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Multirotor Design Optimization Using a Genetic Algorithm

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***Abstract*— An optimization problem based on discrete-integer and continuous variables in order to design a multirotor for a specific application (maximum thrust-to-weight and maximum flight time) is developed. We considered the length and the diameter of the rods as continuous variables and the selection of the motors, propellers, and battery as discrete-integer variables, the components selected are available in common hobby stores. The problem is solved for two cases: maximum thrust-to-weight ratio and maximum flight time. The problem is stated using the aerodynamic principles and mechanical design as well. We used a Genetic Algorithm provided by the Optimization Toolbox in Matlab®, in order to solve the two cases; a brief description of the algorithm is presented. The results showed that the general idea is to consider an hex-rotor in order to obtain the maximum thrust-to-weight ratio and the maximum flight time.**

I. Introduction

In the last years, the study of aerial robotics and its applications shows that the field is still very young. Aerial robotics represent a very interesting and exciting area of unmanned vehicles; involving very dynamic platforms whose size ranges from a few centimeters to several tens of meters, [1], [2]. However, the study of the design process has overtaken due to the easiness of construction that implies.

The current study of UAVs involves novel control techniques, [3], [4]; nevertheless, the process of assembly i.e. a quadcopter consists mainly of a frame with a given number of arms, motors, propellers and a battery in addition to the control system. The combination of these components gives different behaviors of the aircraft, including the thrust-to-weight (T/W) ratio, flight time, total mass, among others.

The selection of the components is based on builder's expertise and this is an expensive task because it is done by trial and error. On the other hand, there are some techniques in order to select the components based on previous studies, e.g. validating the components from web calculators in order to select hardware by expertise, [5], [6].

Design optimization of multirotors has been studied by some authors, [7], [8], [9]. However, the objectives and approaches are different. In [7], the author proposed different quality indexes based on his own expertise in order to select de best components for an index quality desired. In [8], the author focused more on the optimization method rather than the application, he only minimized the total mass of the aircraft and defined constraints in order to select the battery, motors, and propellers. In [9], the authors select some aerodynamic functions in order to select the best combination of components including the number of the rotors.

There are many applications in engineering that are mathematically modeled in terms of mixed discrete-continuous variables. Problems which contain integer, discrete, zero-one and continuous design variables are often referred to as mixed integer nonlinear programming (MINLP) problems, [10].

This type of problems that contains discrete-integer and continuous variables have been used in the solution of engineering design problems, e.g. in [11], an algorithm based on genetic algorithms (GA) with mixed-discrete variables is developed in order to design optimization, they show the advantages of using GA for the solution of mechanical design optimization, e.g. design of reinforced concrete beam, pressure vessel, 25 truss bar, among others.

According to [12], GA is a powerful tool for solving MINLP problems. These methods do not require gradient or Hessian information. GA is broadly applicable stochastic search and optimization techniques that really works for many problems that are very difficult to solve by conventional techniques (Mathematical Programming). According to [13], most engineering problems are optimization problems subject to complex constraints.

In [14], a design for gear box is presented, they use a GA to find the optimal parameters of the gearbox. The mass of the gearbox was reduced under the premise such as strength, stiffness, and vibration resistance.

In [15], a concurrent design of four bar mechanism is developed taking into account the mechanical part and the control part, in order to find the optimal design, the integer-discrete variables play an important role in selecting from a list the motor that provides the best performance of the mechanism in velocity, this is where discrete-integer variables appear; as an optimization tool they used an algorithm based on GA.

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In this paper, an optimization problem based on continuous and discrete-integer variables is proposed, i.e. the length of the rods and their diameter in order to find the overall dimensions of the multirotor for a specific application. In addition, the problem involves the selection of motors, propellers, and battery type. The problem is solved for two cases: the maximum T/W ratio and the maximum flight time. The problem is stated using the aerodynamic principles and mechanical design as well.

II. Optimization Strategies

The general problem is to find the best combination of components that conforms a multirotor, especially the following: motors, propellers, battery, as well as the length of the rods and their diameter. In addition, the number of motors is taken as a constraint in order to find the best solution for each configuration (quad-rotor, hex-rotor, octo-rotor).

A. Frame Design

The frame design consists of finding the length and diameter of the rods based on the classical materials mechanical problem, i.e. a pure bending beam problem; this is because a static optimization problem is considered, so, no additional forces due to acceleration are considered. Fiber carbon for the multirotor frame was chosen because its properties of light-weighting and maximum tensile stress.

1) Profile Design: According to [16], a cylinder profile with a pattern pressed outwards at 90 degrees possesses the lowest drag coefficient which remains nearly constant even at higher Reynolds number, unlike roughened cylinders. Based on this, it just left to define the radius of the rod.

We establish the following mechanical design problem: Design a beam (rod) profile subjected to a single concentrated load P $[N]$ (weight) at its midpoint $0.5L$ $[m]$ (see Fig. 1).

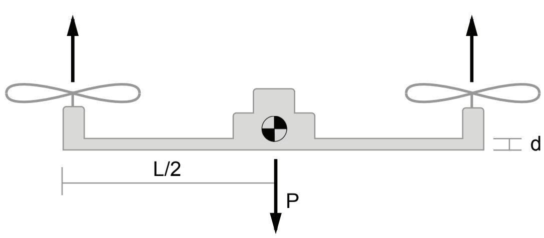

Fig. 1. Free body diagram of a rod.

According to [17], the maximum stress occurs when the bending moment is maximum M_B $[Nm]$, in this particular case is defined as $0.25PL$ where P is the value of the load, in this case, the total mass. The maximum normal stress is defined as follows,

$$\sigma_m = \frac{|M_B|}{S} \tag{1}$$

where S $[m^3]$ is the modulus of the rod; for a cylinder profile the modulus is defined as,

$$S = \frac{\pi d^3}{32}. \tag{2}$$

where d $[m]$ is the diameter of the rod.

2) Total mass: The total mass of the aircraft is the sum of the frame mass, the mass of each motor, the mass of each propeller, the battery mass, and the mass of the avionics (electronic, sensors, cables), this sum is defined as follows:

$$m_T = n_r(m_C + m_R + m_P) + m_B + m_E \tag{3}$$

where m_T $[Kg]$ is the total mass; m_C is the frame mass; m_B is the battery mass; m_R is the motor mass; m_P is the propeller mass; m_E is the avionics mass and n_r is the number of the rods.

Taking into account the material of the rods and their profile, the frame mass can be expressed as a product of the length of the rod L, the area of the profile $A = \pi D^2/4$ $[m^2]$ and the density of the material ρ_m $[Kg/m^3]$, the equation (3) can be rewritten as follows,

$$m_T = n_r\left(0.5\rho_m AL + m_R + m_P\right) + m_B + m_E \tag{4}$$

equation (4) is the total mass of the aircraft.

B. Thrust Force

According to [18], [19] the thrust F_{Th} $[N]$ of each motor is given by the equation (5).

$$F_{Th} = \rho_a C_T n^2 D^4 \tag{5}$$

where ρ_a $[Kg/m^3]$ is the air density; C_T is the thrust coefficient of the propeller, this coefficient is not fixed, but it changes with the angular velocity, it chose the coefficient with zero velocity of displacement and 4000 rpm of angular velocity; n is the angular velocity of the propeller in rps and D $[m]$ is the diameter of the propeller.

C. Flight Time

Low pitch propellers yield less speed, but more torque. Such propellers are desirable in an optimization problem, [9]. On the other hand, the flight time is related to the power propeller P_p $[W]$. According to [18], the power drawn from the propeller is calculated as,

$$P_p = \rho_a C_p n^3 D^5 \tag{6}$$

where C_p is the power coefficient of the propeller, this coefficient is not fixed, but it changes with the angular velocity, it chose the coefficient with zero velocity of displacement and 4000 rpm of angular velocity.

The battery has some parameters that allow to calculate the flight time taking into account the power that propeller draws during the flight, this power can be calculated with equation (6). From the propeller power and battery voltage, the current consumption is calculated from the electrical power equation $P = VI$. So, the flight time h $[hours]$ can be expressed using the parameters of the battery and the propeller power, as follows,

$$\begin{aligned} h &= \frac{C_B}{n_r I} \\ &= \frac{V_B C_B}{n_r P_p} \\ &= \frac{V_B C_B}{n_r \rho C_p n^3 D^5} \end{aligned} \tag{7}$$

where I $[A]$ is the current consumption due to the propeller, C_B is the battery capacity in mAh and V_B $[V]$ is the voltage of the battery.

Another important parameter is the "C-rating", this parameter defines how much current battery can safely draw up continuously, e.g. if the battery capacity is $1000mAh$ with C-rating of 20, it means that battery can safely draw up to $20A$ continuously.

D. Number of Motors

The number of motors can be considered as a constraint or as a design variable, this last in order to find the optimal number of motors for a specific application. We compute a parametrization in order to find the constraints due to the diameter of the propellers.

In Fig. 2, it is shown how the diameter of the propellers acts as a constraint for the length of the rod (L).

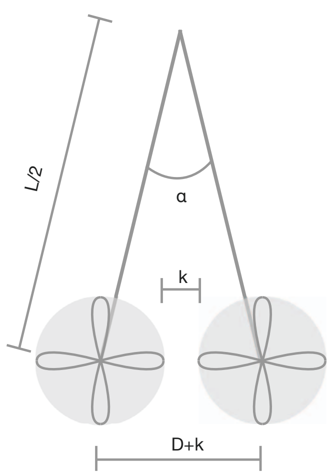

Fig. 2. Diagram for L parametrization.

We can define the lower limit of the design variable L from Fig. 2, this limit is a function of D and n_r, as given in equation (8).

$$L \geq 2\frac{\sin\left(90 - \frac{180}{n_r}\right)}{\sin\left(\frac{360}{n_r}\right)}(D + k) \tag{8}$$

where $k \geq 0$ is an offset parameter.

E. Design Constraints

The constraints of the problem were defined using the previous equations. We considered the restrictions from the electrical part and the mechanical part as well. The constraints are the following:

- The propeller power has to be less than the maximum motor power.
- C-rating of the battery has to be enough for the current consumption due to all rotors.
- Maximum normal stress has to be less than the maximum stress permissible of the material.
- The length of the rods has to be proportional to the diameter of the propellers.
- Maximum and minimum weight.
- Maximum and minimum length of the rod.
- Maximum and minimum diameter of the rod.

III. Optimal Problem Statement

The optimization problem is a mixed variables problem, i.e. it involves discrete and continuous variables, and to be more precise discrete-integer variables. The last is because the algorithm has to select from a list of available components each component in order to find the best combination possible. In general, optimization problem can be stated as:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & F(x) \\ \text{subject to} \quad & h_i(x) = b_i,\ i = 1, \ldots, m. \\ & g_j(x) \leq b_j,\ j = 1, \ldots, n. \end{aligned}$$

where: $F(x)$ is the objective function, x is a set of design variables, $h_i(x)$ are the equality constraints and $g_j(x)$ are the inequality constraints.

The vector of design variables is given by:

$$x = \left[d_1, d_2, ..., d_n, c_1, c_2, ..., c_n\right]$$

where d_i are the discrete-integer variables and c_i are the continuous ones.

A. Discrete and Continuous Variables

Discrete variables correspond to the selectable components from tables that contain the available components in common places e.g. APC [20] and HobbyKing [21], they are motor, propeller, and battery.

The available motors are listed in Table I. In order to find the best combination of components, it requires the following characteristics: Kv $[rpm/V]$ value that refers to the rpm constant of a motor, i.e. it is the number of revolutions per minute that the motor will turn when one volt is applied with no load attached to the motor, the maximum power of the motor and the maximum current.

The available propellers are listed in Table II. Coefficients C_T and C_P were taken from the given in [20].

TABLE I
AVAILABLE MOTORS

i	Weight (Kg)	Kv	Max Power (W)	Current (A)
1	0.144	400	350	15.8
2	0.068	800	285	15.4
3	0.144	340	333	15
4	0.144	480	364	16.4
5	0.192	335	623	28
6	0.108	700	560	30
7	0.200	580	444	20

TABLE II
AVAILABLE PROPELLERS

j	Diameter (m)	C_t	C_p	Weight (Kg)
1	0.2032	0.1524	0.0760	0.007
2	0.2286	0.1166	0.0476	0.011
3	0.2286	0.1479	0.0696	0.009
4	0.254	0.1123	0.0442	0.015
5	0.254	0.1207	0.0517	0.015
6	0.2794	0.1043	0.0392	0.017
7	0.3048	0.0995	0.0358	0.022
8	0.3302	0.0901	0.0306	0.024

The available batteries are listed in Table III. The selection of the battery according with the selected motors is a common mistake because the C-rating of the battery is not taking into account.

TABLE III
AVAILABLE BATTERIES

k	Voltage (V)	Capacity (mAh)	C-rating	Weight (Kg)
1	14.8	6200	40	0.770
2	14.8	5000	60	0.756
3	14.8	4000	40	0.594
4	18.5	5000	25	0.645
5	18.5	5800	25	0.720
6	18.5	3700	25	0.493
7	22.2	5800	40	1.08

Continuous variables are the length $[m]$ and the diameter $[m]$ of the rod.

B. Maximizing Thrust/Weight Ratio

The T/W ratio means how many times the aircraft is capable of lifting its own weight. So, the objective function $F(x)$ can be derived from equations (4) and (5) as follows,

$$F(x) = \frac{n_r F_{Th}}{g m_T} \tag{9}$$

where g $[m/s^2]$ is the value of gravity.

Using objective function (9), the flight time can be constrained to the application needs. So, the flight time becomes into a constraint. The constraints defined in Section II-E are taking into account in the optimization problem as well.

C. Maximizing Flight Time

The flight time is expressed by equation (7), so, the objective function $F(x)$ is defined as follows,

$$F(x) = \frac{V_B C_B}{n_r \rho_a C_p n^3 D^5} \tag{10}$$

It can see from equation (10), that the weight is not expressed explicitly, but implicit, i.e. total weight of the multirotor has an impact in the velocity of the motors, so, the velocity n changes in function of the weight. On the other hand, the T/W ratio can be constrained in order to satisfy the application needs. In addition, the constraints defined in Section II-E are taking into account in the optimization problem as well.

D. Genetic Algorithm

GA is an evolutionary optimization method which is an alternative to traditional optimization methods, such as mathematical programming. GA is most appropriate for complex non-linear models where the location of the global optimum is a difficult task and for combinatorial problems. Some engineering problems solved (car suspension design, gear train design, among others) using a GA can be found in [22].

The problems were solved using the GA included in the Optimization Toolbox on Matlab®. The GA consists of five stages: evaluation, selection, crossover, mutation and elitism. First of all, the algorithm generates an initial population of size N with individuals with integer-discrete and continuous variables within the lower and upper limits.

- The *evaluation* operation measures the fitness of each individual solution in the population and assigns it a relative value based on the defining optimization criteria.
- The *selection* operation selects individuals of the current population based on the value of their fitness value in order to develop the next generation.
- The *crossover* operation takes the two selected individuals and combines them about a crossover point thereby creating two new individuals and measures the fitness of each new individual; the number of new individuals created by crossover is specified as a percentage of the initial population (N).
- The *mutation* operation randomly modifies the genes of an individual subject to a small mutation factor, and create new individuals in order to introduce diversity into the population, so, the amount of new individuals created by mutations is a percentage of the initial population (N).
- The *elitism* operation chooses a percentage of the population (N) with the best fitness value to survive

to the next generation without any changes; candidate solutions that are preserved unchanged remain eligible for selection as parents.

On the other hand, in order to handle the constraints the genetic algorithm attempts to minimize a penalty function, not the fitness function. The penalty function includes a term for infeasibility [22]. This penalty function is combined with binary tournament selection to select individuals for subsequent generations. The penalty function value of an individual of a population is:

- If the individual is feasible, the penalty function is the fitness function.
- If the individual is infeasible, the penalty function is the maximum fitness function among feasible individuals of the population, plus a sum of the constraint violations of the (infeasible) point.

Parameters used in GA are listed in Table IV.

TABLE IV
GA PARAMETERS

Parameter	Value
Initial population	5000
Selection mechanism	Roulette Wheel Selection
Crossover percentage	0.8
Mutation percentage	0.2
Elitism percentage	0.1
Maximum generations	1000

In the problem stated in this paper, the search space for discrete variables is finite and its dimension is about 1176 possible combinations. In order to solve the problem and find the optimal combination of elements and the dimensions of the frame, an optimization problem for each possible combination of discrete search space in combined with continuous variables would have to be solved. This task of combining the elements and at the same time finding the dimensions of the frame is achieved by the GA.

IV. RESULTS AND ANALYSIS

In this section, simulation results of both objectives are presented. The first case corresponds in finding the maximum T/W ratio with flight time as a constraint (minimum and maximum desired values of flight time). The second case corresponds in finding the maximum flight time with T/W ratio as a constraint.

A. First Case - Thrust-to-Weight Ratio

In this case, several scenarios in order to understand the optimization process and to validate the results were considered. First, the flight time is another constraint as was mentioned before; it defines three-time lapses: a) $5-10$ minutes, b) $10-20$ minutes and c) $20-25$ minutes.

In addition, the results are presented in three cases; considering as an equality constraint the number of the motors (4, 6 and 8 motors); the results are shown in Table V.

The results give the best combination of different components and the best length and diameter of the frame in order to find the maximum T/W ratio in each case of flight time established. The possible combinations depending on the number of motors suggest that the best combination and frame dimensions that give the maximum T/W ratio are with six motors no matter the flight time application needs. This is reasonable because few motors give a lower thrust than eight, but eight motors mean more weight and more current consumption that lead to requiring a more powerful and heavier battery.

B. Second Case - Flight Time

In this case, the objective is to maximize the flight time with T/W ratio as a constraint according to the application. First, the T/W ratio is another constraint and three ratios were defined: a) $2-3$, b) $3-4$ and c) $4-5$.

In order to compare with the first case, the results are presented in three cases, considering as an equality constraint the number of the motors (three cases: 4, 6 and 8 motors); the results are shown in Table VI.

In this case, the results confirm the previous results about the T/W ratio, i.e. the best combination depending on the number of motors suggest that the best combination and frame dimensions that give the maximum time flight with the T/W ratio established are with six motors.

However, the results for flight time are small because of the T/W ratio established for solving the problem. This last constraint is strong because most of the applications need an extra thrust, for instance, a camera.

V. CONCLUSION

The results in each case lead to the idea that the six motors give the maximum T/W ratio and the maximum flight time. However, the nature of the problem is more complex, so, an hex-rotor cannot be considered as a multipurpose aircraft. The general result of an hex-rotor is not a coincidence and of course depends on the available components in stores.

This is not the overall solution and a unique conclusion but is a first approach to consider not only the mechanical part but the control part as well. The last because the performance of the aerial vehicle will depend on the best combination of components and the structural aspects. The design process based on optimization has to be taken into account from the very start, the overall performance will depend on entirely of the combination of its components. Design optimization helps to saved money and to build the precise machine for a specific application with a highly reconfigurable design.

TABLE V

RESULTS FOR FIRST CASE - THRUST-TO-WEIGHT RATIO

Characteristics	Flight Time $5-10$ Minutes			Flight Time $10-20$ Minutes			Flight Time $20-25$ Minutes		
Number of motors	4 Motors	6 Motors	8 Motors	4 Motors	6 Motors	8 Motors	4 Motors	6 Motors	8 Motors
Flight time (minutes)	5.56	5.0	5.34	10.6	10	10	24.95	21	23.84
Total mass (Kg)	3.33	3.95	3.34	2.78	3.33	3.58	3.95	3.95	3.46
Effective Payload (Kg)	5.9	9.5	6.71	3.45	4.95	3.81	3	3.8	1.26
Length (cm)	95	96	98	88	99	98	97	90	98
Diameter of the rod (cm)	2.2	1.9	1.5	2.2	1.6	1.5	2.4	1.8	1.5
Motor (# Motor)	7	3	6	3	1	4	5	5	4
Propeller (# Propeller)	7	8	2	8	8	2	5	6	2
Battery (# Battery)	1	7	1	6	1	5	7	7	3

TABLE VI

RESULTS FOR SECOND CASE - FLIGHT TIME

Characteristics	T/W ratio $2-3$			T/W ratio $3-4$			T/W ratio $4-5$		
Number of motors	4 Motors	6 Motors	8 Motors	4 Motors	6 Motors	8 Motors	4 Motors	6 Motors	8 Motors
Flight time (minutes)	7.7	7.6	6.42	3.5	5.14	3.46	1.94	3.15	1.72
Total mass (Kg)	3.1	3.32	3.61	2.85	3.12	3.35	2.98	3.14	3.2
Effective Payload (Kg)	3.14	4.17	3.77	5.78	4.41	6.7	8.63	9.8	8.1
Length (cm)	95	93	98	78	97	98	80	97	99
Diameter of the rod (cm)	2.2	1.7	1.5	2.4	1.6	1.5	2.2	1.6	1.7
Motor (# Motor)	3	3	4	1	3	6	7	1	2
Propeller (# Propeller)	8	7	2	8	8	2	8	8	2
Battery (# Battery)	4	5	5	6	4	1	3	4	3

VI. FUTURE WORK

An aspect that must be considered as further research is to proof if the problem is multi-objective; and to state an optimization problem considering the dynamic performance of the system, such as, the performance of the motors.

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