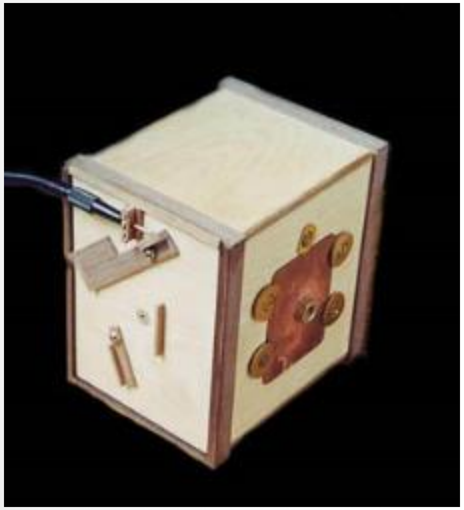
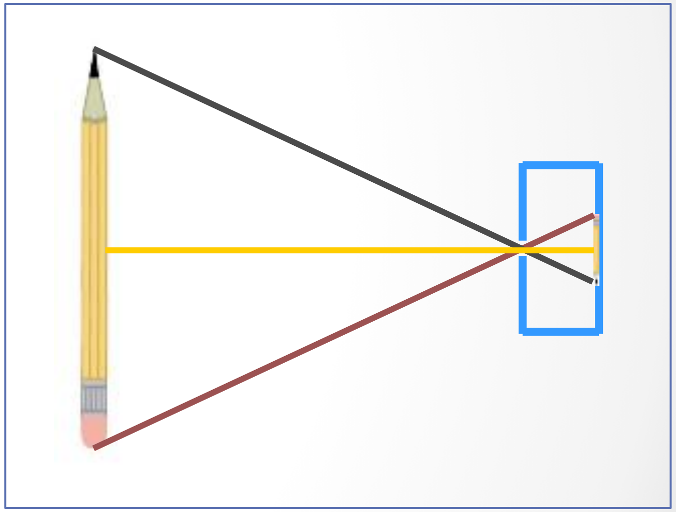
CSE578: Computer Vision

Spring 2020

Imaging and Camera Model





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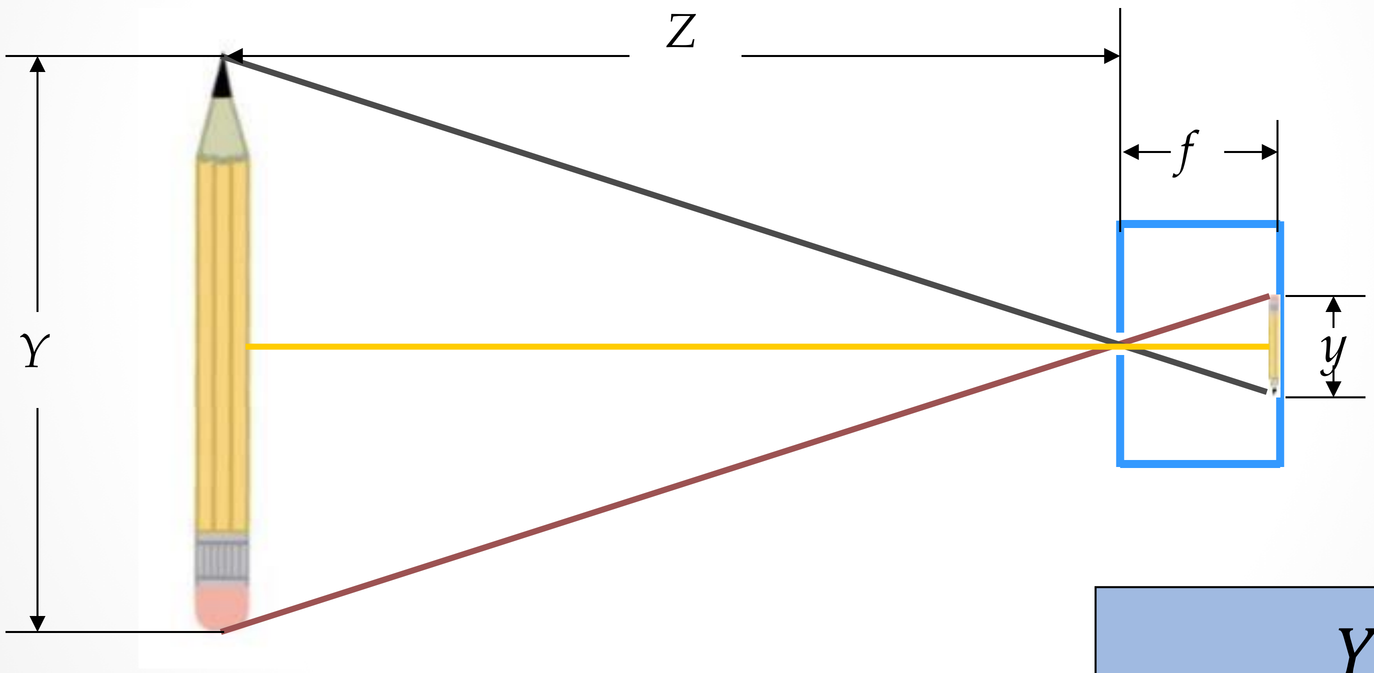
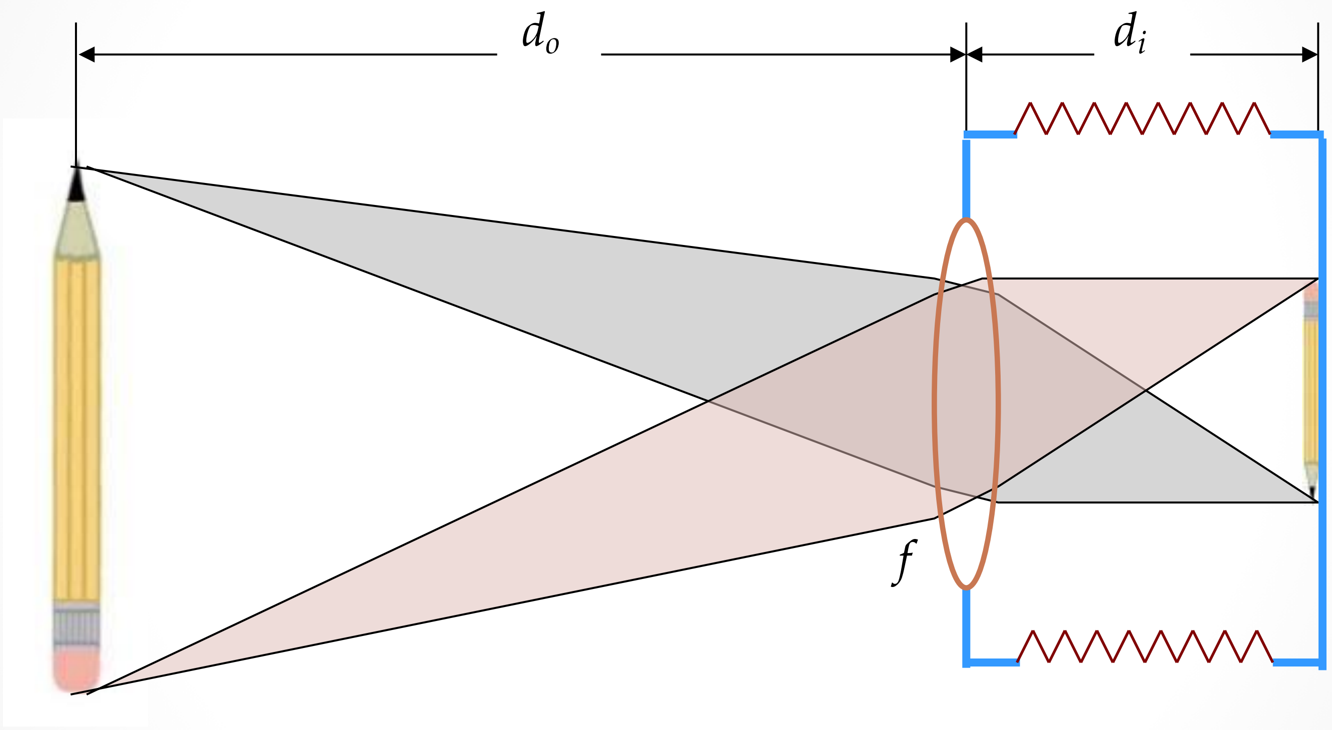
The Pinhole Camera



$$y = f\frac{Y}{Z}$$

Camera with Lens

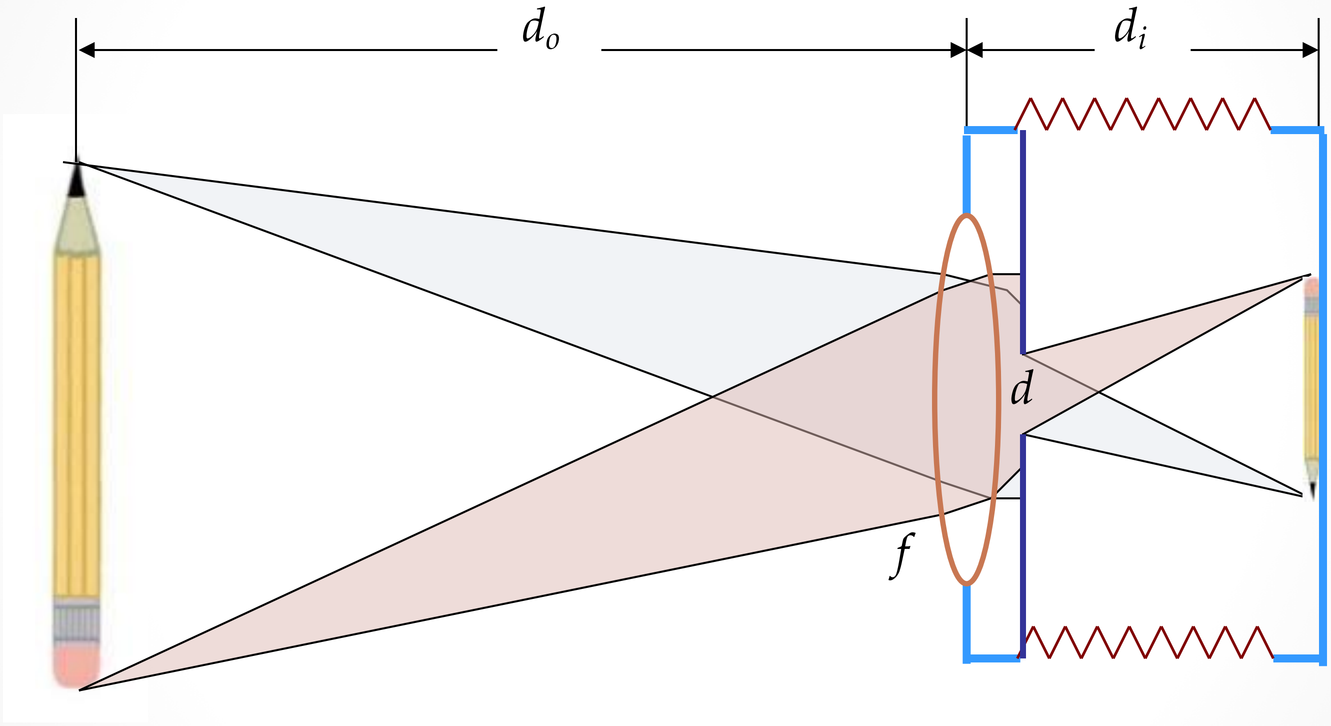


Thin lens equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

$$d_i = f\frac{d_o}{(d_o - f)}$$

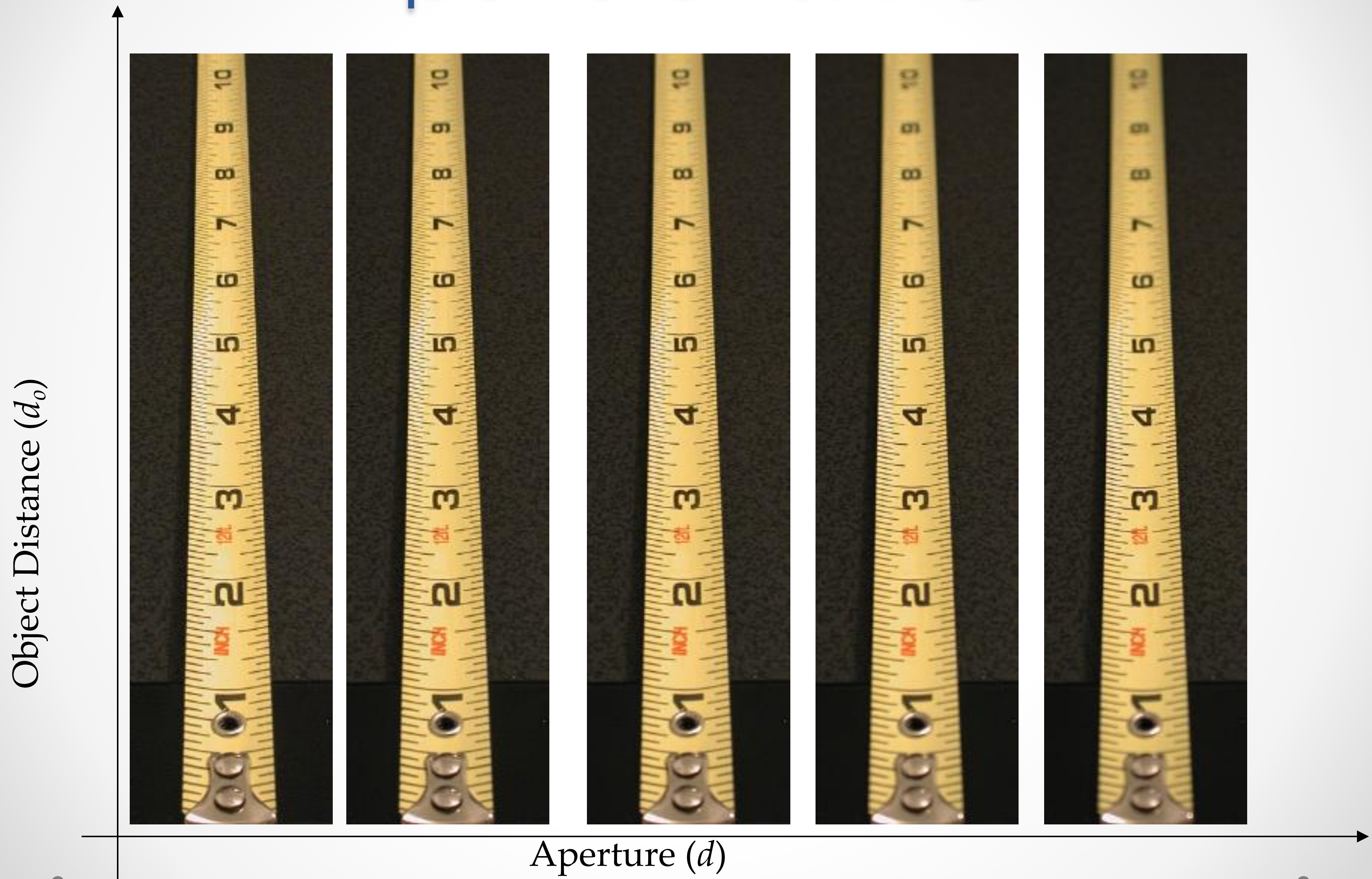
Focus and DOF

Aperture

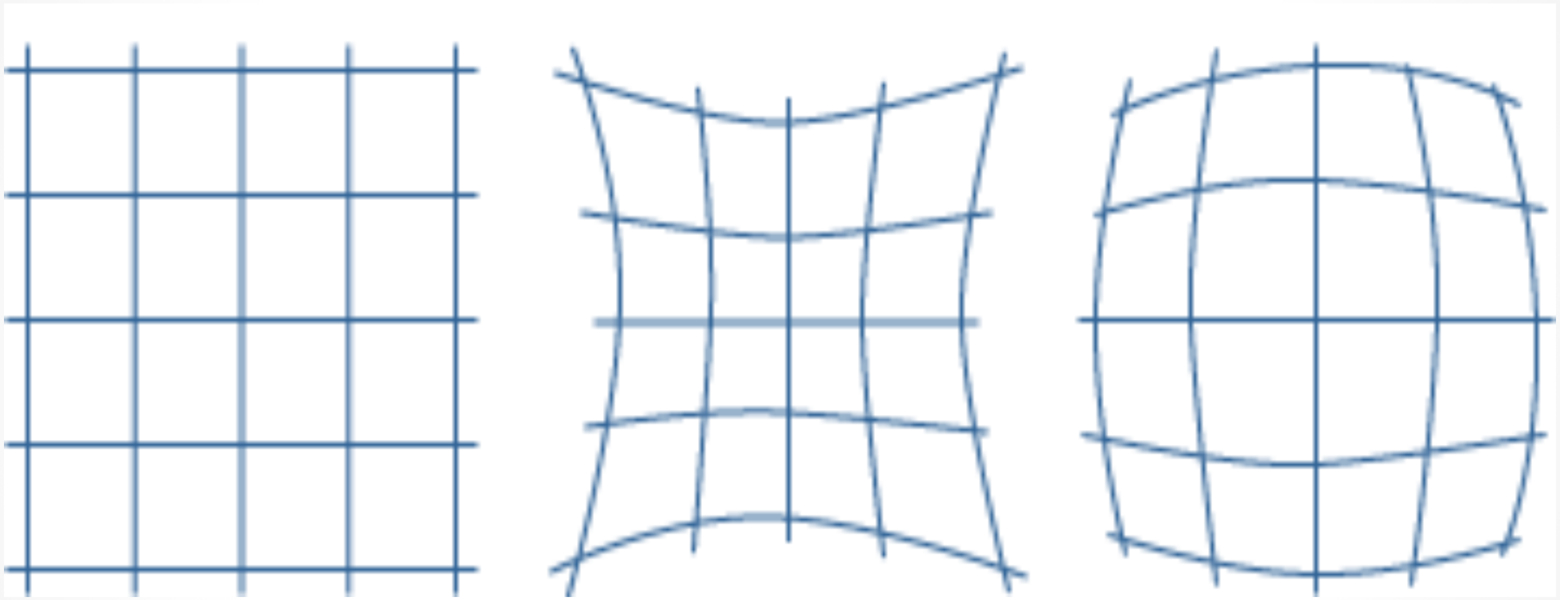


Focal Ratio = $f\,/\,d$

Aperture vs. DOF



Geometric Distortions



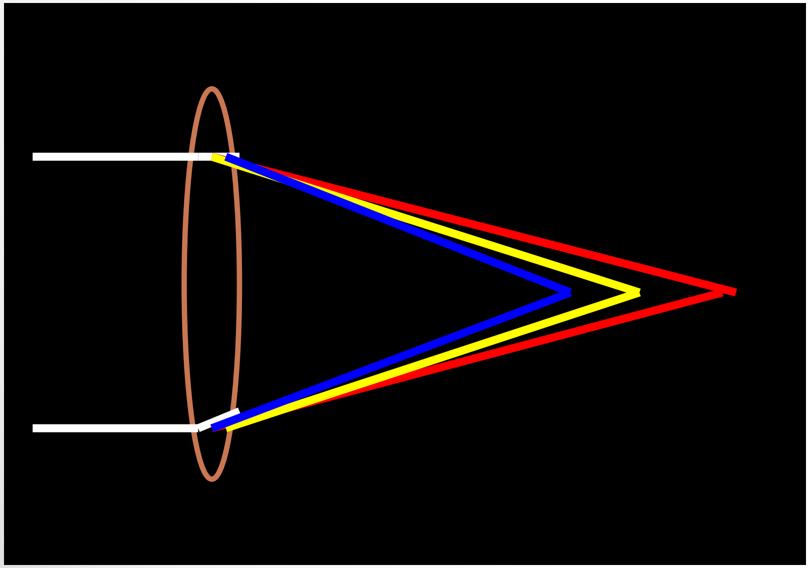
original pincushion barrel

Geometric Distortions

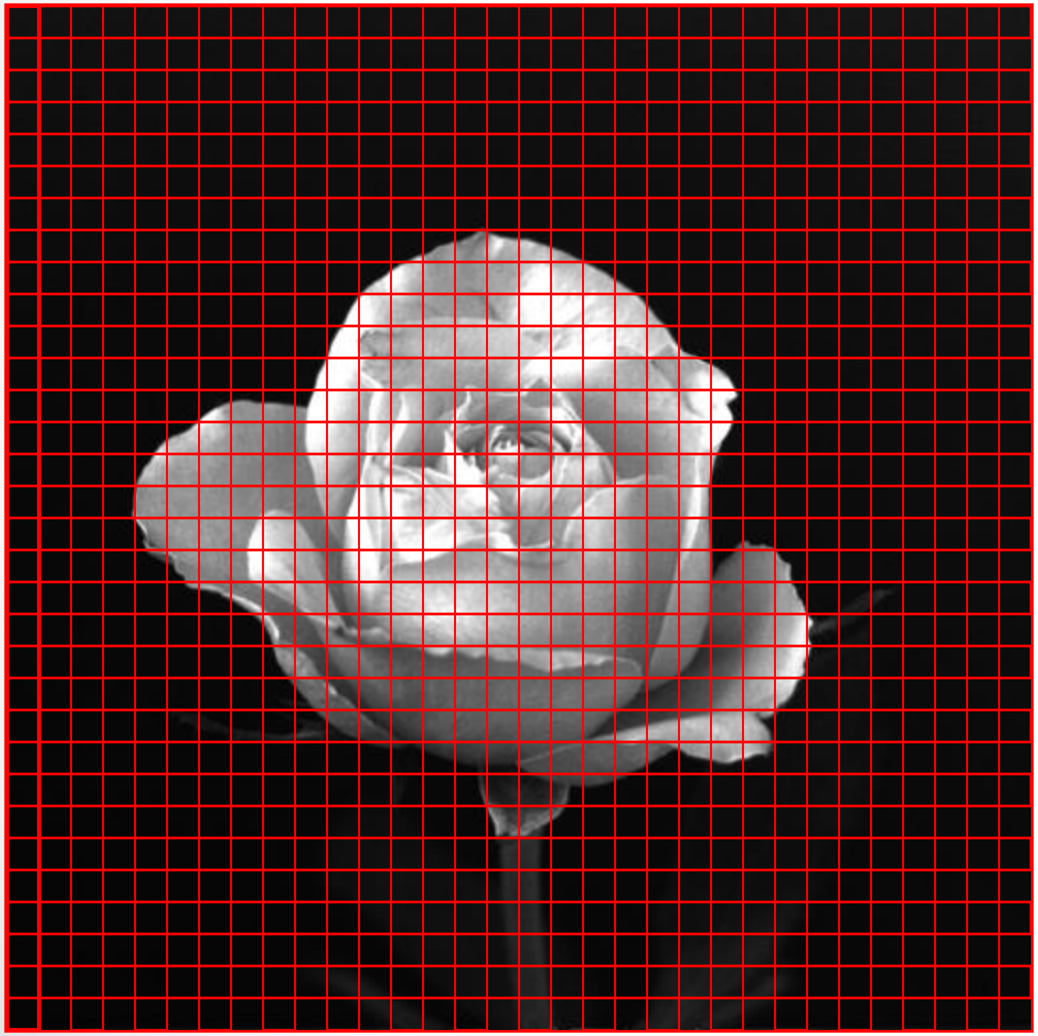
Lens Flare

Chromatic Aberration

Normal lenses diffract different wavelengths to different degree



Sampling an image



Resolution

- The number of samples in an image (number of sensor elements) is referred to as its resolution
- The resolution is typically represented as the product of number of samples in the horizontal and vertical directions in the image. e.g.: 32x32, 256x256, 640x480

<u>Common Resolutions:</u>

NTSC:	648 x 486
Typical Webcam:	1280 x 720
High-end camera (A7RIV):	9504 x 6336
Hubbles Telescope:	1,600 x 1,600

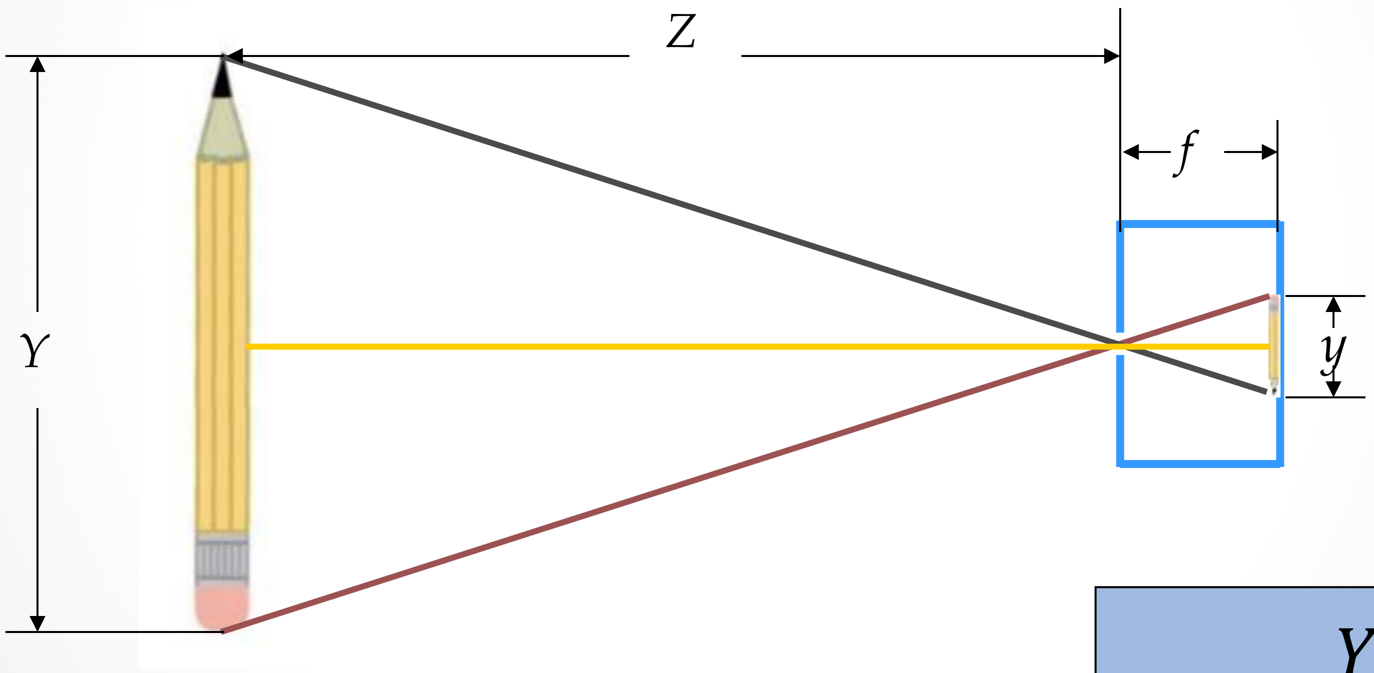
Camera Module: Objectives

- Mathematically model what a camera does
  - Also understand what the model means
- Getting the model for a real-world camera
  - Estimation from real world measurements
- Special imaging configurations with simpler properties
  - Simpler relationships
- General theory on fitting linear models under noisy observations
  - Techniques that work across problems

What does a Camera do?

- Form an image on the 2D image plane of the 3D world visible to it.
- Image is *behind* the lens; the scene is in front.
- 3D world is **projected** down to a 2D plane.
- Significant loss of information as one dimension is dropped.
- Mathematical depiction of this projection ...
-

We Assume a Pinhole Camera

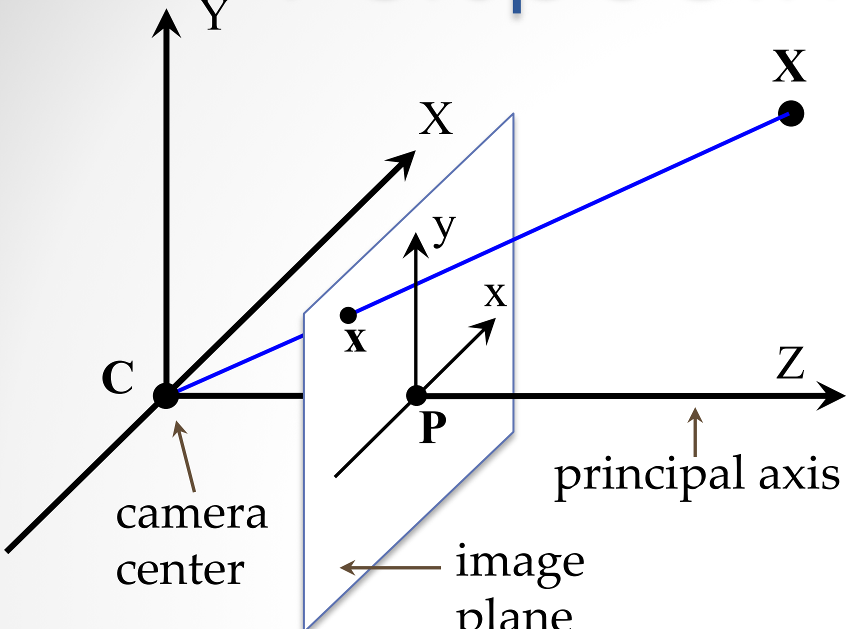


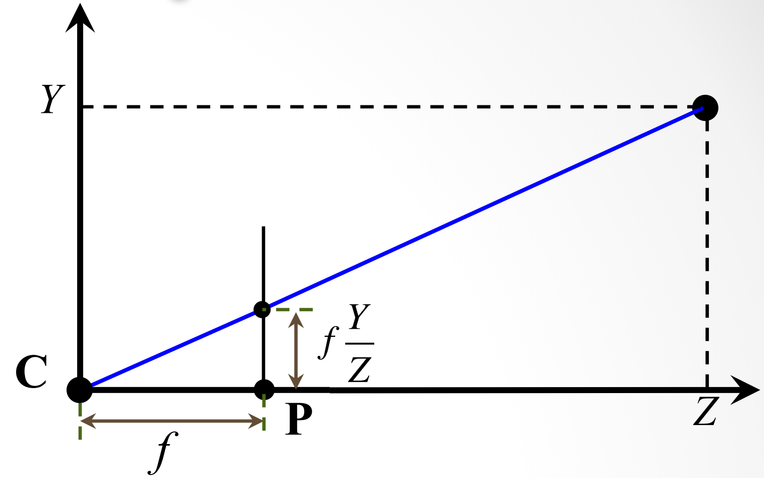
$$y = f\frac{Y}{Z}$$

Problem

- You have a person who is **1.75m** tall standing at a distance of **7m** from a camera. The camera has a focal length of **50mm**. The sensor is 3cm tall and has a resolution of **4000x3000**.
  - Find the height of the person in pixels in an image.
  - If the camera is raised by **1m**, how much does the person move in the sensor (in pixels)?
  - How much does the Sun move in the above case Note: Sun is **150 million kms** away (in pixels)?

Perspective Projection





- Cartesian image coordinates: $x = f\dfrac{X}{Z}, \qquad y = f\dfrac{Y}{Z}$
- In matrix form (homogeneous):

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{PX}$$

Basic Camera Equation

A pinhole camera projects a 3D point $\mathbf{X}_c$ in camera coords to an image point $\mathbf{x}$ via the 3x4 camera matrix $\mathbf{P}$ as:

$$\mathbf{x} = \mathbf{P}\mathbf{X}_c = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [\mathbf{I}|\mathbf{0}]\mathbf{X}_c = \mathbf{K}[\mathbf{I}|\mathbf{0}]\mathbf{X}_c,$$

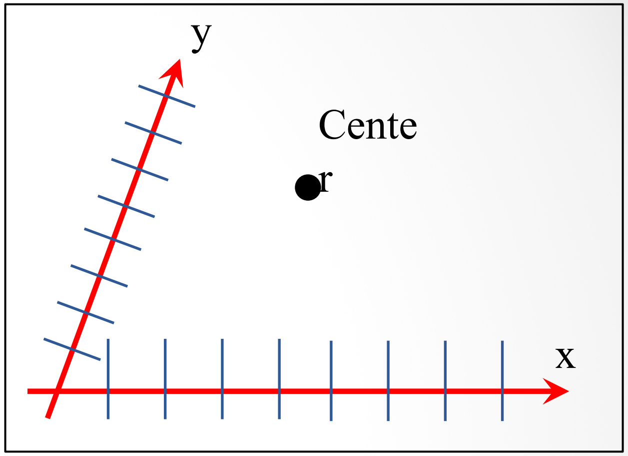
where $\mathbf{K}$ is the internal camera calibration matrix.

Note that:

- The camera is at the origin
- Z is the Camera or Optical axis
- Principal Point: Center of the image
- Focal length in pixel units
- Orthogonal image axes with uniform scale

A General Camera

Image center at (x_0,y_0), Non-orthogonal axes with skew s, and different scales for axes with focal lengths, α_x and α_y.

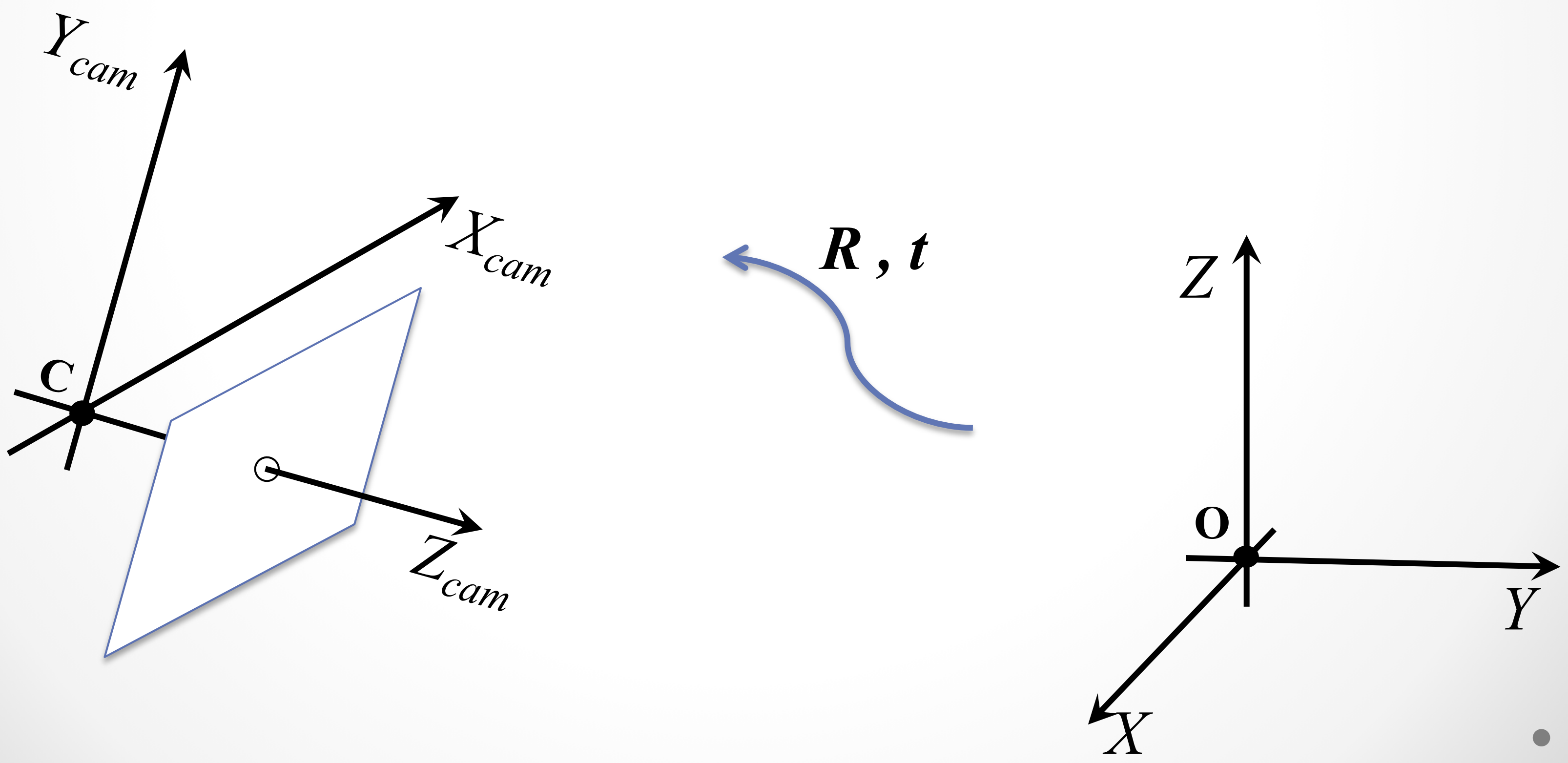


$$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{K}$ an upper diagonal matrix with 5 degrees of freedom.

Moving the Camera from Origin

- General Setting: Camera is not at origin and Z is not the optical axis.
- Camera is at a point C in world coordinates. The camera axes are also rotated by a matrix R.



General Camera Equation

- Camera and world are related by:

$$\mathbf{X_c} = \begin{bmatrix} \mathbf{R} & \mathbf{-RC} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \mathbf{X_w}$$

- 2D projection $\mathbf{x}$ of a 3D point $\mathbf{X_w}$ given by:
  - $\mathbf{x = K[\ I \mid 0\ ]\ X_c = K\ [\ R \mid -RC\ ]\ X_w}$
- $\mathbf{x = PX_w}$; camera matrix $\mathbf{P = [KR \mid -KRC] = [M \mid p_4]}$
- Common K: General K:

$$\begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$