## Unicycle kinematic model:

$$x(t+1) = x(t) + v_x(t)\cos(\theta + \omega(t)\Delta t)\,\Delta t$$

$$y(t+1) = y(t) + v_y(t)\cos(\theta + \omega(t)\Delta t)\,\Delta t$$

$\omega \rightarrow$ angular velocity

$[x, y] \rightarrow$ position co-ordinates.

$v \rightarrow$ linear velocity.

$$\theta_K = \theta_{K-1} + \omega_K \Delta t \quad \rightarrow \text{heading at instant 'k'.}$$

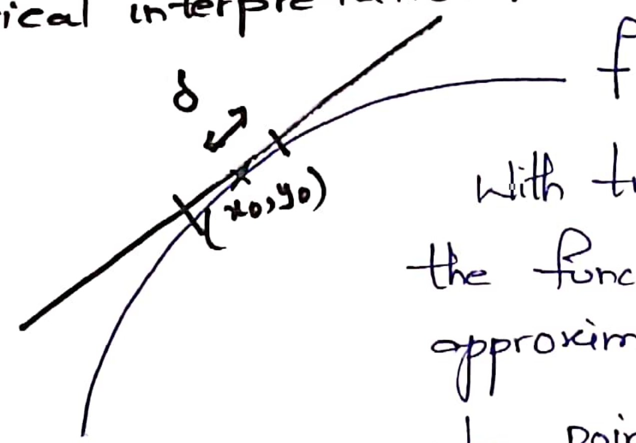
## Convexification by taylor's approximation:

$f(x,y) \longrightarrow$ function with variables $x, y$

multivariate taylor's expansion of the function:

$$\bar{f}(x,y) = f(x_0, y_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0, y_0}(x - x_0) + \left.\frac{\partial f}{\partial y}\right|_{x_0, y_0}(y - y_0)$$

## Geometrical interpretation:




With trust region radius '$\delta$' the function '$f$' is approximated linearly at point $(x_0, y_0)$

Convexification of non-holonomic motion model

$$x(t) = x(t-1) + v(t)\cos(\theta + \omega(t)\Delta t)$$

approximating with initial linear and angular velocities $v_1, v_2, \dots, v_n$ and $\omega_1, \omega_2, \dots, \omega_n$.

Considering 3 linear and angular velocities for the start:

$$x_0 = x_0$$

$$x_1 = x_0 + v_1\cos(\theta_0 + \omega_1 \Delta t)\Delta t$$

$$x_2 = x_1 + v_2\cos(\theta_1 + \omega_2 \Delta t)\Delta t$$

$$\Rightarrow \quad x_2 = x_0 + v_1\cos(\theta_0 + \omega_1\Delta t) + v_2\cos(\theta_1 + \omega_2\Delta t)$$

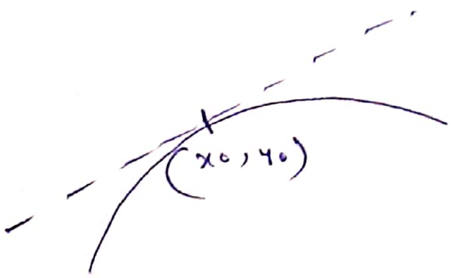
$$\because \theta_1 = \theta_0 + \omega_1 \Delta t$$

final $x_2 = x_0 + v_1\cos(\theta_0 + \omega_1\Delta t)\Delta t + v_2\cos(\theta_0 + (\omega_1+\omega_2)\Delta t)\Delta t$

now this state equations are non linear with respect to $v_1, v_2; \omega_1, \omega_2$. $\therefore$

$\Rightarrow$ in order to Convexify we need to linearize them around a trust region.

$\Rightarrow$ because the closest approximation to a non-linear model / concave model is an affine approximation.




linearization of a concave function

thus linearized state equation of $x_1$:

$\Rightarrow \quad x_1 = x_0 + V_1 \Delta t \cos(\theta_0 + \omega_1 \Delta t)$

Say approximation at $(x_i, y_i)$

$$\frac{\partial x_1}{\partial V_1}\Big|_{(\theta, x_i)} = \cancel{x_0}^{0} + \Delta t \cos(\theta_0 + \omega_1 \Delta t)$$

$$\frac{\partial x_1}{\partial \omega_1}\Big|_{(V_i, \omega_i)} = -V_1 \Delta t \sin(\theta_0 + \omega_1 \Delta t)(\Delta t)$$

$$= -V_1 \Delta t^2 \sin(\theta_0 + \omega_1 \Delta t)$$

thus $x_1\Big|_{(V_i, \omega_i)} = \left[x_0 + V_i \Delta t \cos(\theta_0 + \omega_i \Delta t)\right] + \Delta t \cos(\theta_0 + \omega_i \Delta t)(V - V_i)$

$+ \left(-V_i \Delta t^2 \sin(\theta_0 + \omega_i \Delta t)\right)(\omega - \omega_i)$

thus the function is linear with respect to $\omega_i$ and $V_i$.

$\Rightarrow \quad x_2 = x_0 + V_1 \Delta t \cos(\theta_0 + \omega_1 \Delta t) + V_2 \Delta t \cos(\theta_0 + (\omega_1 + \omega_2)\Delta t)$

By approximation:

term I: $f(V_1, V_2, \omega_1, \omega_2) = x_0 + V_1 \cos(\theta_0 + \omega_1 \Delta t) + V_2 \cos(\theta_0 + (\omega_1 + \omega_2)\Delta t)$

term II: $\Delta t \cos(\theta_0 + \omega_1 \Delta t)(V - V_1) + \cos(\theta_0 + (\omega_1 + \omega_2)\Delta t)\Delta t (V - V_2)$

term III: $\left(-V_1 \Delta t^2 \sin(\theta_0 + \omega_1 \Delta t) - V_2 \Delta t^2 \sin(\theta_0 + (\omega_1 + \omega_2)\Delta t)\right)(\omega - \omega_1)$

$-V_2 \Delta t^2 \sin(\theta_0 + (\omega_1 + \omega_2)\Delta t)\ (\omega - \omega_2)$

$x_2 = \text{term I} + \text{term II} + \text{term III}$

Similarly for y-co-ordinate and higher terms.

Formulating Cost-Function:

~~X = x_0 +~~

$$W = [w_1, w_2, \ldots w_n]$$

$I$ = identity matrix $n \times n$

$$\Theta_0 = \theta_0$$
$$\Theta_1 = \theta_0 + w_1 \Delta t$$
$$\Theta_2 = \theta_0 + (w_1 + w_2) \Delta t$$
$$\Theta_3 = \theta_0 + (w_1 + w_2 + w_3) \Delta t$$

$$\Theta = \theta_0 + \begin{bmatrix} w_1 \\ w_1 + w_2 \\ w_3 + w_2 + w_1 \end{bmatrix} \Delta t$$

$$= \theta_0 + \begin{bmatrix} w_1 & 0 & 0 \\ w_1 & w_2 & 0 \\ w_1 & w_2 & w_3 \end{bmatrix} \Delta t$$

$$= \theta_0 + w_1 \cdot \text{tril}(\text{ones}(3,3)) \Delta t.$$

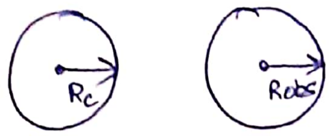
$$X_{1 \times n} \Rightarrow x_0 + V_{1\times n} \cos(\Theta_{1\times n} \Delta t] \Delta t$$

Similarly $Y_{1 \times n}$

Objective fn is written in Quadratic form

as $(X - X_{given})^2 + (Y - Y_{given})^2 + (\theta_1 - \theta_0)^2 + (\theta_f - \theta_n)^2$

## Formulating obstacle avoidance Constraints:




$$(x_c - x_o)^2 + (y_c - y_o)^2 \geq R^2$$

$$R = R_c + R_{obs}.$$

the Car can be reduced to apoint and the net radius to be considered is extended.

## MPC design parameters:

**Prediction Horizon:**

The number of steps for which the control action is predicted. usually 10 - 20 timesteps.

→ predict for a certain number of time steps.

→ Run for 10% of prediction horizon (or) any number of steps.

→ re iterate optimization.