Angular drag on body

It seems textbooks on classical mechanics covers linear drag very well but frequently leaves out angular drag. For example linear air drag is given by,

$$|F_d| = \frac{1}{2} C_{\text{lin}} V^2 \rho A$$

where ρ is air density, A is surface area normal to the velocity V and C_{lin} is linear air drag coefficient which usually ranges for 0.5 to 1.5 for many real world objects in SI units.

I'm looking for similar equation for angular drag. So far, I can only find a little hint in the book Physics for Game Developers which has physics simulation code which implies that the angular drag equation *might be* similar to linear drag equation, i.e.,

$$|T_d| = \frac{1}{2} C_{\text{ang}} \omega^2 \rho A$$

where ω is angular velocity and A is total area that comes across the rotation of body.

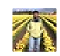
My questions are,

1. Is above equation a good approximation for angular drag in air?
2. What are the typical values for C_{ang} for air? I understand this really depends on object but I just wanted to get some intuition. From the above referenced book it seems that C_{ang} is approximately 10x smaller than C_{lin} for a sphere of same material.
3. If object is rectangular box, is there any better approximation?

fluid-dynamics friction torque drag angular-velocity

edited Jan 14 '17 at 7:50

asked Jan 13 '17 at 0:23



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2 Answers

If you're casting about for a relationship like this, start with dimensional analysis. For example, in

$$|F_d| = \frac{1}{2} C_{\text{lin}} v^2 \rho A$$

we have on the right-hand side units $[v^2] = \text{m}^2\,\text{s}^{-2}$, $[\rho] = \text{kg}\,\text{m}^{-3}$, $[A] = \text{m}^2$. In order to multiply these to make a force, with units $[F] = \text{kg m s}^{-2}$, you need ... nothing. So C_{lin} is dimensionless. (The half is useful as an integration constant for reasons which I can't recall right now.)

Notice that my linear-drag formula is different from the one in your question (v1), by one power of the area --- an error on your end, which I noticed because I was doing dimensional analysis.

The *reason* that this sort of dimensional analysis gets used is because it can be used immediately, knowing nothing about the details of the system. Later on you can build a better model and perhaps come up with a realistic dimensionless constant C for some special cases --- but that sort of algebra typically gives you factors like "two" or "one-half", so guessing $C \approx 1$ is usually not a bad place to start.

Your substitution $v \to \omega$ to find a relationship for torque gives you the wrong units: torque has the same dimension as force *times* distance. I'd expect something like

$$|\tau| \propto R v^2 \rho A = R^3 \omega^2 \rho A$$

where R is the "effective radius" of the dragging part of your rotor that gives it the right speed. If all the drag is caused by some paddle that's very far from the axis of rotation, this expression just turns the ordinary drag force into a torque. However if your drag force is caused by an "arm" with a large radial extent, like a fan blade, then you'll wind up doing the same sort of integral over the radius as a person who's computing moments of inertia. You'll end up with some constant of order one relating the "effective radius" of the fan blade to the actual radius, and some other dimensionless constant $C \sim 1$ related to airflow, as in the linear-drag case.

If your rotating shape doesn't have paddles or protrusions --- a spinning ball, say --- then $C_{\text{rot}} \ll 1$ makes sense:

Thanks for pointing out the error. Your explanation makes lot of sense and I think we can make it more general. At any point on surface we have velocity $v = r \times \omega$. Each point on surface should face same linear drag as if it was having velocity v. So angular drag can be given as $\rho \int_s (r \times \omega)^2 ds$. This gets dimensions right as well ($((\frac{m}{s})^2 m^3 \frac{kg}{m^3})$). We can use same linear drag constant but it would be divided by much larger constant. If you like to expand on this, please free. – ShitalShah Jan 13 '17 at 8:18

The answer turned out to be much more complex than I thought. Here is the dump of my day worth of research on this subject. Below is, of course, simplified view and doesn't consider many other effects in fluid dynamics such as turbulent flows, non-Newtonian fluids etc.

Two types of angular drag

The damping of angular motion in fluid happens due to two main types of drag effects: (1) tangential velocity and (2) normal velocity to the surface. The former is known as shear stress however there doesn't seem to be agreed upon name for the later. To keep in line with standard textbook "Introduction to Fluid Mechanics" p.160, sec 9.3.5, we will refer to it as *friction torque*.

To get an intuitive idea, imagine a cylinder placed at origin with its height aligned with z-axis and circular face on x-y plane. If you rotate this cylinder around x or y axis with angular velocity ω then fluid hitting the shaft would be normal to its surface. If you were to rotate this cylinder around z axis with angular velocity w then there shaft surface is experiencing only tangential velocity.

Linear drag

First lets revisit linear drag because we will use it to compute angular friction drag later. Linear drag force is given by,

$$F_{lin} = \frac{1}{2}\rho v^2 . C_l in. A$$

A fun fact is that majority of textbooks simply states this equation as "fact" never bothering to examine it closely. I wanted to understand how this equation came to existence and so far the intuitive explanation is that force exerted on body moving with relative velocity v can be given in terms on pressure on its exposed area:

$$F_{lin} \propto PA$$

Now the pressure exerted can be given as kinetic energy of the fluid,

$$P = \frac{1}{2}\rho v^2$$

Above is known as dynamic pressure. So if you were wondering how did that $\frac{1}{2}$ slipped in to equation, it traces back to kinetic energy definition. You can also find entertaining introduction to this equation in essay "The velocity dependence of Aerodynamic Drag" by Long and Weiss.

Friction Torque

Its fairly easy to compute friction torque. Each tiny surface area ds of body located at vector r experiences the linear velocity $v = r \times \omega$ where w is angular velocity. So you just take that linear velocity, plug in to equation for linear drag to get the force dF exerted on that tiny area ds. Then you get torque $dT = dF \times r$. Finally integrate over entire exposed surface and you get total friction torque.

Here's the worked example for cylinder (also see: Introduction to Fluid Mechanics, sec 9.3.5, p. 160)

First compute the frictional torque for the circular top and bottom discs with radius r. Note that $r \times \omega$ is simply rw because fluid is hitting right on surface normal.

$$dF_{disc} = dP_{disc}. C_{lin}. dA_{disc} dr$$

$$dF_{disc} = \frac{1}{2}\rho (rw)^2 . C_{lin}. 2\pi r. dr$$

$$T_{disc} = \int_{r=0}^{r=r_0} r. dF_{disc} = \frac{1}{5}\pi C_{lin} \rho \omega^2 r_0^5$$

Now lets find the frictional torque experienced by shaft part of the cylinder with height h. This time around there is no need to do integration because r is fixed over the surface of the shaft.

$$F_{shaft} = P. A. C_{lin}$$

$$F_{shaft} = \frac{1}{2}\rho (r_0\omega)^2 . 2\pi r_0 h. C_{lin}$$

$$T_{shaft} = \pi. \omega^2 r_0^4 h \rho C_{lin}$$

Now lets move on to this other variety. This one is particularly difficult to compute because velocity from body is transferred to surrounding fluid via tangential component at a point on body. So there is this "boundary layer" formed surrounding body which is rotating at same speed as body at the touch point but then its velocity reduces linearly as go you go further out from the body. The big issue is that equations assume limited width of boundary layer and that width appears in denominator of force. That means if you don't get width right, force value can technically become very error prone and even arbitrary! I would suggest to check Wikipedia or here to view the related equations. In the book "Viscous Fluid Flow", 2nd ed, by White, p112 there is a case for cylinder that has boundary width of infinite size (which also requires cylinder to be of infinite height) that would yield the drag force as,

$$F_{shear} = 2\pi\mu\omega rh$$

where μ is dynamic viscosity coefficient which for air at room temperature is 1.8×10^{-5}.

(see derivation here)

End notes

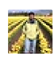
I'm looking to use all these in practice but the calculations for arbitrary real-world shapes might certainly get hard. Also notice that we don't take in to account all the effects that different shapes likes concave or wings etc can produce. From looking at purely applied point of view, it seems that drag due to shear would be very small because of not only very small constant μ for air but also the fact it depends only on first power of ω and second power of r. So I might ignore it all together for practical purposes as I don't have any shapes that will amplify its effect. For frictional drag, one approximation might be to compute area "component" in each axis to form "area vector" and then do coefficient wise multiplication with velocity vector (in body frame) twice. Here velocity vector may simply be approximated by multiplying angular velocity with average encompassing radius of body in axis's direction.

Another quick note: So how does physics simulation engines handle angular drag computation? So far I have see that they take easy way out: Just define some constant like 0.01 and reduce angular velocity by that amount each time step. This eliminates surprises like forever rotating body but this just simply isn't right, obviously.

Word of caution: I'm not expert on fluid dynamics so please do not view above as an official statement from an expert and please feel free to comment if you see any mistakes! It would be super awesome if some expert in the field can validate this answer, of course!

edited Jan 14 '17 at 8:40

answered Jan 14 '17 at 7:33



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