Multiple-view Geometry 5

Bundle Adjustment

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Bundle Adjustment

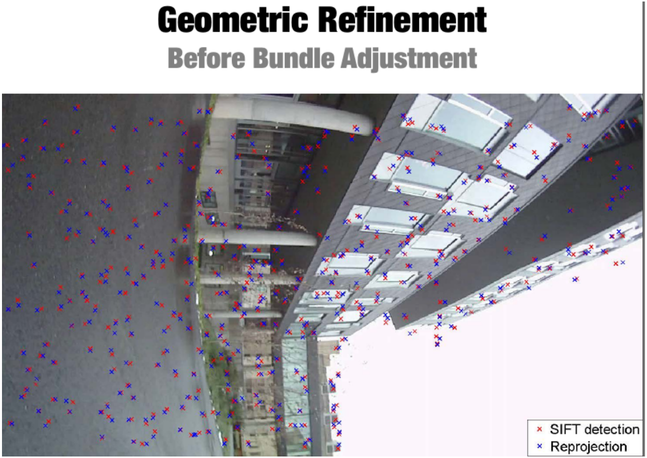
Bundle Adjustment

- Bundle adjustment is a **non convex** optimization problem that solves for camera parameters and 3D point locations of the environment.
- To reduce the noise and improve the accuracy of points and camera parameters, we run an optional iterative refinement step, called bundle adjusment.
- This is a non-linear optimization problem wherein we minimize the sum of the squared reprojection errors of the reconstructed N 3D points over M images.

$$arg \min_{X_j, P_i} \sum_{i=1}^{M} \sum_{j=1}^{N} \|x_{ij} - P_i X_j\|^2$$

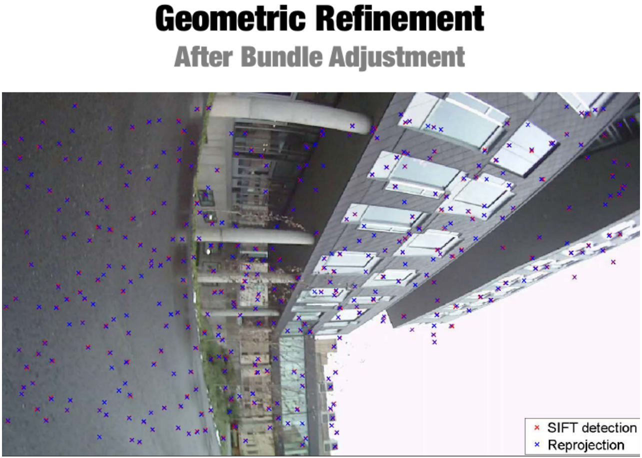
where P_i is projection matrix of the ith view, x_{ij} is the image(pixel locations) of the jth 3D point in the ith view.

Bundle adjustment



Courtesy: Jianbo Shi

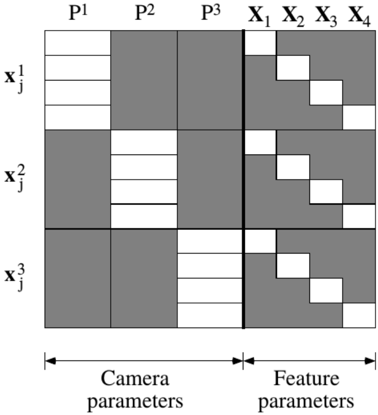
Bundle adjustment



Courtesy: Jianbo Shi

Bundle Adjustment

Structure of the Jacobian for a typical BA problem,



Here x_j^i is the projection of the j^{th} 3D point in the i^{th} image. Each x_j^i has u and v for image pixel locations.

Bundle Adjustment

Dimensions of the Jacobian matrix will be (2*M*N) x (12*M + 3*N)

- There are M images(views) having projections of N 3D points each (**assuming all the 3D points are visible in all views**). We multiply this with 2 as we have u and v which determine the location of the pixel coordinates (2*M*N).
- For each view we have to refine the projection matrix. Since there are M views the total elements are 12*M. Additionally, 3*N is added as there are N 3D points and each of them have 3 elements.
- The total parameters to refine for the bundle adjustment problem in this case is 11*M (projection matrix having 11 DOF) + 3*N.

Bundle Adjustment

Gauss-Newton iterative minimization

- Suppose we are given a hypothesized functional relation Y = f(P) where Y is the true value and we need to find P such that Y = f(P) + ϵ where $||\epsilon||_2$ is minimized.
- At every iteration we wish to find ΔP that minimizes $||\epsilon||_2$ using taylor series expansion and taking only the 1st order terms.

$$Y = f(P + \Delta P)$$

$$Y = f(P) + \frac{\delta f(P)}{\delta p_{11}^1}\Delta p_{11}^1 + \frac{\delta f(P)}{\delta p_{12}^1}\Delta p_{12}^1 + ... = f(P) + J\Delta P$$

$$D = Y - f(p) = J\Delta P$$

$$D = J\Delta P$$

$$J^T D = J^T J\Delta P$$

$$\Delta P = (J^T J)^{-1} J^T D$$

Bundle Adjustment

Gradient descent and Levenberg-Marquardt iterative minimization

- **Gradient descent** update rule for error D^2 is the following:

$$\Delta P = \frac{\delta D^2}{\delta P} = -2 \frac{\delta f(P)}{\delta P}^T D$$

$$\lambda \Delta P = J^T D$$

Note: D = Y - f(P). The λ compensates for the scalars obtained by differentiating.

- **Levenberg–Marquardt** iterative minimization combines Gauss Newton and Gradient descent.

$$(J^T J + \lambda I)\Delta P = J^T D$$

$$\Delta P = (J^T J + \lambda I)^{-1} J^T D$$

Study material

References

- Iterative minimization: Multiple-View Geometry - Richard Hartley, Andrew Zisserman
- Bundle adjustment: Cyrill Stachniss

Study material
Courses

- Photogrammetry I & II - Cyrill Stachniss
- Multiple-view Geometry - Daniel Cremers
- Vision algorithms for mobile robots - Davide Scaramuzza