# CSE483-Mobile Robotics

## End-semester exam Monsoon 2019 November 18<sup>th</sup>

Maximum points: 40 Duration: 180 minutes

#### Instructions

- This is an **open-book** exam. You are allowed to use any paper notes or textbooks that you have brought with you.
- Laptops, tablets, or smartphones are NOT allowed. You also cannot collaborate with other students.
- Your answers must be concise and to-the-point. Verbosity will NOT fetch you additional marks.
- Sufficient space has been provided for each question. Using additional sheets are discouraged, if you need them you're probably doing something wrong.
- You do NOT get credit for replicating whatever is present in the textbook or your notes. Please do not fill your answer scripts with excerpts from such sources.
- Use the last page for rough work or for any of your answers, if necessary.
- State your assumptions clearly if there is any ambiguity with the question(s).

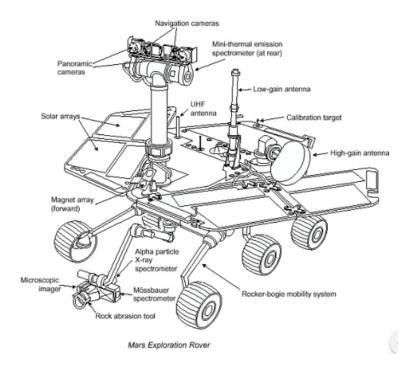
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Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total

Q1) A mars rover, equipped with cameras and various scientific instruments, is tasked to autonomously explore the surface of mars and collect data. Ignoring all the scientific instruments, and considering only the stereo navigation camera, systematically describe in words how you'd implement the localization, mapping, and planning modules to achieve this task. A high-level description of the algorithms in sufficient. (5 points)



Q2) Consider two calibrated cameras  $C_1$ ,  $C_2$  that are looking at a scene. The relative orientation R, t between the two cameras, and an up-to-scale 3D point cloud of the scene,  $\{X_i\}$ , were estimated by applying our well known linear 8-point and triangulation algorithms. We now wish to refine these estimates by applying bundle adjustment. (a) Write down the cost function to minimize. (1 point) (b) Why is the cost function squared? (1 point) (c) How can the cost function be written in linear form? (1 point) (d) What are the dimensions of the Jacobian? Show the structure of the Jacobian in block form. (2 points)

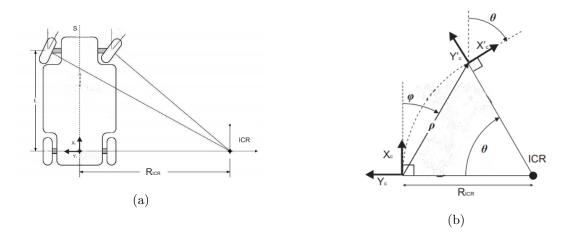
Q3) Suppose a 2D robot exists in space and it measures its distance from a set of n known landmarks located at  $l_1, l_2, ..., l_n$  using a range sensor. Each distance measurement  $r_i$  is given as  $r_i = ||\mathbf{x} - \mathbf{l_i}|| + \eta_i$ , where  $\mathbf{x}$  is the robot location and  $\eta_i$  is some unknown noise. Given these noisy measurements, describe in detail how you'd estimate the location of the robot. Derive any intermediate quantities needed. (5 points)

Q4) A sensor platform, consisting of a stereo camera and an inertial measurement unit (IMU), moves freely through a 3D scene. We identify the pose of the platform as the pose of the IMU. The rotation and translation of the camera relative to the IMU is known and is given by  $R_c^I$  and  $t_c^I$  respectively. We wish to estimate the pose of the platform with respect to some fixed world frame, using the sensor measurements as the platform moves around via an extended Kalman filter.

The scene contains known point landmarks on the floor which are observed by the stereo camera, as  $y_k = [u_{left}, v_{left}, u_{right}, v_{right}]^T$ , for every landmark at every time step k. The 3D locations of these landmarks are known in the world frame. The angular velocity and translational velocity of the platform,  $\omega_k$ ,  $v_k$ , relative to the IMU, are obtained from the IMU measurements for every time step k.

(a) What is the state vector and its dimension? (1 point) (b) Derive the translational and rotational motion models for the platform. (*Hint:*  $\Psi_k = \omega_k dt$ , where  $\Psi_k$  is the angle-axis representation for the rotation.) (2 points) (c) Derive a suitable observation model for the stereo camera. (2 points) (d) What are the dimensions of the observation noise covariance? (1 point) (e) What are the dimensions of the Kalman gain for a single landmark? (2 points) (f) Derive the Jacobian for the observation model. (2 points)

**Q5)** Consider a non-holonomic car that is equipped with a camera. The camera axes are denoted by  $X_c$  and  $Y_c$ , and is placed such that  $X_c$  is perpendicular to the rear-wheel axis as shown in figure (a). Due to the motion constraints of the car, the camera's motion can be perceived to be a circular motion, as shown in figure (b).  $X'_c$  and  $Y'_c$  are the camera axes in motion, whose origin with respect to the first frame, in polar coordinates, is at  $\rho, \phi$  and is rotated by  $\theta$ .



(a) Compute the essential matrix E for this configuration. (3 points) (b) How many parameters does it have? How many corresponding image points are required for estimating this E? It is not eight. (2 points)

**Q6)** (a) Consider a holonomic disc-shaped robot with diameter, say d. The robot is initially at start location and is required to reach the goal location while avoiding obstacles. In order to avoid collision, the robot must maintain at least l distance (where l = d/2) from its center to the obstacle surface. It considers to have reached the goal if its center coincides with the goal location. Write an RRT (unidirectional, and not goal biased) algorithm for the robot to find a path from start to goal. (3 points) (b) State the likely problems faced by basic RRT and goal biased RRT in the given environment. Also suggest improvements to be done to the algorithm to resolve these problems. (2 points)

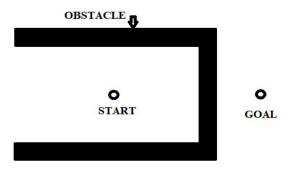


Figure 2: Robot environment

Q7) Consider a drone that starts from a location  $(x_0, y_0, z_0)$  which strives to reach  $(x_g, y_g, z_g)$ . (a) Formulate a quadratic goal reaching cost function that computes a set of controls to reach the goal. (1 points) (b) What are the constraints you may want to add to the above cost function. Do not worry about the obstacle avoidance constraint. Write down all other constraints that you feel are essential. Explain these constraints. (1 points) (c) Consider an obstacle of radius R, centered at  $(x_{ob}, y_{ob}, z_{ob})$ . Formulate an obstacle avoidance constraint that enables optimizing over a sequence of controls to eventually reach the goal. (1 points) (d) Linearize the obstacle avoidance constraint. (2 points)

### Extra space

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