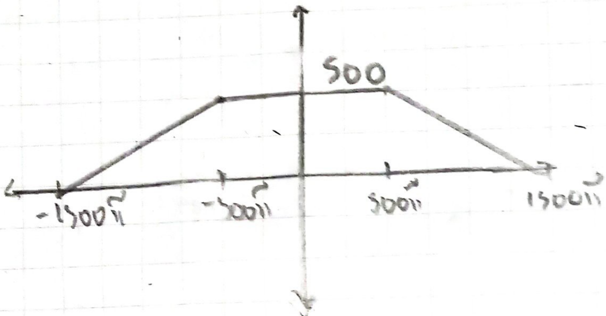
Tarea 8

1.

a. Realizamos un trapecio mediante la multiplicación de dos cuadrados

$$\frac{\sin(500\pi t)}{\pi t} \cdot \frac{\sin(1000\pi t)}{\pi t}$$

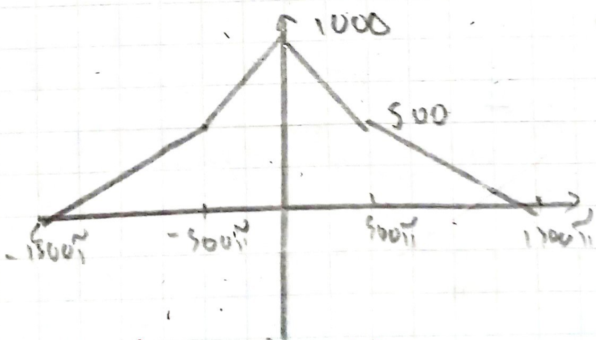
Multiplicamos por 2pi para que nos de altura 1000π



Sumamos un triangulo de la forma

$$c(t) = \left(\frac{\sin(250\pi t)}{\pi t}\right)^2$$

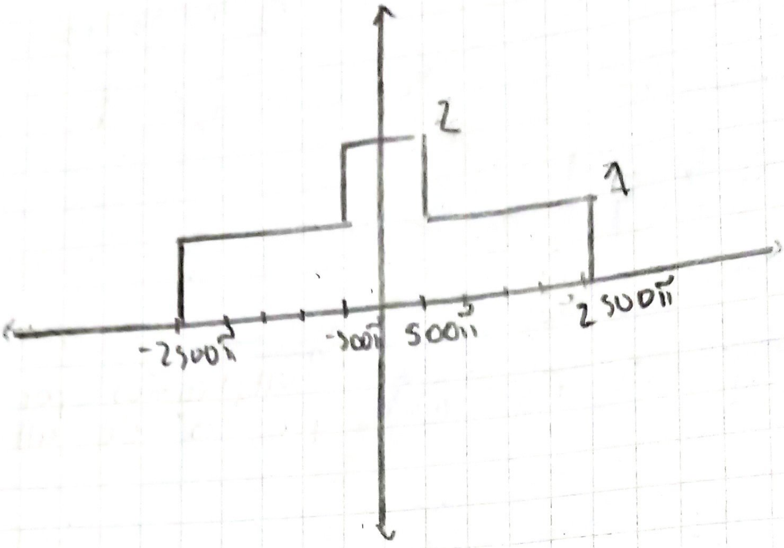
$$h(t) = 2\pi\left(\frac{\sin(500\pi t)}{\pi t} \cdot \frac{\sin(1000\pi t)}{\pi t}\right) + 4\pi\left(\frac{\sin(250\pi t)}{\pi t}\right)^2$$

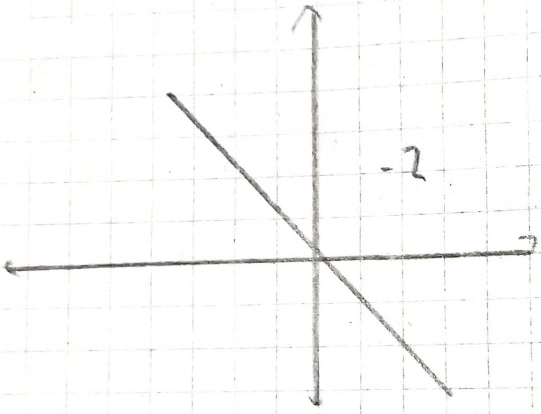


$$h(t) = 2\pi\left(\frac{\sin(500\pi t)}{\pi t} \cdot \frac{\sin(1000\pi t)}{\pi t}\right) + 4\pi\left(\frac{\sin(250\pi t)}{\pi t}\right)^2$$

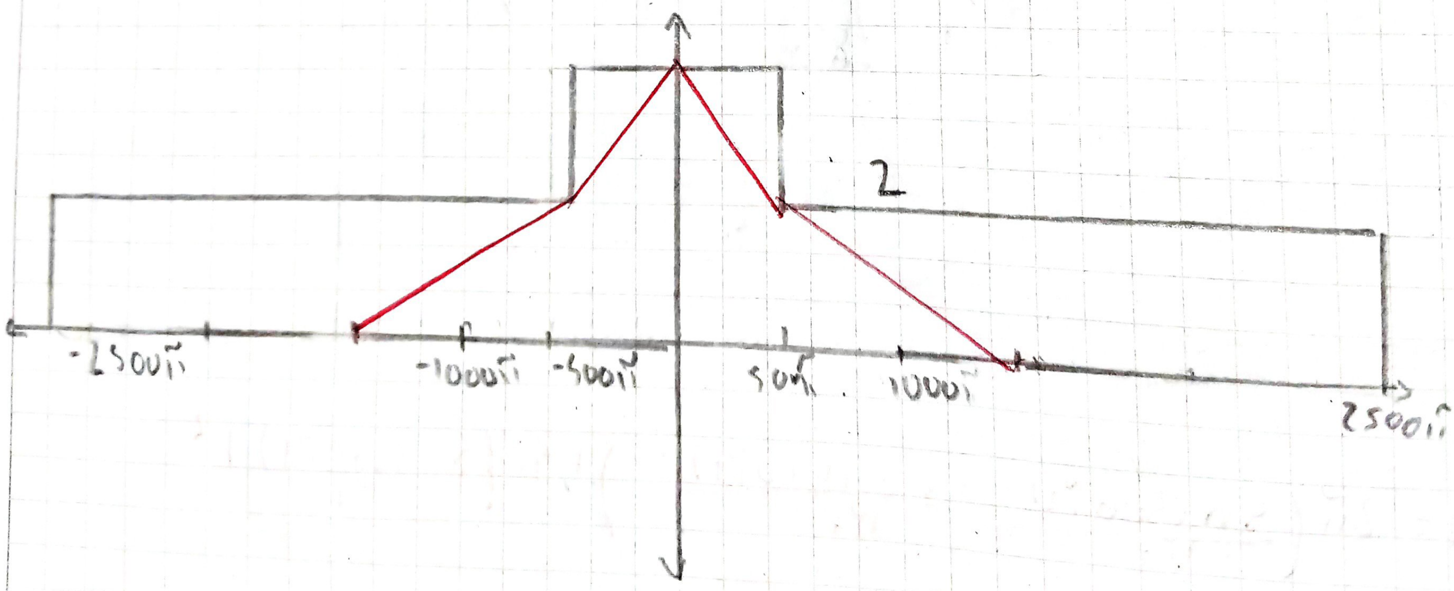
$$h(t) = 2\pi\left(\frac{\sin(500\pi(t-3))}{\pi(t-3)} \cdot \frac{\sin(1000\pi(t-3))}{\pi(t-3)}\right) + 4\pi\left(\frac{\sin(250\pi(t-3))}{\pi t}\right)^2$$

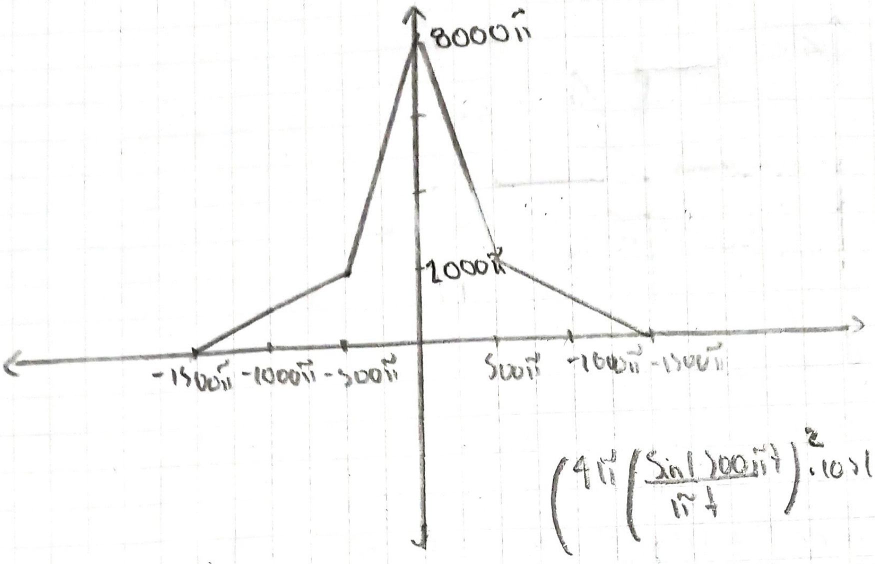
b.


2
1
-2500π
-500π
500π
2500π


-2

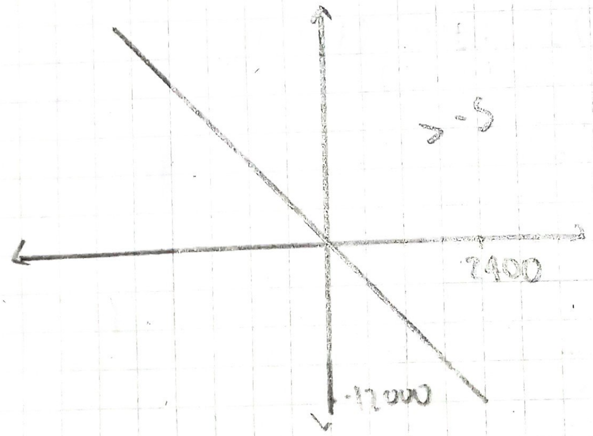
c.


2
-2500π
-1000π
-500π
500π
1000π
2500π



$$\left(4\pi\left(\frac{\sin(1000\pi t)}{\pi t}\right)^2 \cdot \cos(1000\pi t)\right) + \left(\frac{2000\pi \cdot \sin(500\pi t)}{\pi t}\right)$$

$$+16\pi\left(\frac{\sin(2500\pi t)}{\pi t}\right)^2$$



d Si h(t) es la señal normal 1 a, calcule:

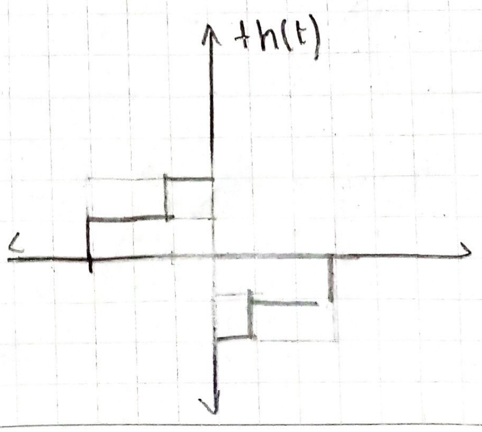
$E = \int_{-\infty}^{\infty} |th(t+3)|^2 dt$ El termino corresponde a la derivada, calculamos la derivada.

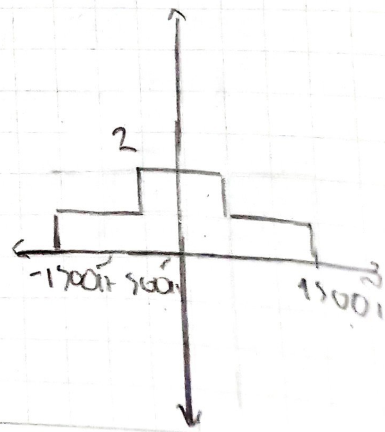
Primer tramo, P=1

Segundo, P=2

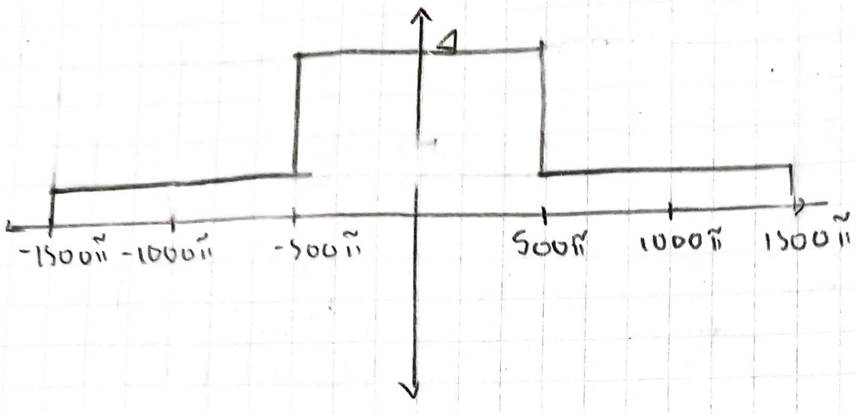
Tercero, P=-2

Cuarto, P=-1





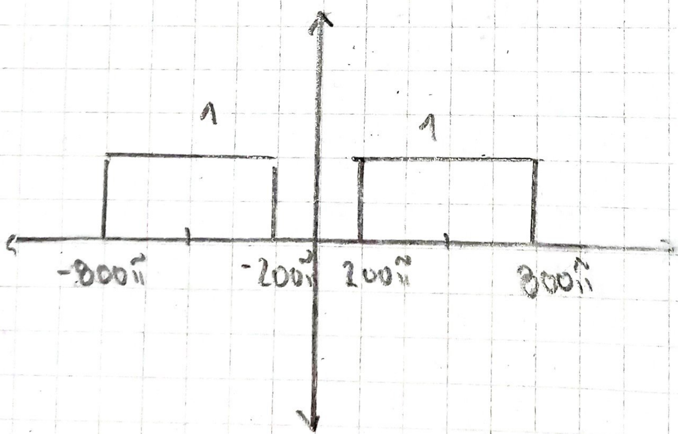
Note como el adelanto no afecta el calculo

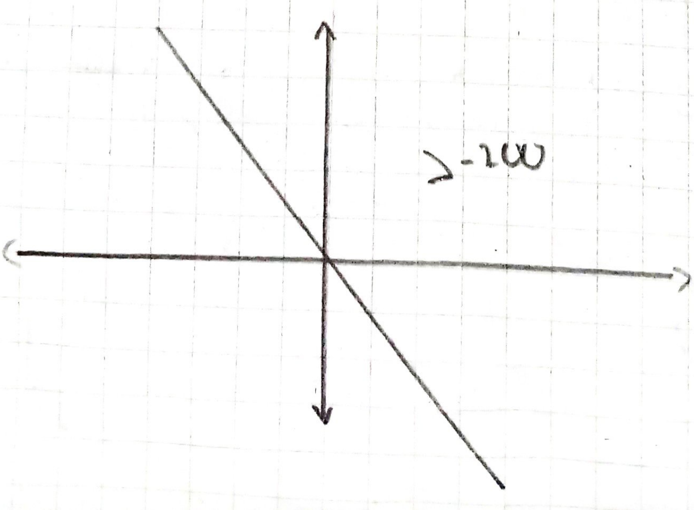


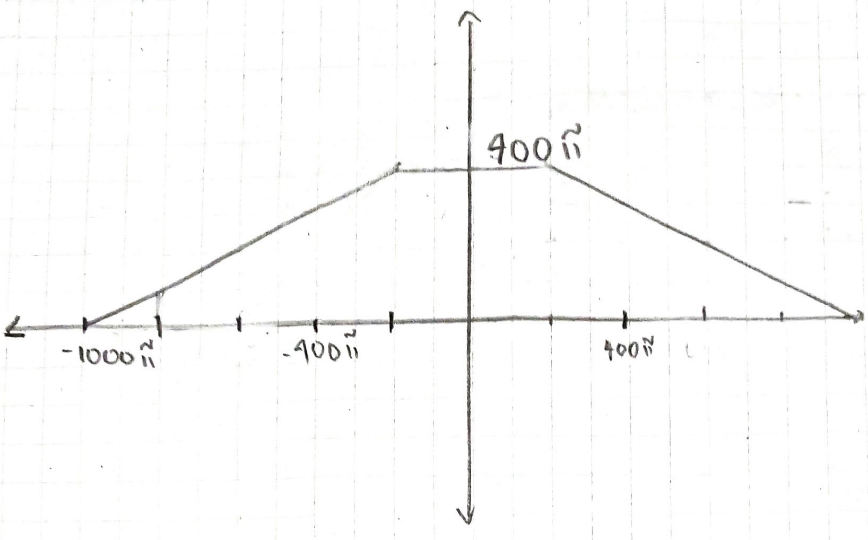
Calculamos el area

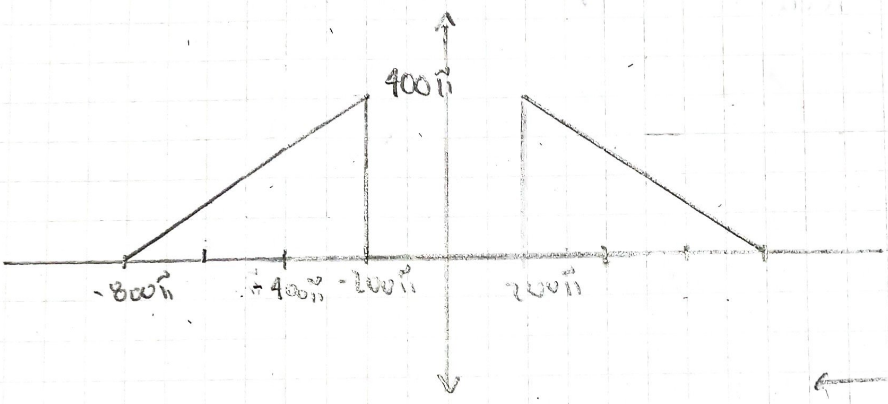
$$E = \frac{1000\pi \cdot 1 + 1000\pi + 1000\pi \cdot 4}{2\pi} = \frac{6000\cancel{\pi}}{\cancel{2\pi}} = 3000$$

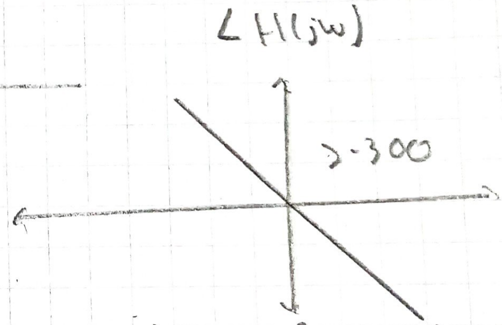
2. $h(t) = 2\cos(600\pi(t-200)) \dfrac{\sin(400\pi(t-200))}{\pi(t-200)}$



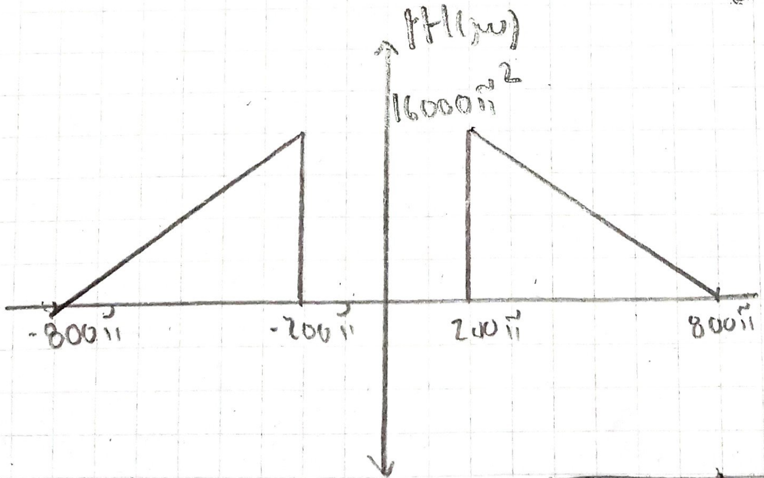








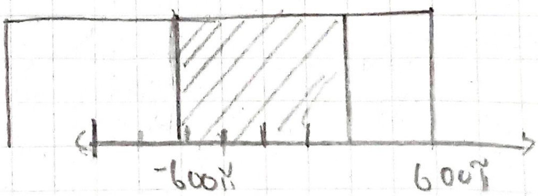
Calculamos $E=\int_{-\infty}^{\infty}|y(t)|^2$



$$E=\frac{\left(\frac{600\pi\cdot 16000\pi^2}{2}+\frac{600\pi\cdot 16000\pi^2}{2}\right)}{2\pi}$$

$$E=\left(\frac{600\pi\cdot 16000\pi^2}{4\pi}\right)\cancel{2}$$

$$E=\frac{600\pi\cdot 16000\pi^2}{2\pi}$$

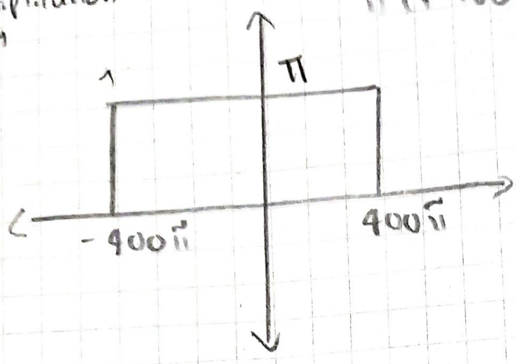


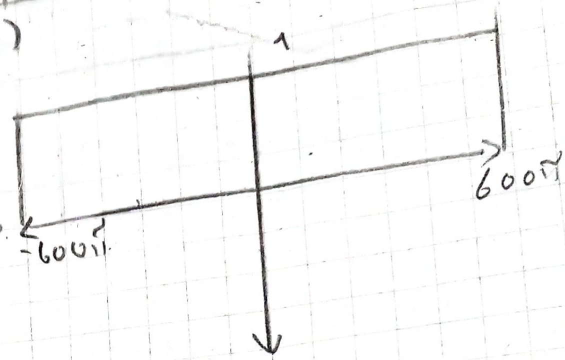
$E = 47374101{,}13$

6. $x(t) = \frac{\sin(400\pi(t-100))}{\pi(t-100)} * \frac{\sin(600\pi(t-200))}{\pi(t-200)}$

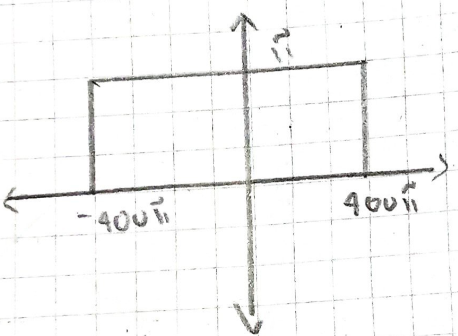
Convolución en tiempo es multiplicación en frecuencia

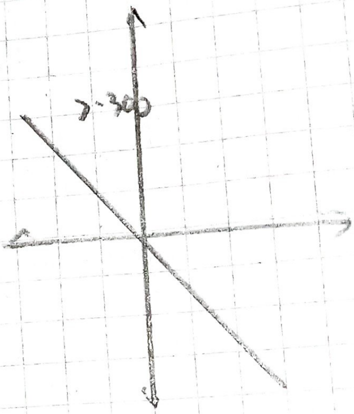
$\frac{\sin(400\pi(t-100))}{\pi(t-100)}$

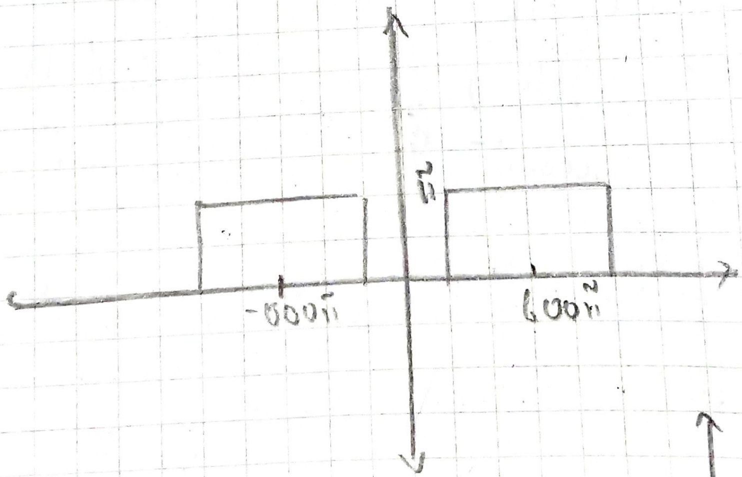


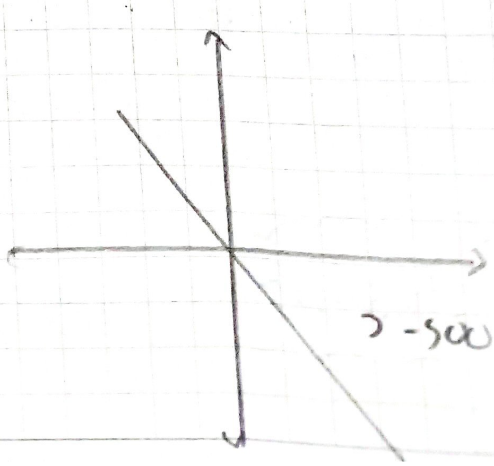


El resultado sera



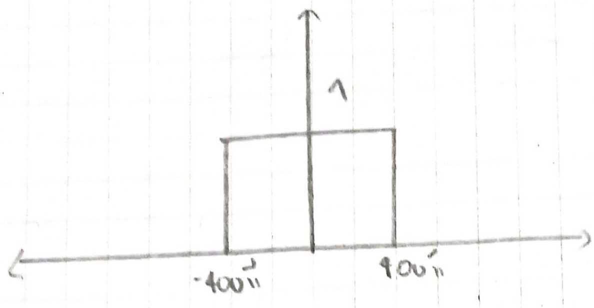


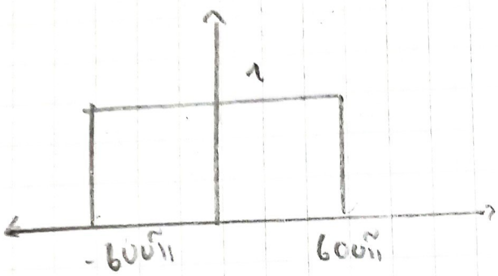


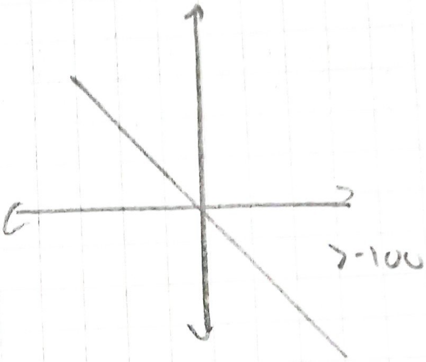


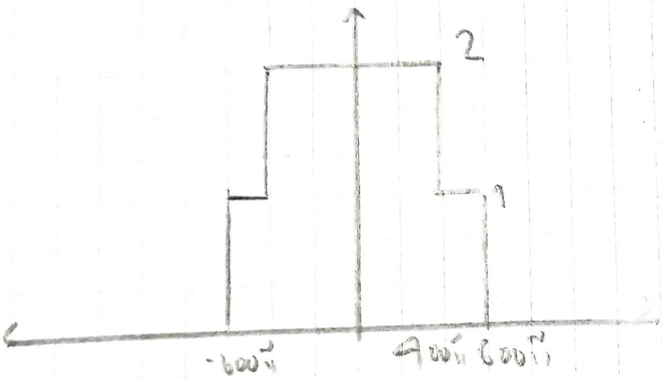
C.

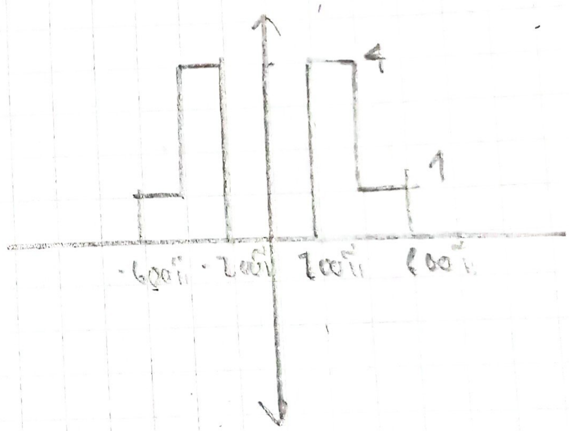
$$x(t) = \frac{\sin(400\pi(t-100))}{\pi(t-100)} + \frac{\sin(600\pi(t-100))}{\pi(t-100)}$$



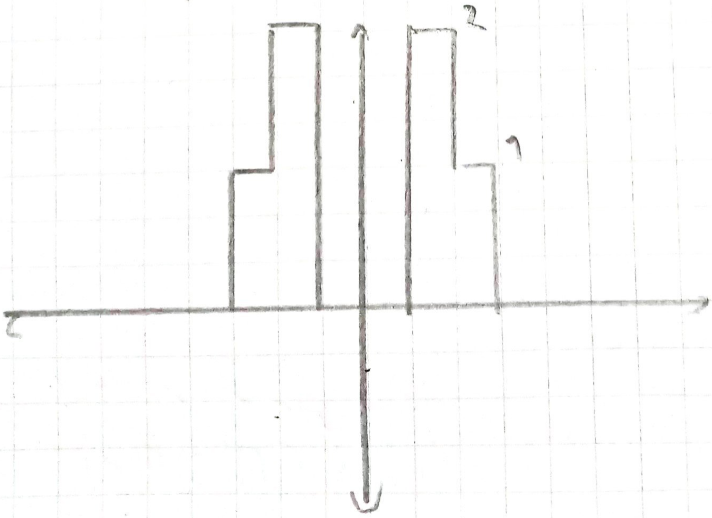








Multiplicamos por la fase resultando en



$$E = \frac{200\pi + 800\pi + 200\pi + 800\pi}{2\pi}$$

$$E = 1000$$

3.

a. $x(t) = 100\pi\delta(t-4) + 400\pi \frac{\sin(400\pi(t-4))}{\pi(t-4)} - 2\pi\left(\frac{\sin(100\pi(t-4))}{\pi(t-4)}\right)^2$

$x(t) \overset{FT}{\longleftrightarrow} X(j\omega)$

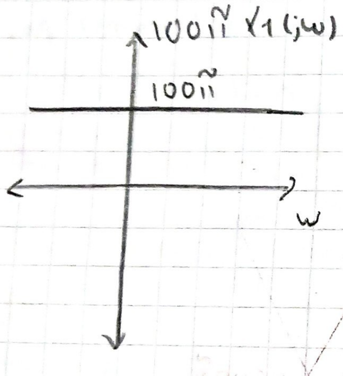
Note que el primer termino es un impulso retrasado, el segundo un cuadrado y el tercero un triangulo

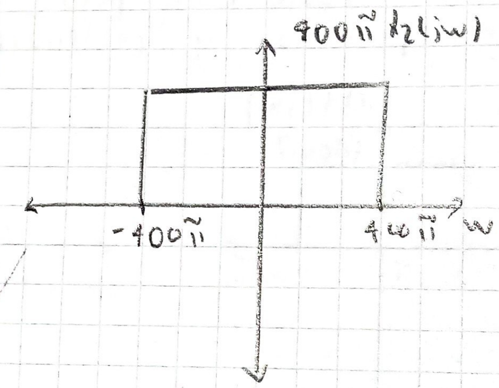
Todas las señales estan retrasadas a razón de 4 así que

$x_1(t) = \delta(t)$, $x_2(t) = \frac{\sin(400\pi t)}{\pi t}$, $x_3(t) = \left(\frac{\sin(100\pi t)}{\pi t}\right)^2$

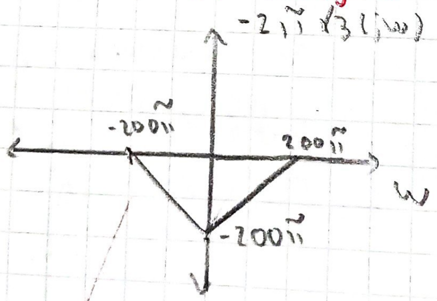
$X(j\omega) = e^{-j\omega 4}\left(100\pi \cdot X_1(j\omega) + 400\pi(j\omega) - 2\pi X_3(j\omega)\right)$

Graficamos las señales

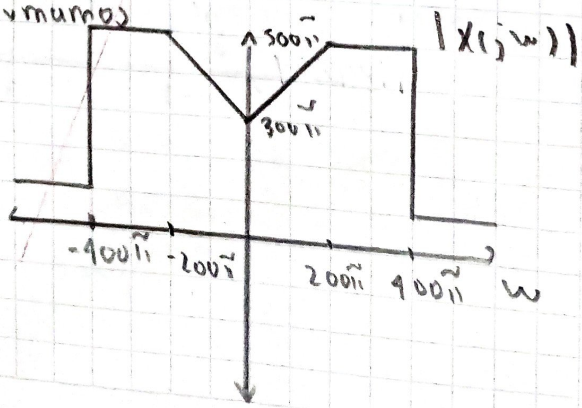




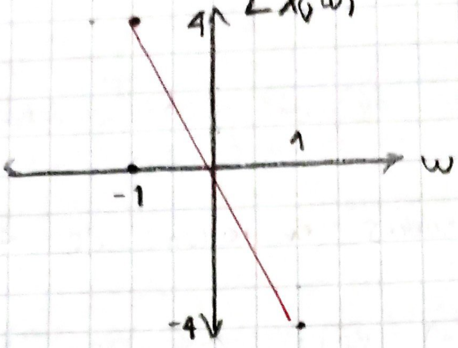
Note que el simbolo del triangulo es negativo con lo cual



Así pues, sumamos y obtenemos la señal resultante:

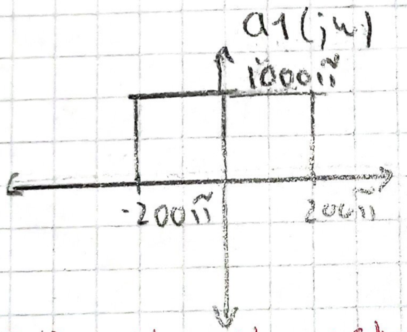


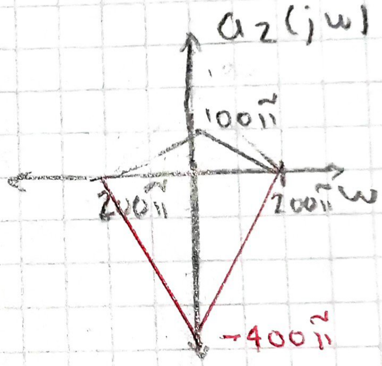
Como mencionamos antes se presenta un retraso de 4



6. $y(t) = 1000\pi \dfrac{\sin(200\pi(t-10))}{\pi(t-10)} - 4\pi \left(\dfrac{\sin(100\pi(t-10))}{\pi(t-10)} \right)^2$

La primera señal es cuadrado mientras que el segundo es un triángulo, así pues graficamos sin las constantes multiplicadas y posteriormente retrasamos por 10

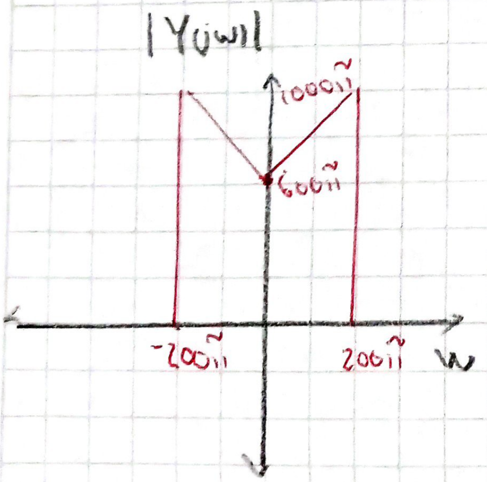


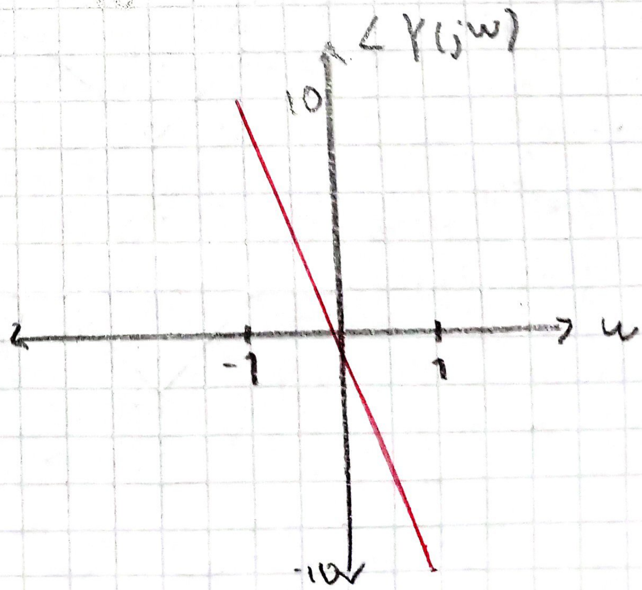


Ahora factorizamos el retraso

$Y(jw) = e^{-jw10}\left(1000\pi\, a_1(jw) - 4\pi\, a_2(jw)\right)$

Se presenta un retraso de 10





C. La salida esta definida por

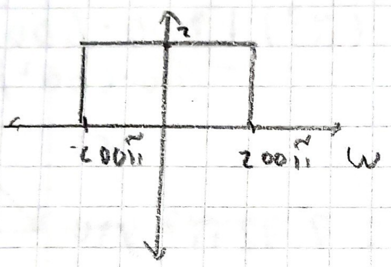
$$y(j\omega) = X(j\omega) \cdot H(j\omega)$$

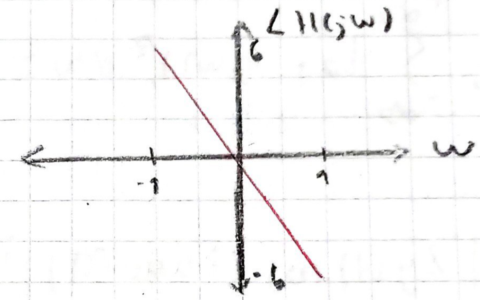
y la magnitud y fase

$$|y(j\omega)| = |X(j\omega)| \cdot |H(j\omega)|$$

$$\angle Y(j\omega) = \angle X(j\omega) \cdot \angle H(j\omega)$$

La señal se multiplica por dos y una fase o retraso de 6





Por lo anterior

$$h(t) = 2 \cdot \frac{\sin(200\pi(t-6))}{\pi(t-6)}$$

d. $\int_{-\infty}^{\infty} |2y(t)\cos(2000\pi t)|^2 dt$

Definimos la señal

$$a_1 = 2y(t) \cdot \cos(2000\pi t)$$

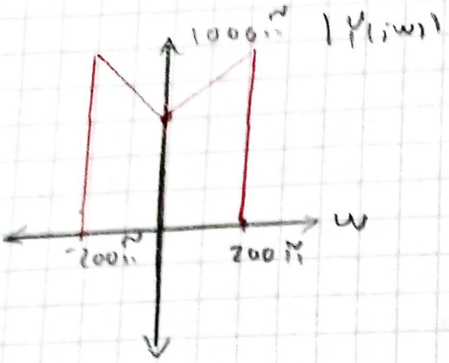
Aplicamos parseval

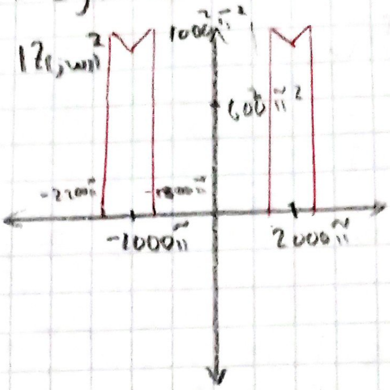
$$\int_{-\infty}^{\infty} |a_1(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |a_1(j\omega)|^2 d\omega$$

Aplicamos corrimiento en frecuencia

$$x(t) \cdot \cos(\omega_0 t) \xleftrightarrow{FT} \frac{1}{2}\left(X(j(\omega-\omega_0)) + X(j(\omega+\omega_0))\right)$$

Para lo cual $a_1 = 2y(t)\cos(700\pi t) = Y(j(w-200\pi)) + Y(j(w+200\pi))$





Así pues

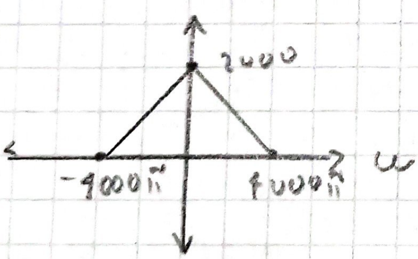
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |a_1(jw)|^2\, dw = \frac{1}{2}(\pi^2)(800\cdot 600^2 + 400\cdot 1000^2 - 400\cdot 600^2)$$

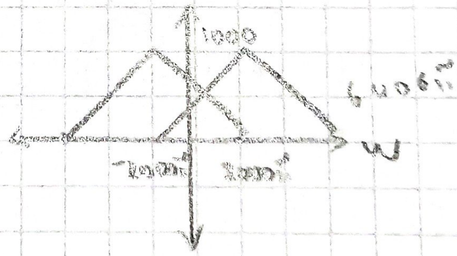
$$\int_{-\infty}^{\infty} |2y(t)\cos(2000\pi t)|^2\, dt = 2.72\pi^2 \times 10^8$$

4. a. $h(t) = h_1(t-82)$

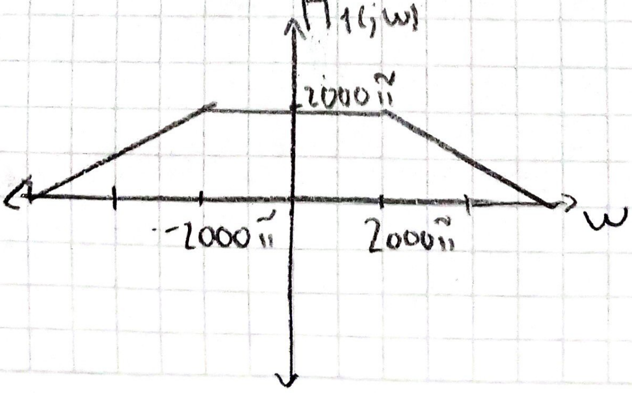
donde $h_1(t) = 2\pi\left(\frac{\sin(2000\pi t)}{\pi t}\right)^2 \cos(2000\pi t)$

Son dos triangulos por el coseno y un triangulo





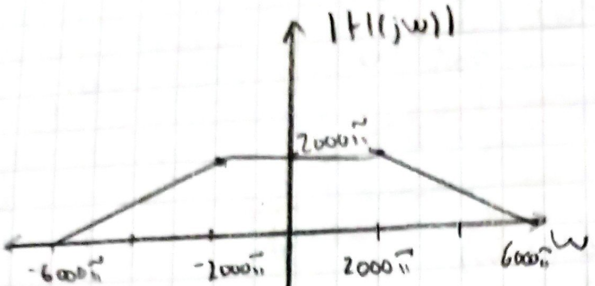
Sumamos y multiplicos por 2π

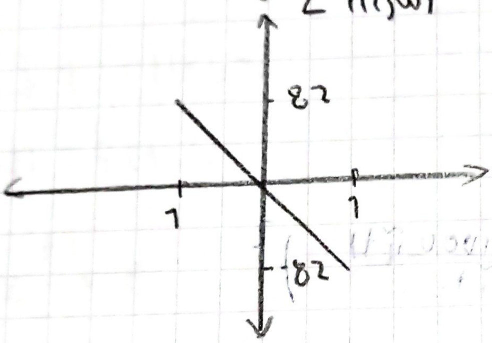


Aplicamos el retraso

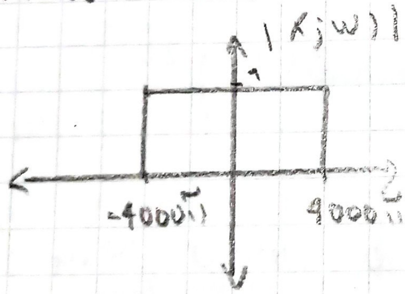
$h(t) = h_1(t-82) \xleftrightarrow{FT} e^{-j\omega(82)} \cdot H_1(j\omega) = H(j\omega)$

Realizamos la grafica $\angle H(j\omega) = -82\,\omega$





6. Si tenemos $x(t)$ este es un cuadrado

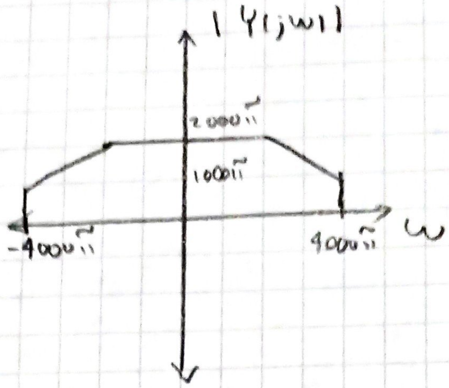


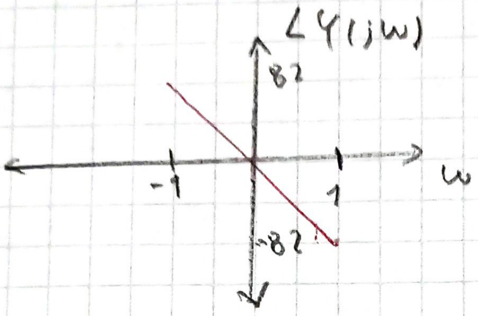
No hay fase

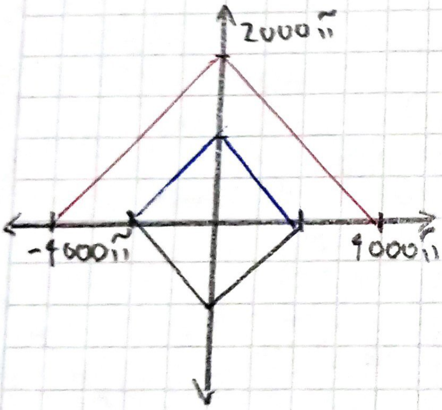
Sabemos que $|Y(j\omega)| = |X(j\omega)| \cdot |H(j\omega)|$

$\angle Y(j\omega) = \angle X(j\omega) + \angle H(j\omega) = \angle H(j\omega)$

$$|Y(j\omega)| = \begin{cases} |H(j\omega)|, & -4000\pi \le \omega \le 4000\pi \\ 0 & \text{de lo contrario} \end{cases}$$



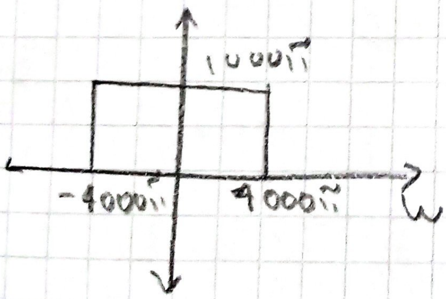




$$\pi\left(\frac{\sin(1000\pi t)}{\pi t}\right)^2$$

$$\pi\left(\frac{\sin(2000\pi t)}{\pi t}\right)^2$$

$$-\pi\left(\frac{\sin(1000\pi t)}{\pi t}\right)^2$$



Así que la salida sería

$$y(t) = \pi\left(\frac{\sin(2000\pi(t-82))}{\pi(t-82)}\right)^2 - \pi\left(\frac{\sin(1000\pi(t-82))}{\pi(t-82)}\right)^2$$

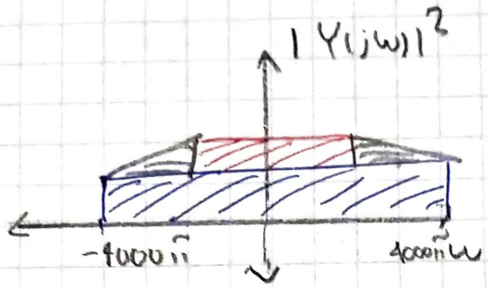
$$+1000\pi\,\frac{\sin(4000\pi(t-82))}{\pi(t-82)}$$

C. Calculamos por parseval

$$E = \int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega$$

$$E = \frac{1}{2\pi}\left(8000\pi \cdot 1000^2\pi^2 + 4000\pi(2000^2\pi^2 - 1000^2\pi^2) + (2000\pi)(2000^2\pi^2 - 1000^2\pi^2)\right)$$

$$E = 1.3\pi^2 \cdot 10^{10}$$



5. $h(t) = h_1(t-2) - \frac{1}{100} h_2(t-2)$

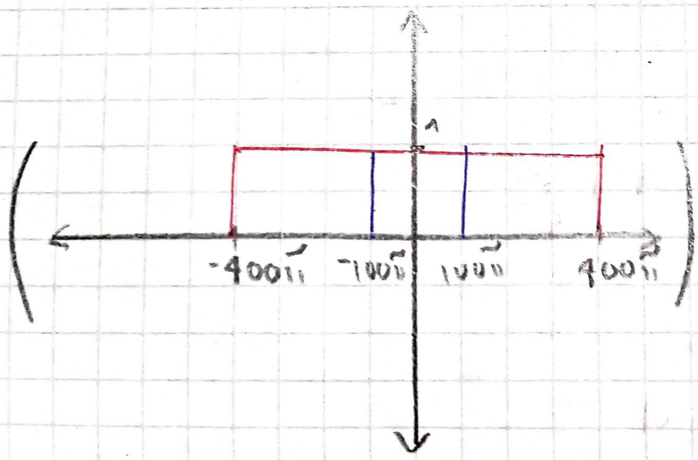
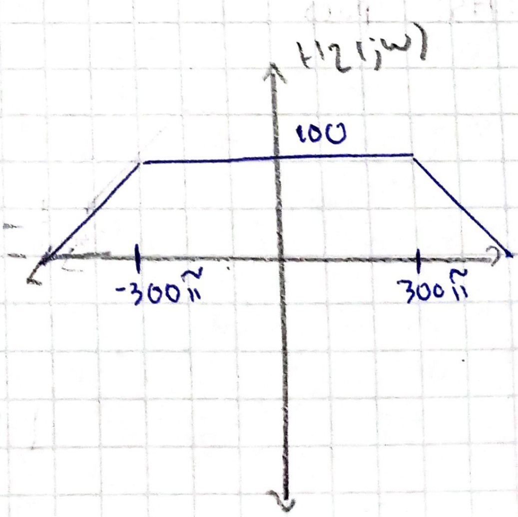
$h_1(t) = \delta(t)$

$h_1(t) \xleftrightarrow{FT} 1 = H_1(j\omega)$

y $h_1(t-2) = \delta(t-2)$

$h_1(t-2) \xleftrightarrow{FT} e^{-j\omega 2} \cdot H_1(j\omega)$ $\quad x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega)$

$h_2(t) = \frac{\sin(400\pi t)}{\pi t} \cdot \frac{\sin(100\pi t)}{\pi t}$

$\frac{1}{2\pi}$  = 

$h_2(t-2) \longleftrightarrow e^{-j\omega 2} H_2(j\omega)$

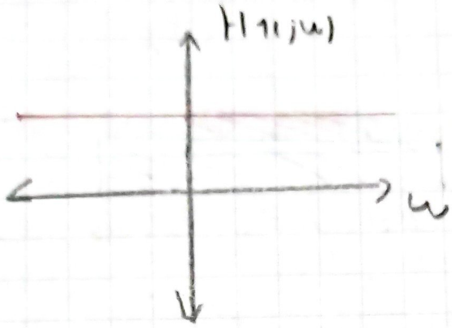
Por linealidad

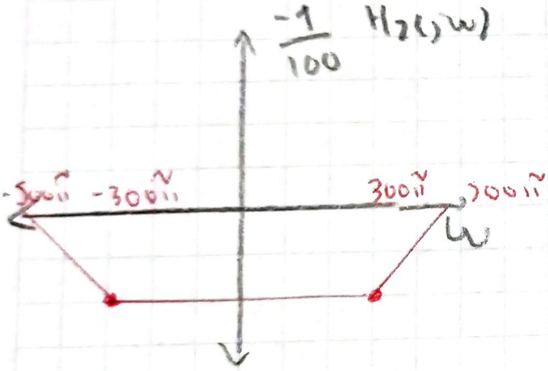
$$H(jw) = e^{-jw2}\left(H_2(jw) - \frac{H_2(jw)}{100}\right)$$

Con $H_1(jw)$ y $H_2(jw)$ son reales

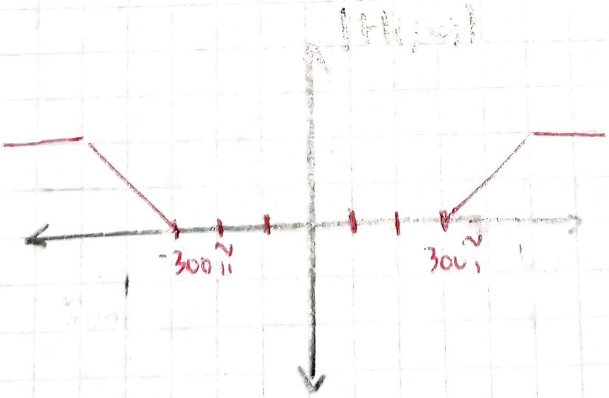
$$|H(jw)| = \left|H_1(jw) - \frac{H_2(jw)}{100}\right|$$

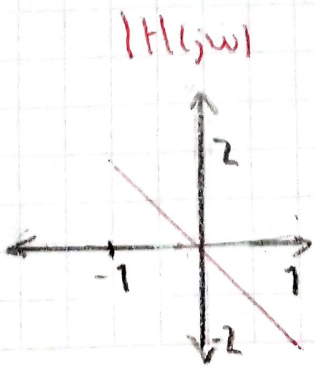
$$\angle H(jw) = -2w$$





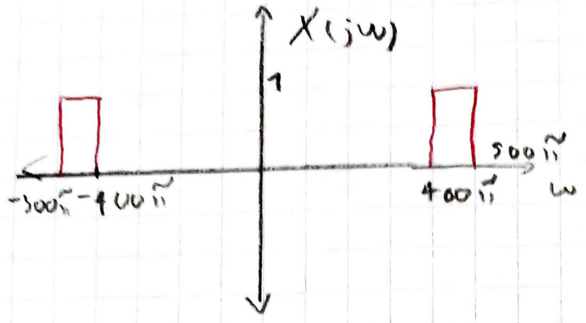
Así pues





6. $x(t) = 2\cos(450\pi t) \cdot \frac{\sin(50\pi t)}{\pi t}$

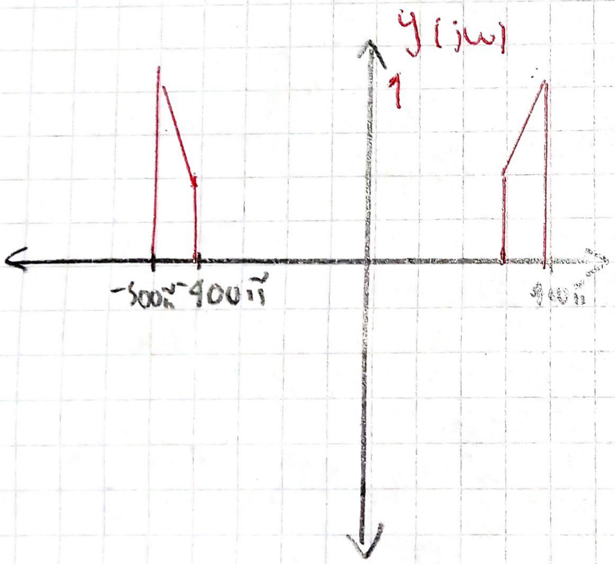
Es un cuadrado en -450π y 450π



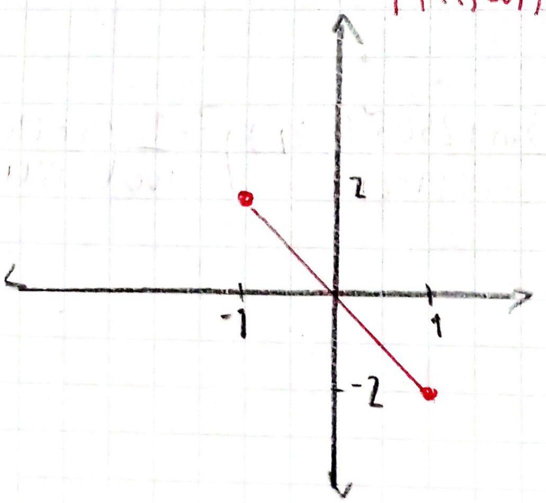
$|Y(jw)| = |X(jw)| \cdot |H(jw)|$

$\angle Y(jw) = \angle X(jw) + \angle H(jw) = -2w$

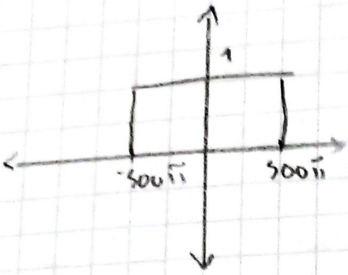
multiplicamos las dos señales



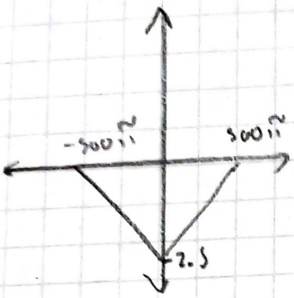
|H(jw)|



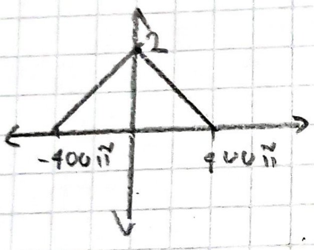
Dividimos la señal



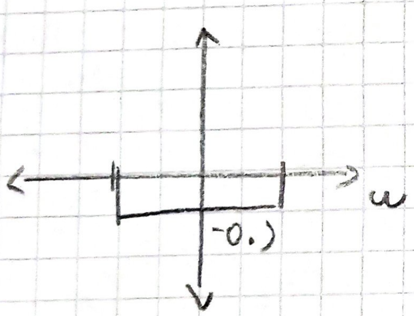
$$\frac{\sin(500\pi t)}{\pi t}$$



$$-\frac{1}{100}\left(\frac{\sin(250\pi t)}{\pi t}\right)^2$$



$$\frac{1}{100}\left(\frac{\sin(200\pi t)}{\pi t}\right)^2$$



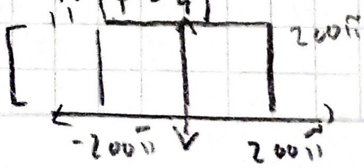
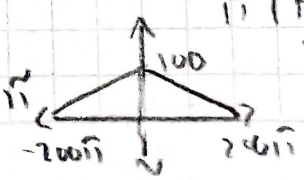
$$-0.5\,\frac{\sin(400\pi t)}{\pi t}$$

Aplicamos el corrimiento de -2

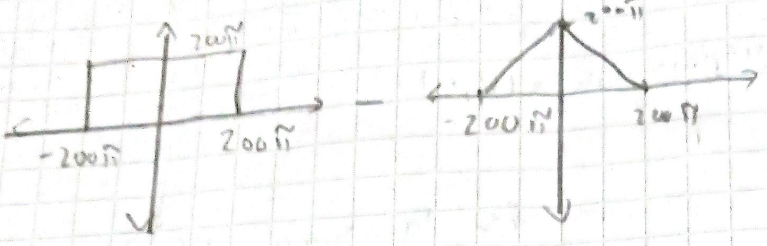
$$y(t) = \frac{\sin(500\pi(t-2))}{\pi(t-2)} - \frac{1}{100}\left(\frac{\sin(250\pi(t-2))}{\pi(t-2)}\right)^2 + \frac{1}{100}\left(\frac{\sin(200\pi(t-2))}{\pi(t-2)}\right)^2 - 0.5\left(\frac{\sin(400\pi(t-2))}{\pi(t-2)}\right)$$

6.

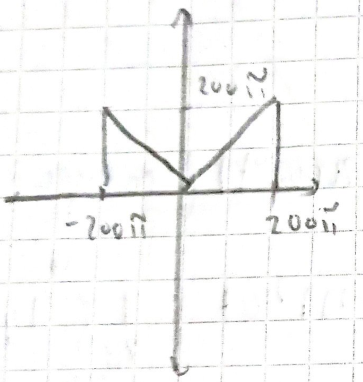
a. $x(t) = 200\pi\,\frac{\text{sen}(200\pi(t-4))}{\pi(t-4)} - 2\pi\left(\frac{\text{sen}(100\pi(t-4))}{\pi(t-4)}\right)^2$

$x(t) \xleftrightarrow{FT}$ [ -2π ] e^{-jw4}

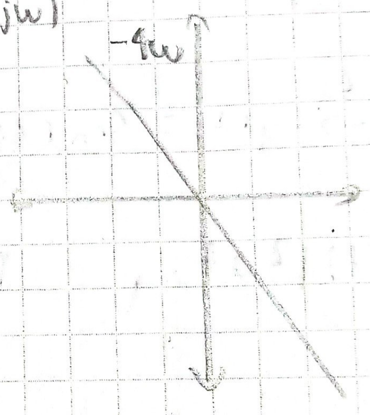
$|X(j\omega)|$



$$|X(j\omega)| = \begin{cases} 200\pi - |\omega + 200\pi| = -\omega, & \omega \in [-200\pi, 0] \\ 200\pi - (-\omega - 200\pi)) = +\omega, & \omega \in (0, 200\pi] \\ 0 & \text{de lo contrario} \end{cases}$$

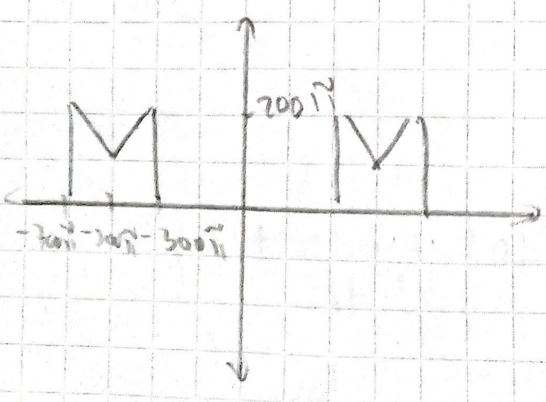


$\angle X(j\omega)$

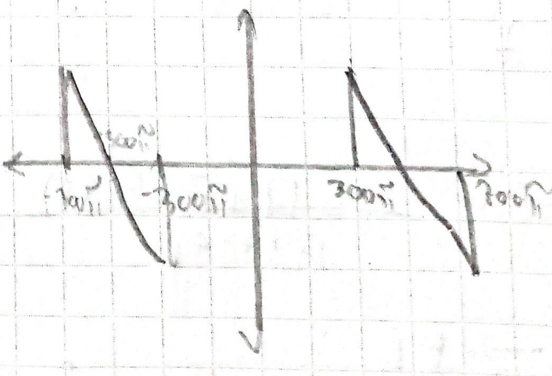


b) $z(t) = x(t) \cdot 2\cos(500\pi t)$

$z(t) \overset{FT}{\longleftrightarrow} |Z(j\omega)|$

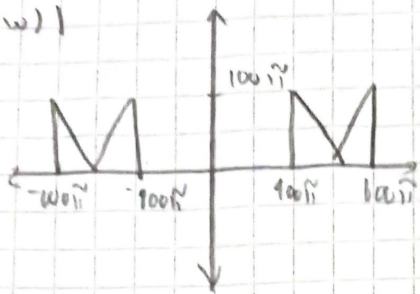


$\angle Z(j\omega)$



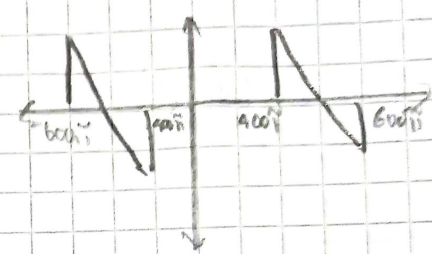
$y(t) \overset{FT}{\longleftrightarrow} Y(jw) = Z(jw) \cdot H(jw)$

$|Y(jw)|$



$\angle H(jw)$: $P_1(0,0)$, $P_2(400\pi, -12000\pi)$

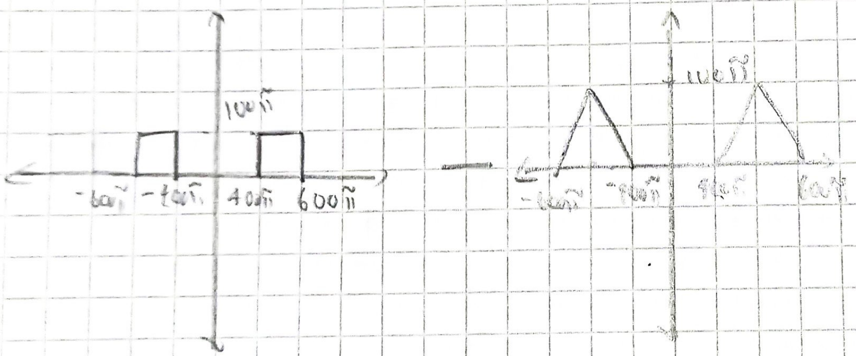
$\angle Y(jw)$



Pendiente = -30 $\quad \angle H(jw) = -30w$

$\angle Y(jw) = -34w$

d $\quad Y(jw)$



$$y(t) = 2\cos(500\pi(t-34))\left[\frac{100\pi \,\mathrm{sen}(100\pi(t-34))}{\pi(t-34)} - 2\pi\left(\frac{\mathrm{sen}(20\pi(t-34))}{\pi(t-34)}\right)^2\right]$$

e $\quad h(t) \to H(jw) \overset{FT}{\longleftrightarrow} h(t) = 2\cos(500\pi(t-30))\,\frac{\mathrm{sen}(100\pi(t-30))}{\pi(t-30)}$

7.a Separamos la señal

$h(t) = 2h_1(t-4) - h_2(t-4)$

$h_1(t) = \delta(t)$

$h_1(t) \xleftrightarrow{FT} H_1(j\omega) = 1$

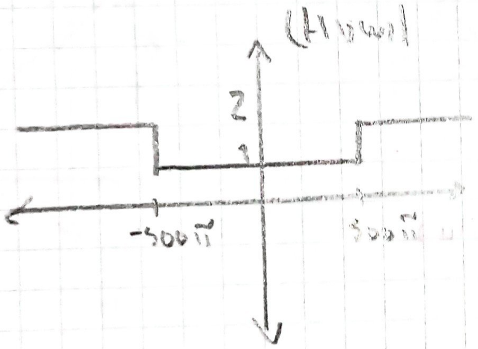
Realizamos la factorización del retraso

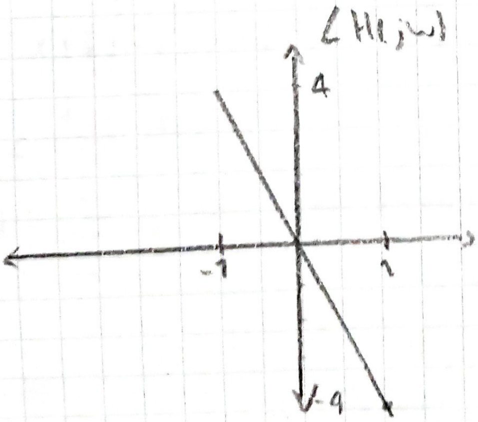
$H(j\omega) = e^{-j\omega 4}(2H_1(j\omega) - H_2(j\omega))$

$|H(j\omega)| = |2H_1(j\omega) - H_2(j\omega)|$

$\angle H(j\omega) = -4\omega$

Restamos entonces un escalón a un DC



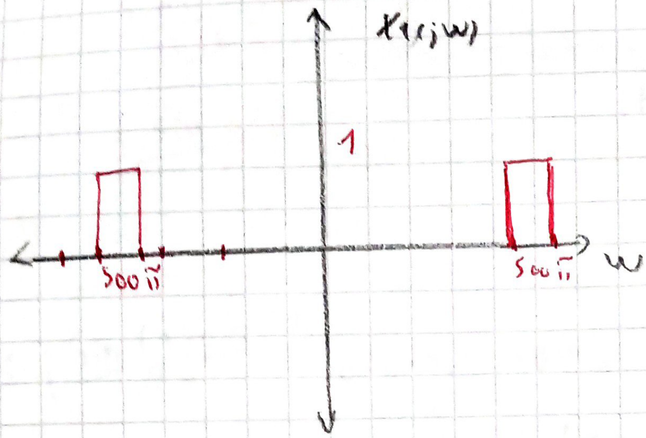


b. Ahora si la entrada

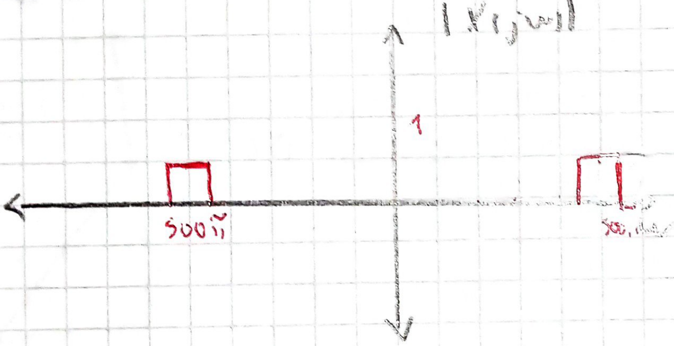
$$x(t) = 2\frac{\sin(50\pi(t-8))}{\pi(t-8)} \cos(500\pi(t-8))$$

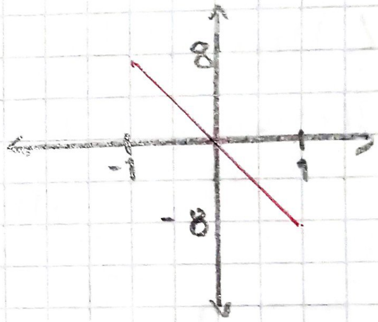
cuadrado

cuadrado en -500π y 500π

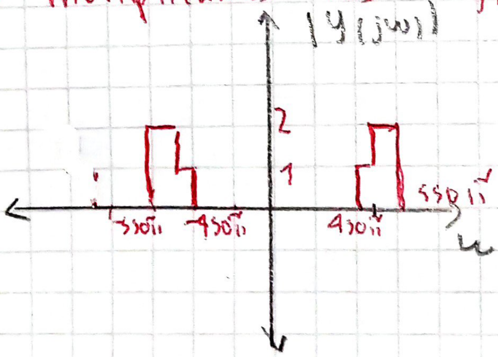


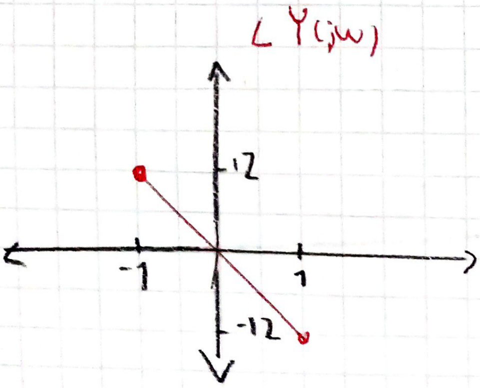
$x(t) = x_1(t-8) \xleftrightarrow{F} e^{-jw8} \cdot X_1(jw)$





Así pues multiplicamos las gráficas





Note que son dos cuadrados sumados con la cual

$$Y(j\omega) \xleftrightarrow{FT} y(t) = \frac{\sin(50\pi(t-12))}{\pi(t-12)} \cdot 2\cos(500\pi(t-12))$$

$$+ \frac{\sin(25\pi(t-12))}{\pi(t-12)} \cdot 2\cos(525\pi(t-12))$$