

Waypoint Planning for Autonomous Underwater Vehicles with Terrain Relative Navigation

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Abstract— Planning routes, defined by a series of commanded waypoints between a start and goal location, for return-to-site missions using Autonomous Underwater Vehicles with Terrain Relative Navigation is particularly challenging due to the dependence of the navigation estimate on the path flown. Waypoints must be selected to ensure the vehicle has an accurate TRN navigation estimate to arrive at the intended target site with a high probability. This probability can be estimated using Monte Carlo simulation and used to optimize the vehicle route. An Upper Confidence Bound can also be used in the optimization to decrease the number of simulations when compared to a brute force search. For the example return-to-site mission at Portuguese Ledge, the optimization significantly increases the expected probability of ending at the target site when compared the straight line route.

Index Terms—TRN, AUV, Planning, Waypoints, Monte Carlo, UCB

I. INTRODUCTION

Presented is a new method for defining an optimal route for an autonomous underwater vehicle (AUV) for a return-to-site mission using terrain-relative navigation (TRN) as its navigation solution. The objective of the mission is to arrive at a goal location that is specified on a map, for example to perform repeated scientific observations of that site. The challenge is that using TRN causes the accuracy of the AUV's navigation solution to be dependent on the route flown. The method presented here generates a commanded route, defined by a set of intermediate waypoints, as the solution to an optimization problem that maximizes the probability that the AUV will reach the goal location.

TRN is the enabling technology for these return-to-site missions due to its ability to provide a position estimate that is map relative rather than one defined in inertial coordinates (e.g. latitude-longitude). This is critical since, while the location of a goal site might be well known on a map, the map used may have significant geo-referencing errors and/or the vehicle inertial estimate may have accumulated drift. TRN requires only a map of the terrain and on-board sensors such as a DVL or INS, plus an altimeter or other range measuring device (e.g. sonar). It can provide a drift-free, map relative position that is accurate on the order of the map resolution.

Recently TRN's utility for a return-to-site mission was demonstrated by the Monterey Bay Aquarium Research Insti-



Fig. 1: Dorado Class AUV

tute (MBARI) at the Portuguese Ledge site in Monterey Bay using a Dorado Class AUV, which is pictured in Figure 1. The results of the demonstration are shown in Figure 2. After the TRN estimate converges, the vehicle is able to follow the planned route and fly directly over the target site. Full results from this demonstration are presented in [1]. Without the TRN estimate, the vehicle would have missed the target site by 10s of meters.

Figure 3 shows another run at Portuguese Ledge. In this run, the AUV did not succeed in flying over its goal site. The difference between these runs is that in the first run, the AUV flew over information rich terrain which enabled TRN to provide an accurate estimate of the AUV's position. In the second run, the AUV flew primarily over flat, information poor terrain, and hence did not succeed in following the commanded route. These results demonstrate the interaction of the route flown and the probability that the AUV will reach its goal state.

One way to define the route followed by an AUV is to choose a set of intermediate waypoints for it to follow. To ensure a high probability of mission success, a TRN return-to-site mission requires a planned set of waypoints that trades traditional performance metrics (i.e. distance traveled) against the expected accuracy of the navigation solution. The offline

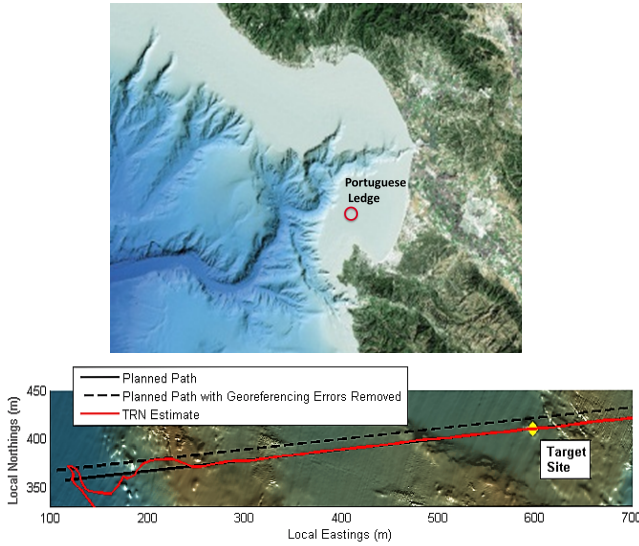


Fig. 2: TRN Return-to-Site Demo at Portuguese Ledge

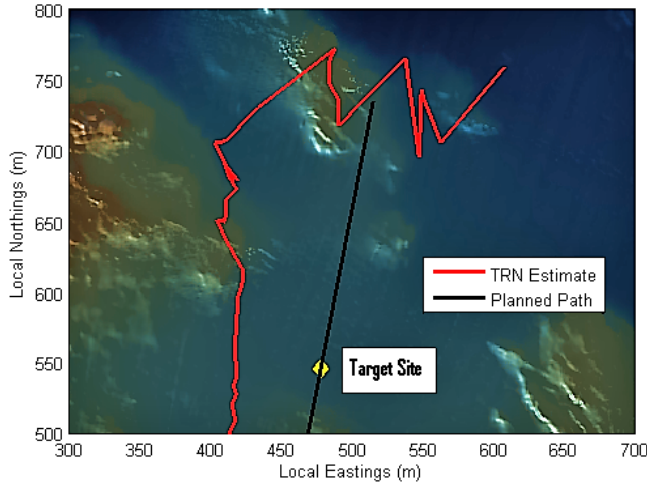


Fig. 3: Unsuccessful TRN Return-to-Site Run at Portuguese Ledge

technique presented in this paper is designed to optimize the probability of reaching the intended goal site.

II. BACKGROUND

Picking waypoints when using TRN-based navigation is more challenging than basic planning problems because the performance metric is both probabilistic and stochastic. Graph based searches usually rely on a metric or heuristic that satisfies the triangle inequality (i.e. distance traveled) or one that can be back propagated, both of which are not the case for the chosen probabilistic performance metric. Also, without a probability distribution assumption (i.e. Gaussian), iterative approaches are computationally intractable for this problem because these methods rely on repeated calculation of their metric. Finally, TRN's use of a stochastic position estimation

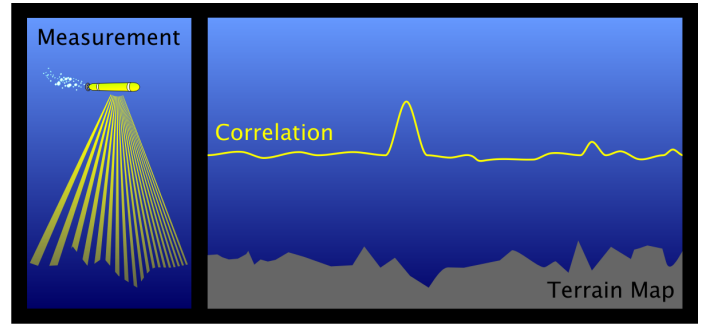


Fig. 4: TRN Sonar Correlation from a Single Measurement

filter makes prediction of the probabilistic performance metric even more difficult to calculate in closed-form. The stochastic TRN filter requires the method is able to estimate the probabilistic performance metric used in the optimization.

A. Terrain Relative Navigation

TRN estimates a map relative position using a Bayesian filter. Sonar range measurements are correlated to a known map to predict the vehicle's position. At each time step, the prediction is updated with the motion from the INS and then refined by comparing the actual measurement to predicted range measurements. Due to the initial inertial drift of the vehicle and the potential geo-referencing errors in the map, the filter is initialized with a large initial (uniform) uncertainty. The predicted measurements encompassed in this area of uncertainty are highly nonlinear and linearization assumptions of traditional filter approaches (i.e. Kalman Filter) are invalid. Instead TRN is accomplished using a particle filter, which is a stochastic, non-parametric filter. The vehicle position estimate is represented as a set of particles randomly spread on an area the map. The particle filter is appropriate for TRN because it is able to handle the non-linear observation model, often leading to multi-modal distributions. Figure 4 shows one correlation of a single measurement to the map. The multi-modal TRN distribution resulting from this correlation evolves through time as the vehicle travels. The distribution relies on both the full topography of terrain encompassed by the particle spread and also the actual sonar measurements. The TRN estimate is only used in the closed loop control if the covariance of the weighted particle position estimates are below a gateway threshold. More information about particle filters in general can be found in [2] and its application to TRN in [3]. Since the TRN estimate is path dependent, enough particles are spread within the region of uncertainty to ensure there are particles estimates in the vicinity of the actual vehicle position. Since particle sampling and motion are stochastic, the TRN filter performance can be difficult to predict from a single evaluation of a route.

III. METHOD

The proposed method for choosing waypoints maximizes the probability that the AUV arrives at its intended goal. Specifically, starting from a stochastic initial region, the

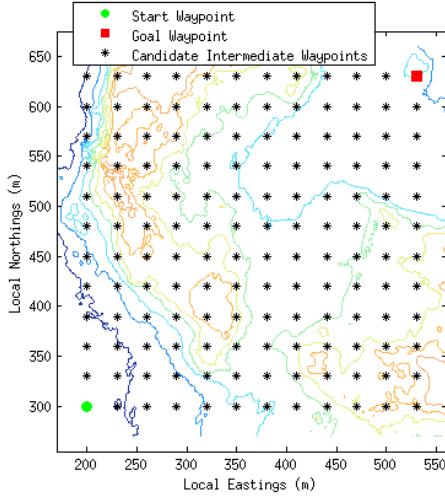


Fig. 5: Mission Waypoints

method maximizes the probability that the AUV's final position is within a specified radius of the goal site (the value function) by changing the commanded route of the vehicle. This route is defined by a fixed start waypoint, a fixed goal waypoint and one or more intermediate waypoint positions (free variables), all of which are defined on a given map. An example is shown in Figure 5. The start waypoint and goal waypoint are fixed locations on the map, and free variable choices are shown as the intermediate waypoints.

The stochastic nature of the TRN navigation filter and the analytic intractability of this performance metric function motivates the use of a Monte Carlo (MC) sampling method to approximate the optimization's value function. MC methods estimate a function by averaging a set of samples. As the number of samples increases, the mean value approaches the true value. The TRN route planning method is based on the Monte Carlo Tree Search (MCTS). MCTS is an algorithm that has been shown to be effective on solving decision making and game playing problems, even when other solution methods are not [4]. Two major benefits over other MC methods from MCTS that are used in the proposed method are: use of an Upper Confidence Bound (UCB) and organization into a tree structure. Supplementing the value function with an UCB reduces the total number of simulations required for the optimization. The tree structure increases search efficiency by breaking down a large decision into a series of smaller decisions.

The method can be broken down into two major phases as shown in Figure 6. Using information about the map, vehicle, and mission, potential waypoint locations are sampled and formed into a tree. Next, this tree is used in an MC optimization that outputs an optimal route and its corresponding success probability.

A. Waypoint Sampling and Tree Formation

Prior to running the optimization, a finite set of waypoints is sampled from the map and formed into a search tree as

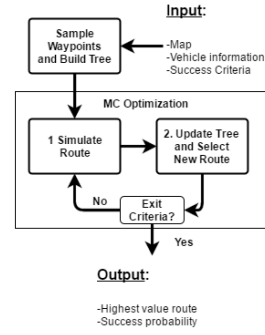


Fig. 6: TRN Route Planning Method

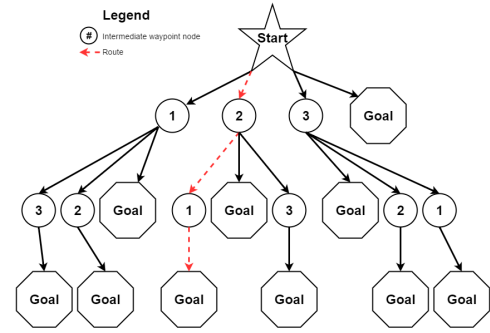


Fig. 7: Example Tree with 3 possible waypoint locations and 4 waypoint max route length

show in Figure 7. Using a sampling approach to build the waypoint tree allows for planning on larger sized maps without increasing the total number of routes. Each node on the tree corresponds to a north/east waypoint position on the map. This position is not unique to all other nodes in the tree, but the position is unique to all other nodes with the same parent node. In the example tree, all the #1 nodes correspond to the same north/east coordinate in the map, but they arrive at this waypoint via different routes. The tree's root node is associated with start waypoint position and every leaf node (and only the leaf nodes) are assigned the goal waypoint position. The number of waypoints in a route is determined by the depth of that leaf node. The total number of routes being tested in the optimization is equal to the total number of leaf nodes and is the largest contributor to the length of time required for the optimization.

B. MC Optimization

Using the tree, the MC optimization algorithm iterates between two steps until it reaches an exit criteria. The first step is to sample the value function for a specific route. Since sampling cannot be done directly, it is accomplished using a MC simulation of the vehicle using the route. The simulation is assigned a value of +1 if the vehicle was within a certain radius of the goal and 0 otherwise. Next, the tree is updated using this value to decide the next route to simulate. The tree is updated backwards, starting with the leaf node. The value of a leaf node is equal to the mean estimate of all prior runs

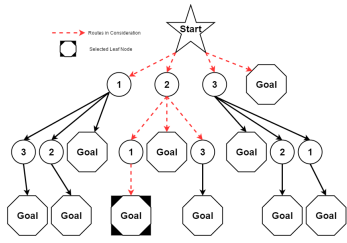


Fig. 8: Route Selection as a Series of Smaller Decisions

plus UCB. The value of a non-leaf node is updated to be the maximum value of all its children nodes. The subsequent route to simulate is chosen by traversing down the tree via the maximum valued child node until a leaf node is reached. The nodes effected during the route selection is illustrated in Figure 8. These two MC optimization steps continues until it reaches one of the terminal conditions.

1) *Upper Confidence Bound*: The use of an UCB allows for a higher proportion of simulations to be run on routes with a higher estimate. An equally valid approach would be to try routes an equal number of times, but the amount of simulations required to ensure a statistically significant result would be significantly higher. An alternative approach would be to simulate the route with the highest estimated value (greedy approach); however, this approach may miss a high value route due to a "unlucky" initial sample.

The use of an UCB in the value function mitigates this issue by balancing exploitation and exploration. The value function, $V(r)$, supplemented with UCB used in the optimization is shown in Eqn 1.

$$\bar{V}(r) = \frac{1}{n_j} \sum_{i=1}^{n_j} x_i(r) + c \sqrt{\frac{2 \ln(N)}{n_j(r)}} \quad (1)$$

The first term is an estimation of the success probability value for a specific route, r , that is being estimated through repeated MC sampling. It is the average value of results from specific simulations, $x_i(r)$, over all $n_j(r)$ simulations of that route. Inclusion of this term biases the route selection towards higher value routes, similar to the greedy approach. The second UCB term promotes exploration of routes that have been simulated a low number of times, n_j , when compared to the total number of simulations performed, N . The exploration constant, c , is the weighting parameter between those two objectives. As both N and n_j increase, the second term approaches 0. The goal of the optimization is to find the route that maximizes \bar{V} as the number of simulations increases.

2) *Terminal Conditions*: The optimization algorithm can be run for a fixed length of time, fixed number of simulations or until a "suitable" route is found. While the first two terminal conditions allow for temporal constraints on how long the algorithm takes, the third terminal condition can be set so that the algorithm terminates when a route is found to meet the mission's needs. This can be based on a minimum threshold for a successful mission probability and a high amount of certainty of this value. One downside of this terminal condition

is that there is no guarantee of such a route existing and if it does exist, the number of simulations needed to find it is unknown. Also, if the threshold condition is too low, the algorithm may terminate earlier than needed returning a less than optimal route. When any of the terminal conditions are reached, the algorithm returns the route with the highest estimated probability of success along with the measure of certainty of the estimate.

IV. RESULTS

A. Setup-up

The results presented are for an example return-to-site mission at Portuguese Ledge in Monterey Bay. The route is intended to be flown at a constant depth between the start and the goal waypoints locations shown in Figure 5. The vehicle location is initialized uniformly random within a 100m square box around the start location. The chosen optimization performance metric is the probability of ending within 2m horizontal distance of the goal waypoint. The set of waypoints used in the optimization are sampled in a uniform grid as shown in Figure 5. The optimization uses simplified dynamics, control, and measurement models as inputs to a TRN particle filter. The optimization termination criterion is a fixed number of total simulations.

B. Discussion of Single Waypoint Results

The progression of the optimization for a single intermediate waypoint route displayed in Figures 9, 10, and 11 demonstrates the benefits of using an UCB. The 7 waypoints with the highest estimated value are displayed on the map. After 1000 simulations, the percentage of simulations is spread fairly evenly between all 143 possible intermediate waypoint routes due to a high uncertainty in the value for each. After 4000 simulations, the optimization has started to narrow the search down to one of two areas. Although they account for 5% of the 143 potential routes in this example, the 7 highest value routes were simulated 10% of the time at this point in the optimization. At the conclusion of 16000 simulations, the optimization converged the search to one of the waypoints in the north west corner of the search area. These 7 top value runs accounted for 15% of the simulations. Also, for comparison, over 4x's the number of simulations would be required for a brute force search to achieve the same precision on optimal route value.

The optimization returned the highest value route through the north west corner as shown in Figure 12. A MC analysis using 1000 runs for this route was compared to the straight line route. The analysis used a higher fidelity simulation running vehicle TRN code. The optimal single waypoint route has a significantly higher success probability than the straight route. This is evident both in the spread of the final position errors in Figure 13 and the maximum circular error probability in Figure 14. To achieve the same success probability, the maximum acceptable error for the straight line path would need to be increased to over 3m. Alternately, the single waypoint route achieves an 80% success probability at this 3m

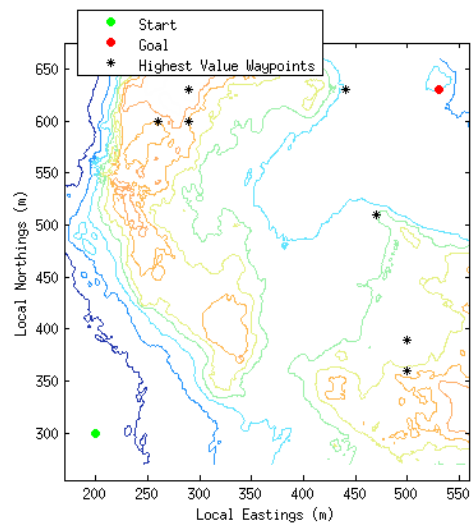
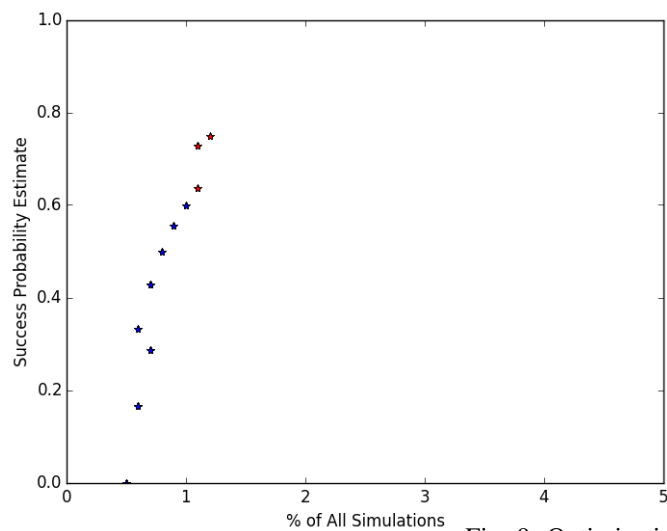


Fig. 9: Optimization at 1000 Simulations

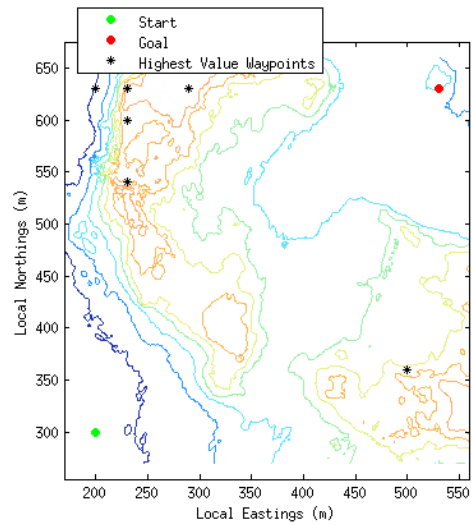
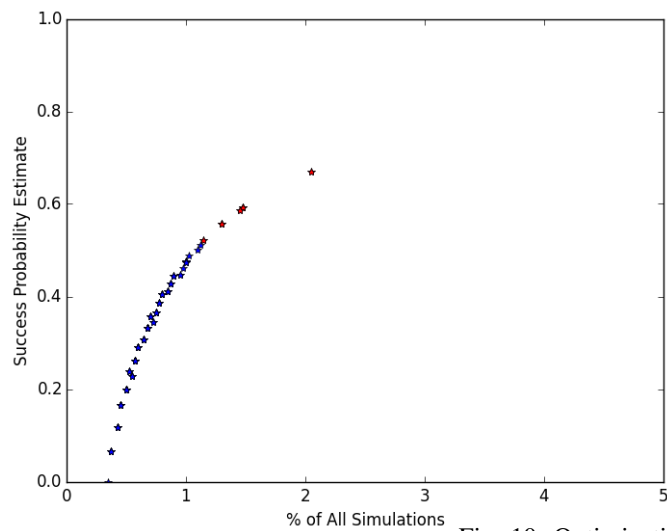


Fig. 10: Optimization at 4000 Simulations

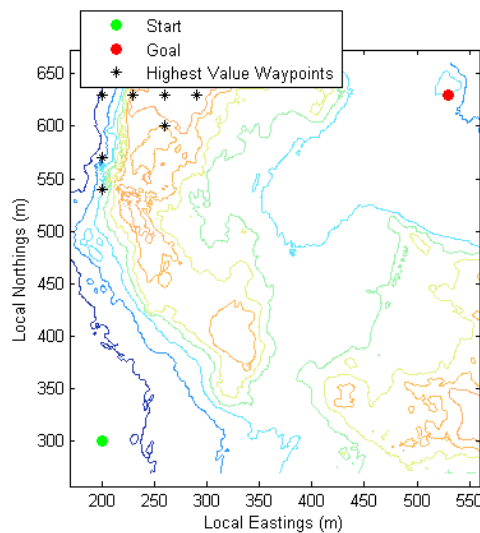
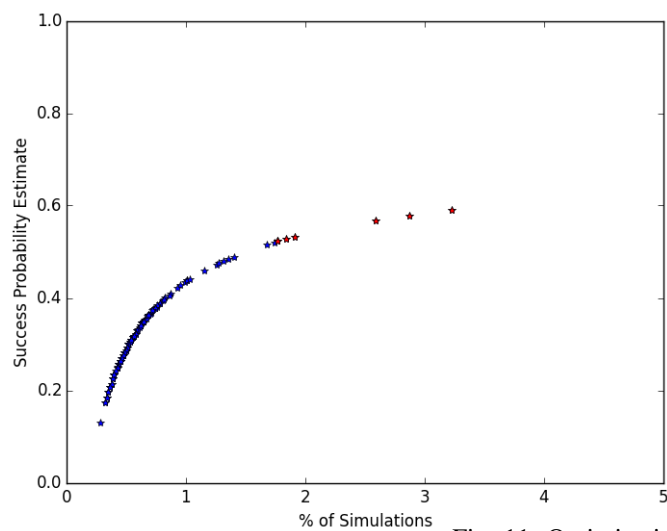


Fig. 11: Optimization at 16000 Simulations

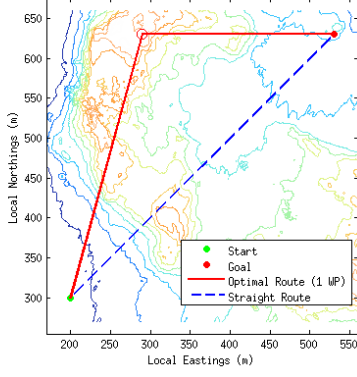


Fig. 12: Resulting Optimal Path

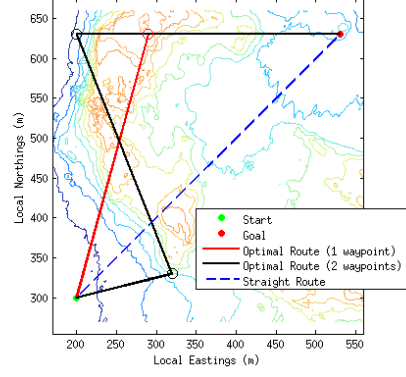
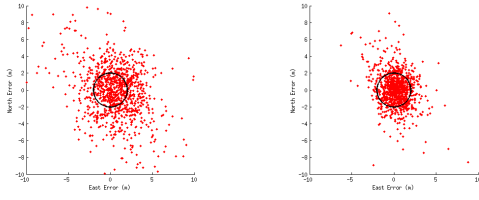


Fig. 15: Resulting Optimal Paths (2 Waypoints)



(a) Straight Line Route (b) Optimal Route (1 WP)

Fig. 13: MC Analysis Results (1000 Simulations)

horizontal error. The additional route length allowed longer travel over higher information terrain, improving the TRN position estimate.

C. Expansion to Multiple Intermediate Waypoints

The probability for a maximum horizontal error of 2m can also be improved in this example problem by expanding the search to routes with two waypoints, as shown in Figures 15 and 16. While the method expands to higher number of intermediate waypoints, the total number of all possible waypoint combinations also greatly increases. This leads to a higher number of simulations required for the optimization. For this reason, this two waypoint optimization was demonstrated using only a limited set of two waypoint routes. Even with the limited set, the additional freedom of a second

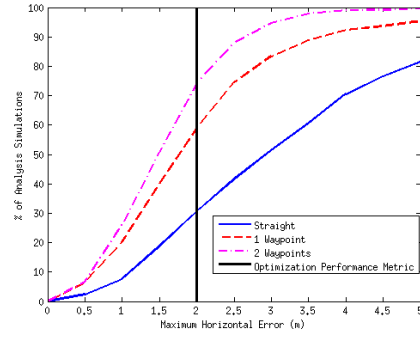


Fig. 16: Final Maximum Horizontal Error Results Analysis (2 Waypoints)

waypoint is able to achieve higher success probabilities than the single waypoint route.

V. SUMMARY

A method was presented to plan optimal waypoint routes for an AUV return-to-site mission using TRN by maximizing the probability of being within a goal region around the site of interest. The analytically intractable probability was approximated and optimized through the use of MC simulations. With the addition of an UCB, the algorithm reduced the total number of simulations required when compared to a brute force search. Routes were planned and analyzed using computer simulations for a mission in Monterey Bay with sea trials planned later this year to verify results. Although this method was applied for two waypoint routes, future work is being focused on improving the scalability of the algorithm when expanding to multiple waypoint routes.

VI. ACKNOWLEDGMENTS

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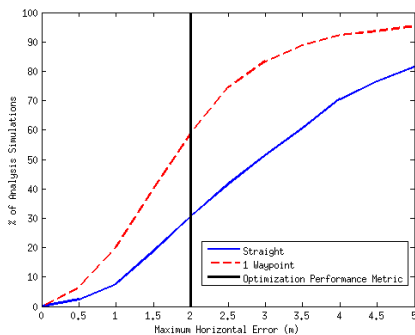


Fig. 14: Final Maximum Horizontal Error Results Analysis

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