$l_2 = \sqrt{a_2^2 + d_3^2}$$
$$l_3 = \sqrt{(R - a_0)^2 + (z - d_0)^2}$$
$$\theta_2 = \arccos\left(\frac{l_1^2 + l_2^2 - l_3^2}{2l_1l_2}\right) - \arcsin\left(\frac{2a_2}{l_1}\right) - \arcsin\left(\frac{a_2}{l_2}\right)$$
$$\theta_1 = \frac{\pi}{2} + \arctan 2\,(z - d_0, R - a_0) - \arccos\left(\frac{l_1^2 + l_3^2 - l_2^2}{2l_1l_3}\right) - \arcsin\left(\frac{2a_2}{l_1}\right)$$

1.4 Inverse Kinematics - Overarm Pose

The following section describes the analytical solution for the Reach Alpha 5 overarm pose.

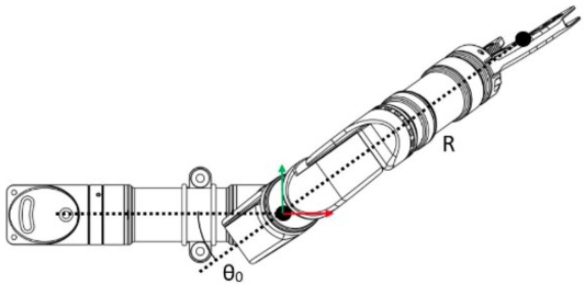

Figure 5: Reach Alpha 5 - Calculation of θ_0 overarm pose

From Figure 5, θ_0 is given by,

$$\theta_0 = \arctan 2\,(y, x)$$

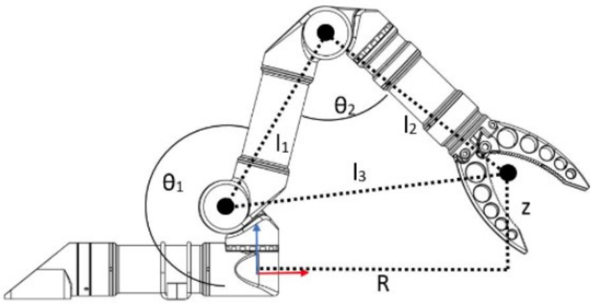

Figure 6: Reach Alpha 5 - Calculation of θ_1 and θ_2 overarm pose

Finally, from Figure 6, θ_1 and θ_2 are given by,

$$R = \sqrt{x^2 + y^2}$$
$$l_1 = a_1$$
$$l_2 = \sqrt{a_2^2 + d_3^2}$$
$$l_3 = \sqrt{(R + a_0)^2 + (z - d_0)^2}$$
$$\theta_2 = \arccos\left(\frac{l_1^2 + l_2^2 - l_3^2}{2l_1l_2}\right) - \arcsin\left(\frac{2a_2}{l_1}\right) - \arcsin\left(\frac{a_2}{l_2}\right)$$
$$\theta_1 = \frac{3\pi}{2} - \arctan 2\,(z - d_0, R + a_0) - \arccos\left(\frac{l_1^2 + l_3^2 - l_2^2}{2l_1l_3}\right) - \arcsin\left(\frac{2a_2}{l_1}\right)$$

2 Dynamics

The inertial frames for Reach Alpha 5 are defined in Figure 7.

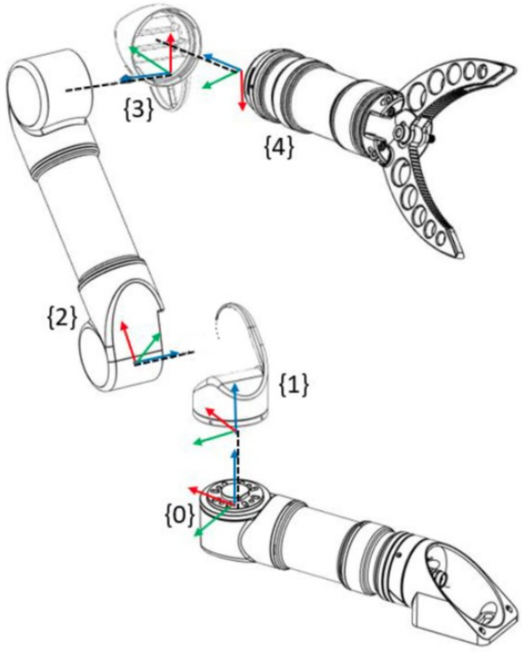

Figure 7: Reach Alpha 5 - Inertial Frame Locations

Such that, inertial frame {0} is the base frame located at the origin of DH frame {0} after the rotation π . Inertial frame {1} is located at the origin of DH frame {0} after the rotation $\theta_0 + \pi$. Inertial frame {2} is located at the origin of DH frame {1} after the rotation $\theta_1 + \theta_c - \pi/2$. Inertial frame {3} is located at the origin of DH frame {2} after the rotation $\theta_2 + \theta_c - \pi/2$. Inertial frame {4} is located at the origin of DH frame {3} after the rotation $\theta_3 + \pi/2$. **Note 1:** the inertial frames shown in Figure 7 are shown in exploded view for diagrammatic purposes and do not represent the actual frame locations.

2.1 Inertial Properties

The inertial properties for Reach Alpha 5 are provided in Table 2. The center of mass (COM) offsets are given with respect to the frames described in Figure 7. For convenience, each inertia tensor is given for the link's output frame (I_{Link}) and the COM aligned with the output coordinate system (I_{COM}). ***Note:*** *the URDF standard expect the inertia tensor to be located at the COM and aligned with the output coordinate system. Additionally, the URDF standard assumes a negative product of inertia convention. The parameters provided in Table 2 use the SolidWorks convention and do not comply with the URDF standard. See the documentation at wiki.ros.org/urdf/XML/link and www.mathworks.com/help/specify-custom-inertia for more details.*

Expressing inertia in different frames The inertia tensor can be expressed with respect to another reference frame through Equation 1. The relationship is defined by a similarity transform for the rotation and the tensor generalization of parallel axis theorem for the translation (MATLAB, 2021).

$$\mathbf{I_B} = \mathbf{R_{A/B}} \mathbf{I_A} \mathbf{R^T_{A/B}} + m[(\vec{P} \cdot \vec{P})\mathbf{E_3} - (\vec{P} \cdot \vec{P})^T] \tag{1}$$

where I_A is the inertia with respect to $\{A\}$, I_B is the inertia with respect to $\{B\}$, $\mathbf{R_{A/B}}$ is the rotation from $\{A\}$ to $\{B\}$, m is the mass, $\vec{P}$ is the translation vector between $\{A\}$ and $\{B\}$, and $\mathbf{E_3}$ denotes a 3x3 identity matrix. Using skew symmetric identities, this relationship can be stated more concisely as,

$$\mathbf{I_B} = \mathbf{R_{A/B}} \mathbf{I_A} \mathbf{R^T_{A/B}} + m(SKEW(\vec{P})^2) \tag{2}$$

where $SKEW(\vec{P})$ denotes the skew-symmetric matrix associated to the position vector $\vec{P}$.

Link	Mass (kg)	COM (mm)	I_{COM} ($kg.mm^2$)	I_{Link} ($kg.mm^2$)
0	0.234	$\begin{pmatrix} -52.425 & -7.120 & -0.037 \end{pmatrix}$	$\begin{pmatrix} 45.012 & 2.832 & -4.115 \\ 2.832 & 393.932 & -0.010 \\ -4.115 & -0.010 & 397.995 \end{pmatrix}$	$\begin{pmatrix} 56.495 & 91.063 & -3.655 \\ 91.063 & 1043.265 & 0.059 \\ -3.655 & 0.059 & 1059.604 \end{pmatrix}$
1	0.161	$\begin{pmatrix} 7.746 & 0.112 & 25.038 \end{pmatrix}$	$\begin{pmatrix} 109.469 & 0.808 & 29.828 \\ 0.808 & 122.362 & 0.544 \\ 29.828 & 0.544 & 43.987 \end{pmatrix}$	$\begin{pmatrix} 209.962 & 0.976 & 60.874 \\ 0.976 & 232.445 & 1.089 \\ 60.874 & 1.089 & 53.581 \end{pmatrix}$
2	0.38	$\begin{pmatrix} 73.563 & -0.091 & -0.734 \end{pmatrix}$	$\begin{pmatrix} 82.697 & -74.836 & -1.866 \\ -74.836 & 847.148 & 0.317 \\ -1.866 & 0.317 & 868.483 \end{pmatrix}$	$\begin{pmatrix} 82.905 & -77.380 & -22.366 \\ -77.380 & 2903.184 & 0.342 \\ -22.366 & 0.342 & 2924.317 \end{pmatrix}$
3	0.142	$\begin{pmatrix} 11.188 & -16.660 & 0.148 \end{pmatrix}$	$\begin{pmatrix} 63.596 & -21.363 & -0.802 \\ -21.363 & 40.651 & 0.337 \\ -0.802 & 0.337 & 75.158 \end{pmatrix}$	$\begin{pmatrix} 102.986 & -47.813 & -0.566 \\ -47.813 & 58.418 & -0.014 \\ -0.566 & -0.014 & 132.308 \end{pmatrix}$
4	0.355	$\begin{pmatrix} -0.030 & -1.482 & -100.881 \end{pmatrix}$	$\begin{pmatrix} 615.921 & 0.067 & -7.916 \\ 0.067 & 625.307 & -0.428 \\ -7.916 & -0.428 & 62.574 \end{pmatrix}$	$\begin{pmatrix} 4233.813 & 0.083 & -6.827 \\ 0.083 & 4242.419 & 52.693 \\ -6.827 & 52.693 & 63.355 \end{pmatrix}$

Table 2: Reach Alpha 5 - Inertial properties (Link 4 includes interlocking jaws in the closed position)

2.2 Buoyancy

Table 3 describes the buoyancy properties of the Reach Alpha 5. The buoyancy properties are described with respect to the inertial frames conveyed in Figure 7. The inertial frames are aligned with the DH frames as described in Section 2.1.

Link	Volume (L)	COB (mm)
0	0.157	$\begin{pmatrix} -53 & -3 & -1 \end{pmatrix}$
1	0.032	$\begin{pmatrix} 2 & -4 & 31 \end{pmatrix}$
2	0.218	$\begin{pmatrix} 76 & 0 & -1 \end{pmatrix}$
3	0.032	$\begin{pmatrix} 2 & -4 & 29 \end{pmatrix}$
4	0.134	$\begin{pmatrix} 0 & 0 & -97 \end{pmatrix}$

Table 3: Reach Alpha 5 - Internal volume displacement and Centre of Buoyancy (COB)

2.2.1 Actuator Properties

The actuator properties for Reach Alpha 5 are provided in Table 4. The input friction F values are approximated from the current-velocity profile. All other properties are taken from the motor/gearbox manufacture data-sheets. ***Note:*** *actuators 0-3 are revolute joints rotating the reference frames described in Figure 7. Actuator 4 is the prismatic (linear) joint to actuate the end-effector.*

Actuator	R [Ω]	L [μH]	F [Nmm]	η_{max}[%]	K_t[Nm/A]	K_v[V/rad/s]	Gr
0	3.22	231	3.752±10%	86	0.0134	0.0116	1754.4
1	3.22	231	3.752±10%	86	0.0134	0.0116	1754.4
2	3.22	231	3.752±10%	86	0.0134	0.0116	1754.4
3	38.6	1280	0.469±15%	77	0.0271	0.0545	340.4
4	38.6	1280	0.469±15%	77	0.0271	0.0545	348.7

Table 4: Reach Alpha 5 - Actuator Properties