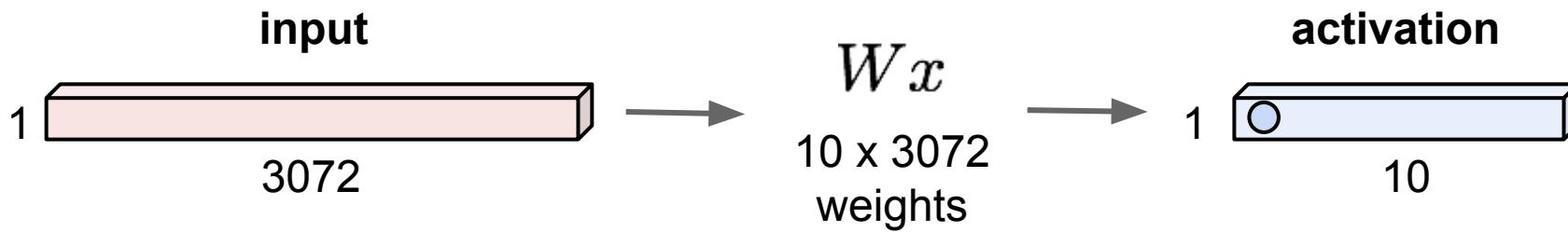


Lecture 5: Convolutional Neural Networks

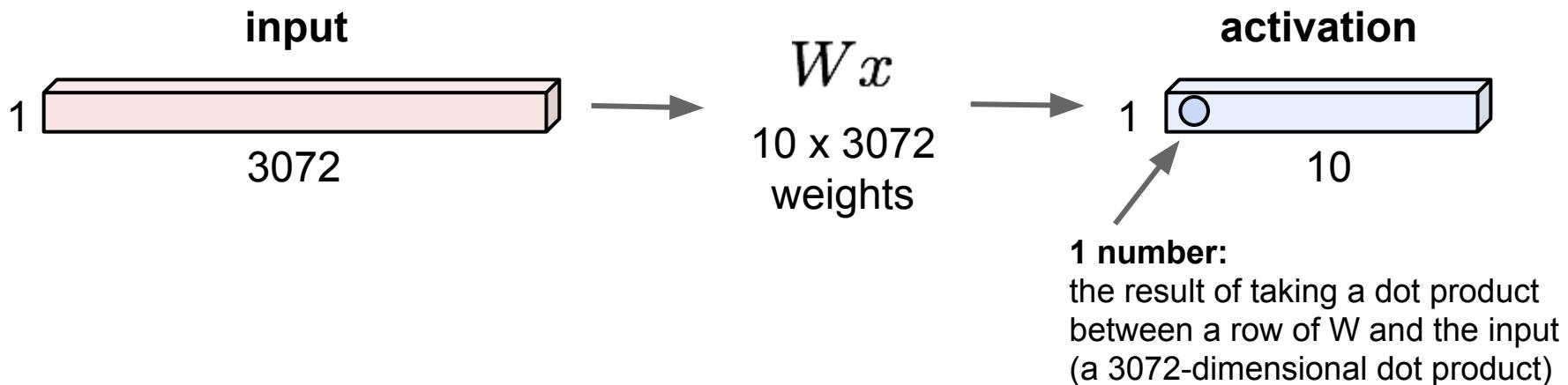
Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



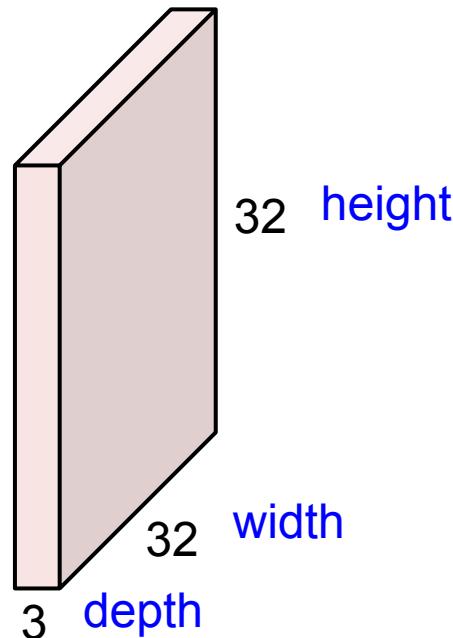
Fully Connected Layer

32x32x3 image -> stretch to 3072×1



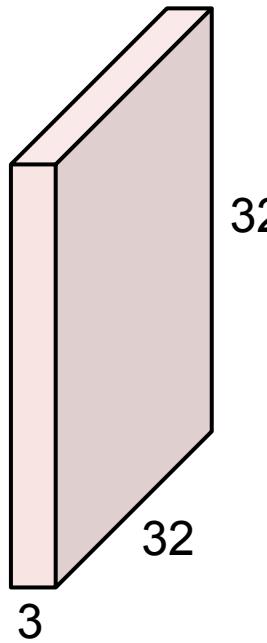
Convolution Layer

32x32x3 image -> preserve spatial structure

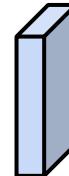


Convolution Layer

32x32x3 image



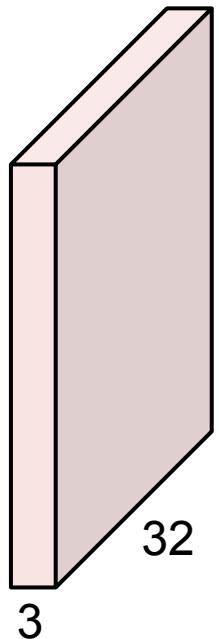
5x5x3 filter



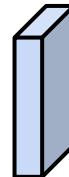
Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolution Layer

32x32x3 image



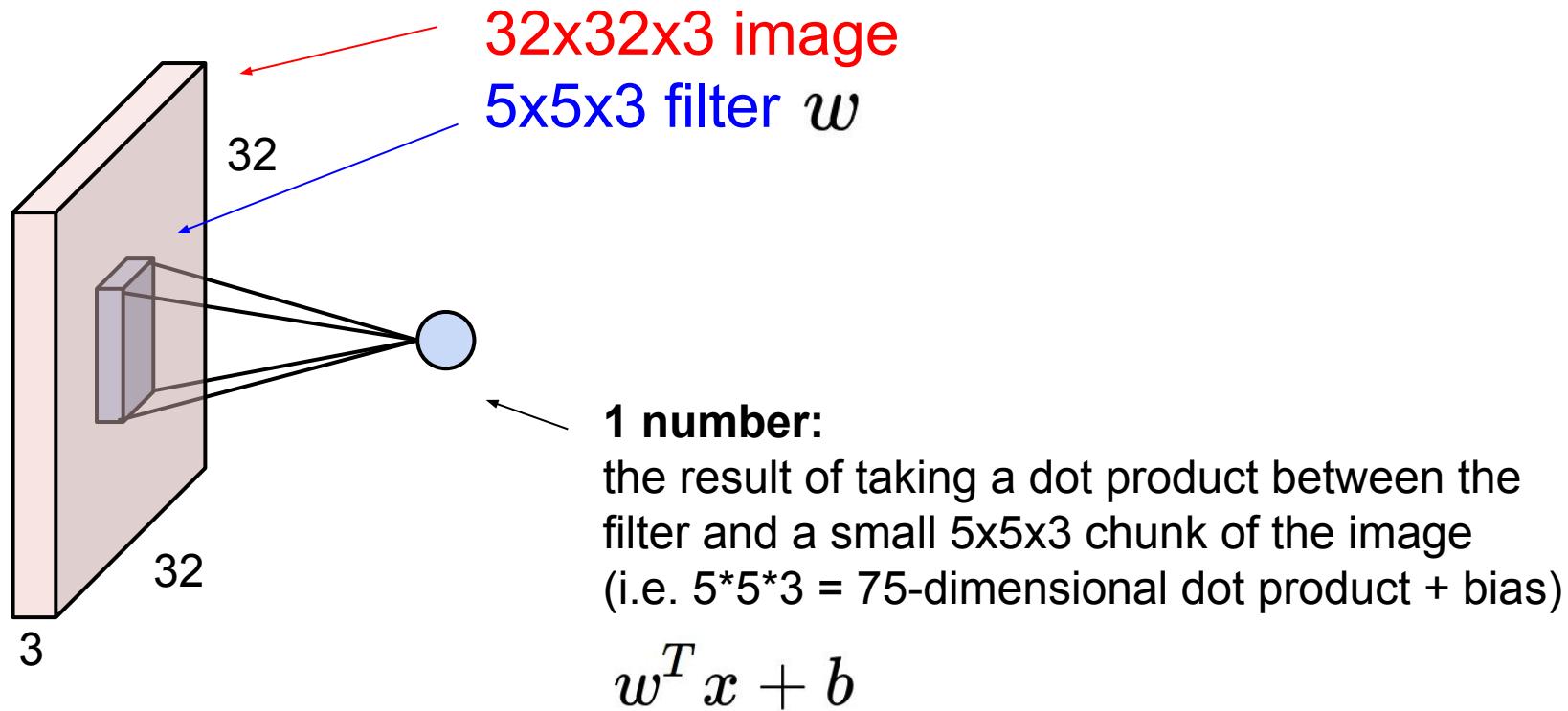
5x5x3 filter



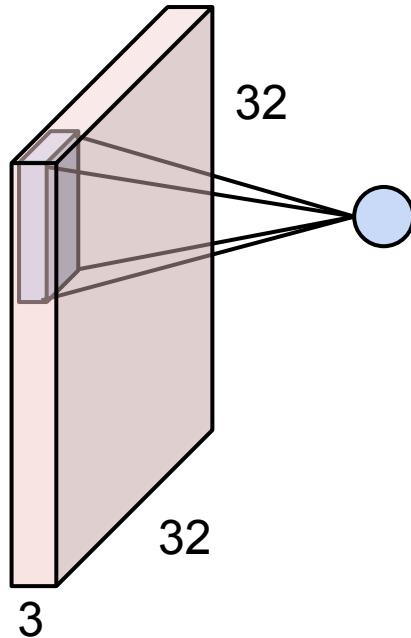
Filters always extend the full depth of the input volume

Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

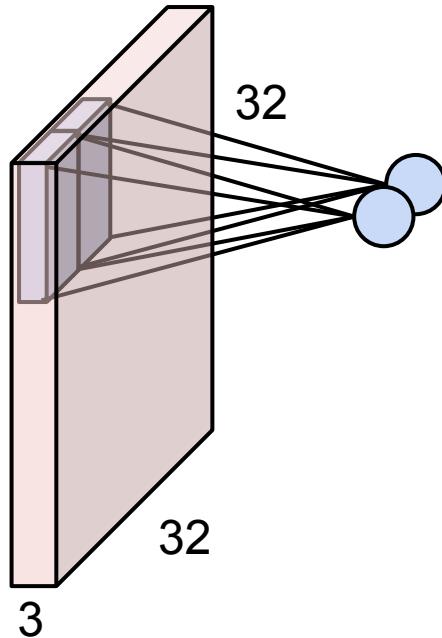
Convolution Layer



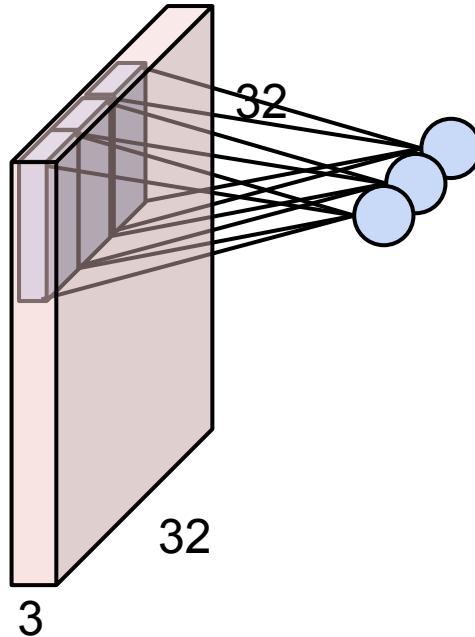
Convolution Layer



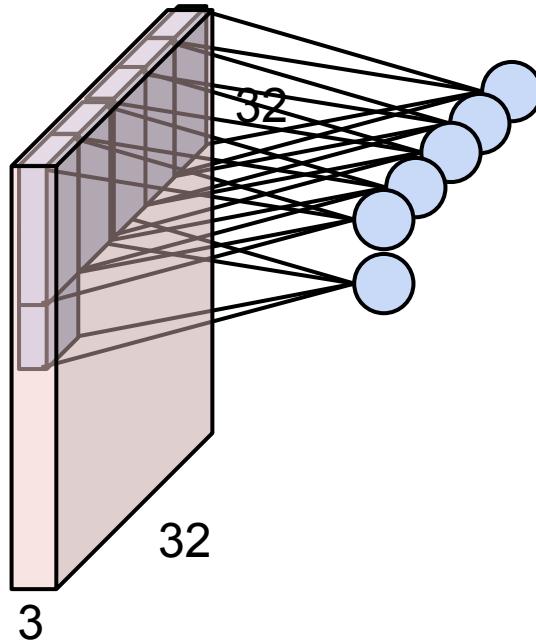
Convolution Layer



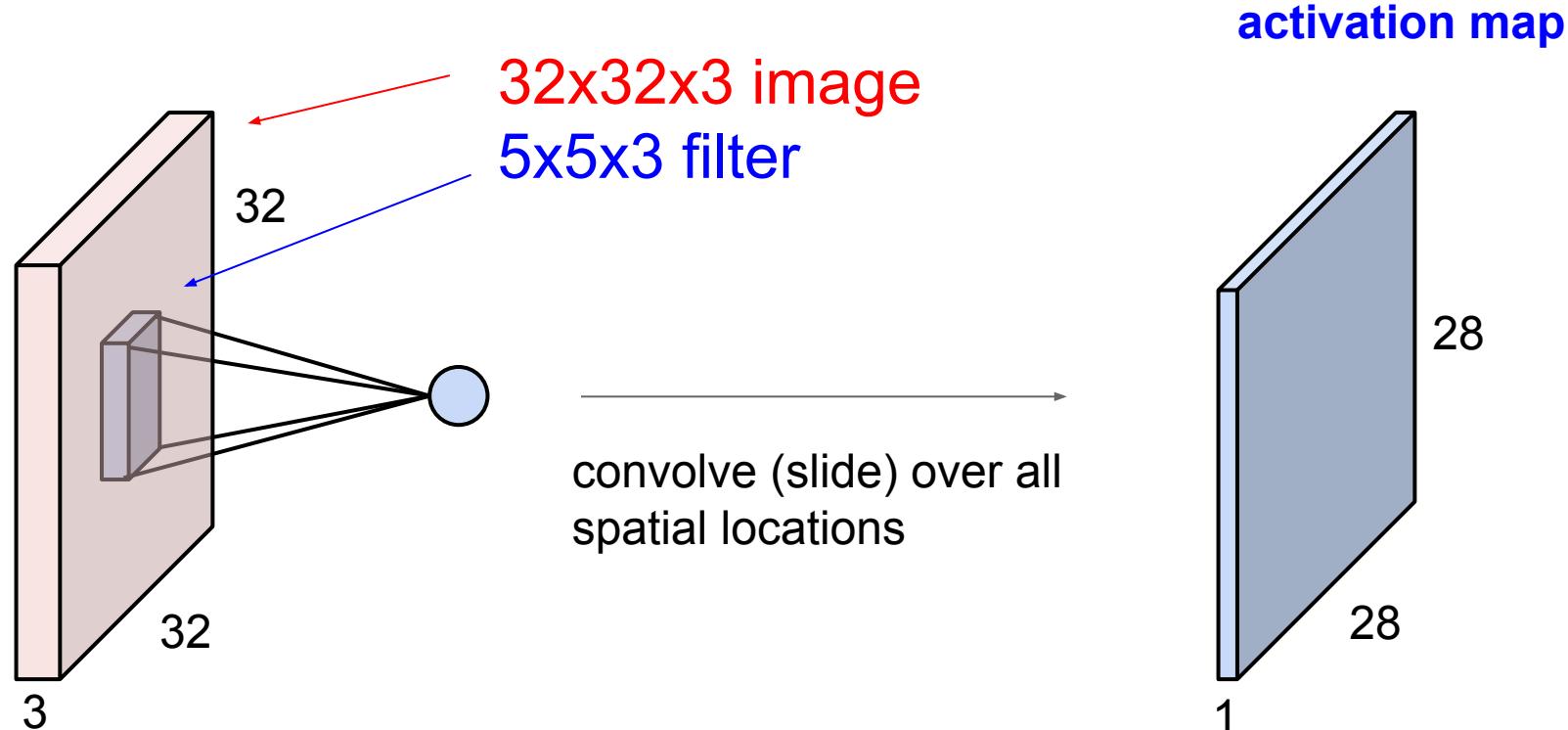
Convolution Layer



Convolution Layer

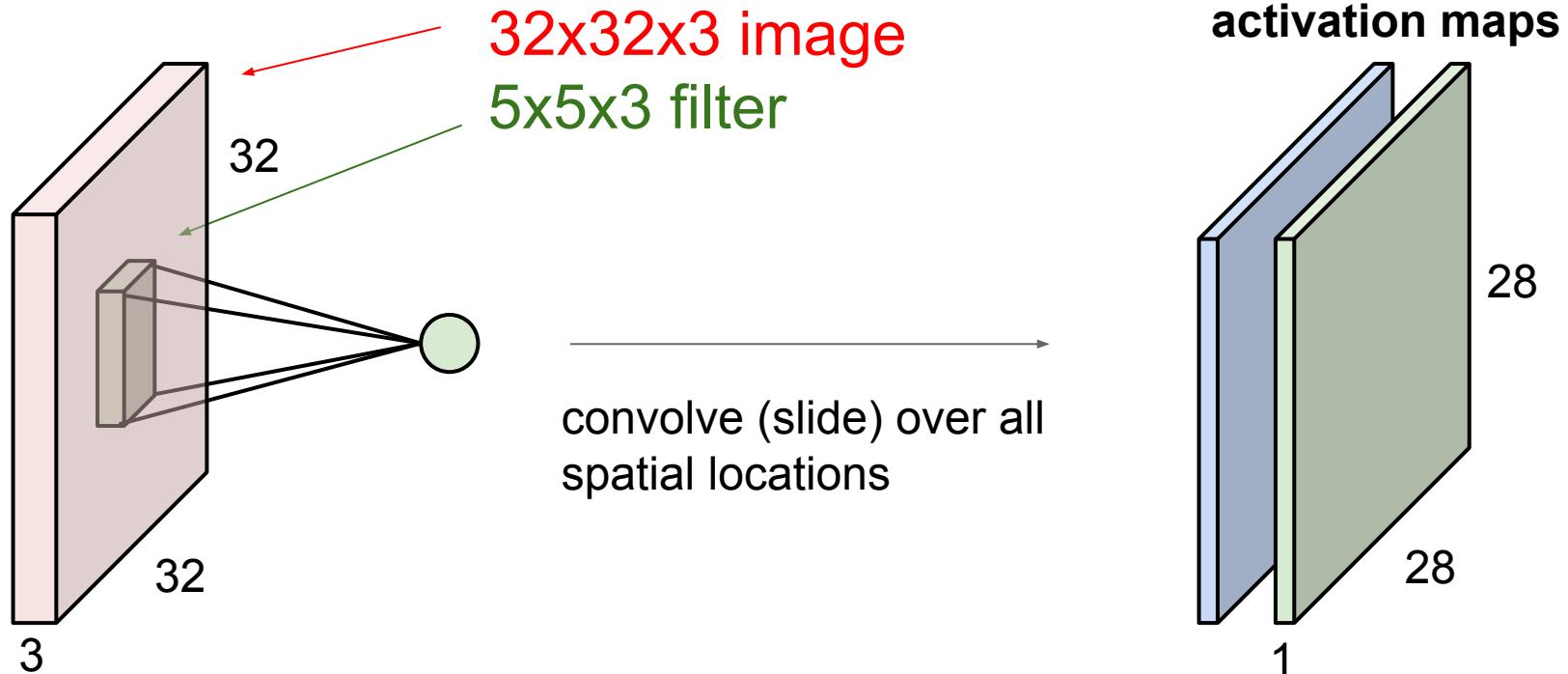


Convolution Layer

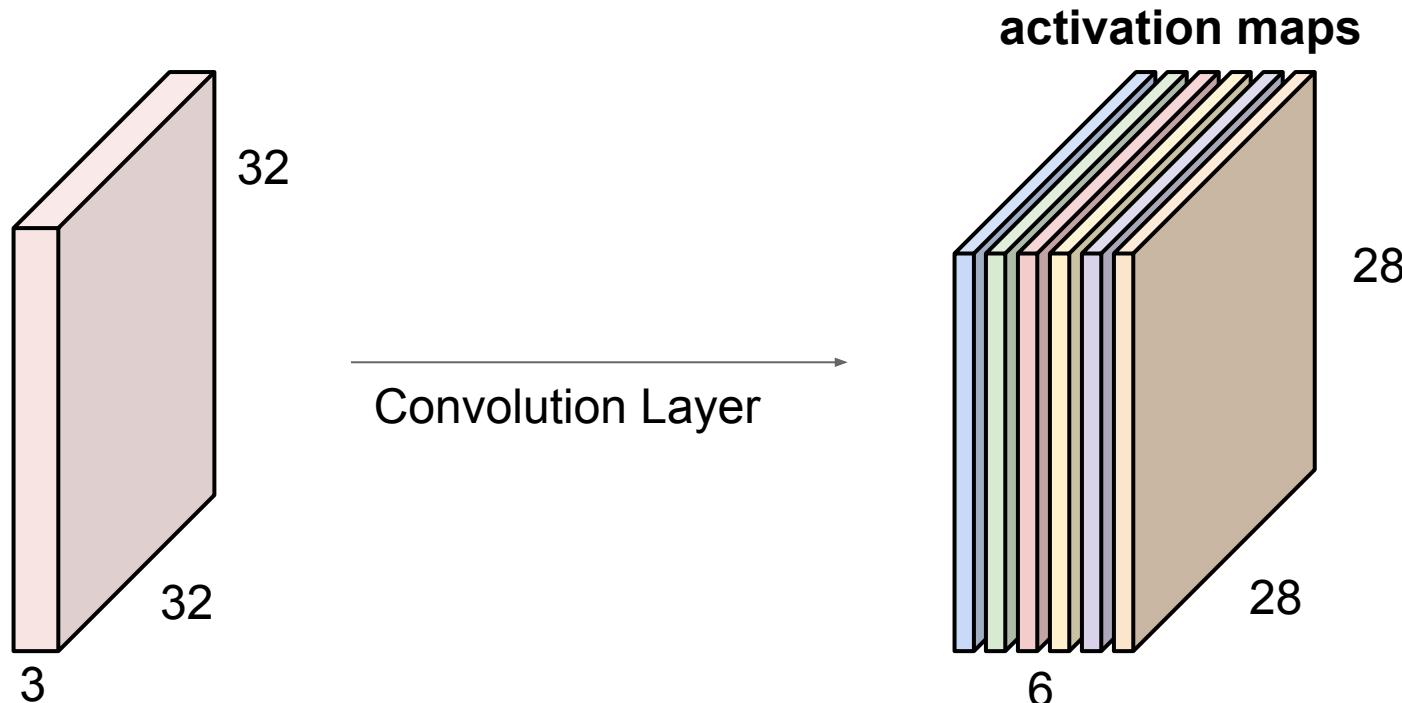


Convolution Layer

consider a second, green filter

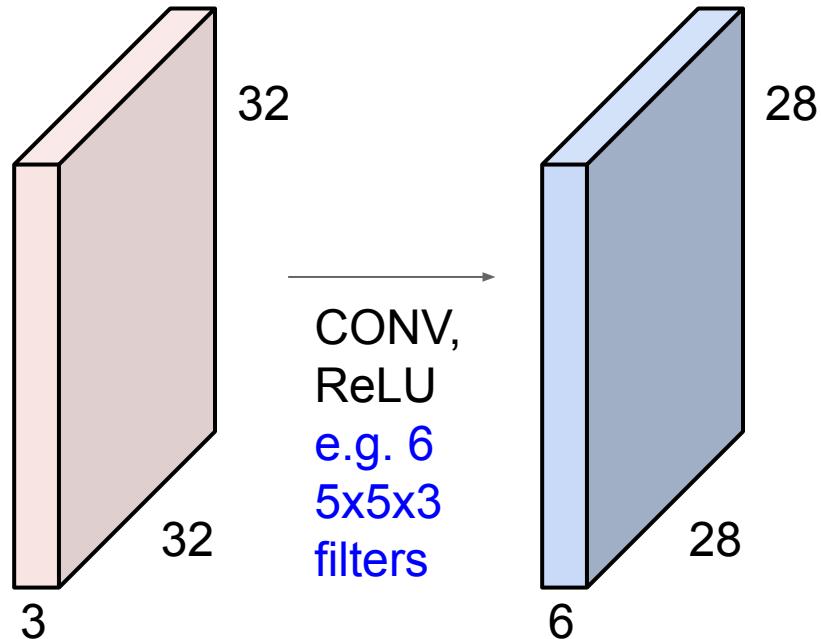


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

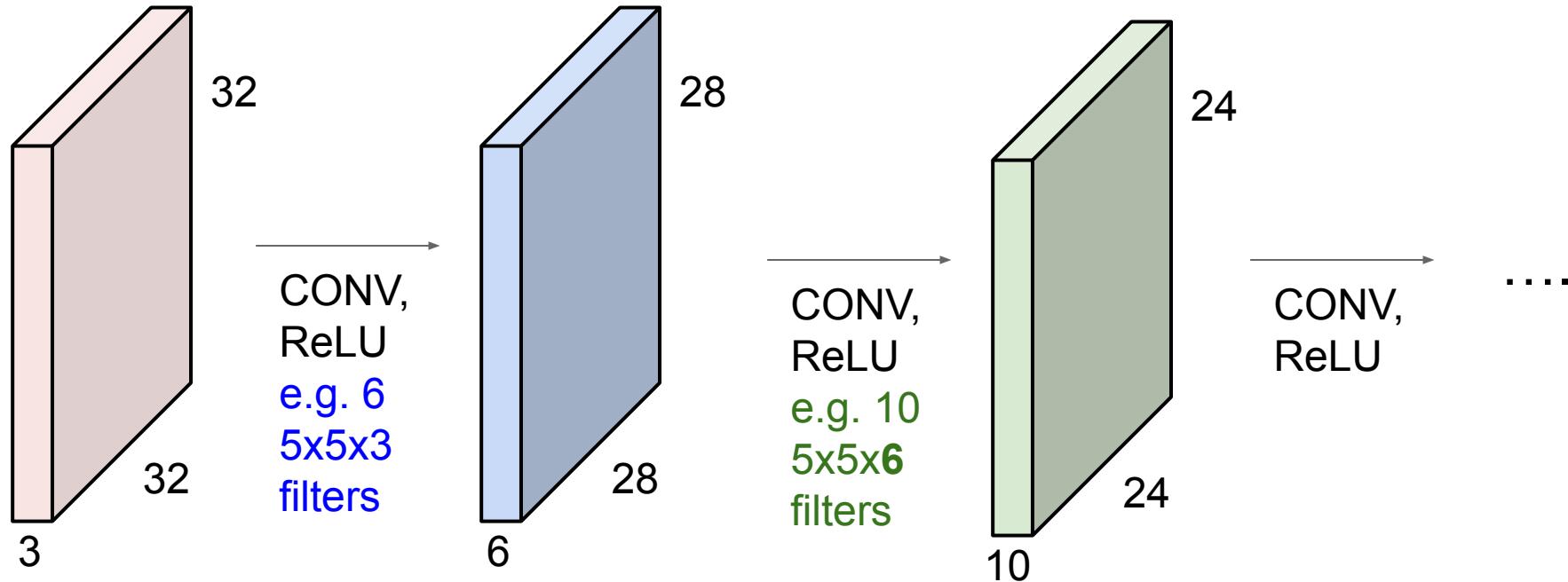


We stack these up to get a “new image” of size $28 \times 28 \times 6$!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



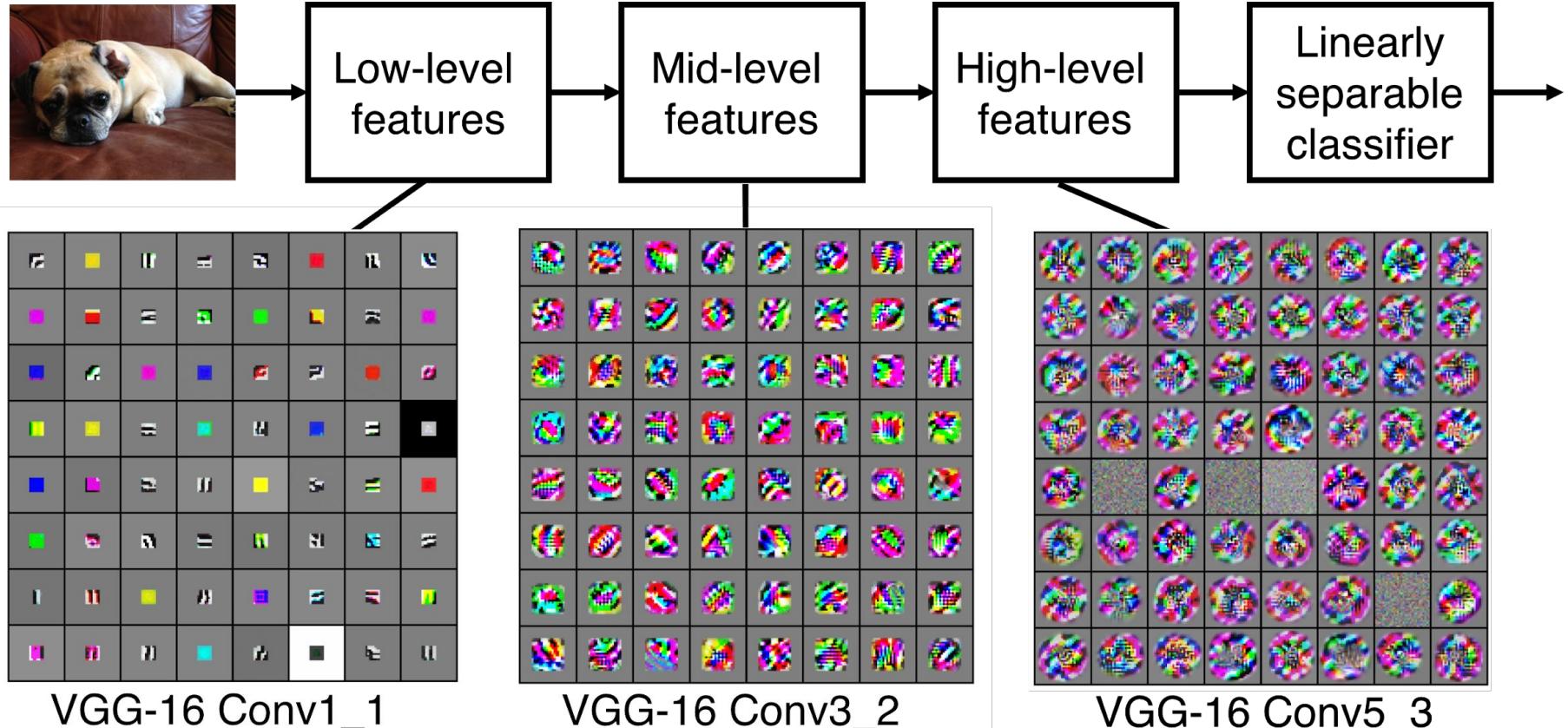
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



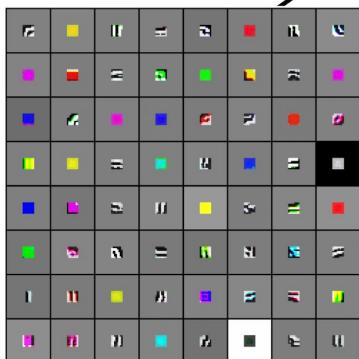
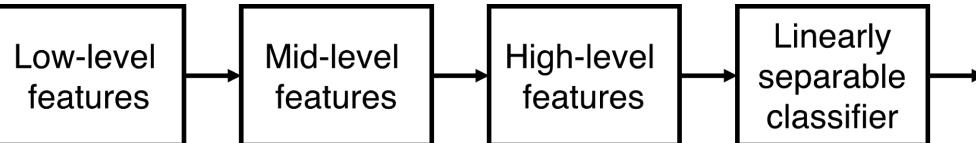
Preview

[Zeiler and Fergus 2013]

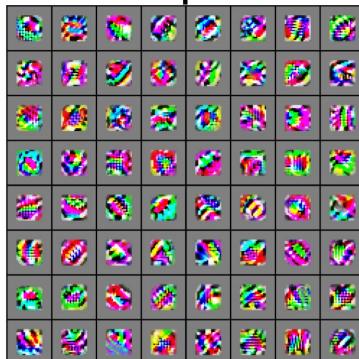
Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].



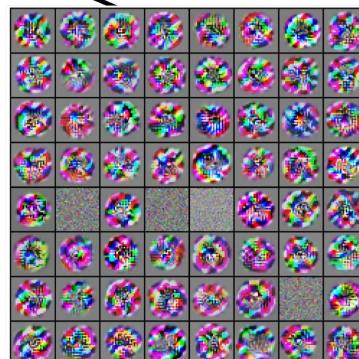
Preview



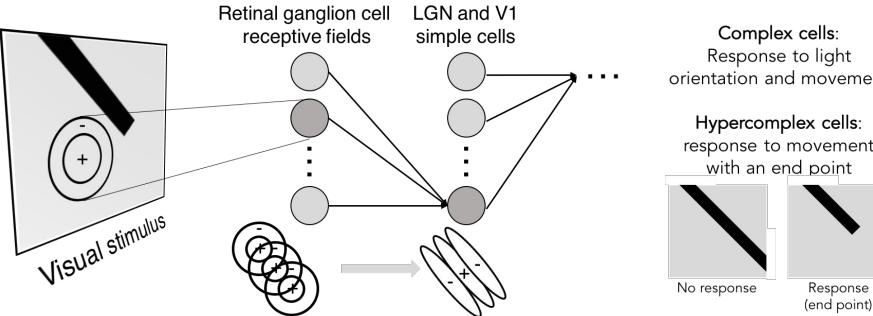
VGG-16 Conv1_1



VGG-16 Conv3_2

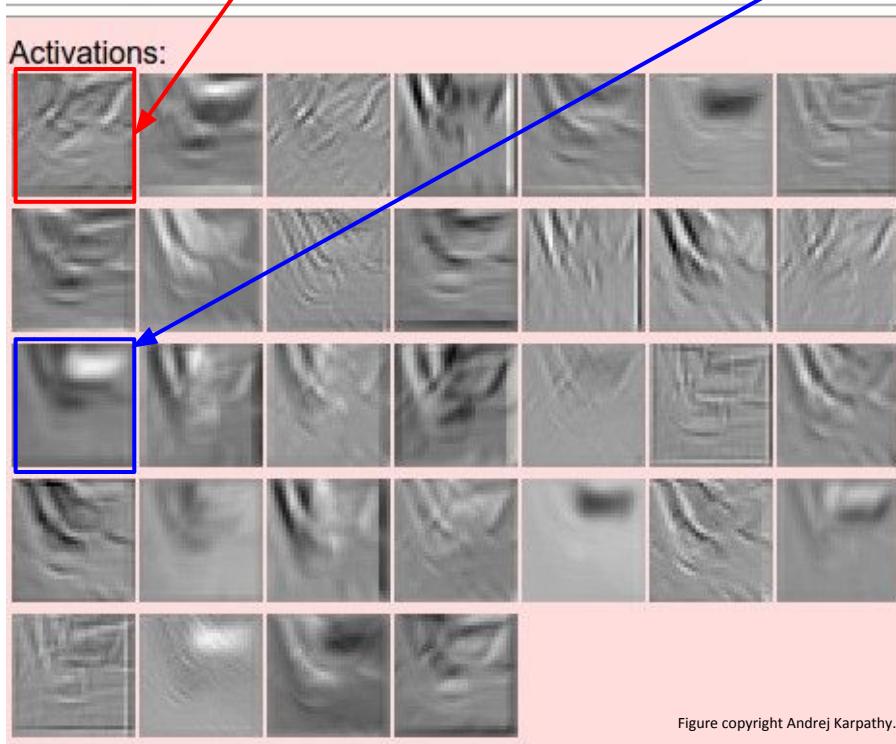


VGG-16 Conv5_3





one filter =>
one activation map



example 5x5 filters
(32 total)

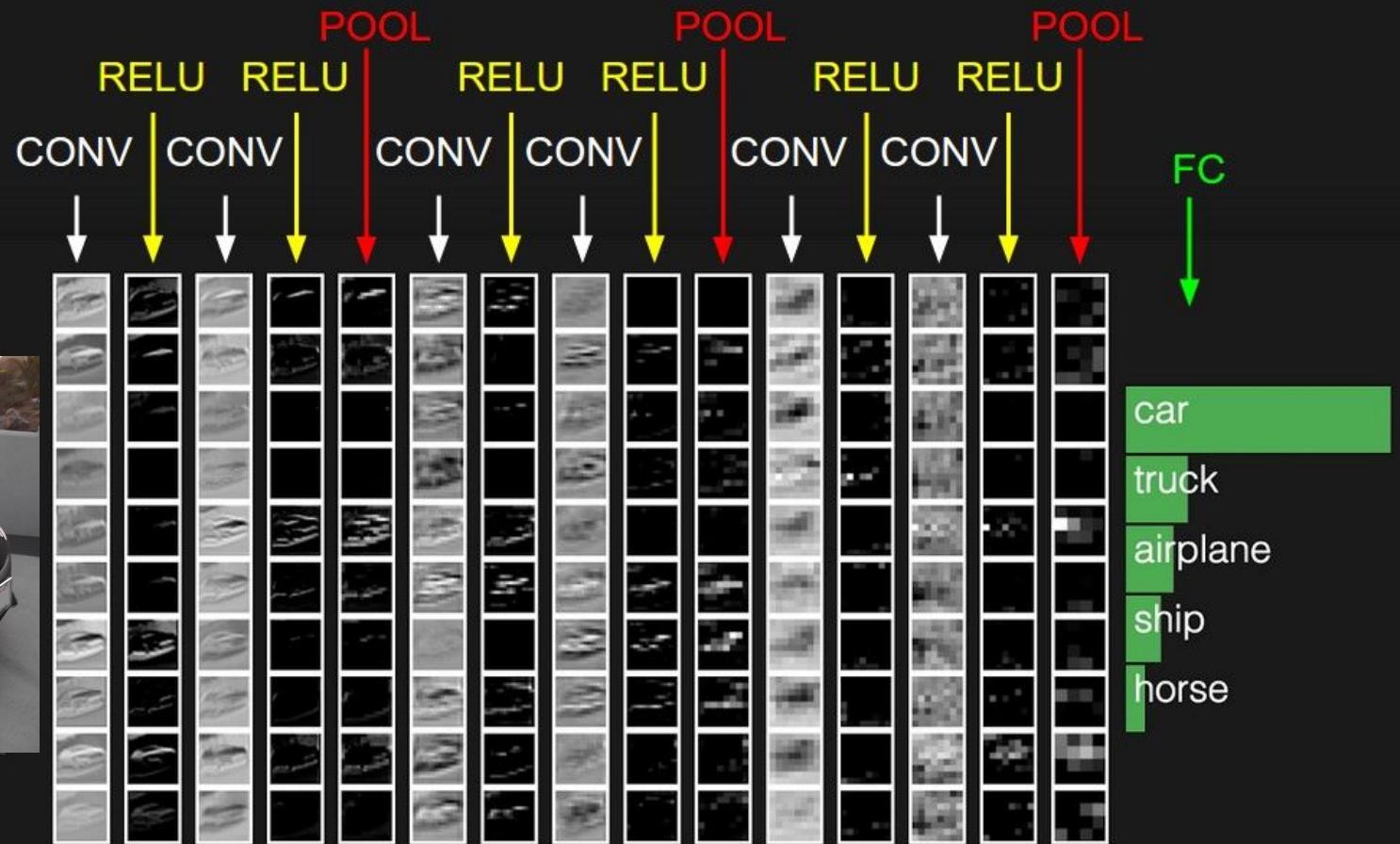
We call the layer convolutional
because it is related to convolution
of two signals:

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1, n_2] \cdot g[x - n_1, y - n_2]$$

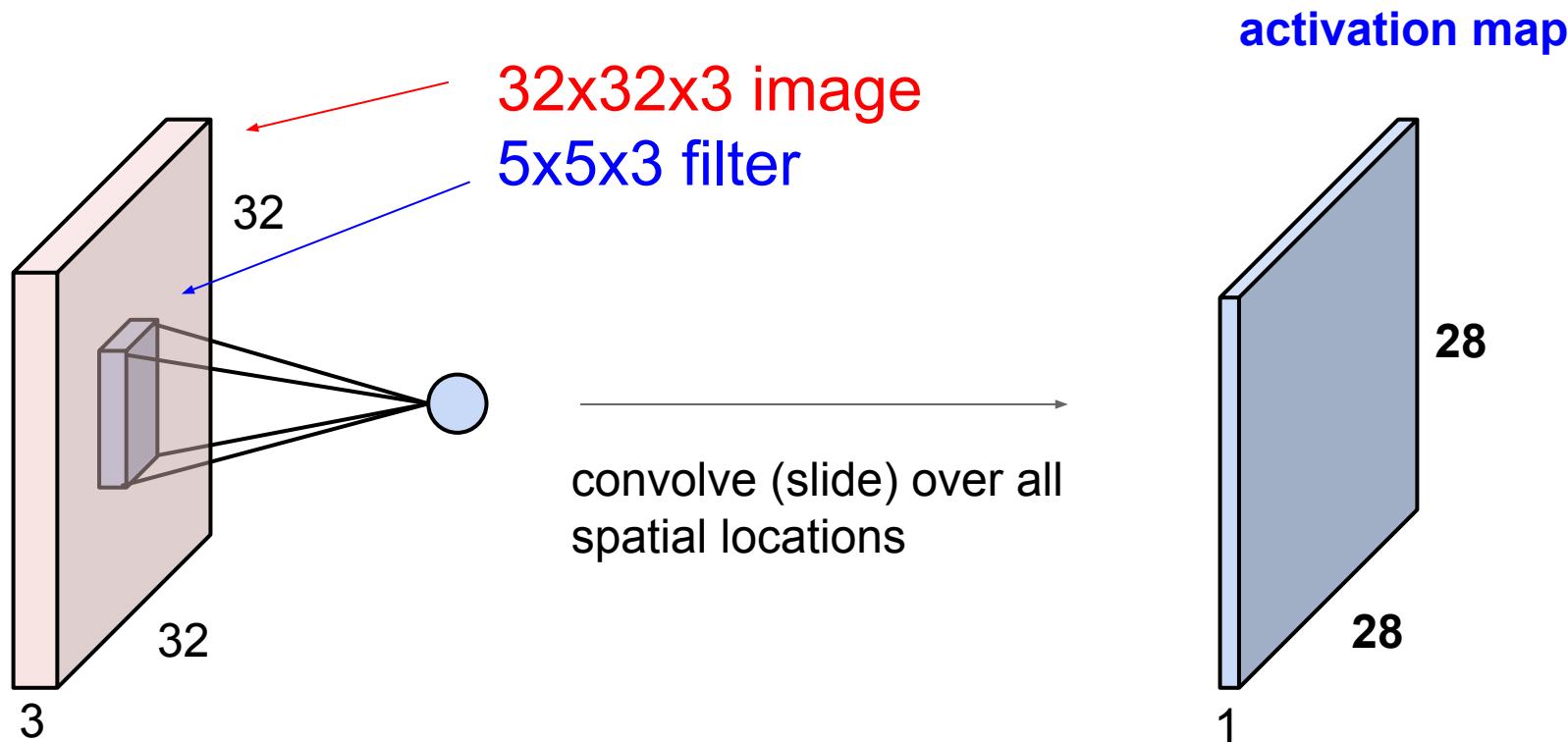


elementwise multiplication and sum of
a filter and the signal (image)

preview:

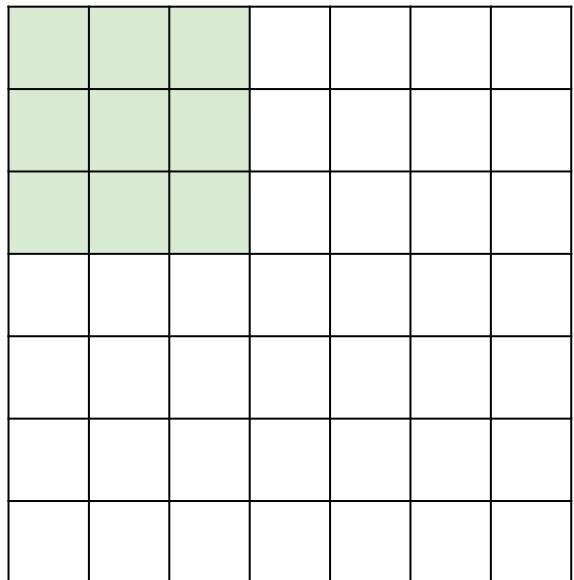


A closer look at spatial dimensions:



A closer look at spatial dimensions:

7

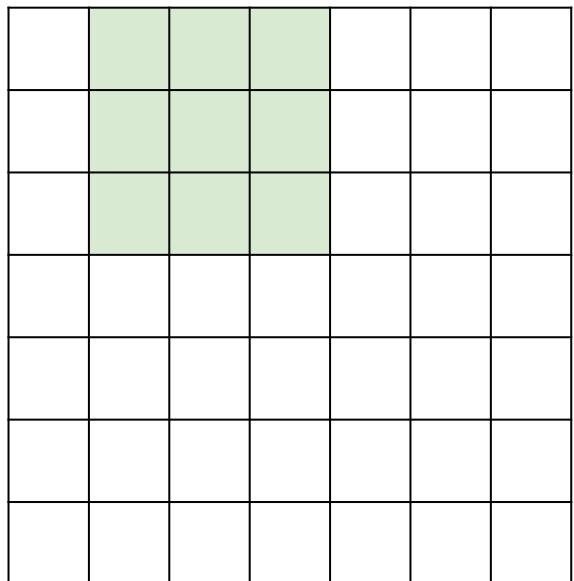


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

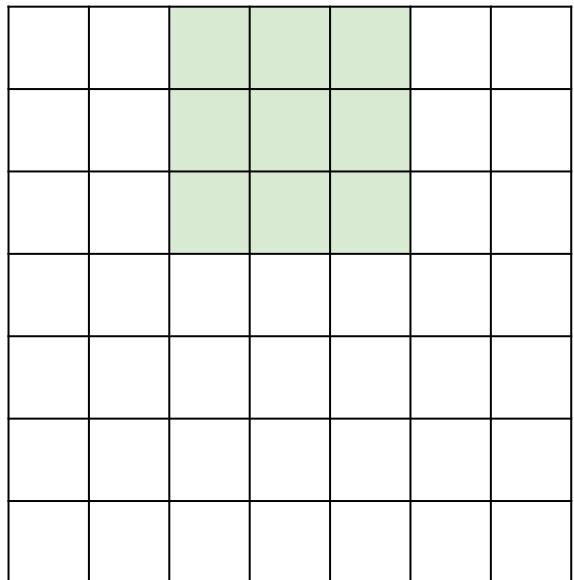


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

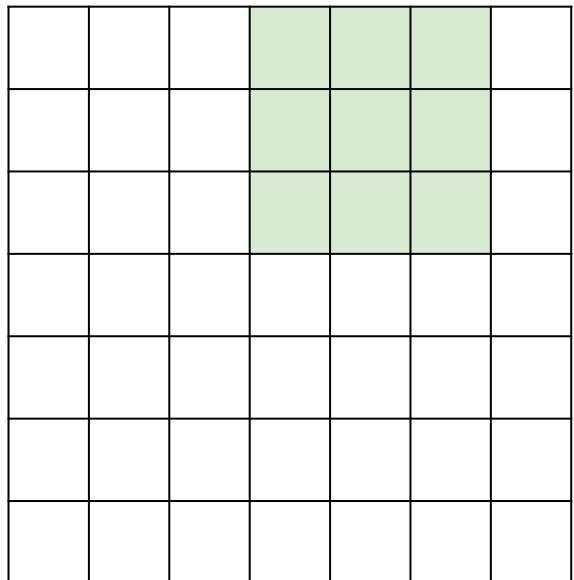


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7

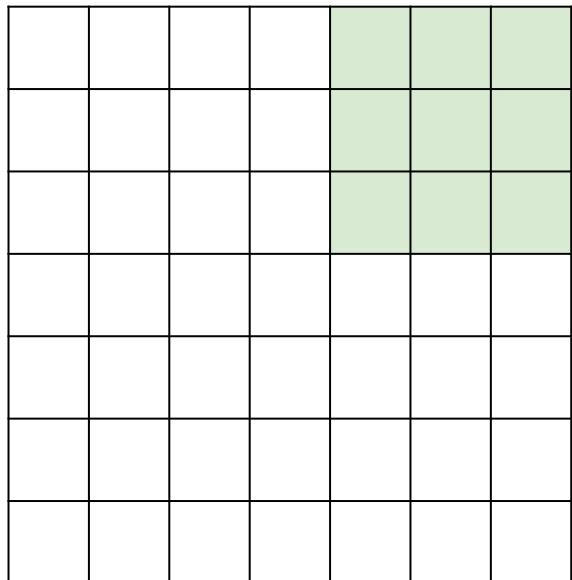


7x7 input (spatially)
assume 3x3 filter

7

A closer look at spatial dimensions:

7



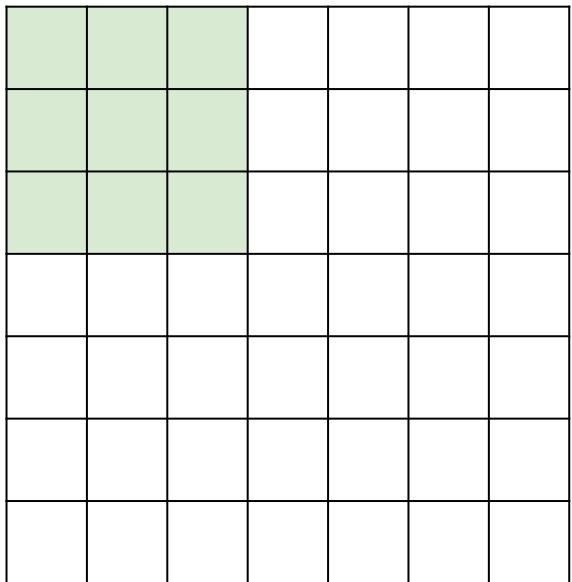
7x7 input (spatially)
assume 3x3 filter

=> 5x5 output

7

A closer look at spatial dimensions:

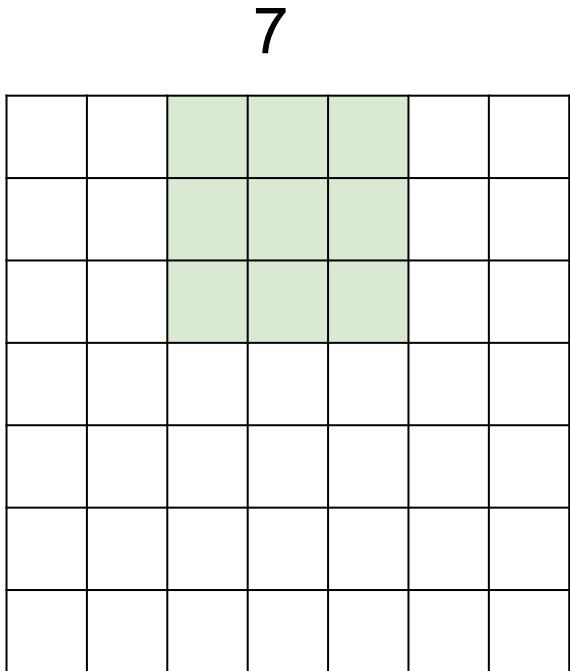
7



7

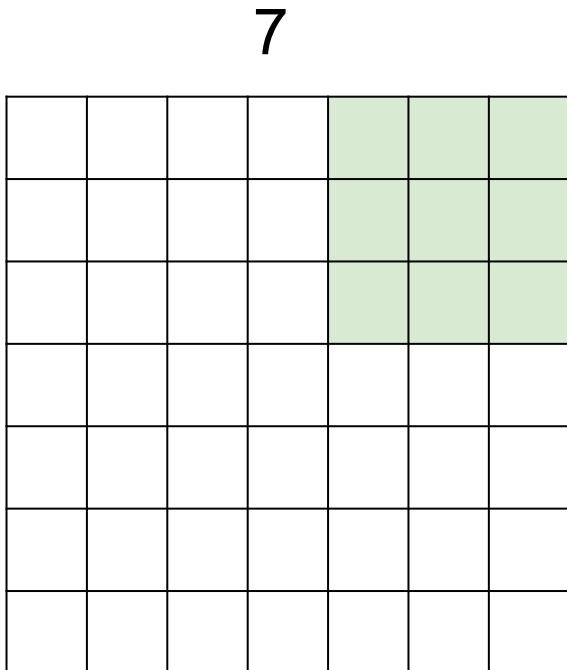
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:

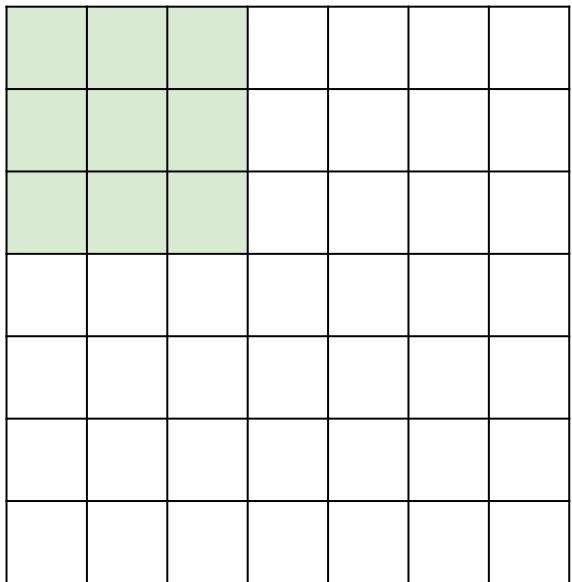


7

7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:

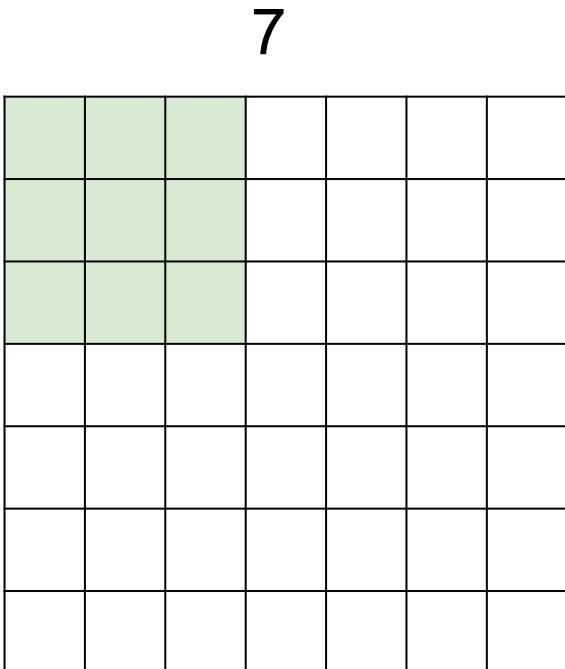
7



7

7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

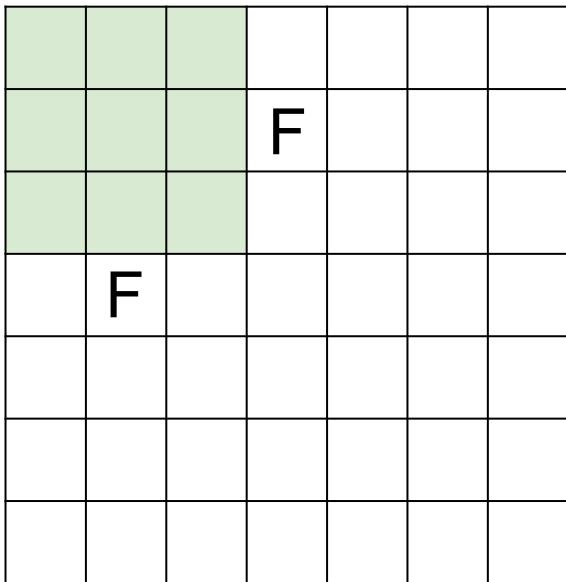
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

doesn't fit!
cannot apply 3x3 filter on
7x7 input with stride 3.

N



N

Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:

$$\text{stride 1} \Rightarrow (7 - 3)/1 + 1 = 5$$

$$\text{stride 2} \Rightarrow (7 - 3)/2 + 1 = 3$$

$$\text{stride 3} \Rightarrow (7 - 3)/3 + 1 = 2.33 : \backslash$$

In practice: Common to zero pad the border

0	0	0	0	0	0		
0							
0							
0							
0							

e.g. input 7x7

3x3 filter, applied with **stride 1**

pad with 1 pixel border => what is the output?

(recall:)

$$(N - F) / \text{stride} + 1$$

In practice: Common to zero pad the border

0	0	0	0	0	0		
0							
0							
0							
0							

e.g. input 7x7

3x3 filter, applied with stride 1

pad with 1 pixel border => what is the output?

7x7 output!

(recall:)

$$(N + 2P - F) / \text{stride} + 1$$

In practice: Common to zero pad the border

0	0	0	0	0	0		
0							
0							
0							
0							

e.g. input 7x7

3x3 filter, applied with stride 1

pad with 1 pixel border => what is the output?

7x7 output!

in general, common to see CONV layers with stride 1, filters of size $F \times F$, and zero-padding with $(F-1)/2$. (will preserve size spatially)

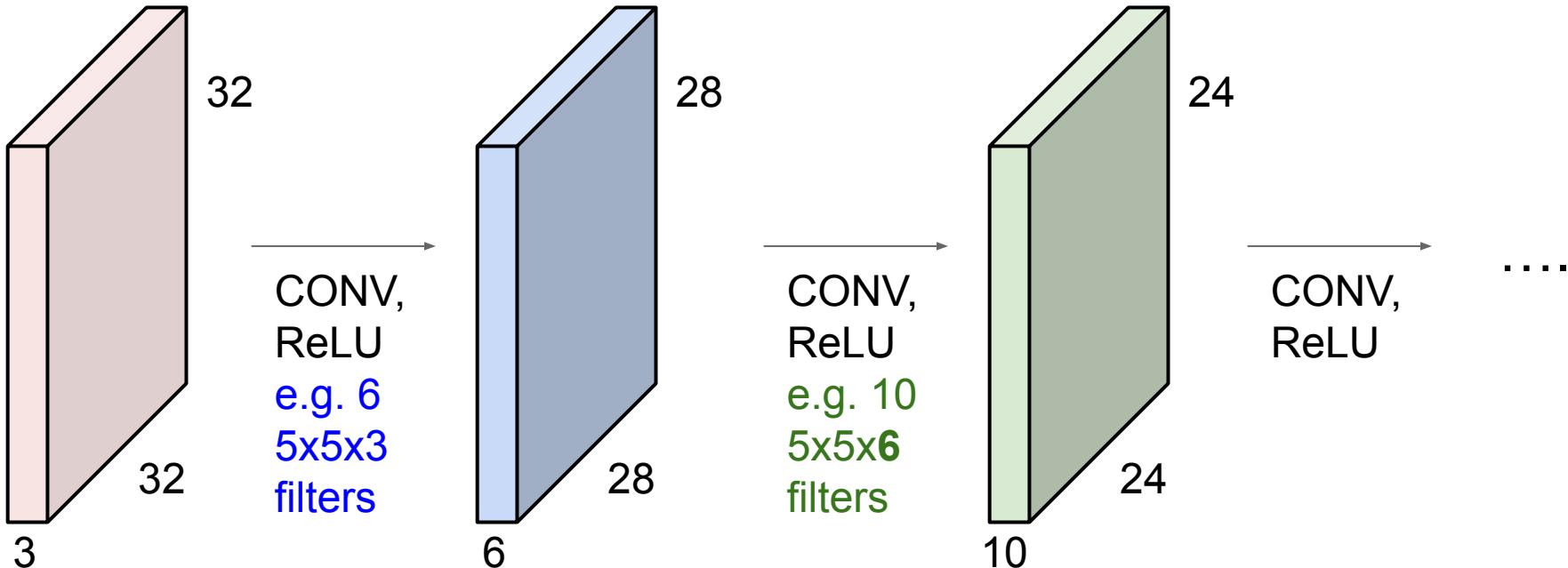
e.g. $F = 3 \Rightarrow$ zero pad with 1

$F = 5 \Rightarrow$ zero pad with 2

$F = 7 \Rightarrow$ zero pad with 3

Remember back to...

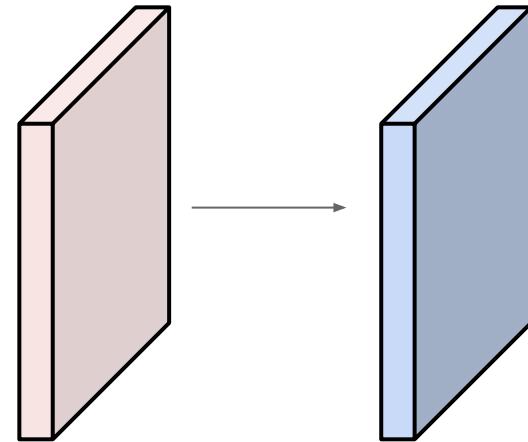
E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially!
(32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.



Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

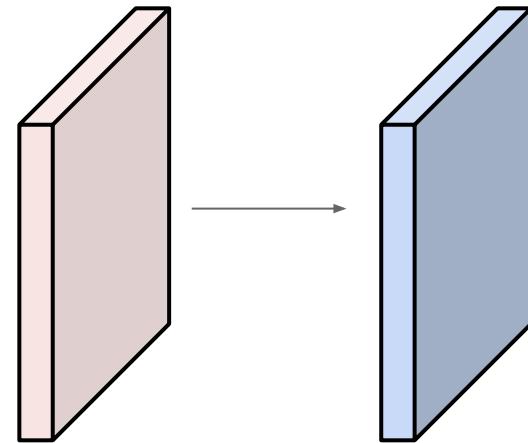


Output volume size: ?

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad **2**



Output volume size:

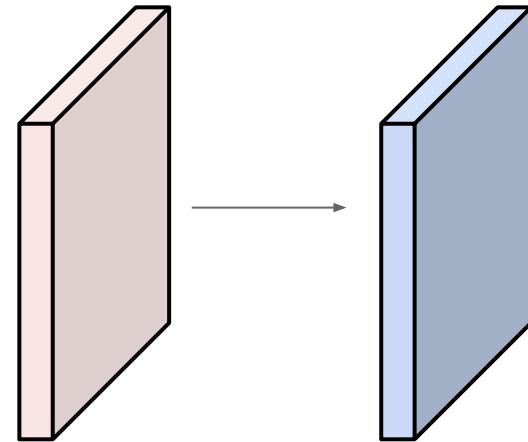
$(32+2*2-5)/1+1 = 32$ spatially, so

32x32x10

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

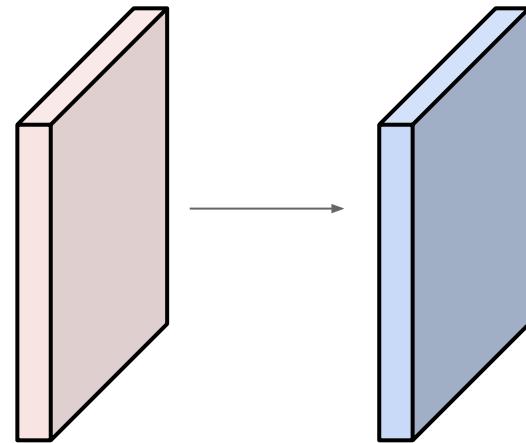


Number of parameters in this layer?

Examples time:

Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2



Number of parameters in this layer?

each filter has $5*5*3 + 1 = 76$ params (+1 for bias)
 $\Rightarrow 76*10 = 760$

Convolution layer: summary

Let's assume input is $W_1 \times H_1 \times C$

Conv layer needs 4 hyperparameters:

- Number of filters K
- The filter size F
- The stride S
- The zero padding P

This will produce an output of $W_2 \times H_2 \times K$

where:

- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

Number of parameters: F^2CK and K biases

Convolution layer: summary

Let's assume input is $W_1 \times H_1 \times C$

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

This will produce an output of $W_2 \times H_2 \times K$

where:

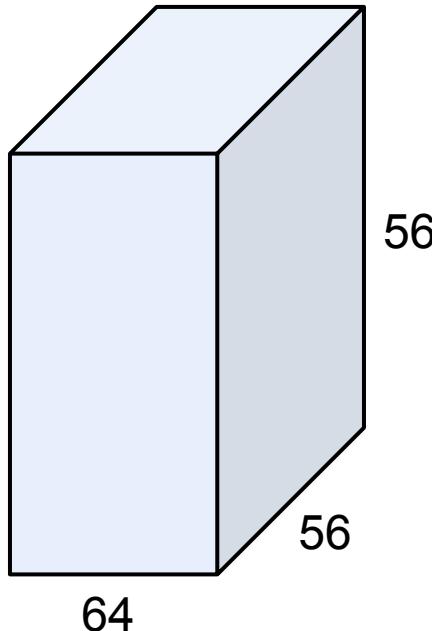
- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

Number of parameters: F^2CK and K biases

Common settings:

- K** = (powers of 2, e.g. 32, 64, 128, 512)
- $F = 3, S = 1, P = 1$
 - $F = 5, S = 1, P = 2$
 - $F = 5, S = 2, P = ?$ (whatever fits)
 - $F = 1, S = 1, P = 0$

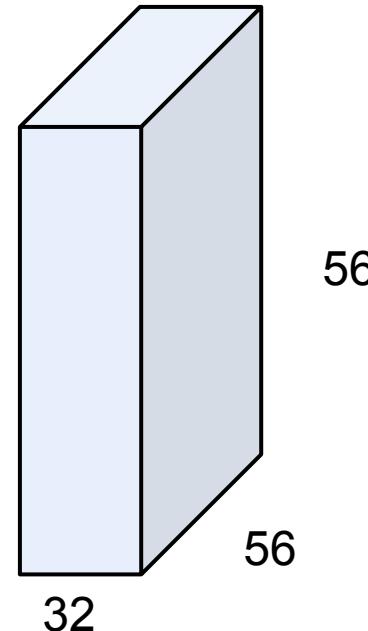
(btw, 1x1 convolution layers make perfect sense)



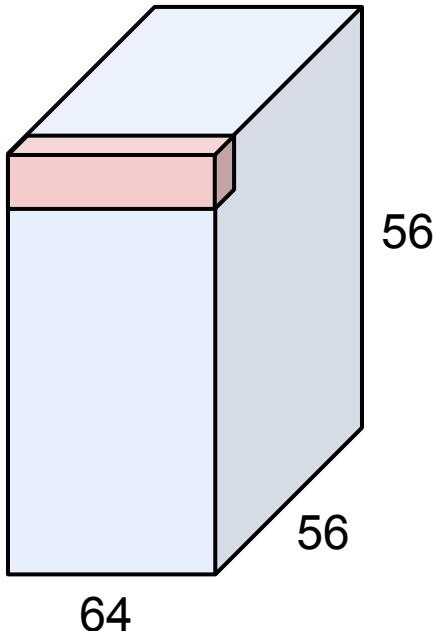
1x1 CONV
with 32 filters

→

(each filter has size
1x1x64, and performs a
64-dimensional dot
product)



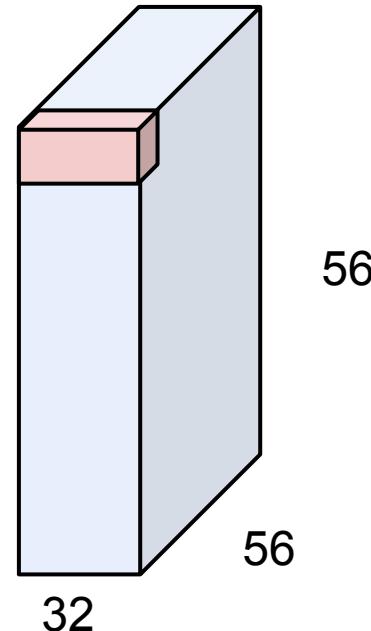
(btw, 1x1 convolution layers make perfect sense)



1x1 CONV
with 32 filters

→

(each filter has size
 $1 \times 1 \times 64$, and performs a
64-dimensional dot
product)



Example: CONV layer in PyTorch

Conv2d

```
CLASS torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0,  
dilation=1, groups=1, bias=True)
```

[SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) * \text{input}(N_i, k)$$

where $*$ is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is width in pixels.

- `stride` controls the stride for the cross-correlation, a single number or a tuple.
- `padding` controls the amount of implicit zero-paddings on both sides for `padding` number of points for each dimension.
- `dilation` controls the spacing between the kernel points; also known as the à trous algorithm. It is harder to describe, but this [link](#) has a nice visualization of what `dilation` does.
- `groups` controls the connections between inputs and outputs. `in_channels` and `out_channels` must both be divisible by `groups`. For example,
 - At `groups=1`, all inputs are convolved to all outputs.
 - At `groups=2`, the operation becomes equivalent to having two conv layers side by side, each seeing half the input channels, and producing half the output channels, and both subsequently concatenated.
 - At `groups= in_channels`, each input channel is convolved with its own set of filters, of size: $\left\lfloor \frac{C_{\text{out}}}{C_{\text{in}}} \right\rfloor$.

The parameters `kernel_size`, `stride`, `padding`, `dilation` can either be:

- a single `int` – in which case the same value is used for the height and width dimension
- a `tuple` of two `ints` – in which case, the first `int` is used for the height dimension, and the second `int` for the width dimension

[PyTorch](#) is licensed under [BSD 3-clause](#).

Example: CONV layer in Keras

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

Conv2D

[source]

```
keras.layers.Conv2D(filters, kernel_size, strides=(1, 1), padding='valid', data_format=None, d:
```

2D convolution layer (e.g. spatial convolution over images).

This layer creates a convolution kernel that is convolved with the layer input to produce a tensor of outputs. If `use_bias` is True, a bias vector is created and added to the outputs. Finally, if `activation` is not `None`, it is applied to the outputs as well.

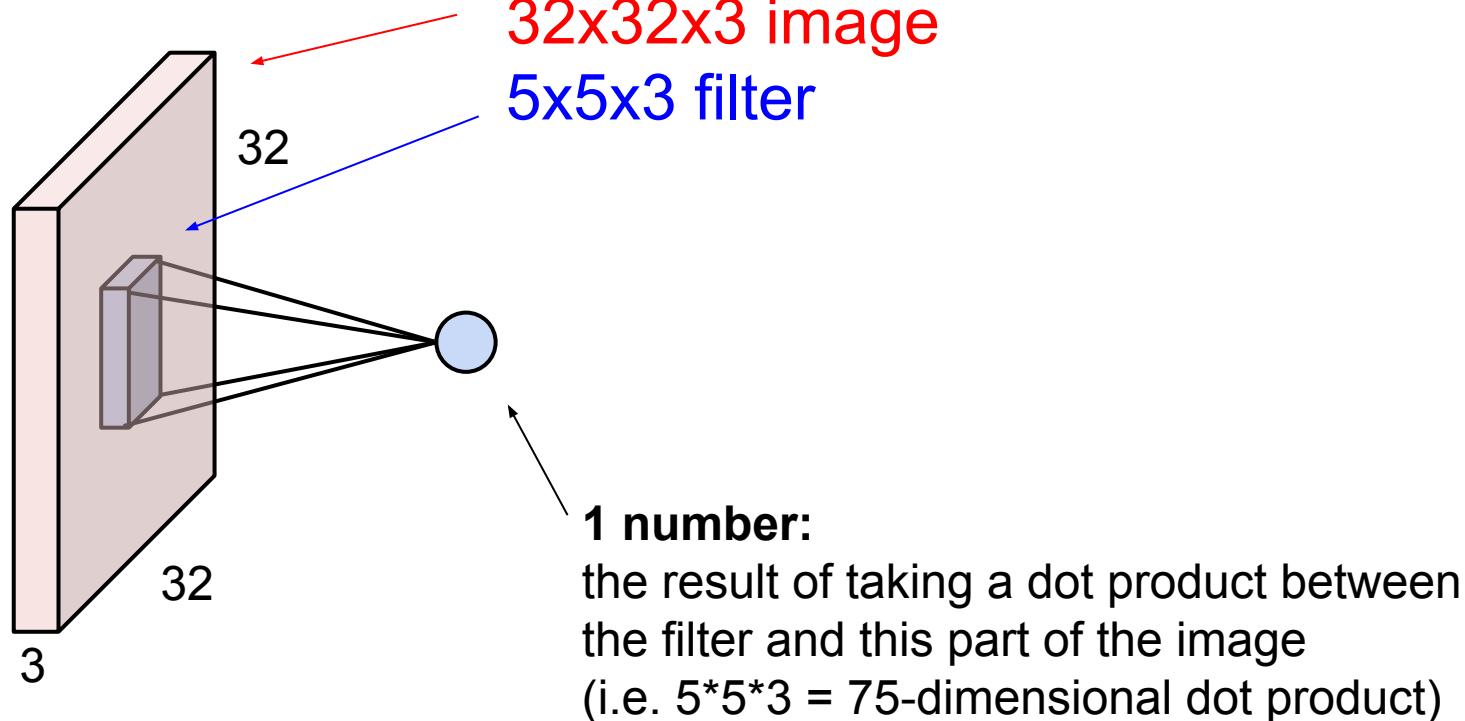
When using this layer as the first layer in a model, provide the keyword argument `input_shape` (tuple of integers, does not include the batch axis), e.g. `input_shape=(128, 128, 3)` for 128x128 RGB pictures in `data_format="channels_last"`.

Arguments

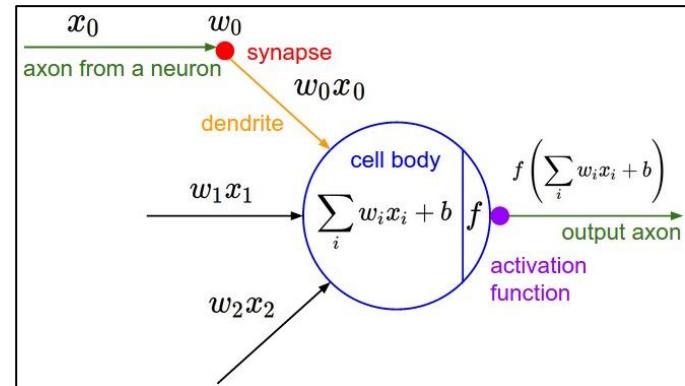
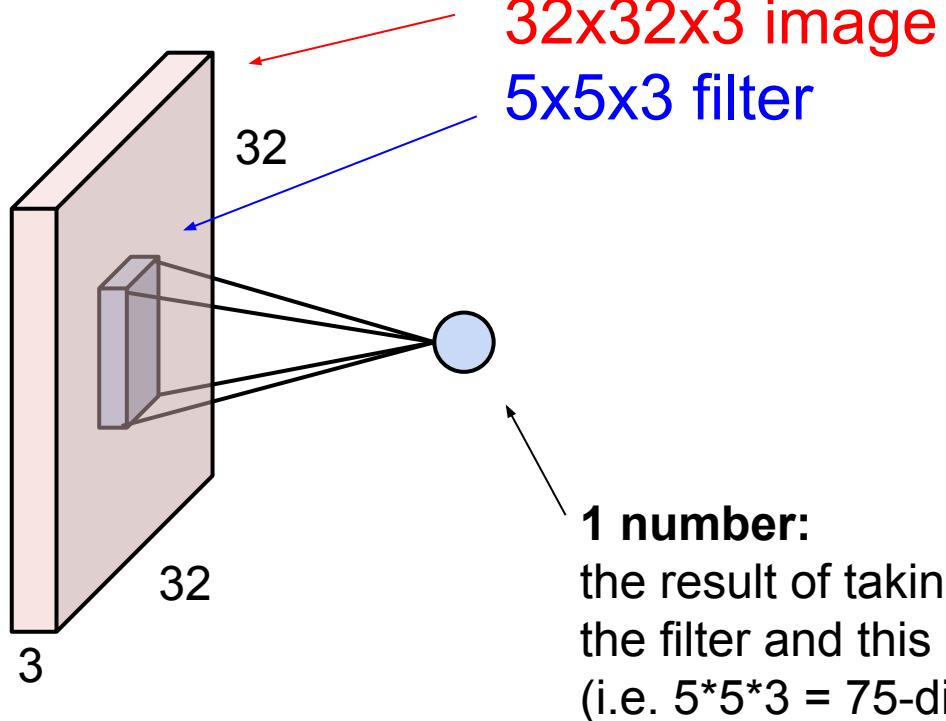
- **filters**: Integer, the dimensionality of the output space (i.e. the number of output filters in the convolution).
- **kernel_size**: An integer or tuple/list of 2 integers, specifying the height and width of the 2D convolution window. Can be a single integer to specify the same value for all spatial dimensions.
- **strides**: An integer or tuple/list of 2 integers, specifying the strides of the convolution along the height and width. Can be a single integer to specify the same value for all spatial dimensions. Specifying any stride value != 1 is incompatible with specifying any `dilation_rate` value != 1.
- **padding**: one of `"valid"` or `"same"` (case-insensitive). Note that `"same"` is slightly inconsistent across backends with `strides` != 1, as described here
- **data_format**: A string, one of `"channels_last"` or `"channels_first"`. The ordering of the dimensions in the inputs. `"channels_last"` corresponds to inputs with shape `(batch, height, width, channels)` while `"channels_first"` corresponds to inputs with shape `(batch, channels, height, width)`. It defaults to the `image_data_format` value found in your Keras config file at `~/.keras/keras.json`. If you never set it, then it will be `"channels_last"`.

Keras is licensed under the [MIT license](#).

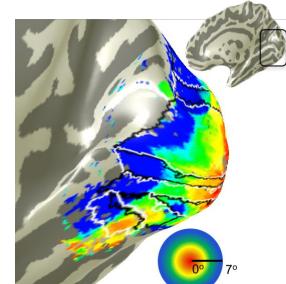
The brain/neuron view of CONV Layer



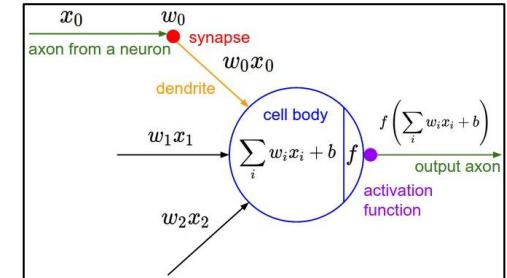
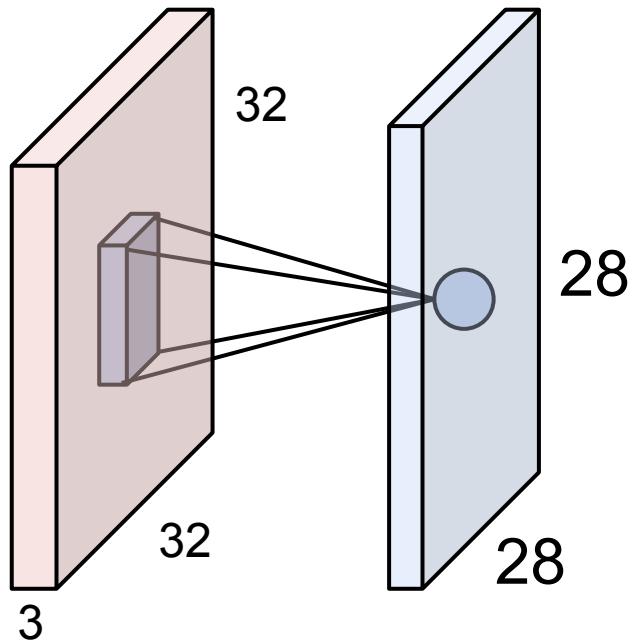
The brain/neuron view of CONV Layer



It's just a neuron with local connectivity...



The brain/neuron view of CONV Layer

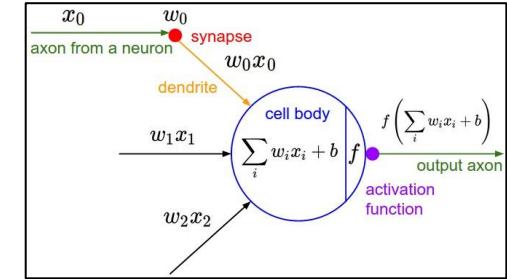
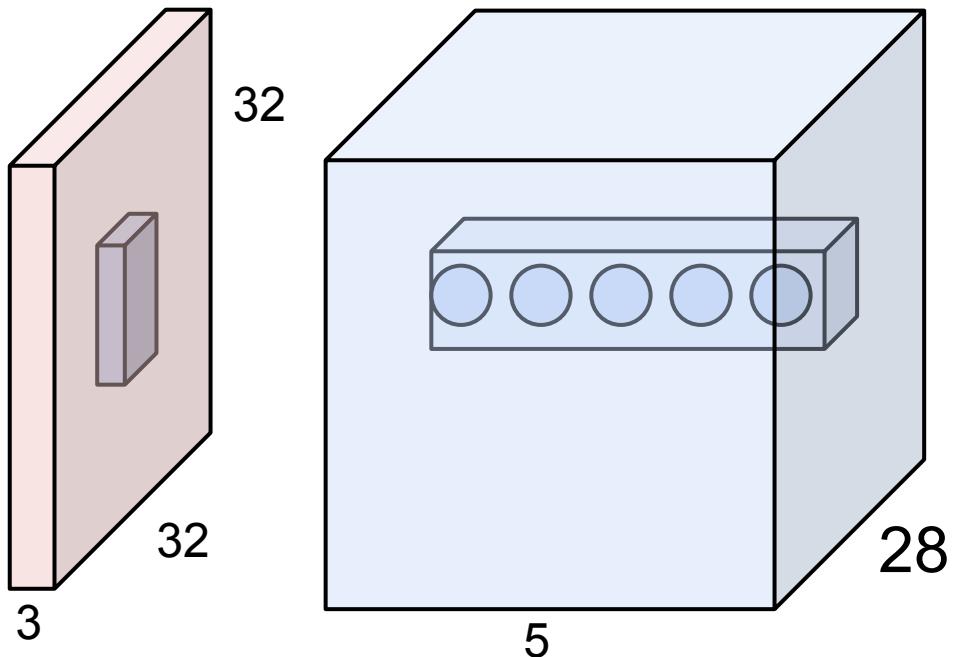


An activation map is a 28x28 sheet of neuron outputs:

1. Each is connected to a small region in the input
2. All of them share parameters

“5x5 filter” -> “5x5 receptive field for each neuron”

The brain/neuron view of CONV Layer



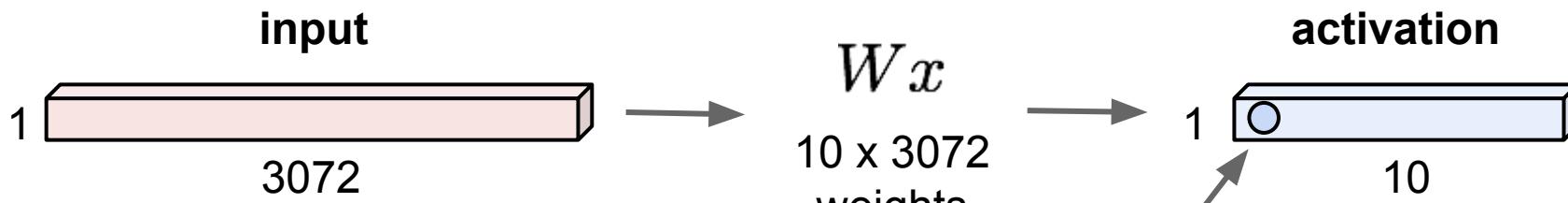
E.g. with 5 filters,
CONV layer consists of
neurons arranged in a 3D grid
($28 \times 28 \times 5$)

There will be 5 different
neurons all looking at the same
region in the input volume

Reminder: Fully Connected Layer

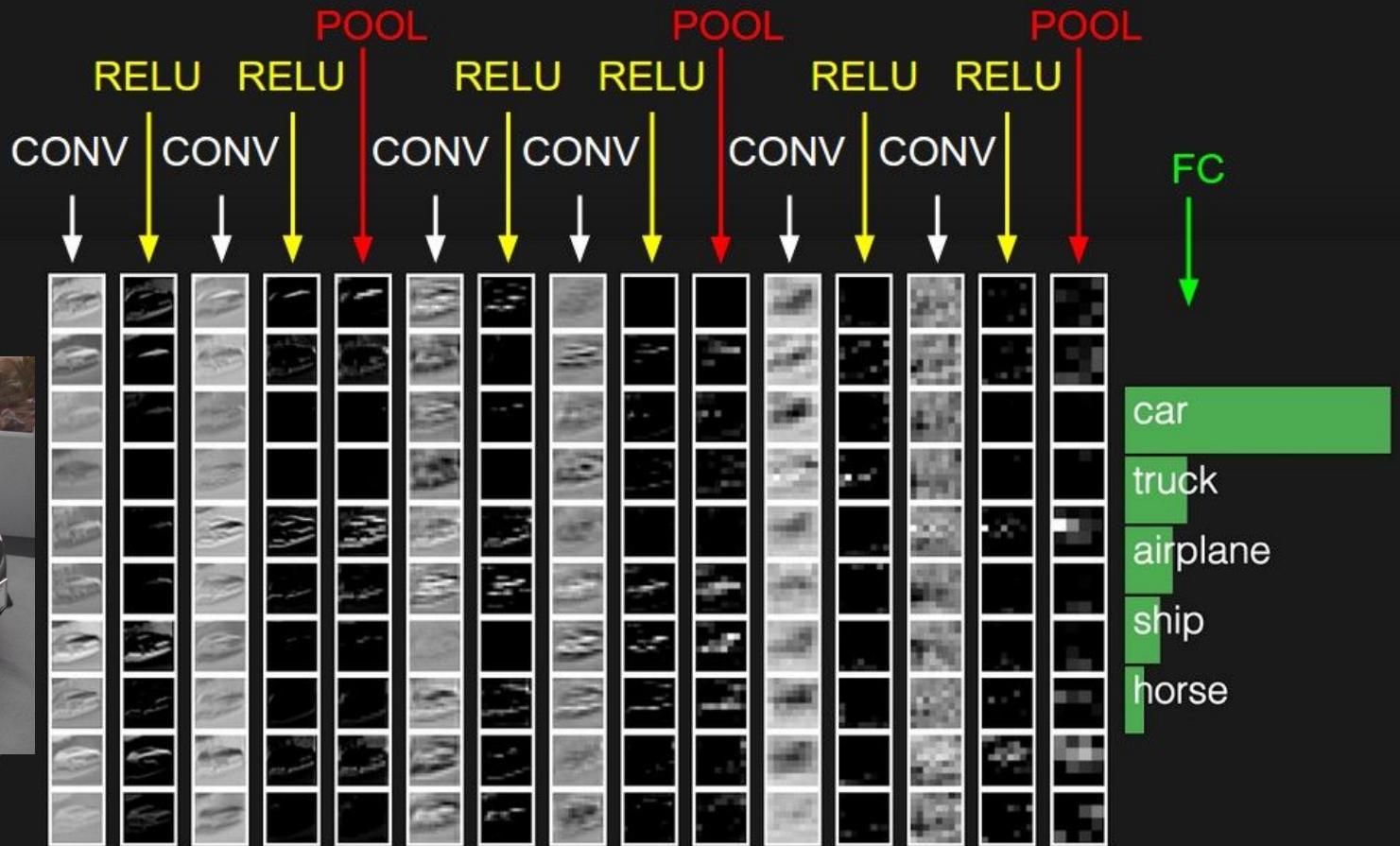
32x32x3 image -> stretch to 3072 x 1

Each neuron
looks at the full
input volume



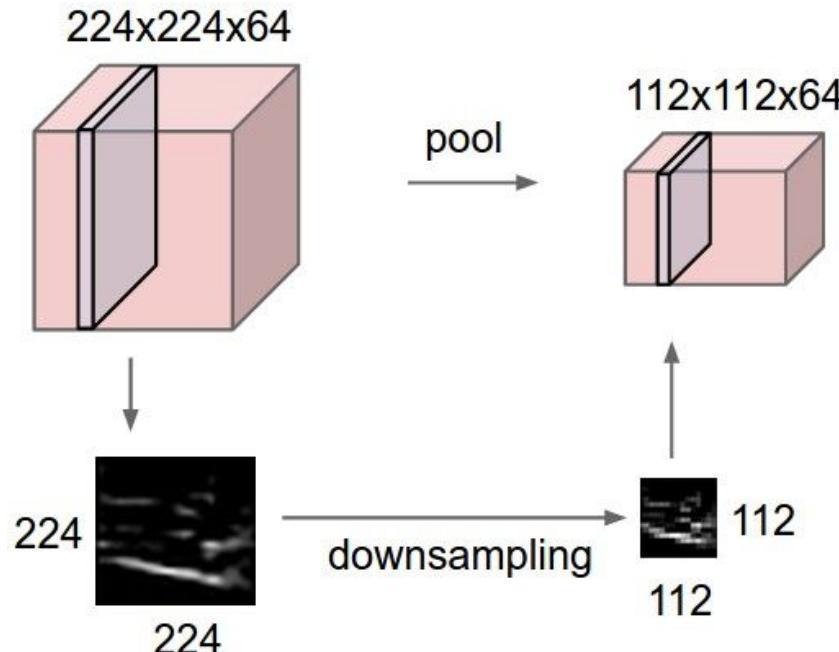
1 number:
the result of taking a dot product
between a row of W and the input
(a 3072-dimensional dot product)

two more layers to go: POOL/FC

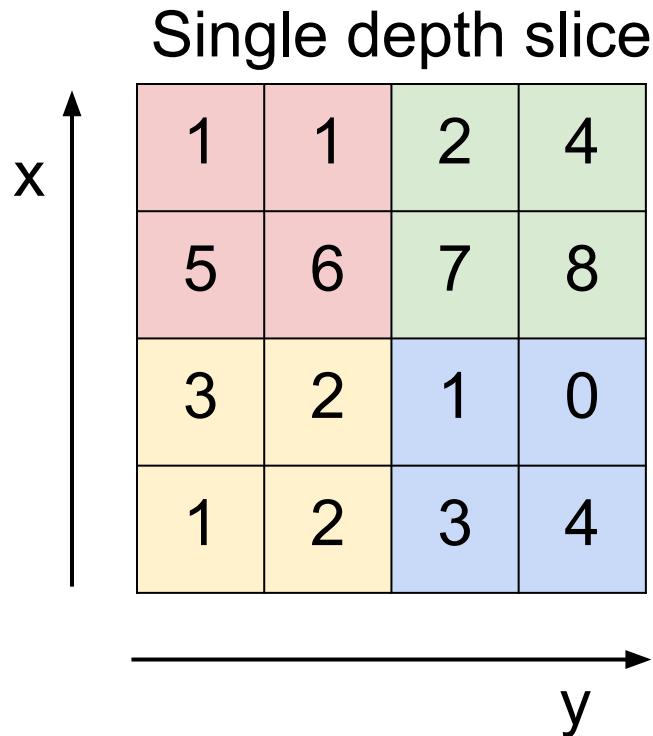


Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



MAX POOLING



max pool with 2x2 filters
and stride 2

6	8
3	4

Convolution layer: summary

Let's assume input is $W_1 \times H_1 \times C$

Conv layer needs 2 hyperparameters:

- The spatial extent **F**
- The stride **S**

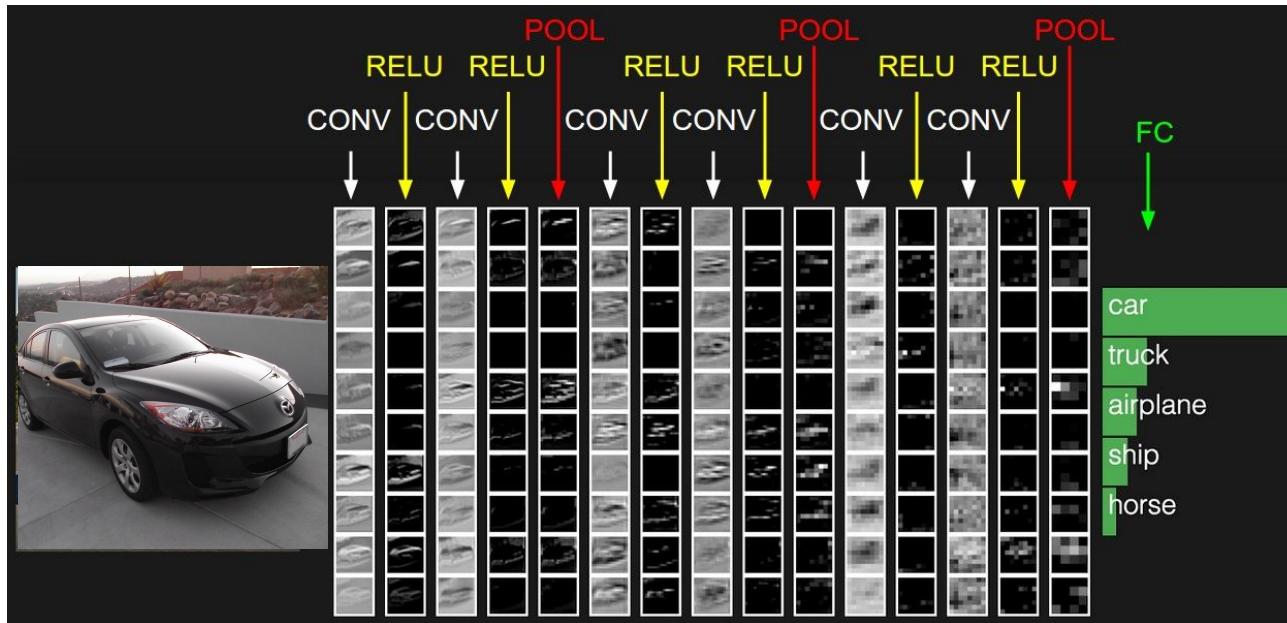
This will produce an output of $W_2 \times H_2 \times C$ where:

- $W_2 = (W_1 - F)/S + 1$
- $H_2 = (H_1 - F)/S + 1$

Number of parameters: 0

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Historically architectures looked like
 $[(\text{CONV-RELU})^* \text{N-POOL?}]^* \text{M-(FC-RELU)}^* \text{K,SOFTMAX}$
where N is usually up to ~5, M is large, $0 \leq K \leq 2$.
 - but recent advances such as ResNet/GoogLeNet have challenged this paradigm