Quadrotor Dynamics

A friendly introduction

1 PHYSICS

1.1 History

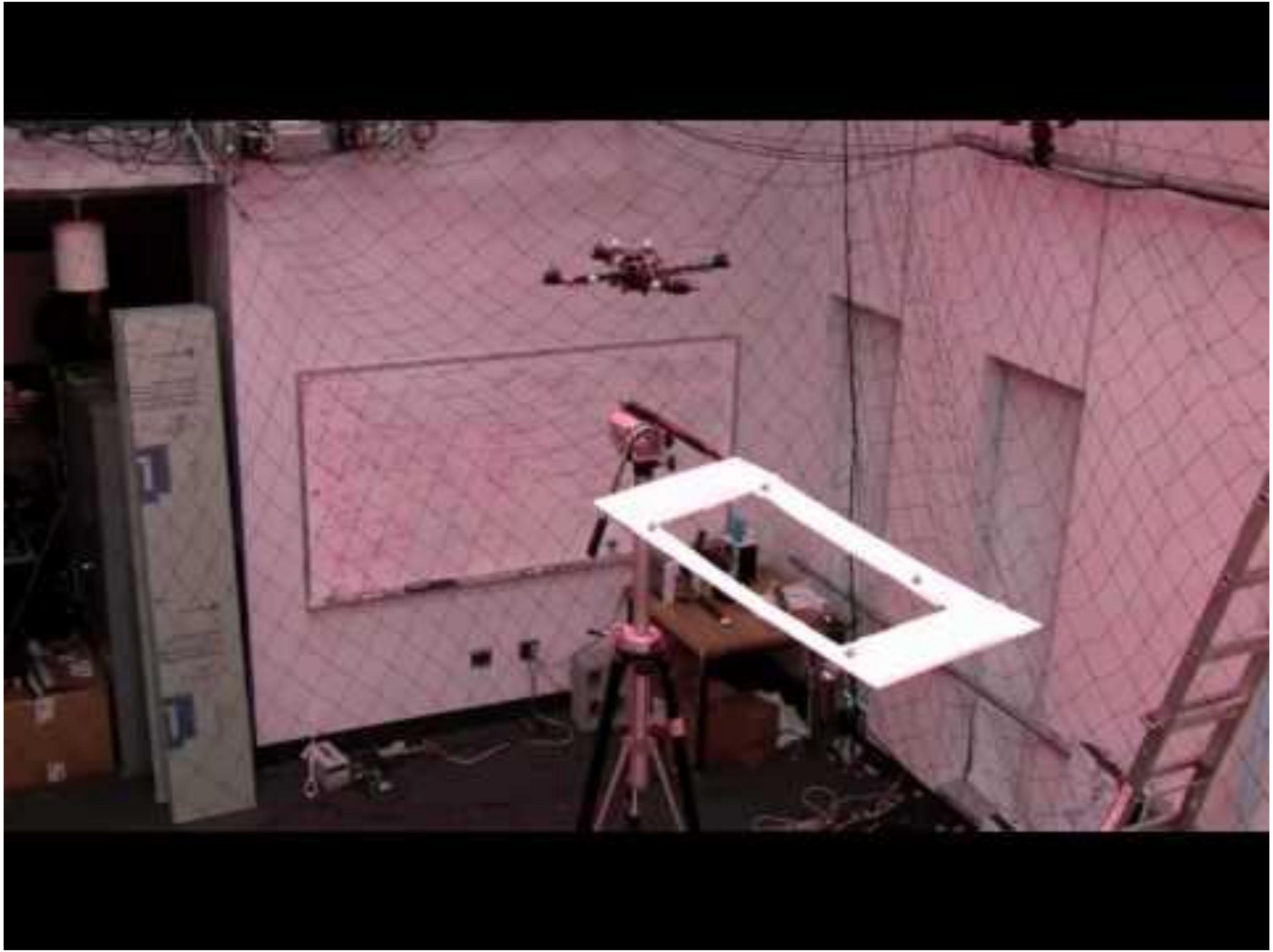
Aristotle said a bunch of stuff that was wrong. Galileo and Newton fixed things up. Then Einstein broke everything again. Now, we've basically got it all worked out, except for small stuff, big stuff, hot stuff, cold stuff, fast stuff, heavy stuff, dark stuff, turbulence, and the concept of time.

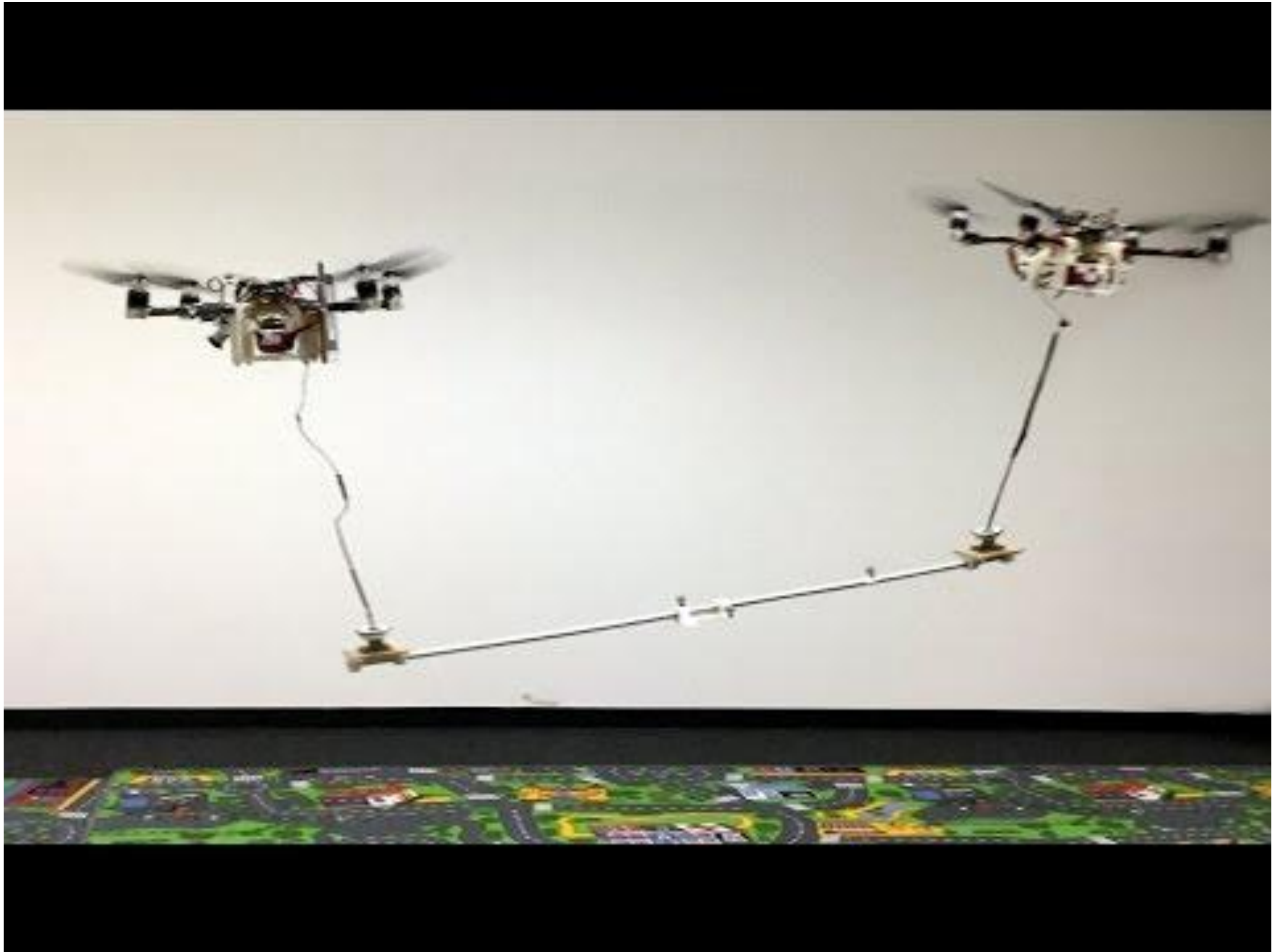
Thanks to these great personalities!

- Aristotle
- Hellenic Dynamics : Archimedes
- Copernicus
- Brahe
- The Renaissance : Kepler
- The formation of classical mechanics : Galileo
- Descartes
- Newton
- Analytical mechanics : Leibniz
- Analytical mechanics : Euler
- Maupertuis
- D’Alembert
- Lagrange
- Einstein
- Erwin Schrodinger, Werner Heisenberg, Max Born

1. Why Mathematical Modelling

- It represents the physical world
- To conduct prediction and analysis
- Carry out experiments with pen and paper
- Eliminates ambiguity, necessary to code





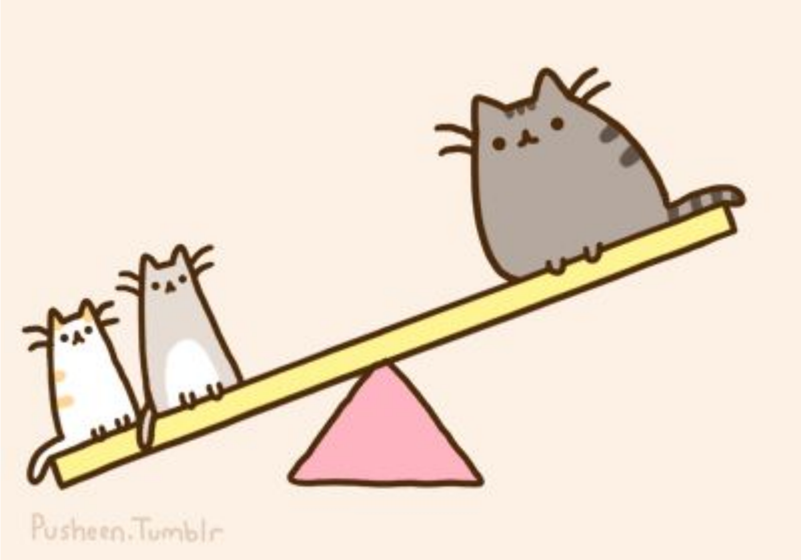

3m/s
0.25x

2. Kinematics v/s Dynamics

- Kinematics is the study of motion WITHOUT considering the cause of motion
- Dynamics considers the forces and moments in its study of motion

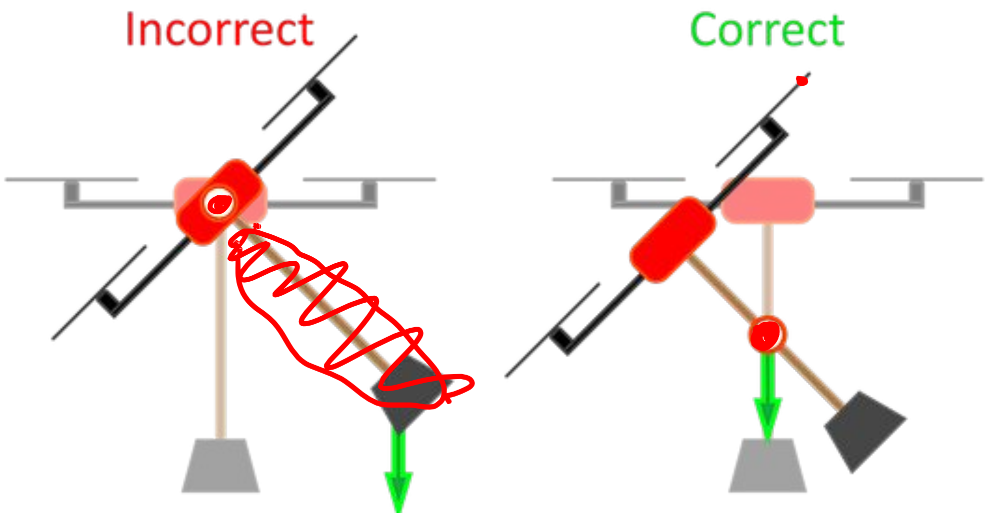
We use rigid body dynamics to study the motion of quadrotors (with rigid frames)

3. Rotation happens w.r.t fixed point



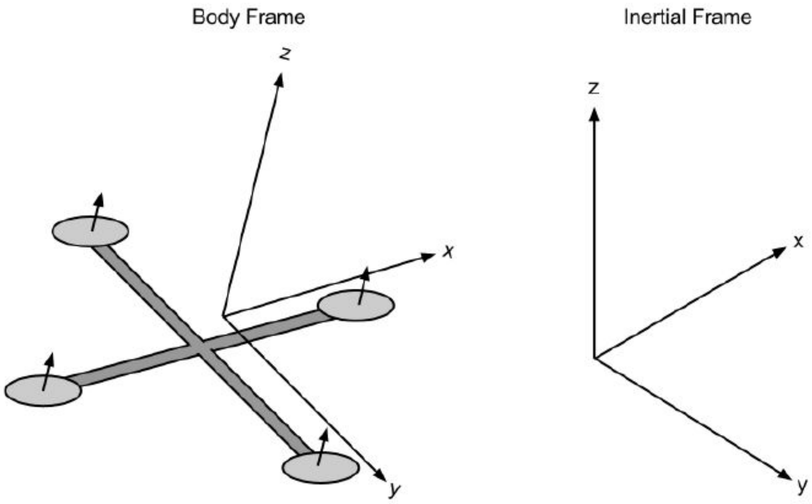
What about in free space?

a. Rotation happens at Center of gravity. Why? : Principle of least action
b. Equations of motion taken w.r.t. CoG
c. CoG : isolates translational and rotational equations of motion



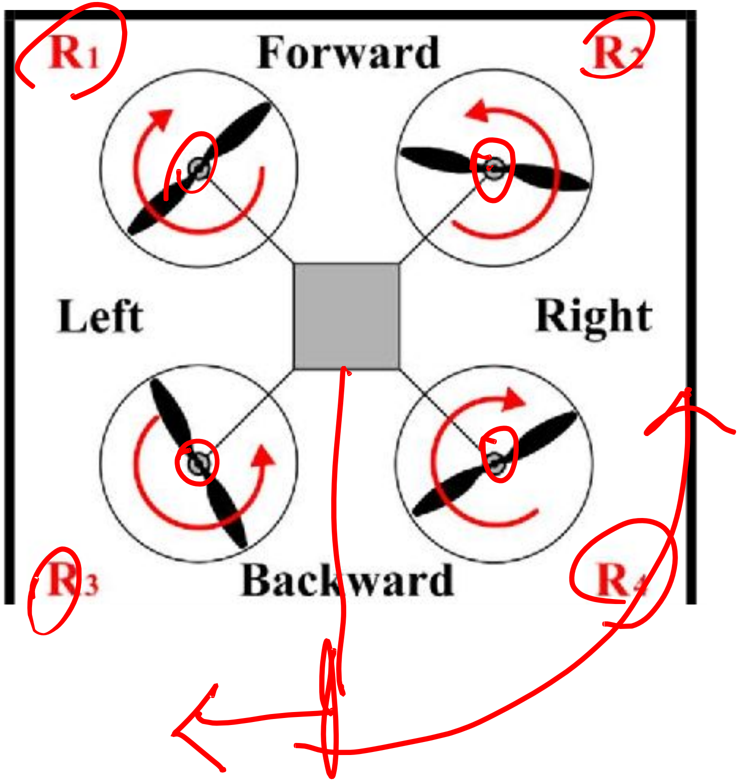
5. Reference Frames

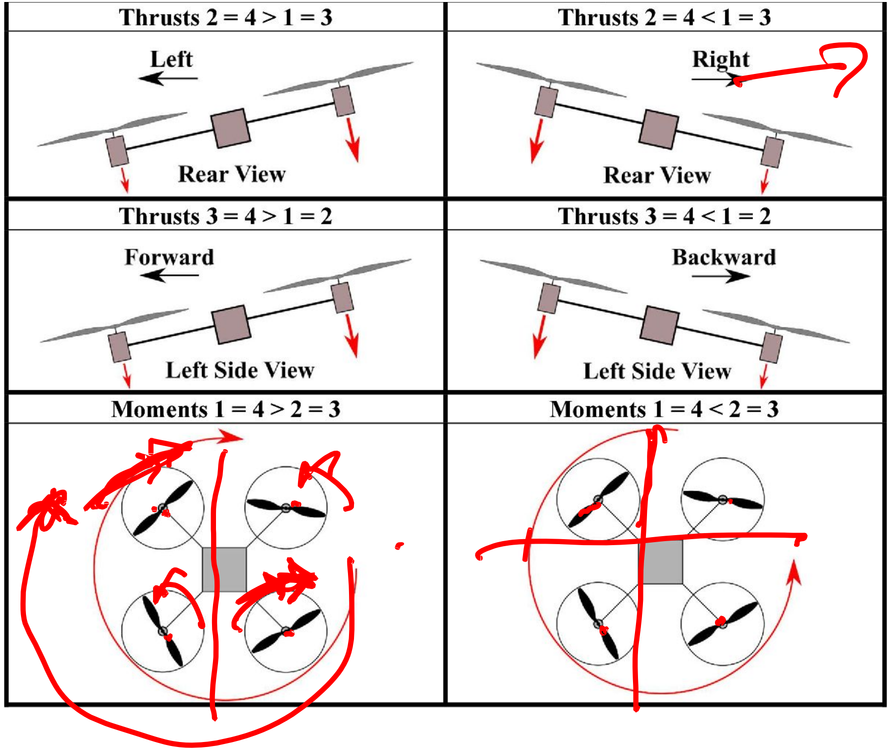
- Inertial and body frames
- Velocity in translation & rotation (https://www.youtube.com/watch?v=d00XI_UTKQo)



Quadcopter Body Frame and Inertial Frame

6. How does a drone.... drone?

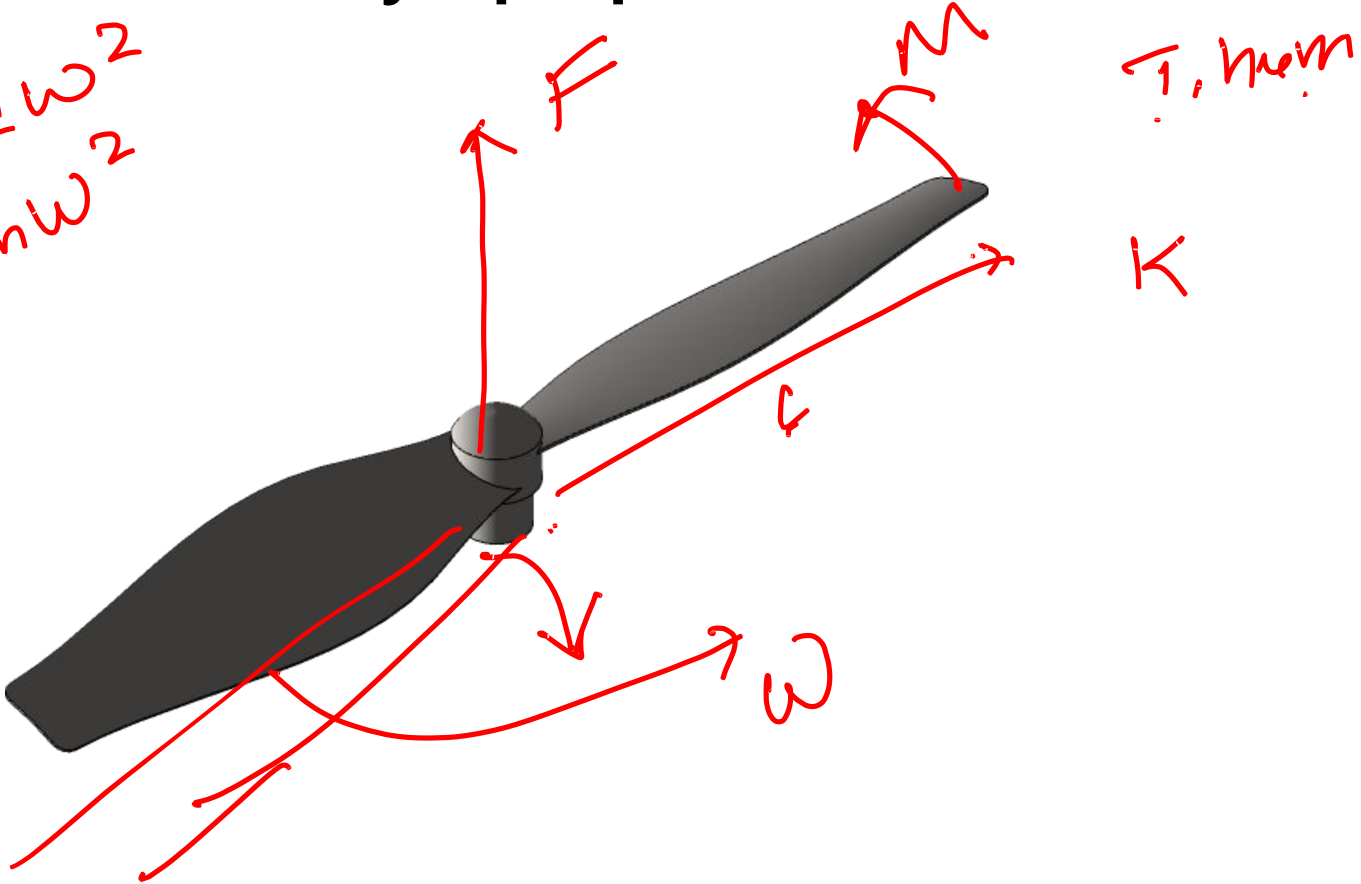




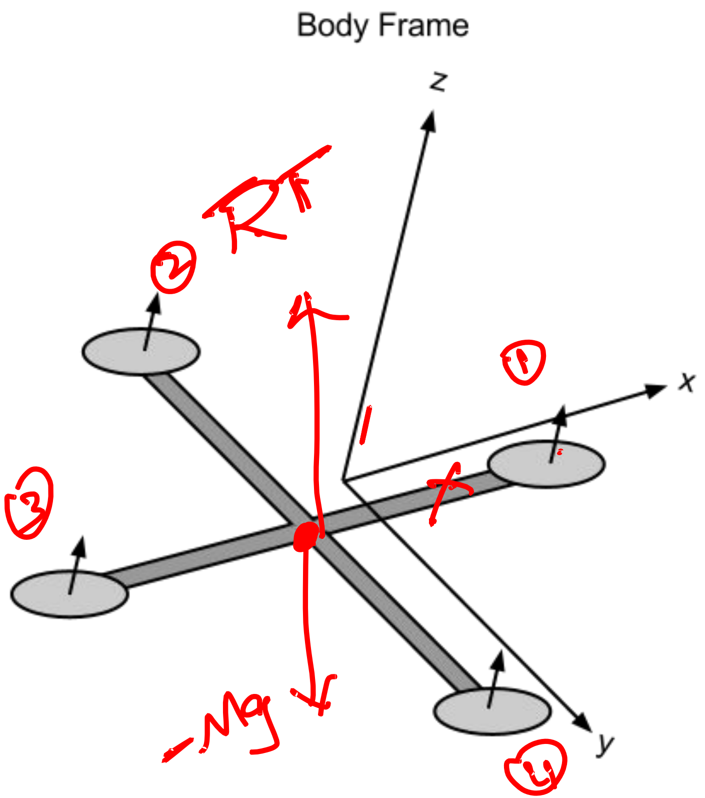
6. Forces produced by a propellor

$F = K_f \omega^2$

$M = K_m \omega^2$



Forces and moments on a quadcoptor



$$F_i = k_f \omega_i^2 \; ; \; M = k_m \omega_i^2$$

$$\hat{z}_b$$

$$T = \begin{bmatrix} 0 \\ 0 \\ F_i \end{bmatrix}$$

Eq of motion: Translations

$$T = \sum_{i=1}^{4} f_i = k \sum_{i=1}^{4} \omega_i^2, \quad \boldsymbol{T}^B = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix};$$

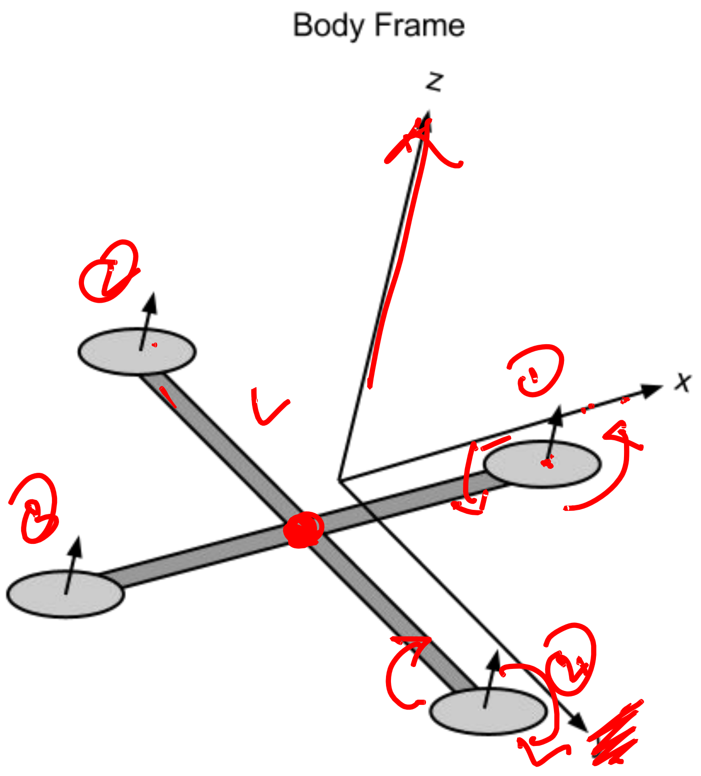
body

$$m\ddot{\boldsymbol{\xi}} = \boldsymbol{G} + \boldsymbol{R}\boldsymbol{T}_B,$$

$-mg$

World frame

~~Forces and~~ moments on a quadcoptor



$f_1 \cdot 0 + f_3 \cdot 0 +$

$f_2 l + f_4 l$

x: $l(-K_f \omega_2^2 + K_f \omega_4^2)$

y: $l(-K_f \omega_1^2 + K_f \omega_3^2)$

z: $K_m(+\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)$

Eq of motion: Rotations

$$\tau_B = \begin{bmatrix} Lk({\omega_1}^2 - {\omega_3}^2) \\ Lk({\omega_2}^2 - {\omega_4}^2) \\ b\left({\omega_1}^2 - {\omega_2}^2 + {\omega_3}^2 - {\omega_4}^2\right) \end{bmatrix}$$

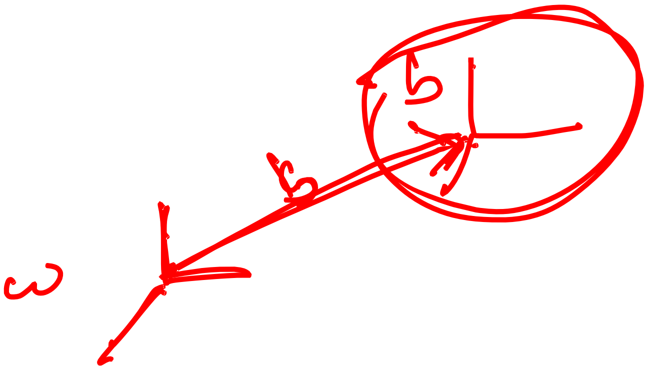
$$(I\dot{\omega}) + \omega \times (I\omega) = (\tau_B)$$

Transport Theorem

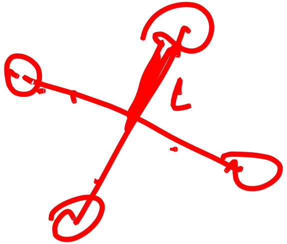
- Transport theorem helps us to connect rate of change of a vector in one reference frame with the same vector in a different frame

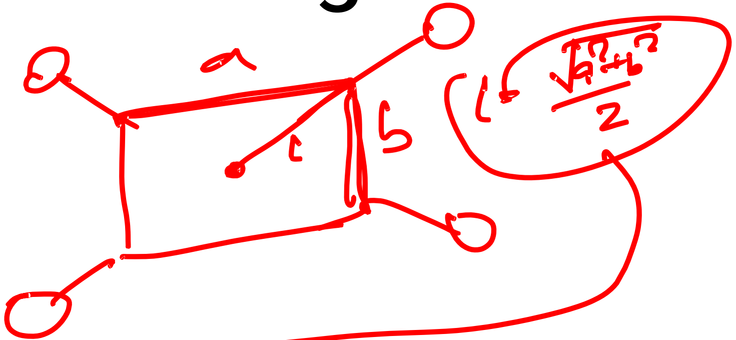
$$ {}^{A}\frac{d\underline{b}}{dt} = {}^{B}\frac{d\underline{b}}{dt} + {}^{A}\underline{\omega}^{B} \times \underline{b} $$

- Proof



How will these equations of motion change for the drone shown





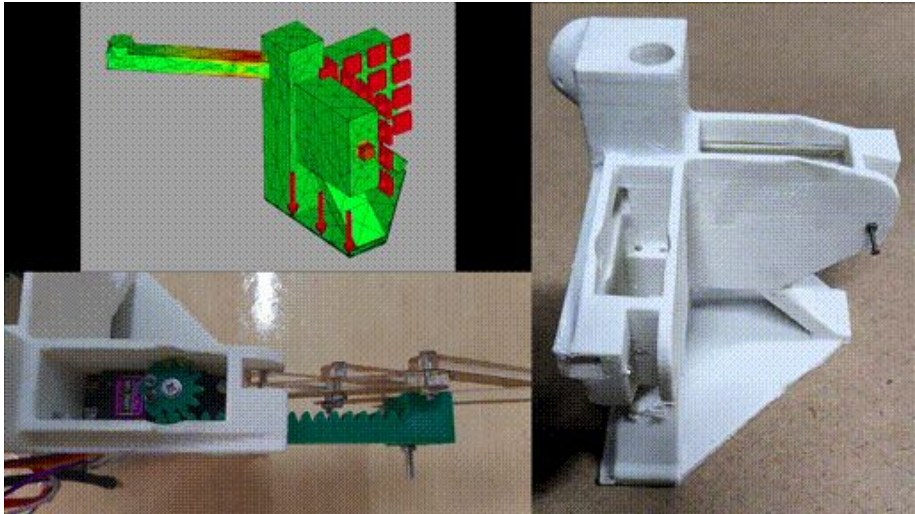
$$T = \sum_{i=1}^{4} f_i = k \sum_{i=1}^{4} \omega_i^2, \quad \boldsymbol{T}^B = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix},$$

$$m\ddot{\boldsymbol{\xi}} = \boldsymbol{G} + \boldsymbol{R}\boldsymbol{T}_B,$$

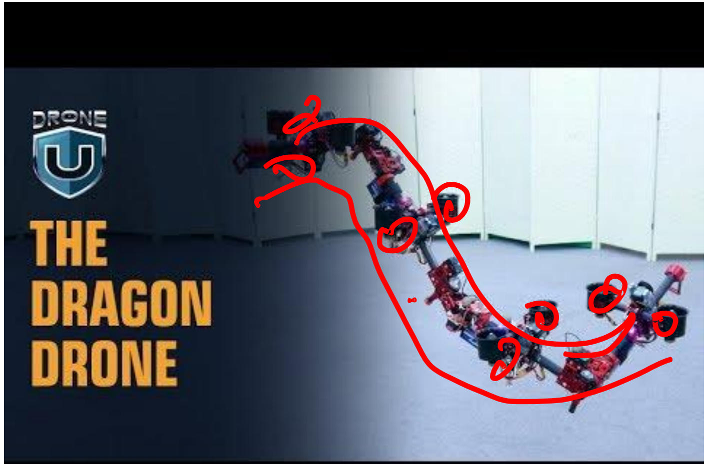
$$\tau_B = \begin{bmatrix} Lk({\omega_1}^2 - {\omega_3}^2) \\ Lk({\omega_2}^2 - {\omega_4}^2) \\ b({\omega_1}^2 - {\omega_2}^2 + {\omega_3}^2 - {\omega_4}^2) \end{bmatrix}$$

?

$$I\dot{\omega} + \omega \times (I\omega) = \tau$$



Now try writing equations of motion for one the following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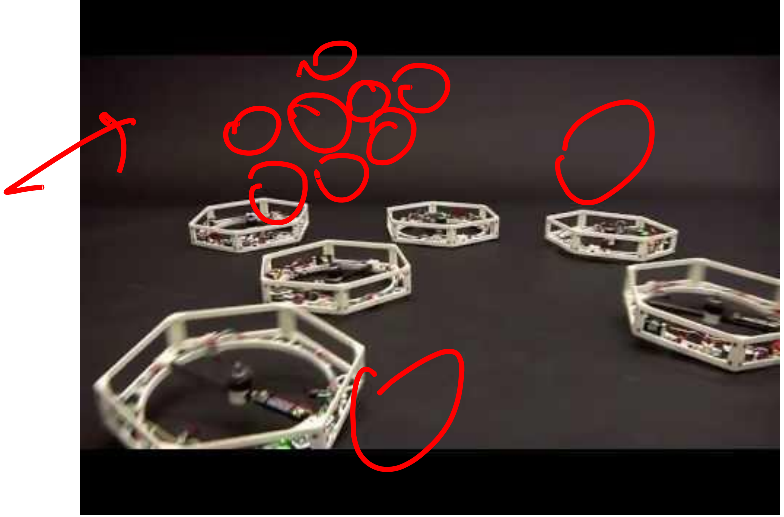
DRAGON
DRONE



Newtonian v/s variational mechanics

Vectorial/Newtonian Mechanics	Analytical/Variational Mechanics
Force and momentum	Kinetic Energy and Work (Potential energy)
Too much attention to coordinate systems	Generalized coordinate system
Fails at very large and very small scales	Consistent
No interesting extensions	Can obtain constants of motion
Constraints induce additional unknowns	Constraints are much easier to deal with

Ted talk by Raffaello D'Andrea

Good reads

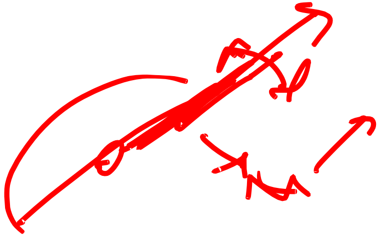
1. Overall drone overview
2. Detailed read on propellor forces

h/w s/w dyn

Tem, hum

Kf

ω, dims, weight



Kf