scipy.interpolate.CubicSpline

class **scipy.interpolate.CubicSpline(***x*, *y*, *axis=0*, *bc_type='not-a-knot'*, *extrapolate=None***)** **[source] (http://github.com/scipy/scipy/blob/v0.18.1/scipy/interpolate/_cubic.py#L352-L770)**

Cubic spline data interpolator.

Interpolate data with a piecewise cubic polynomial which is twice continuously differentiable [R53]. The result is represented as a PPoly (scipy.interpolate.PPoly.html#scipy.interpolate.PPoly) instance with breakpoints matching the given data.

Parameters:

x : *array_like, shape (n,)*

1-d array containing values of the independent variable. Values must be real, finite and in strictly increasing order.

y : *array_like*

Array containing values of the dependent variable. It can have arbitrary number of dimensions, but the length along axis (scipy.interpolate.CubicSpline.axis.html#scipy.interpolate.CubicSpline.axis) (see below) must match the length of x (scipy.interpolate.CubicSpline.x.html#scipy.interpolate.CubicSpline.x). Values must be finite.

axis : *int, optional*

Axis along which *y* is assumed to be varying. Meaning that for `x[i]` the corresponding values are `np.take(y, i, axis=axis)`. Default is 0.

bc_type : *string or 2-tuple, optional*

Boundary condition type. Two additional equations, given by the boundary conditions, are required to determine all coefficients of polynomials on each segment [R54].

If *bc_type* is a string, then the specified condition will be applied at both ends of a spline. Available conditions are:

- 'not-a-knot' (default): The first and second segment at a curve end are the same polynomial. It is a good default when there is no information on boundary conditions.
- 'periodic': The interpolated functions is assumed to be periodic of period `x[-1] - x[0]`. The first and last value of *y* must be identical: `y[0] == y[-1]`. This boundary condition will result in `y'[0] == y'[-1]` and `y''[0] == y''[-1]`.
- 'clamped': The first derivative at curves ends are zero. Assuming a 1D *y*, `bc_type=((1, 0.0), (1, 0.0))` is the same condition.
- 'natural': The second derivative at curve ends are zero. Assuming a 1D *y*, `bc_type=((2, 0.0), (2, 0.0))` is the same condition.

If *bc_type* is a 2-tuple, the first and the second value will be applied at the curve start and end respectively. The tuple values can be one of the previously mentioned strings (except 'periodic') or a tuple *(order, deriv_values)* allowing to specify arbitrary derivatives at curve ends:

- *order*: the derivative order, 1 or 2.
- *deriv_value*: array_like containing derivative values, shape must be the same as *y*, excluding axis (scipy.interpolate.CubicSpline.axis.html#scipy.interpolate.CubicSpline.axis) dimension. For example, if *y* is 1D, then *deriv_value* must be a scalar. If *y* is 3D with the shape (n0, n1, n2) and axis=2, then *deriv_value* must be 2D and have the shape (n0, n1).

extrapolate *: {bool, 'periodic', None}, optional*

If bool, determines whether to extrapolate to out-of-bounds points based on first and last intervals, or to return NaNs. If 'periodic', periodic extrapolation is used. If None (default), extrapolate (scipy.interpolate.CubicSpline.extrapolate.html#scipy.interpolate.CubicSpline.extrapolate) is set to 'periodic' for `bc_type='periodic'` and to True otherwise.

See also:
Akima1DInterpolator (scipy.interpolate.Akima1DInterpolator.html#scipy.interpolate.Akima1DInterpolator), PchipInterpolator (scipy.interpolate.PchipInterpolator.html#scipy.interpolate.PchipInterpolator), PPoly (scipy.interpolate.PPoly.html#scipy.interpolate.PPoly)

Notes

Parameters *bc_type* and interpolate work independently, i.e. the former controls only construction of a spline, and the latter only evaluation.

When a boundary condition is 'not-a-knot' and n = 2, it is replaced by a condition that the first derivative is equal to the linear interpolant slope. When both boundary conditions are 'not-a-knot' and n = 3, the solution is sought as a parabola passing through given points.

When 'not-a-knot' boundary conditions is applied to both ends, the resulting spline will be the same as returned by splrep (scipy.interpolate.splrep.html#scipy.interpolate.splrep) (with `s=0`) and InterpolatedUnivariateSpline (scipy.interpolate.InterpolatedUnivariateSpline.html#scipy.interpolate.InterpolatedUnivariateSpline), but these two methods use a representation in B-spline basis.

New in version 0.18.0.

References

[R53] *(1, 2)* Cubic Spline Interpolation (https://en.wikiversity.org/wiki/Cubic_Spline_Interpolation) on Wikiversity.

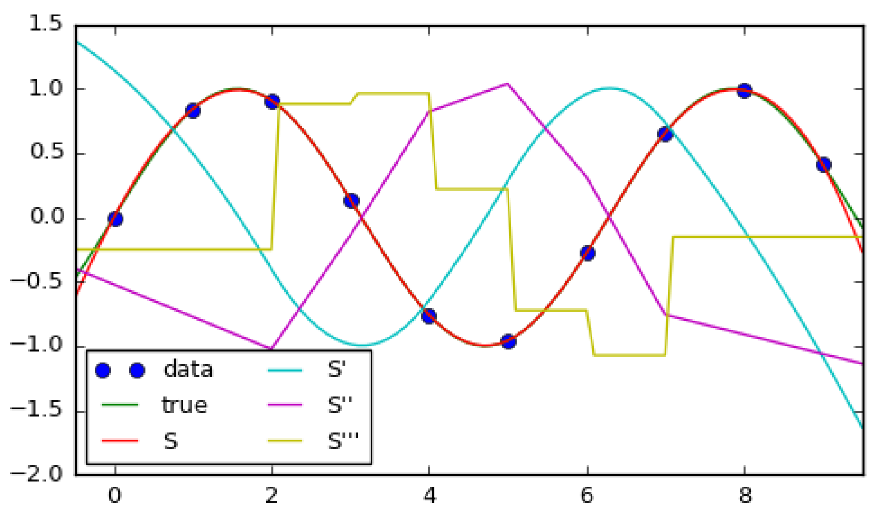
[R54] *(1, 2)* Carl de Boor, “A Practical Guide to Splines”, Springer-Verlag, 1978.

Examples

In this example the cubic spline is used to interpolate a sampled sinusoid. You can see that the spline continuity property holds for the first and second derivatives and violates only for the third derivative.

```
>>> from scipy.interpolate import CubicSpline
>>> import matplotlib.pyplot as plt
>>> x = np.arange(10)
>>> y = np.sin(x)
>>> cs = CubicSpline(x, y)
>>> xs = np.arange(-0.5, 9.6, 0.1)
>>> plt.figure(figsize=(6.5, 4))
>>> plt.plot(x, y, 'o', label='data')
>>> plt.plot(xs, np.sin(xs), label='true')
>>> plt.plot(xs, cs(xs), label="S")
>>> plt.plot(xs, cs(xs, 1), label="S'")
>>> plt.plot(xs, cs(xs, 2), label="S''")
>>> plt.plot(xs, cs(xs, 3), label="S'''")
>>> plt.xlim(-0.5, 9.5)
>>> plt.legend(loc='lower left', ncol=2)
>>> plt.show()
```

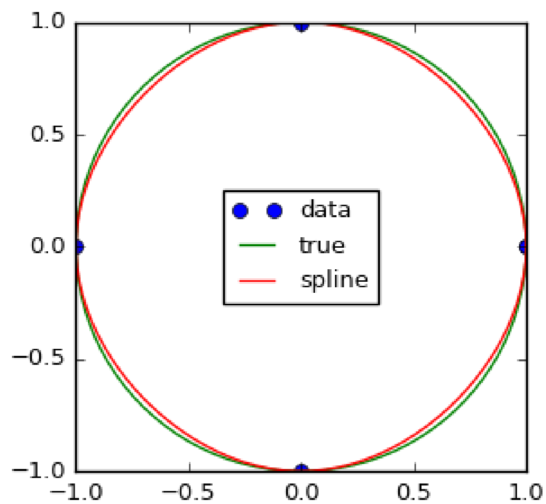
(Source code (../generated/scipy-interpolate-CubicSpline-1.py))



In the second example, the unit circle is interpolated with a spline. A periodic boundary condition is used. You can see that the first derivative values, ds/dx=0, ds/dy=1 at the periodic point (1, 0) are correctly computed. Note that a circle cannot be exactly represented by a cubic spline. To increase precision, more breakpoints would be required.

>>>

```
>>> theta = 2 * np.pi * np.linspace(0, 1, 5)
>>> y = np.c_[np.cos(theta), np.sin(theta)]
>>> cs = CubicSpline(theta, y, bc_type='periodic')
>>> print("ds/dx={:.1f} ds/dy={:.1f}".format(cs(0, 1)[0], cs(0, 1)[1]))
ds/dx=0.0 ds/dy=1.0
>>> xs = 2 * np.pi * np.linspace(0, 1, 100)
>>> plt.figure(figsize=(6.5, 4))
>>> plt.plot(y[:, 0], y[:, 1], 'o', label='data')
>>> plt.plot(np.cos(xs), np.sin(xs), label='true')
>>> plt.plot(cs(xs)[:, 0], cs(xs)[:, 1], label='spline')
>>> plt.axes().set_aspect('equal')
>>> plt.legend(loc='center')
>>> plt.show()
```



The third example is the interpolation of a polynomial y = x**3 on the interval 0 <= x<= 1. A cubic spline can represent this function exactly. To achieve that we need to specify values and first derivatives at endpoints of the interval. Note that y' = 3 * x**2 and thus y'(0) = 0 and y'(1) = 3.

>>>

```
>>> cs = CubicSpline([0, 1], [0, 1], bc_type=((1, 0), (1, 3)))
>>> x = np.linspace(0, 1)
>>> np.allclose(x**3, cs(x))
True
```

Attributes

x	(ndarray, shape (n,)) Breakpoints. The same x (scipy.interpolate.CubicSpline.x.html#scipy.interpolate.CubicSpline.x) which was passed to the constructor.
c	(ndarray, shape (4, n-1, ...)) Coefficients of the polynomials on each segment. The trailing dimensions match the dimensions of *y*, excluding axis (scipy.interpolate.CubicSpline.axis.html#scipy.interpolate.CubicSpline.axis). For example, if *y* is 1-d, then `c[k, i]` is a coefficient for `(x-x[i])**(3-k)` on the segment between `x[i]` and `x[i+1]`.
axis	(int) Interpolation axis. The same axis (scipy.interpolate.CubicSpline.axis.html#scipy.interpolate.CubicSpline.axis) which was passed to the constructor.

Methods

__call__ (scipy.interpolate.CubicSpline.__call__.html#scipy.interpolate.CubicSpline.__call__)(x[, nu, extrapolate])	Evaluate the piecewise polynomial or its derivative.
derivative (scipy.interpolate.CubicSpline.derivative.html#scipy.interpolate.CubicSpline.derivative)([nu])	Construct a new piecewise polynomial representing the derivative.
antiderivative (scipy.interpolate.CubicSpline.antiderivative.html#scipy.interpolate.CubicSpline.antiderivative)([nu])	Construct a new piecewise polynomial representing the antiderivative.
integrate (scipy.interpolate.CubicSpline.integrate.html#scipy.interpolate.CubicSpline.integrate)(a, b[, extrapolate])	Compute a definite integral over a piecewise polynomial.
roots (scipy.interpolate.CubicSpline.roots.html#scipy.interpolate.CubicSpline.roots)([discontinuity, extrapolate])	Find real roots of the the piecewise polynomial.

Previous topic

scipy.interpolate.Akima1DInterpolator.roots (scipy.interpolate.Akima1DInterpolator.roots.html)

Next topic

scipy.interpolate.CubicSpline.__call__ (scipy.interpolate.CubicSpline.__call__.html)