Dynamics and Control - II

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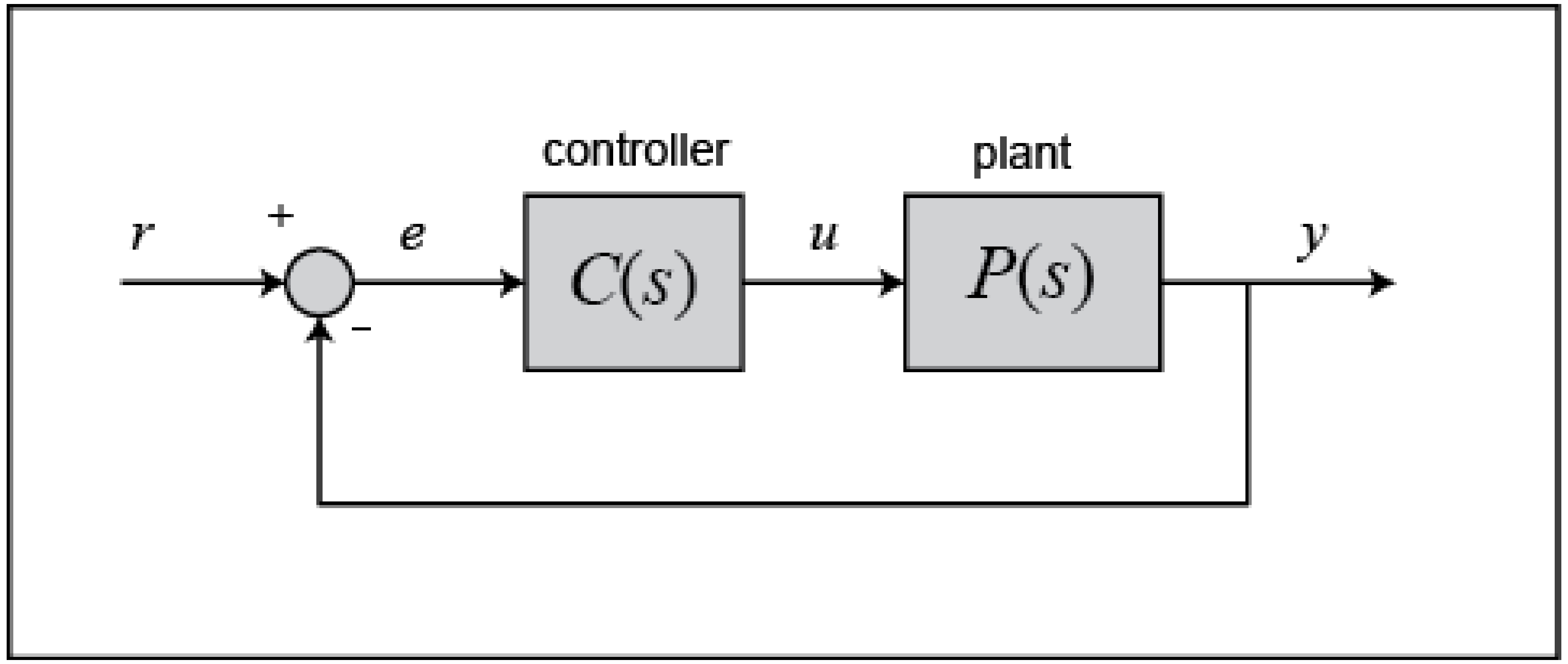
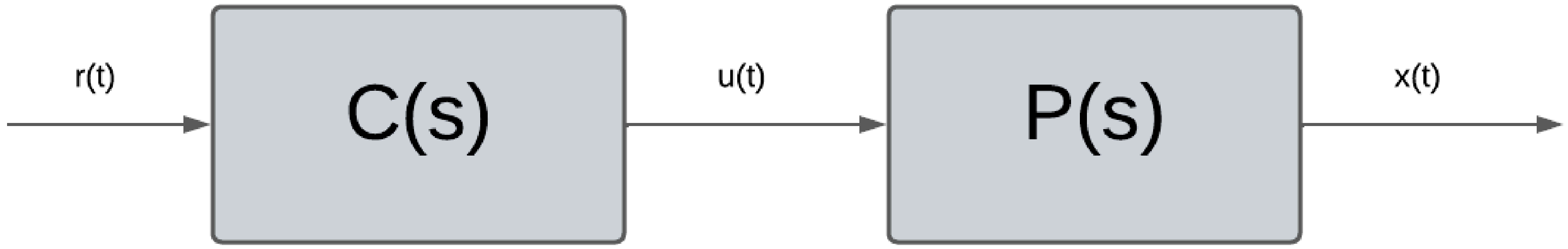
Plant and Controller



- Plant refers to any process that takes in an input and produces an output
- Relation between input and output is governed by the plant dynamics
- A controller is a plant which is designed to influence the (main) plant to obtain desirable results

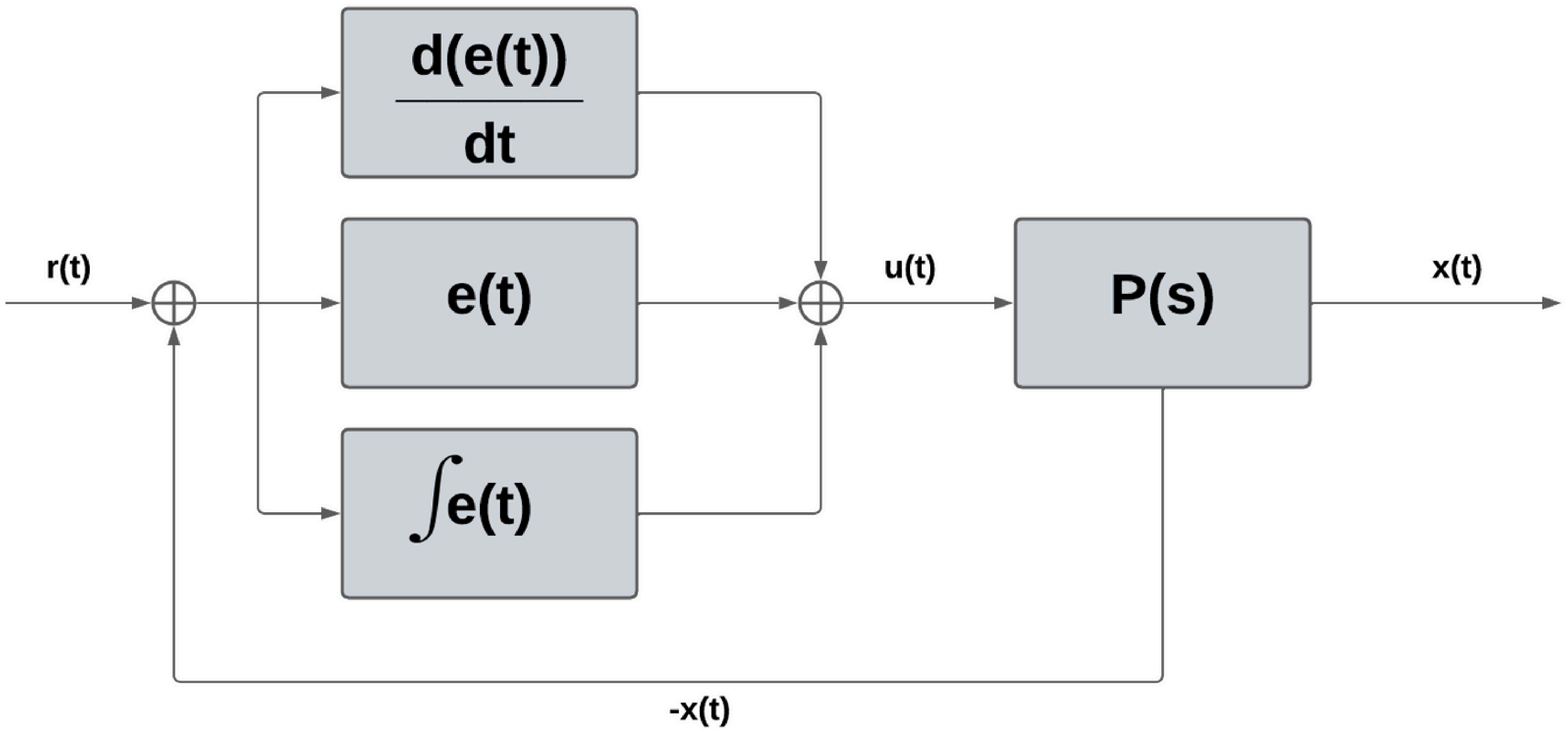
Open Loop Controller



- A controller without any feedback from the plant
- Generally uses information about plant dynamics to compute u(t)

Passively stable system

PID Controller



$$u(t) = kp * e(t) + kd * \frac{d(e(t))}{dt} + ki * \int e(t)$$

PID Controller

Proportional

- Responsible for making the system reach desired reference "asymptotically"
- Small values lead to a slower rise time and sluggish response.
- Large values lead to overshoot

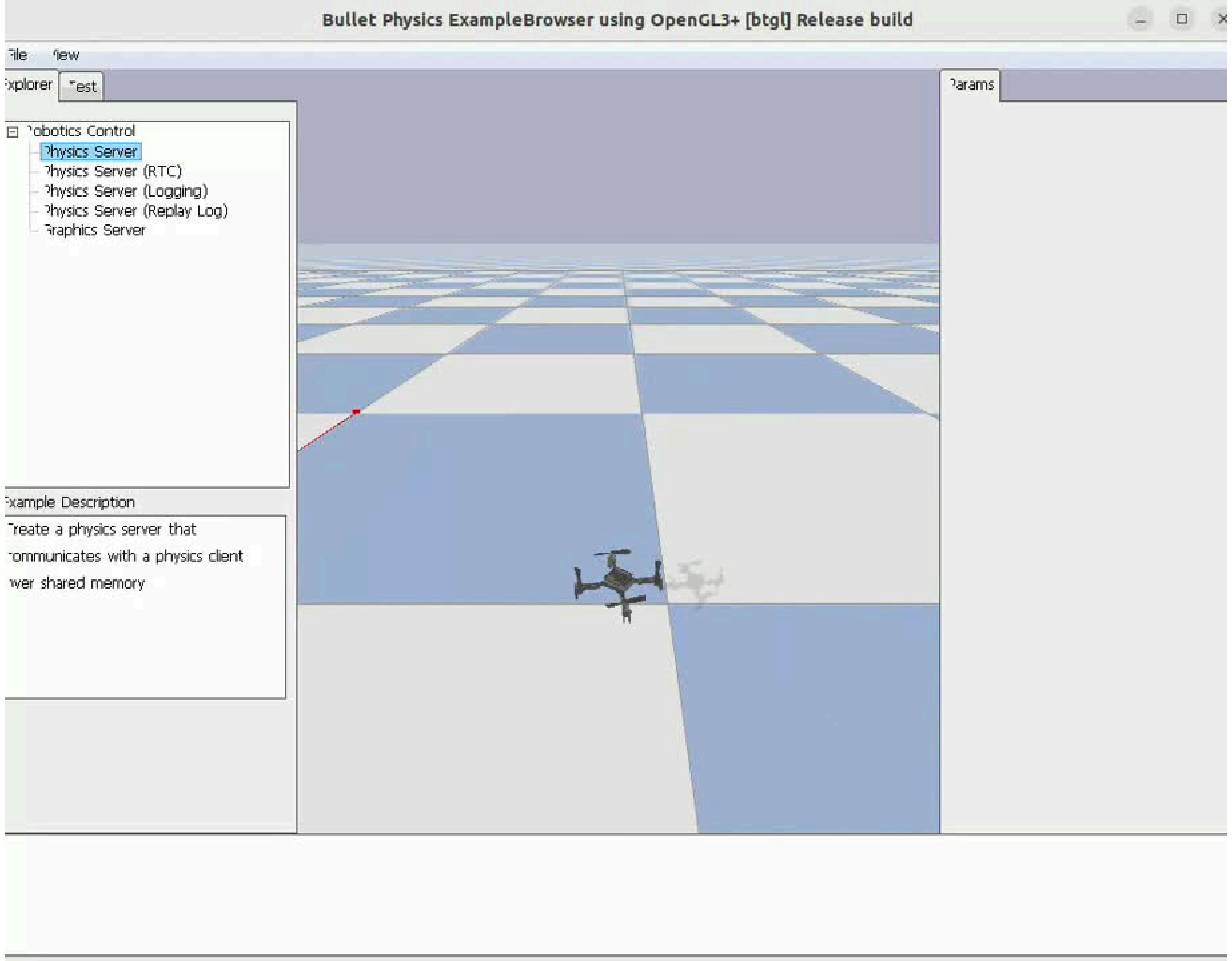
Integral

- Eliminates the steady state error present in the system.
- Small values lead to a slower removal of steady state error, generally safer
- Large values might lead to "integral windup", causing overshoot and oscillations, also saturations

Derivative

- Dampens the system by reacting to rate of error.
- Small values lead to under-damped behavior.
- Large values might lead to noisy and over-damped behavior, reducing rise time and performance

PID Controller



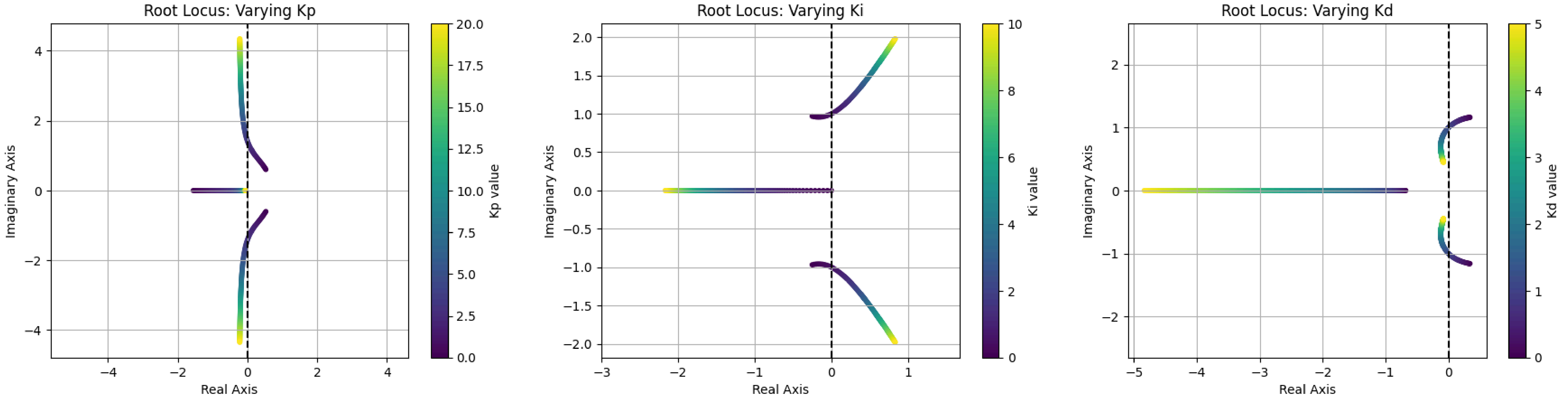
Control theory recap

$$H(s) = \frac{N(s)}{D(s)} = K\frac{(s-z_1)(s-z_2)\ldots(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_{n-1})(s-p_n)}$$

- Poles: Values of s that make the denominator D(s)=0
- Zeros: Values of s that make the numerator N(s) = 0

- Poles determine the system stability, natural frequencies, and response speed
- Zeros influence the shape and overshoot of the response

PID Controller: Cautionary tale about tuning gains



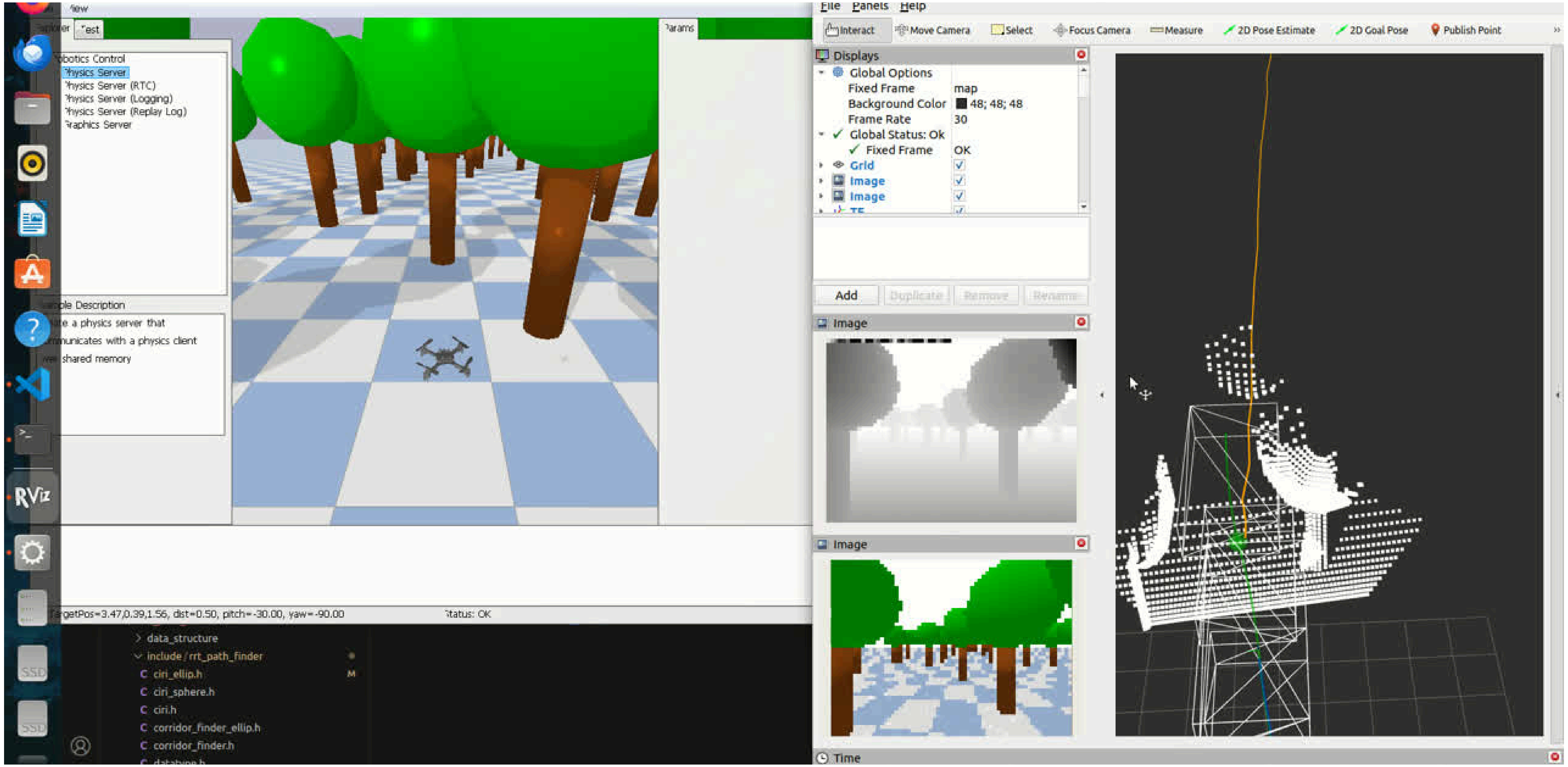
PID Controller: Cautionary tale about tuning gains

Before

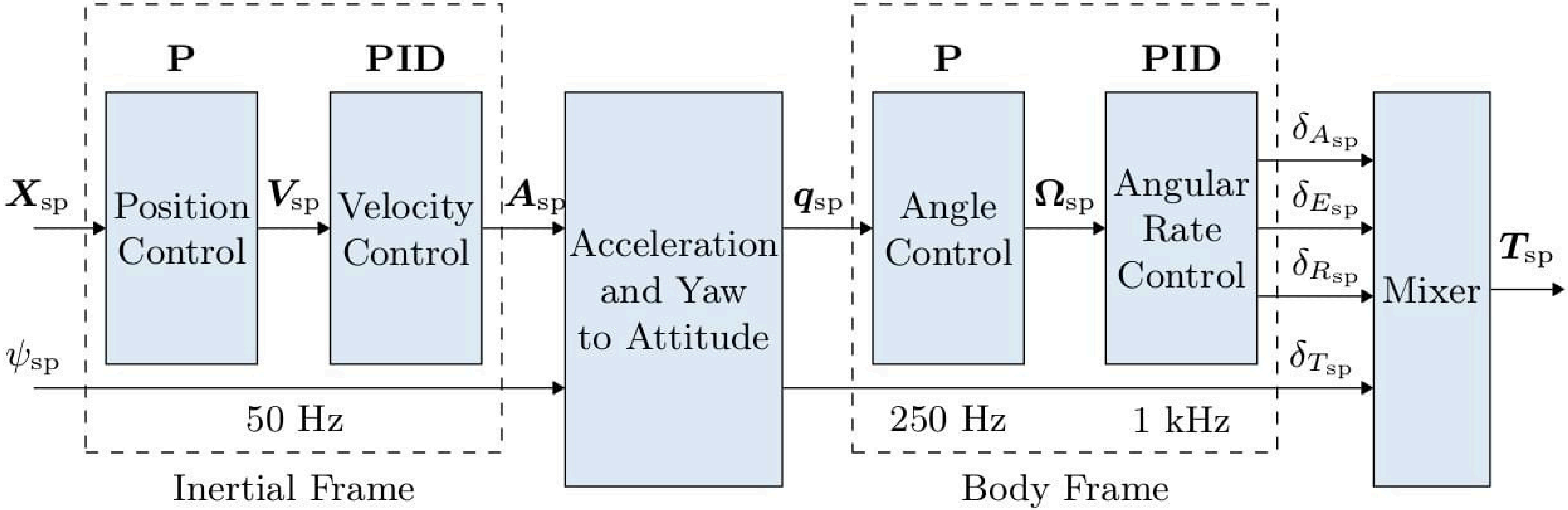
PID Controller: Cautionary tale about tuning gains

After

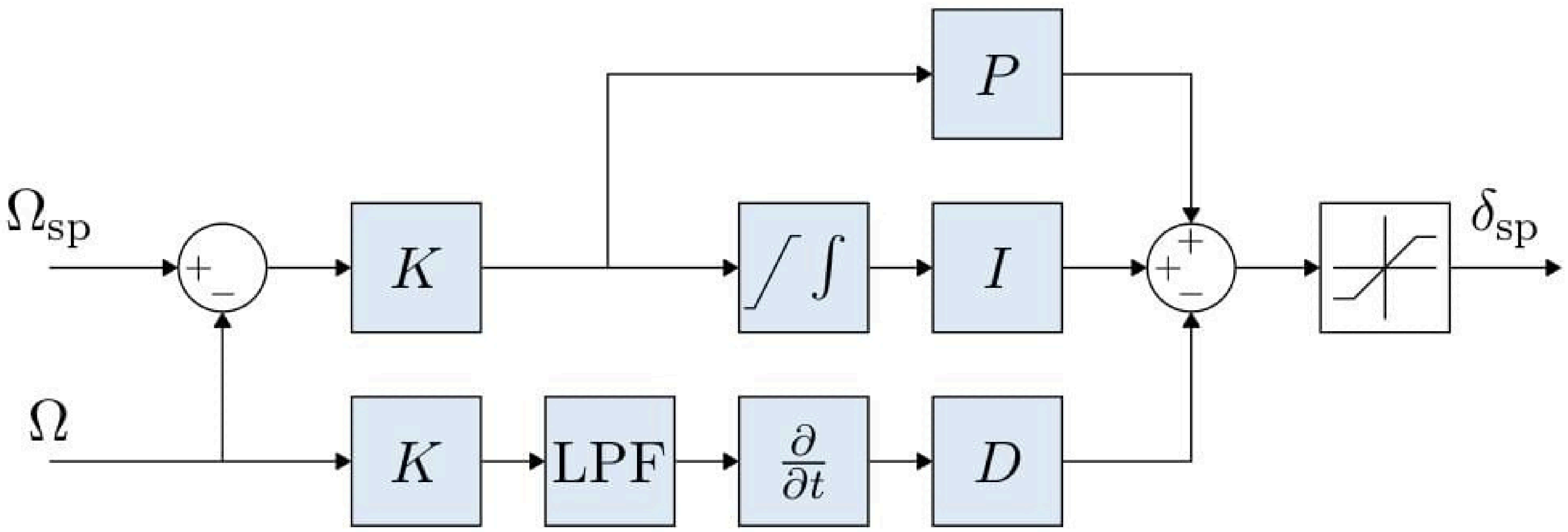
Multirotor Controller

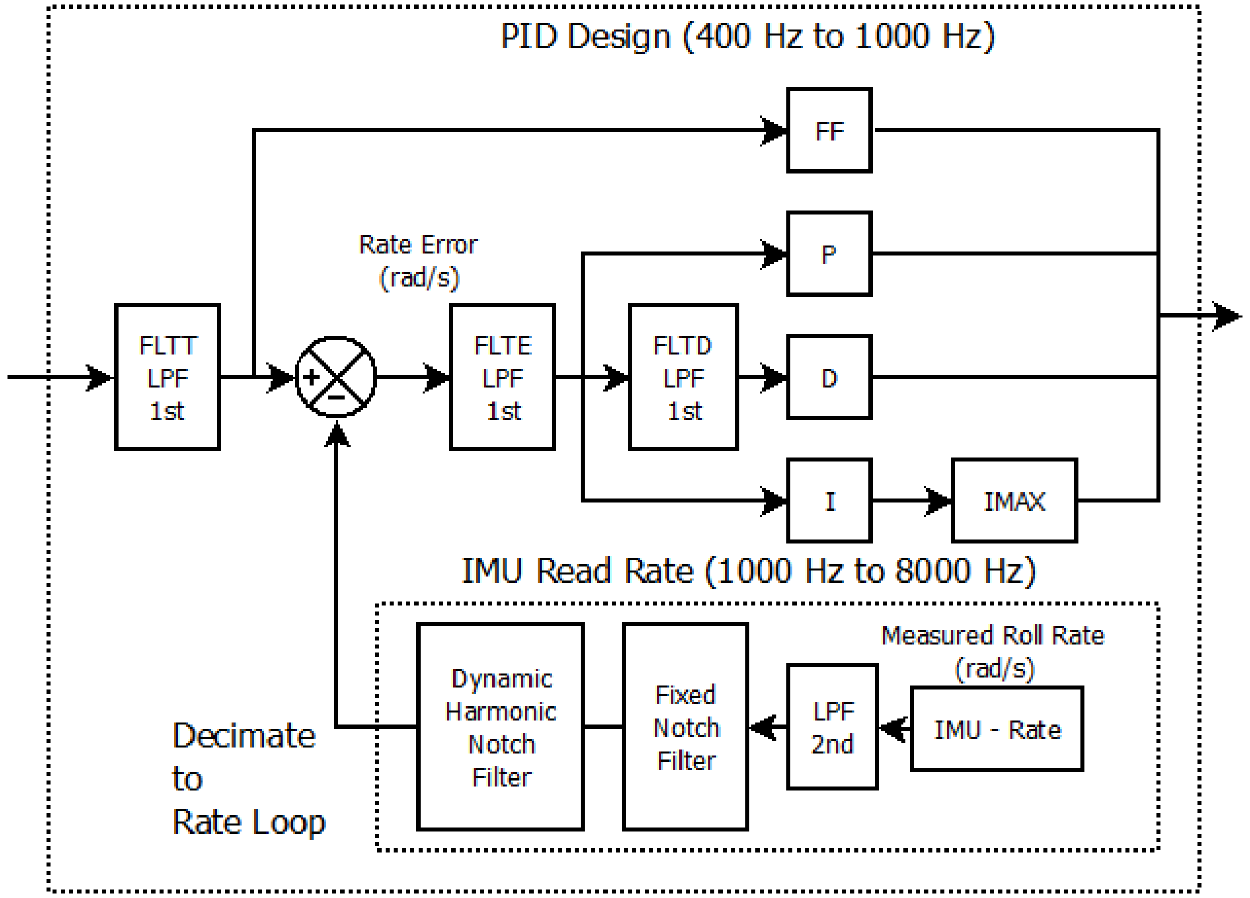


Multirotor Controller



Multirotor Controller: Inner loop rate tracking





Discrepancy: Angular rate derivative gain applied on sensor measurement rather than error between reference and measurement

Multirotor Controller: Acceleration and Yaw to quaternion reference

$$\mathbf{z}_b = \frac{-\mathbf{a}_{des}}{\|\mathbf{a}_{des}\|}$$

$$\mathbf{x}_c = \begin{bmatrix} \cos\psi_{des} \\ \sin\psi_{des} \\ 0 \end{bmatrix}$$

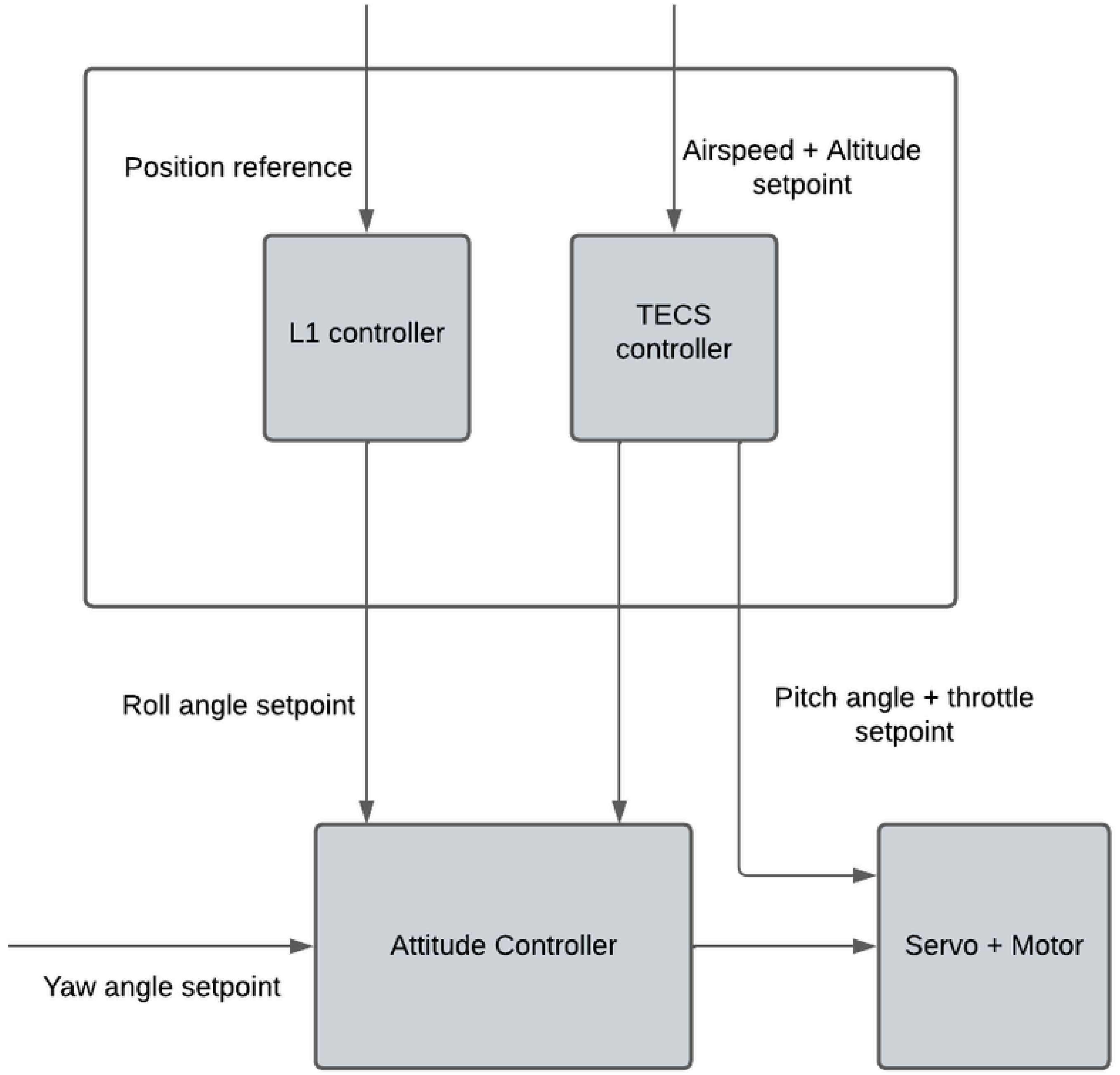
$$\mathbf{y}_b = \frac{\mathbf{z}_b \times \mathbf{x}_c}{\|\mathbf{z}_b \times \mathbf{x}_c\|}$$

$$\mathbf{x}_b = \mathbf{y}_b \times \mathbf{z}_b$$

$$\mathbf{R}_{des} = \begin{bmatrix} \mathbf{x}_b & \mathbf{y}_b & \mathbf{z}_b \end{bmatrix}$$

Note: Implementation present in Ardupilot AC_PosControl.cpp

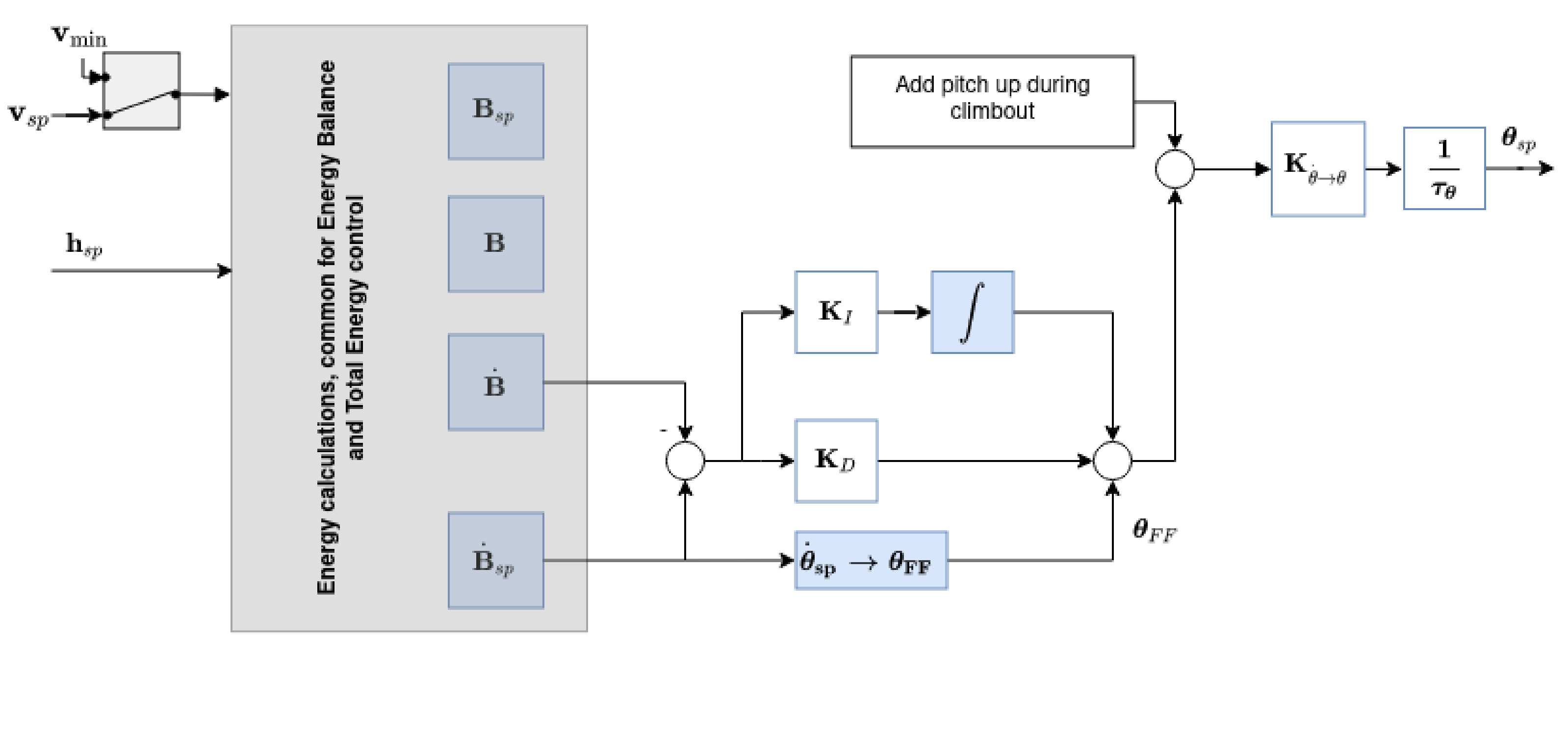
Fixed Wing Controller



- Position controller consisting of TECS and L1 controller for controlling longitudinal and lateral motion respectively
- Attitude references obtained are tracked via a cascaded PID, similar to multirotor

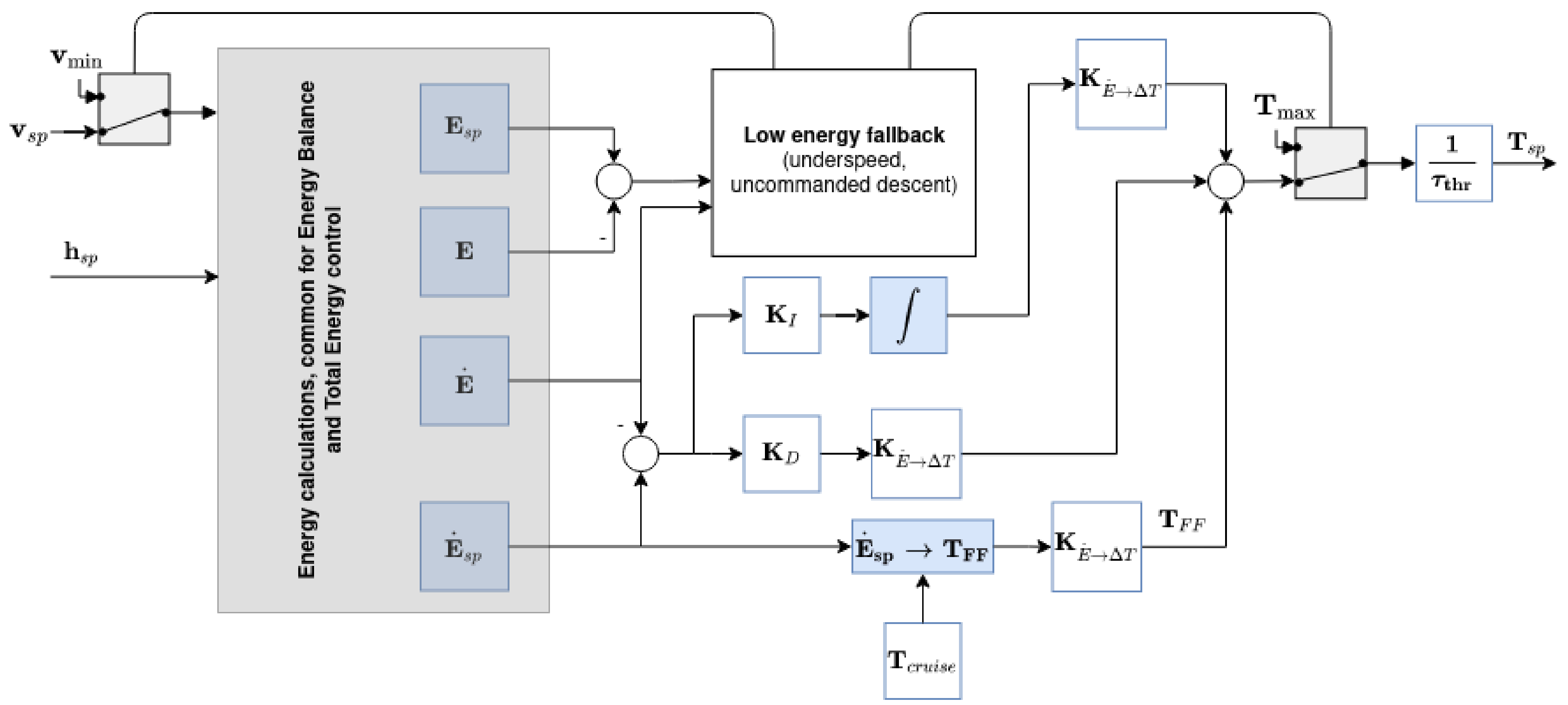
Fixed Wing Controller: In Action

Fixed Wing Controller: TECS



$\mathbf{h}_{sp}$	height setpoint (a.m.s.l) [m]	$\mathbf{B}_{sp}$	energy balance setpoint [m^2/s^2]	$\mathbf{K}_I$	pitch integrator gain	τ_θ	pitch time constant
$\mathbf{V}_{sp}$	TAS airspeed setpoint [m/s]	$\mathbf{B}$	energy balance [m^2/s^2]	$\mathbf{K}_D$	pitch damping gain	θ_{FF}	balance rate feedforward
		$\dot{\mathbf{B}}_{sp}$	energy balance rate setpoint	$\mathbf{K}_{\dot{B}\to\theta}$	energy balance rate to pitch gain	θ_{sp}	pitch setpoint
		$\dot{\mathbf{B}}$	energy balance rate [m^2/s^3]				

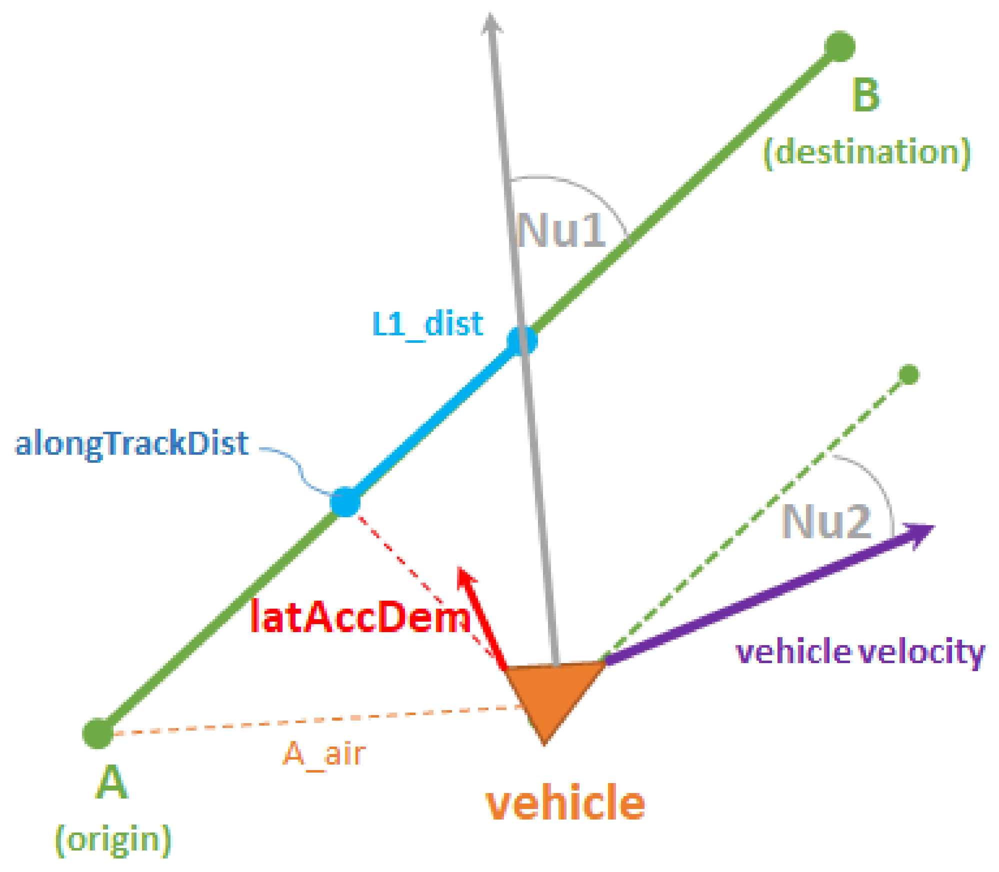
Pitch controller (Energy balancing)



$\mathbf{h}_{sp}$	height setpoint (a.m.s.l) [m]	$\mathbf{E}_{sp}$	energy setpoint [m^2/s^2]	$\mathbf{K}_I$	throttle integrator gain	τ_{thr}	throttle time constant
$\mathbf{V}_{sp}$	TAS airspeed setpoint [m/s]	$\mathbf{E}$	energy [m^2/s^2]	$\mathbf{K}_D$	throttle damping gain	$\mathbf{T}_{FF}$	Feedforward throttle
		$\dot{\mathbf{E}}_{sp}$	energy rate setpoint	$\mathbf{K}_{\dot{E}\to\Delta T}$	energy rate to throttle gain	$\mathbf{T}_{cruise}$	cruise throttle
		$\dot{\mathbf{E}}$	energy rate [m^2/s^3]			$\mathbf{T}_{sp}$	throttle setpoint

Throttle controller (Energy Addition)

Fixed Wing Controller: L1

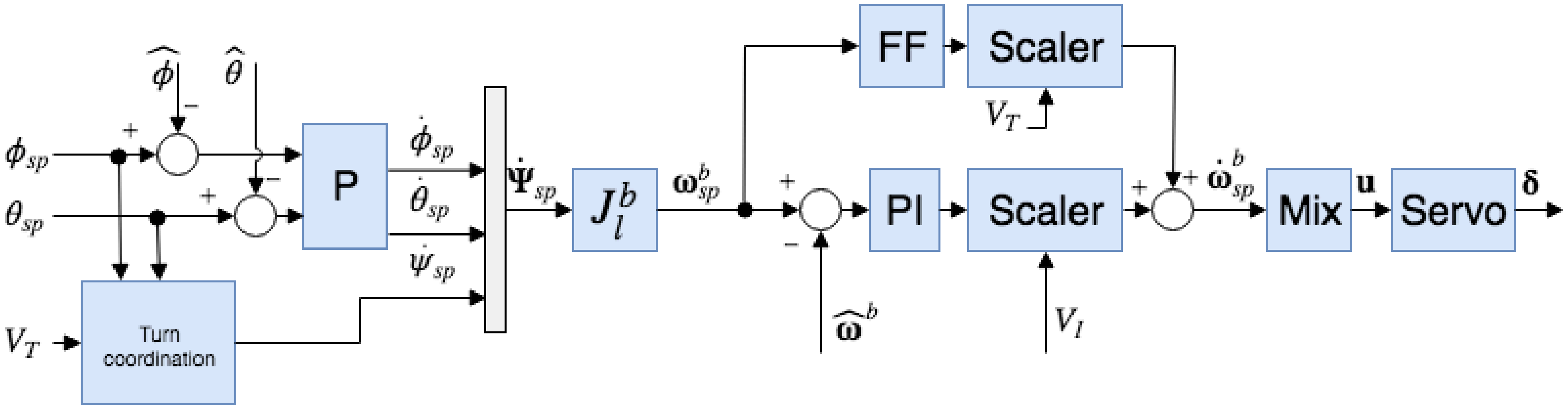


- L1 distance is a hyperparameter, defined as a look ahead distance for the UAV.
- Angle between LOS between L1 point and UAV and the UAV ground velocity is referred as η.

$$a_{\text{lat}} = 2\frac{V^2}{L_1}\sin(\eta)$$

$$\phi_{\text{cmd}} = \arctan\left(\frac{a_{\text{lat}}}{g}\right)$$

Fixed Wing Controller: Attitude Controller



$\mathbf{\Psi} = [\phi \;\; \theta \;\; \psi]^T$ - attitude vector (NED)

$\boldsymbol{\omega}^b$ - body rate vector (FRD)

$\mathbf{u}$ - actuators output

V_T - true airspeed

V_I - indicated airspeed

$\mathbf{J}_l^b$ - Jacobian matrix from local to body

$\boldsymbol{\delta}$ - actuator deflection

P - proportional gain

PI - proportional + integral controller

FF - feed-forward gain

Mix - mixer (control allocation)

Scaler - Scales controllers outputs using airspeed

$(\dot{x})$ - derivative of x

$\widehat{(x)}$ - estimated value of x (EKF)

$(x)_{sp}$ - setpoint of x

LQR control