Multi-View Geometry 4

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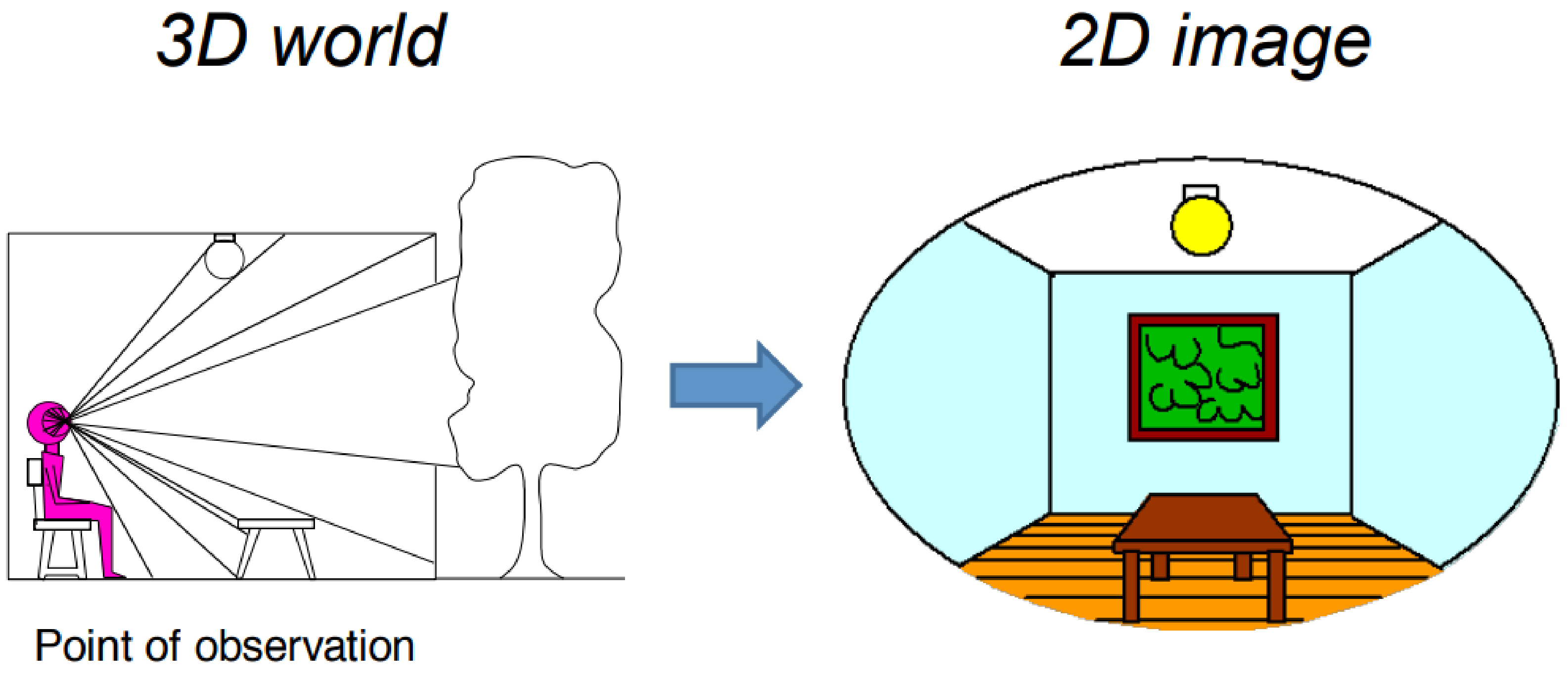
Slides adapted from CMU 16-385 Computer Vision Course

The role of cameras in perception

- We as humans have 5 sensory organs. Robots?
- How much information can cameras give?
- Recall the properties of a camera.

Camera – A dimensionality reduction machine



What’s the problem with 2D information?

Intrinsic Parameter
(intrinsic matrix)

Extrinsic Parameter
(extrinsic matrix)

$$\lambda \begin{bmatrix} x_c \\ y_c \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & x_{c0} \\ 0 & f_y & y_{c0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r11 & r12 & r13 & t_x \\ r21 & r22 & r23 & t_y \\ r31 & r32 & r33 & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

degree of freedom
(for the depth)

Point in
Captured Iamage

focal length f, center of the image c0

rotaional component r

translate components t

Point in Marker
Coodinate

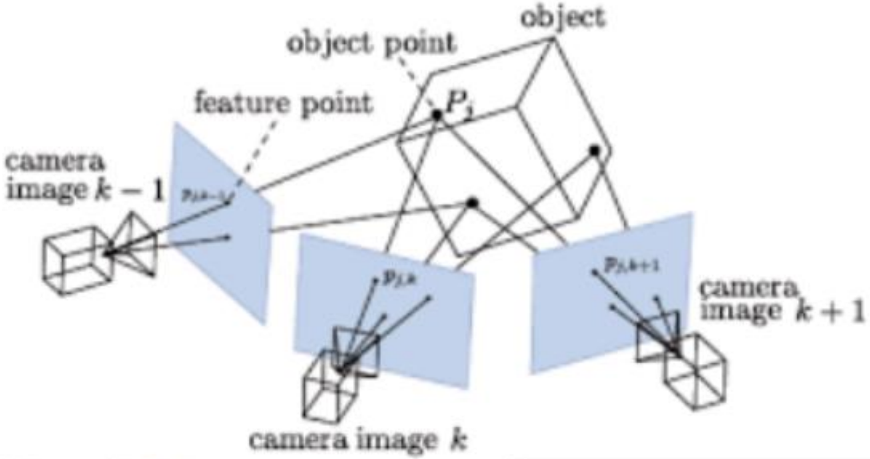
Point in Camera Coordinate

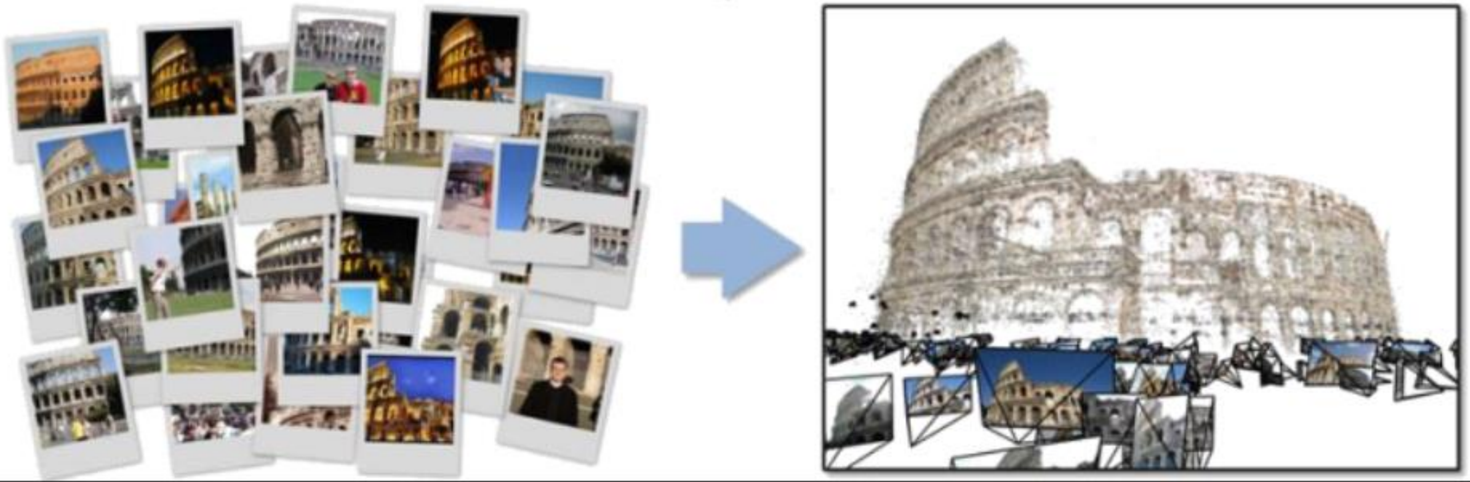
Point in Captured Image coordinate

Why is 3D information important?

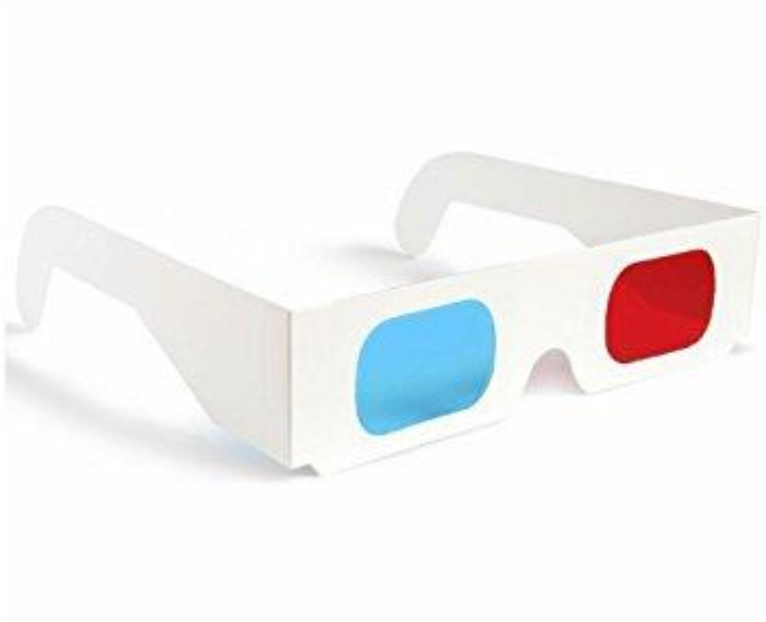
- 3D information -> Depth
- Perception of the environment
- Gauge how close/far away.
- Applications: Self-Driving

If not one then how many?

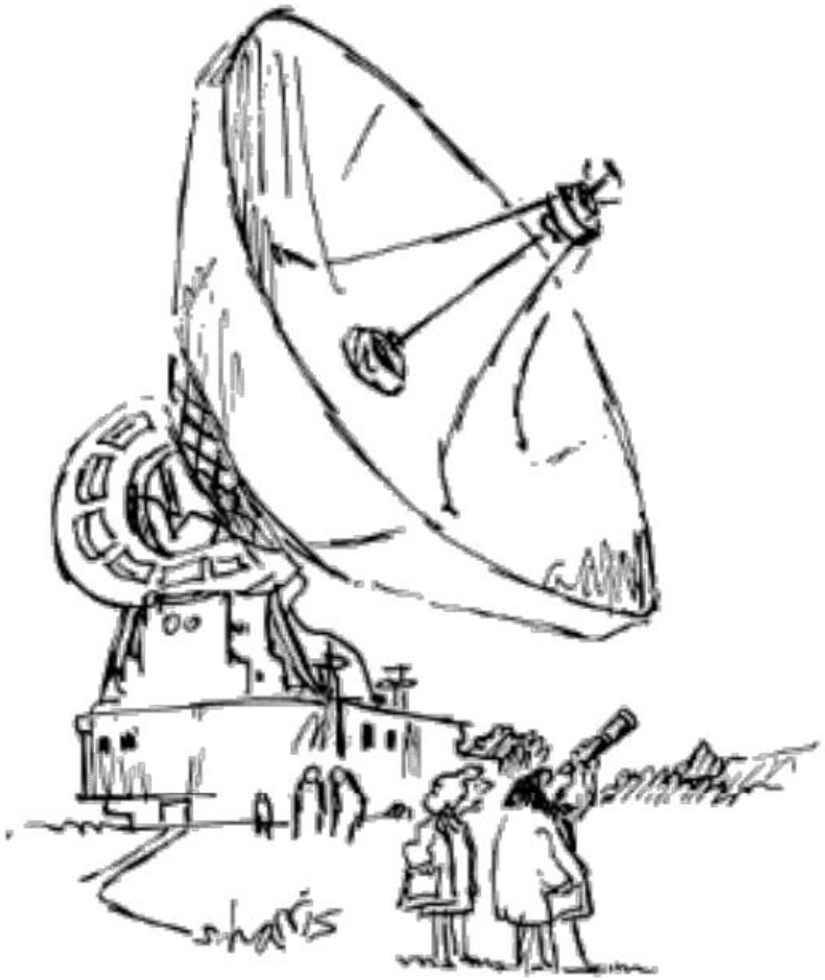




Enter Stereo-Vision!



Two is better than one



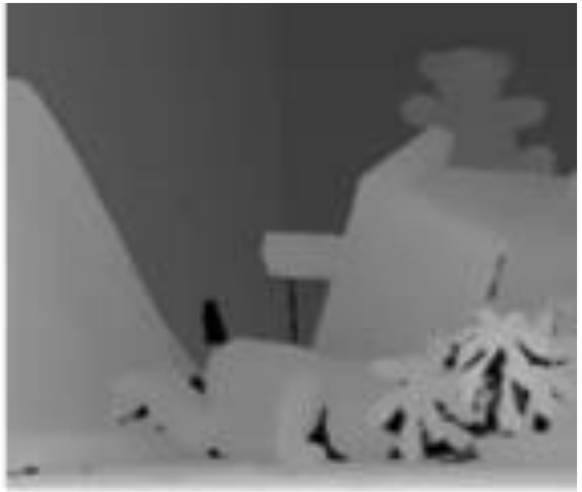
"Just checking."

Biomimicry

- Hold your index finger an arm’s length away.
- Look at it through the left eye keeping the right eye closed.
- Now look at it through the right eye keeping the left one closed.
- You will perceive a shift - this is called as stereo disparity and the brain uses it heavily to infer depth!
- Can we model this problem?

Objects that are close move more or less?

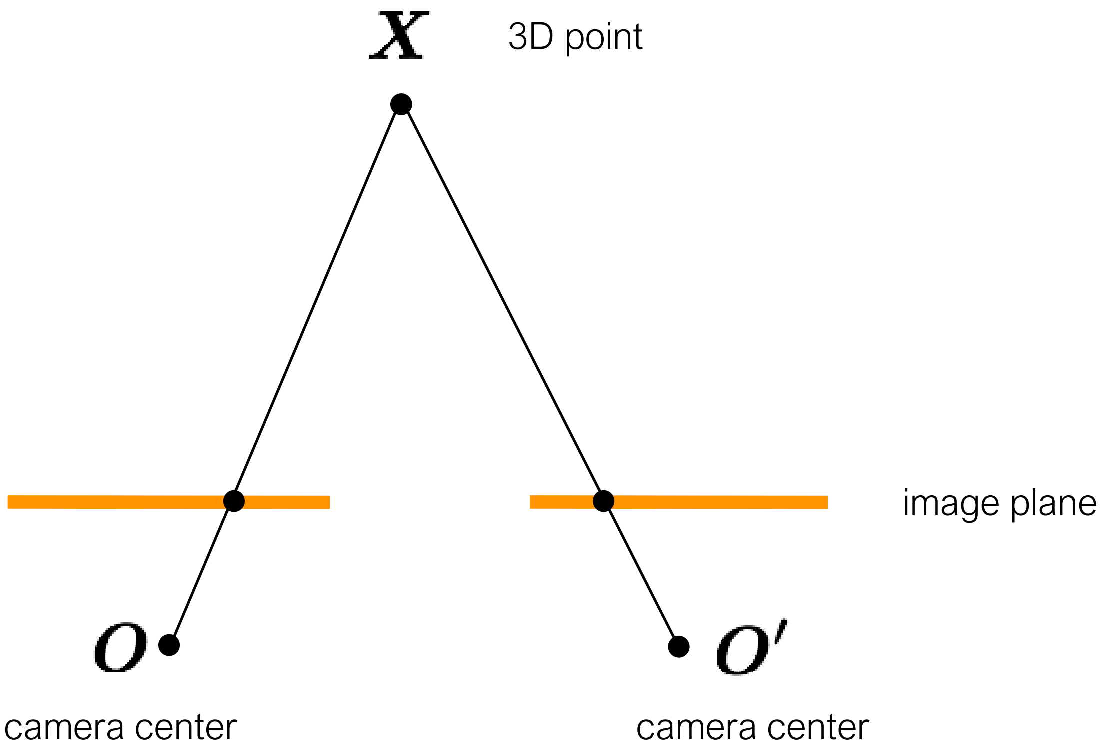
The amount of horizontal movement is inversely proportional to …



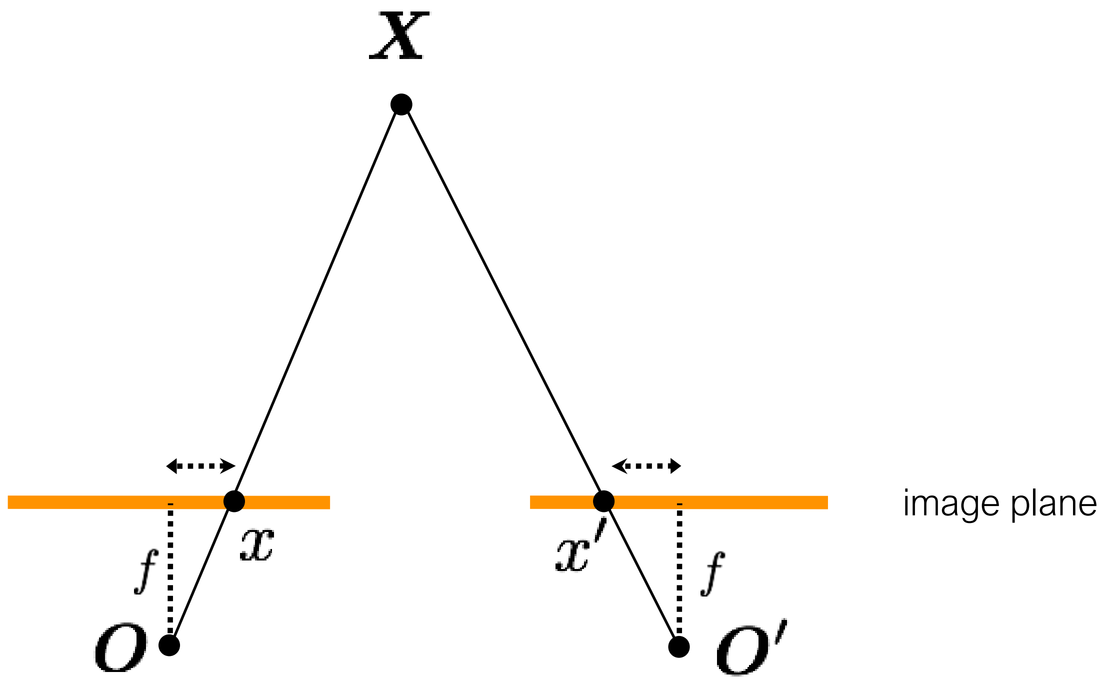
The amount of horizontal movement is inversely proportional to …

… the distance from the camera.

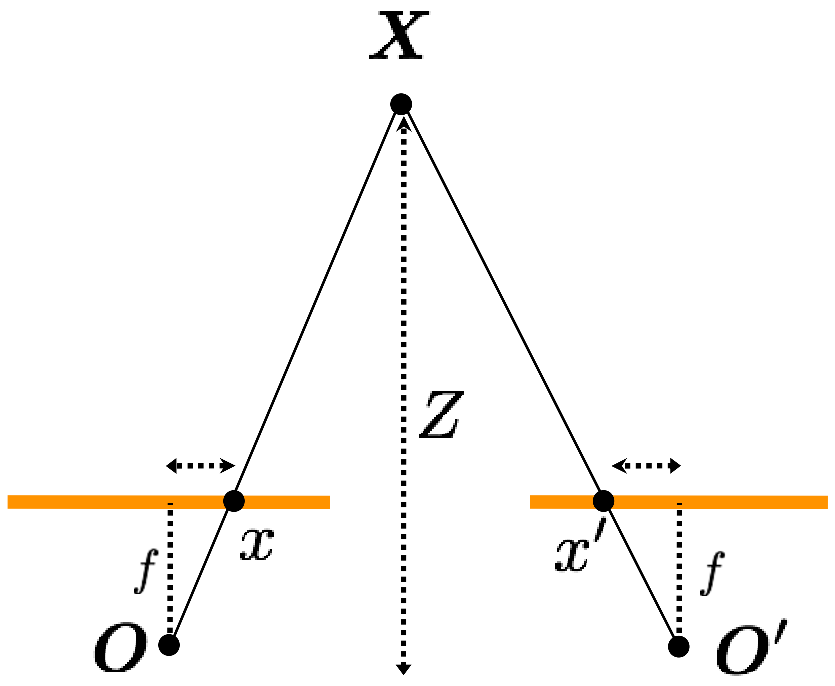
More formally…

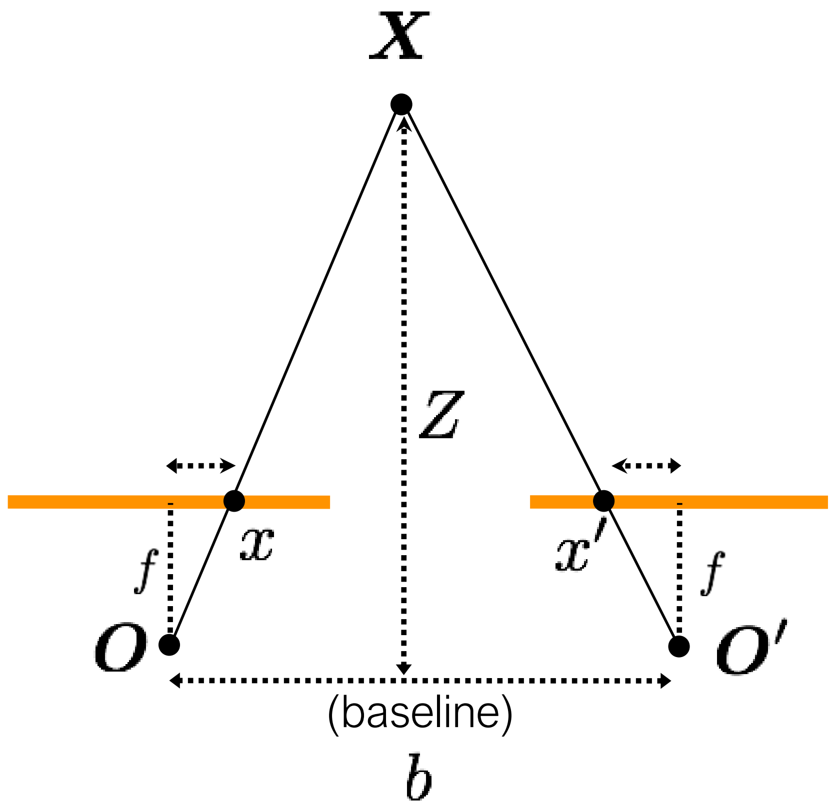


X
3D point
image plane
O
camera center
O'
camera center

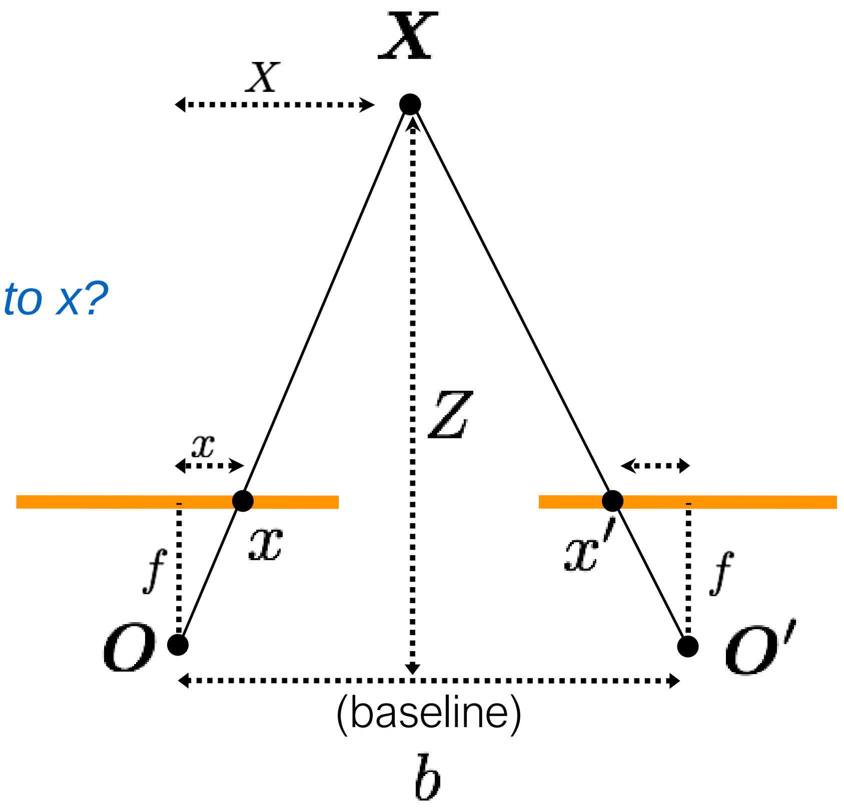


X
x
x'
f
f
O
O'
image plane

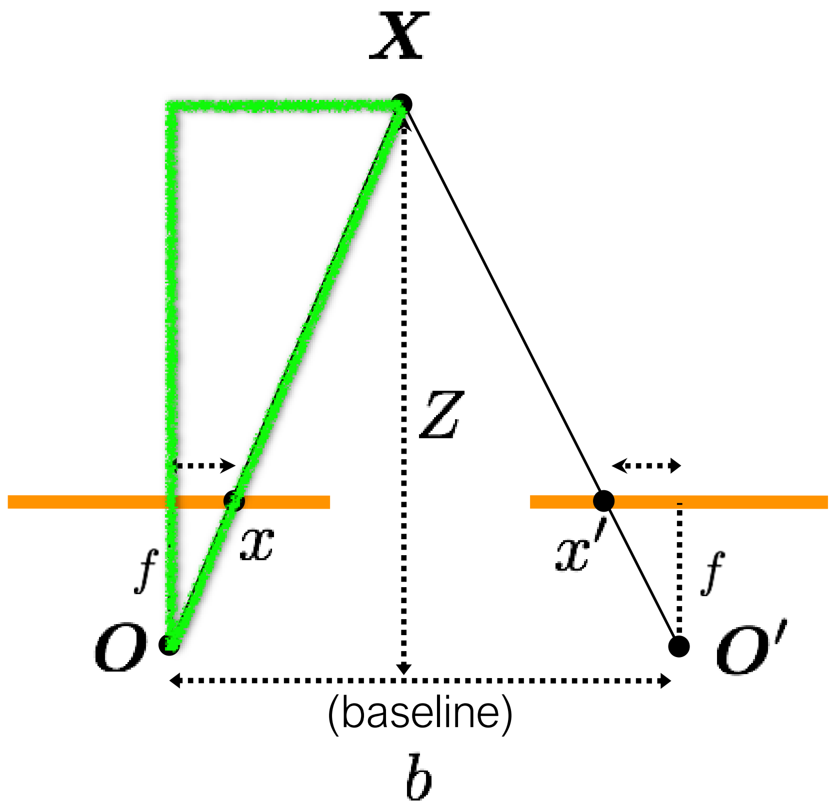

$\boldsymbol{X}$
Z
x
x'
f
f
$\boldsymbol{O}$
$\boldsymbol{O}'$


$\boldsymbol{X}$
Z
x
x'
f
f
$\boldsymbol{O}$
$\boldsymbol{O}'$
(baseline)
b

How is X related to x?



$$\frac{X}{Z} = \frac{x}{f}$$



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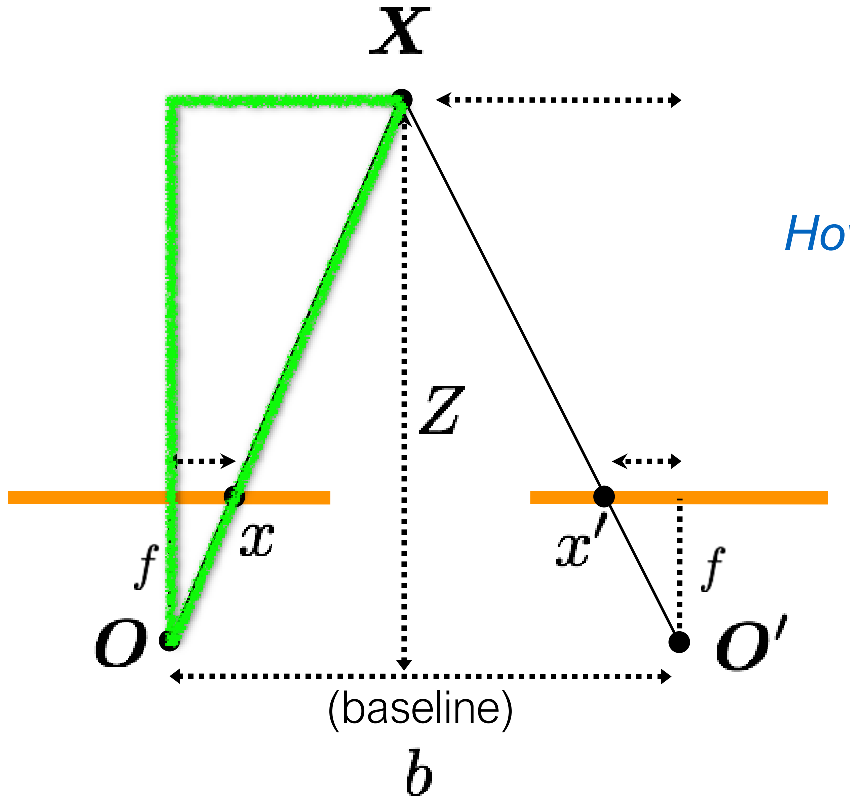
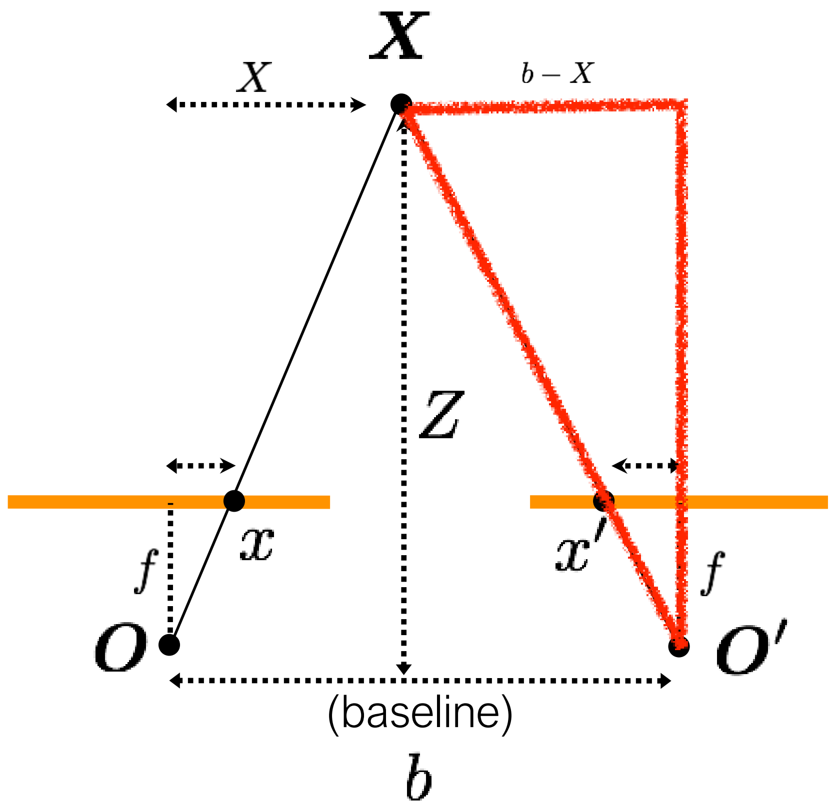


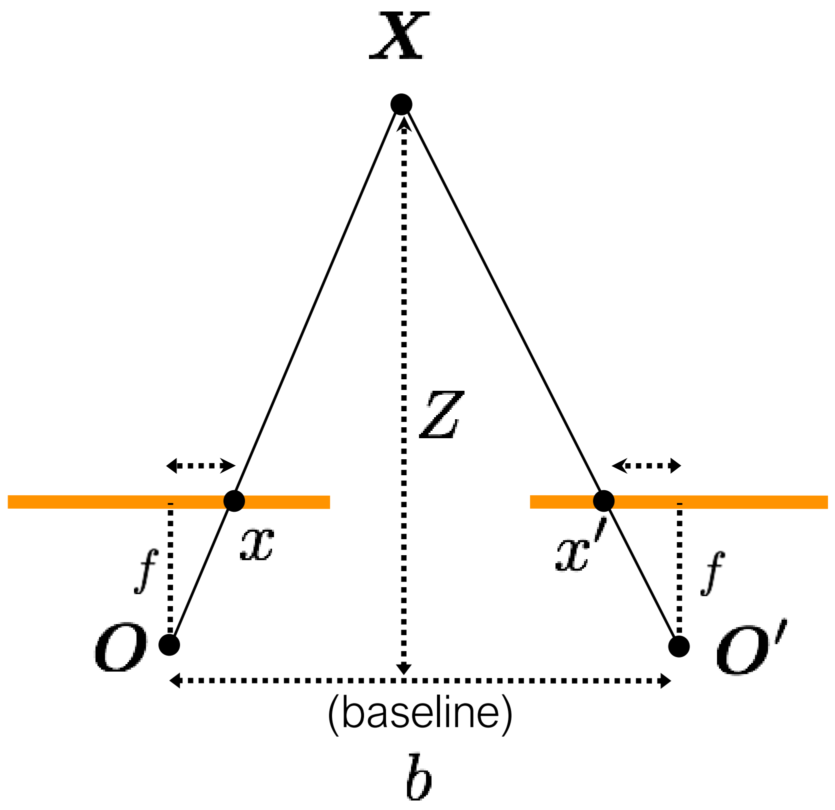
How is X related to x'?

$$\frac{X}{Z} = \frac{x}{f}$$



$$\frac{b - X}{Z} = \frac{x'}{f}$$

$$\frac{X}{Z} = \frac{x}{f}$$



$$\frac{b - X}{Z} = \frac{x'}{f}$$

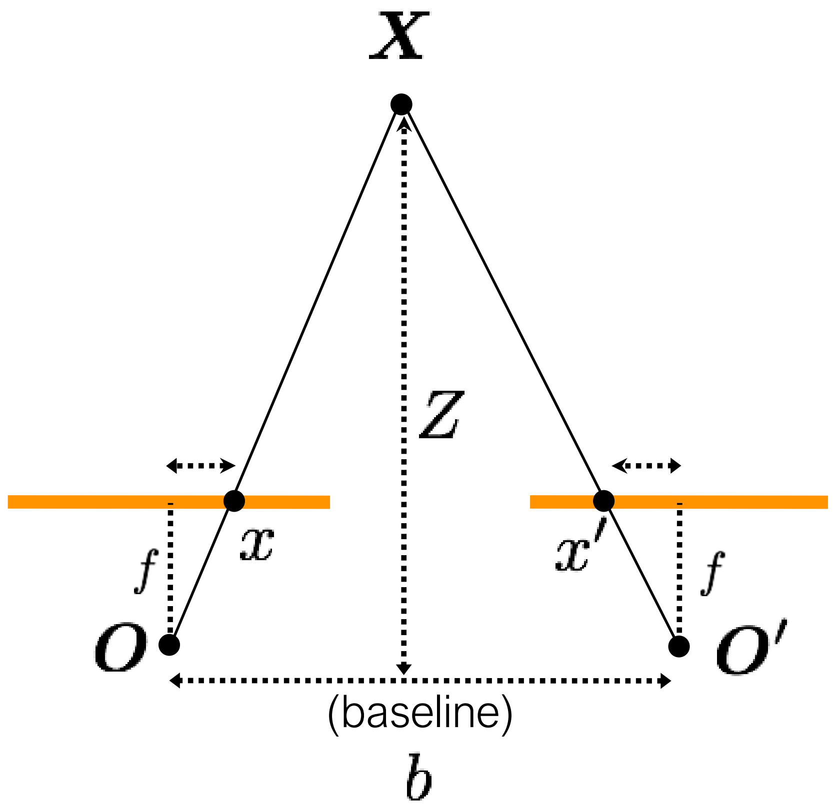
Disparity

$$d = x - x'$$

(wrt to camera origin of image plane)

$$= \frac{bf}{Z}$$

$$\frac{X}{Z} = \frac{x}{f}$$



$$\frac{b - X}{Z} = \frac{x'}{f}$$

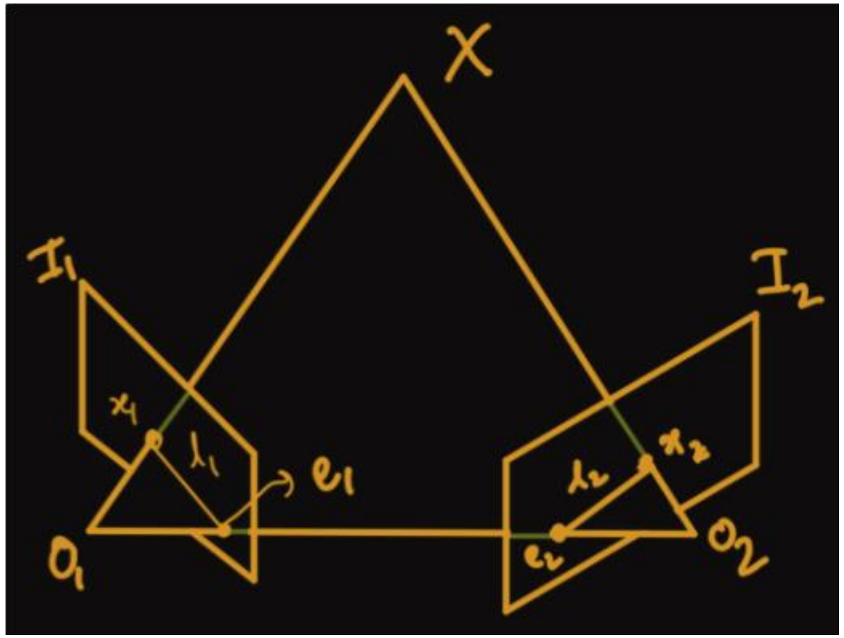
Disparity

$$d = x - x'$$
$$= \frac{bf}{Z}$$

inversely proportional
to depth

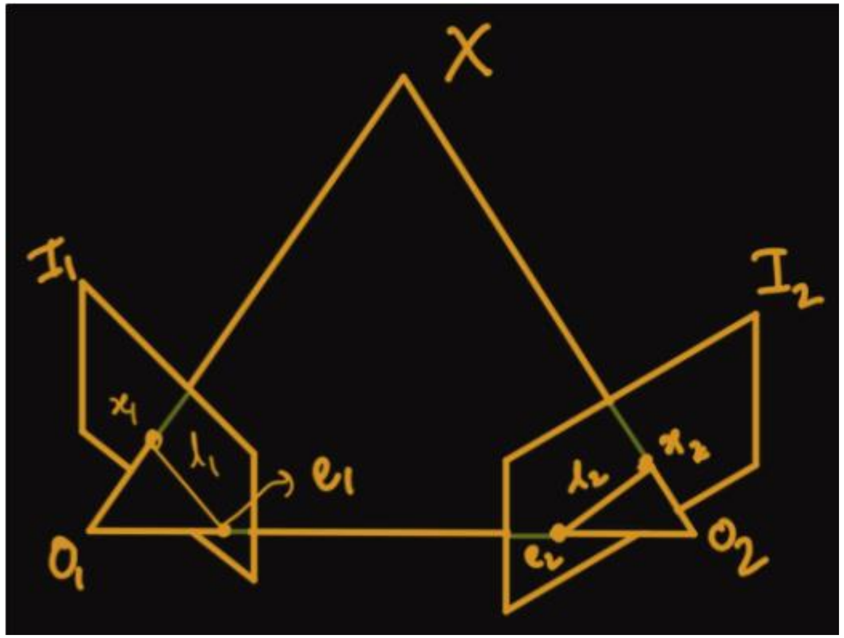
Doesn’t this look familiar?

- 2D-2D correspondences.
- Epipolar Geometry.
- Stereo Vision is a special case.
- Correspondences between images?

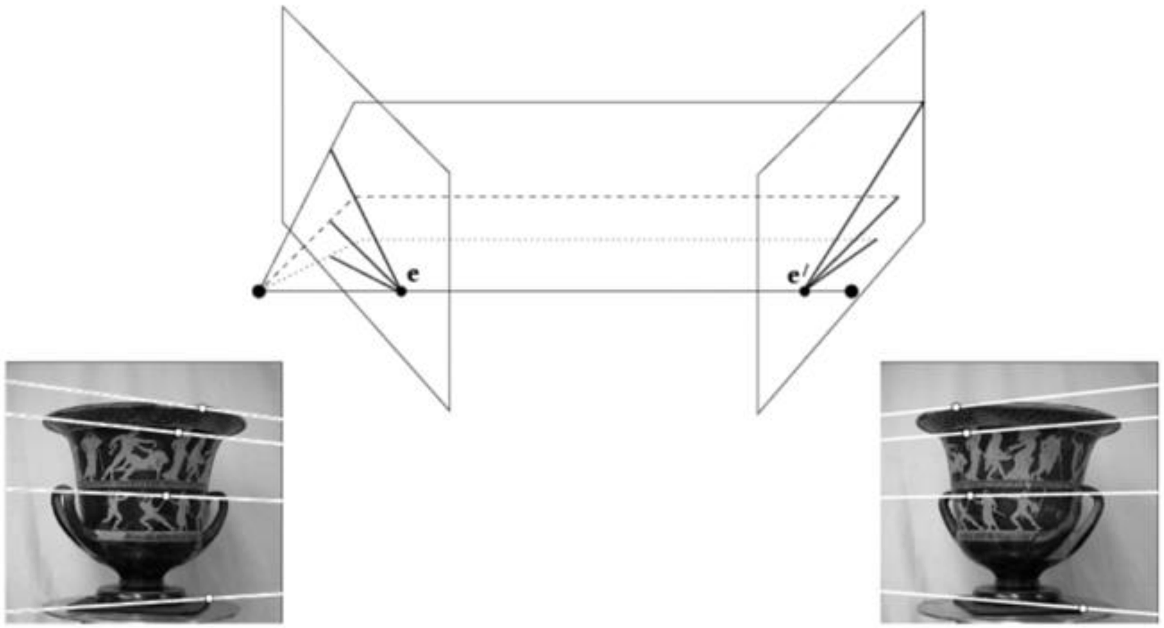


Recall the epipolar constraints

- O1, O2 and X lie on a plane – epipolar plane
- x1, x2 are projections of X - correspondences
- e1, e2 are the epipoles – images of O2, O1
- l1, l2 are the epipolar lines – 1D search
- Epipolar lines pass through the epipoles
- Essential matrix – relationship between planes

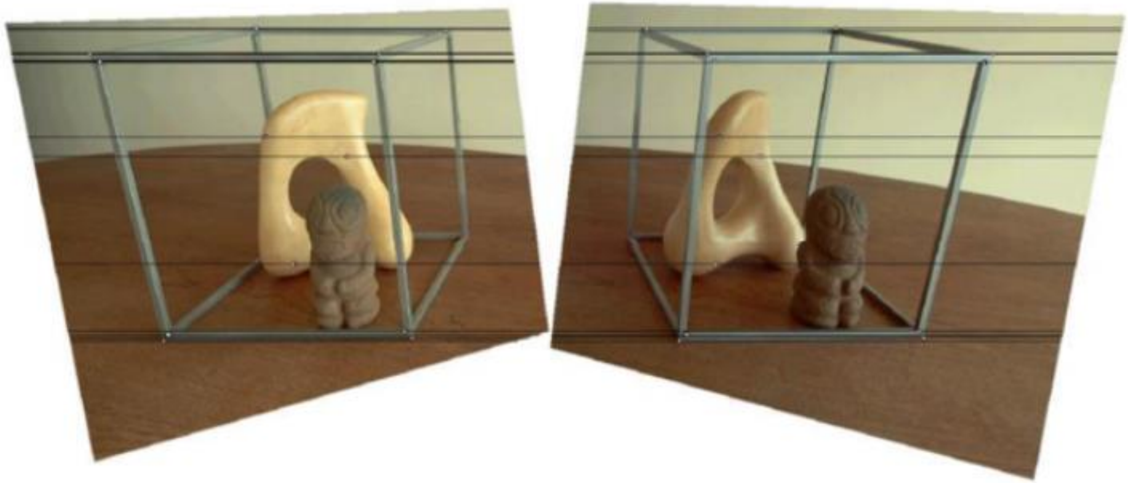


Visualising the epipolar lines



What have we achieved? Problems?

How can you make the epipolar lines horizontal?



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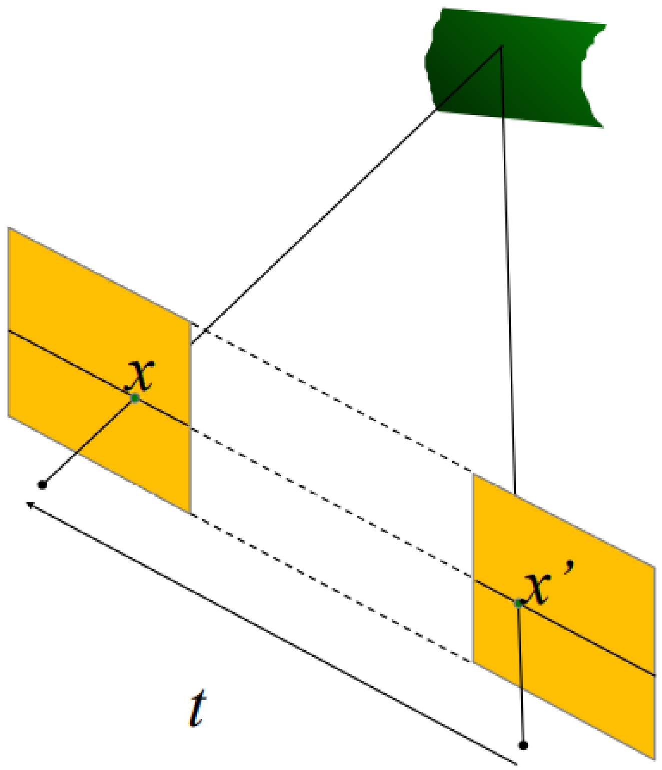
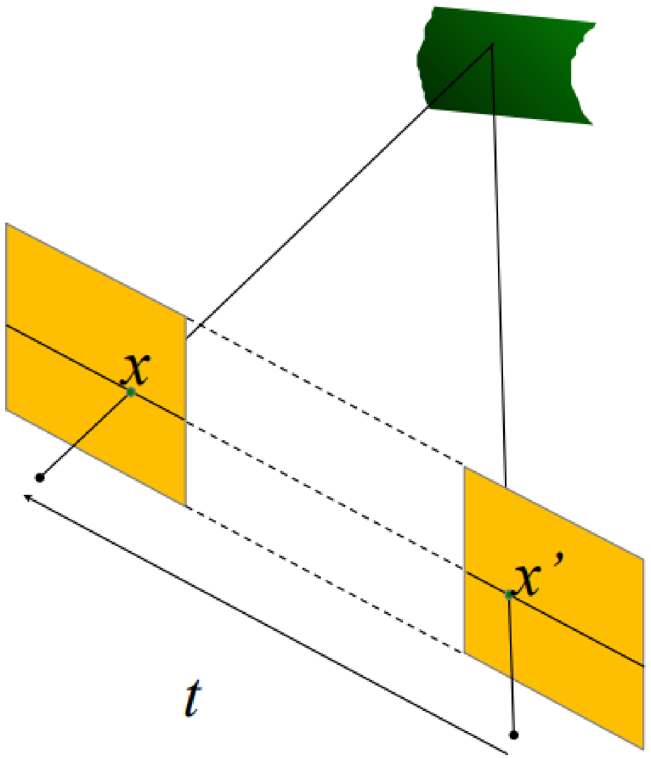


When this relationship holds:

$$\boxed{R = I \qquad t = (T, 0, 0)}$$

How can you make the epipolar lines horizontal?



When this relationship holds:

$$\boxed{R = I \qquad t = (T, 0, 0)}$$

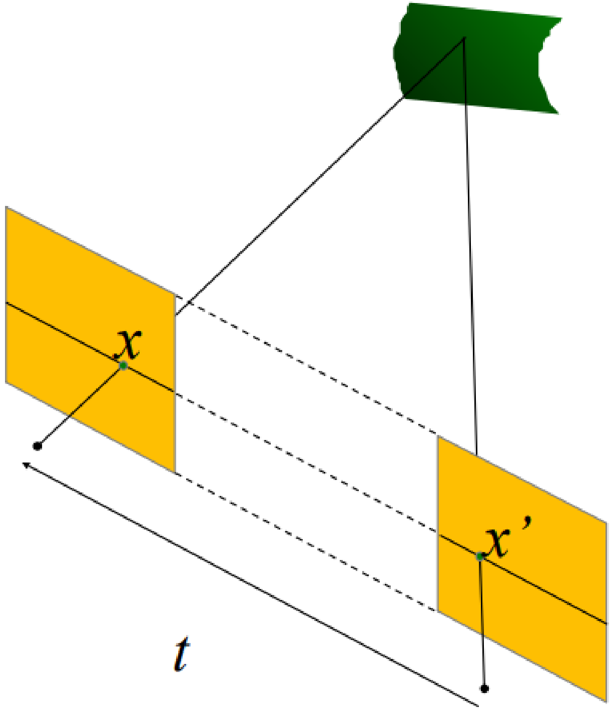
Let's try this out…

$$E = t \times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

This always has to hold

$$x^T E x' = 0$$

How can you make the epipolar lines horizontal?



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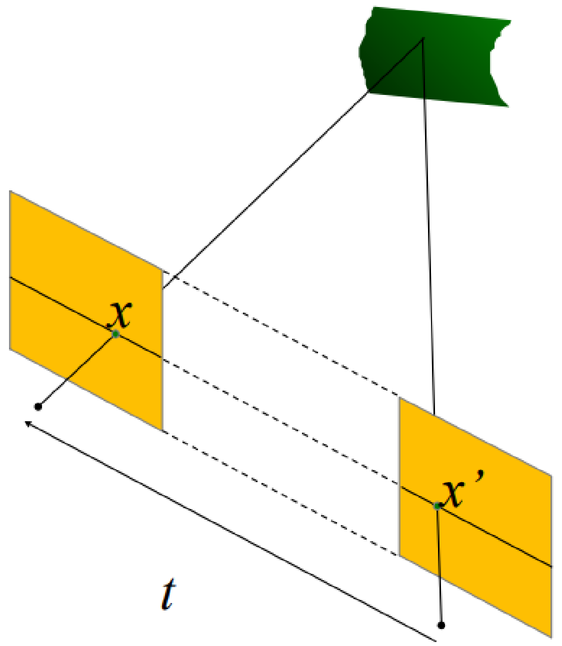
Write out the constraint

$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0$$

How can you make the epipolar lines horizontal?



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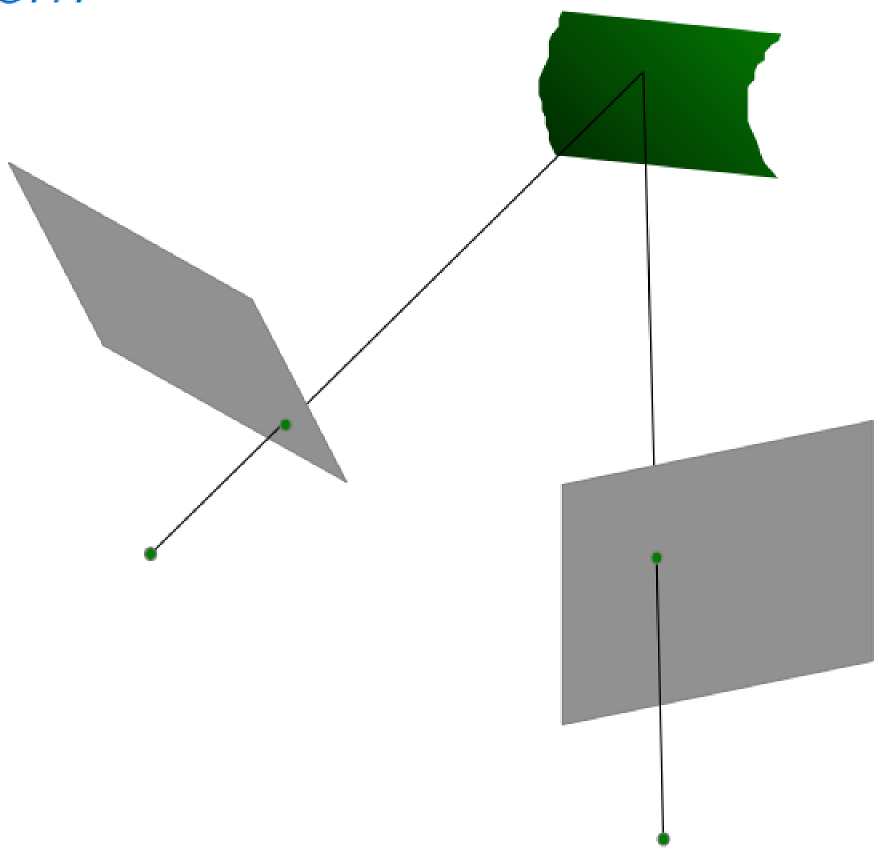
The image of a 3D point will always be on the same horizontal line

$$Tv = Tv'$$

y coordinate is always the same!

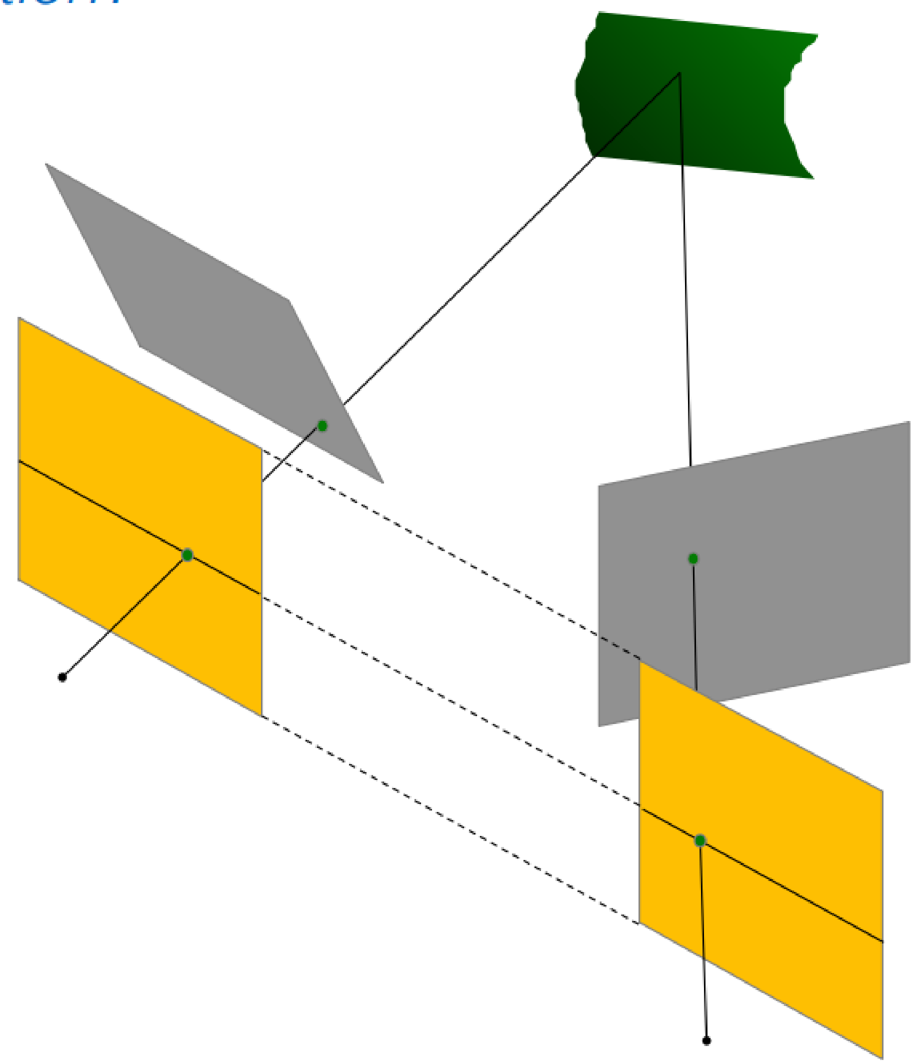
Implications?

What is stereo rectification?



What is stereo rectification?

Reproject image planes onto a common plane parallel to the line between camera centers



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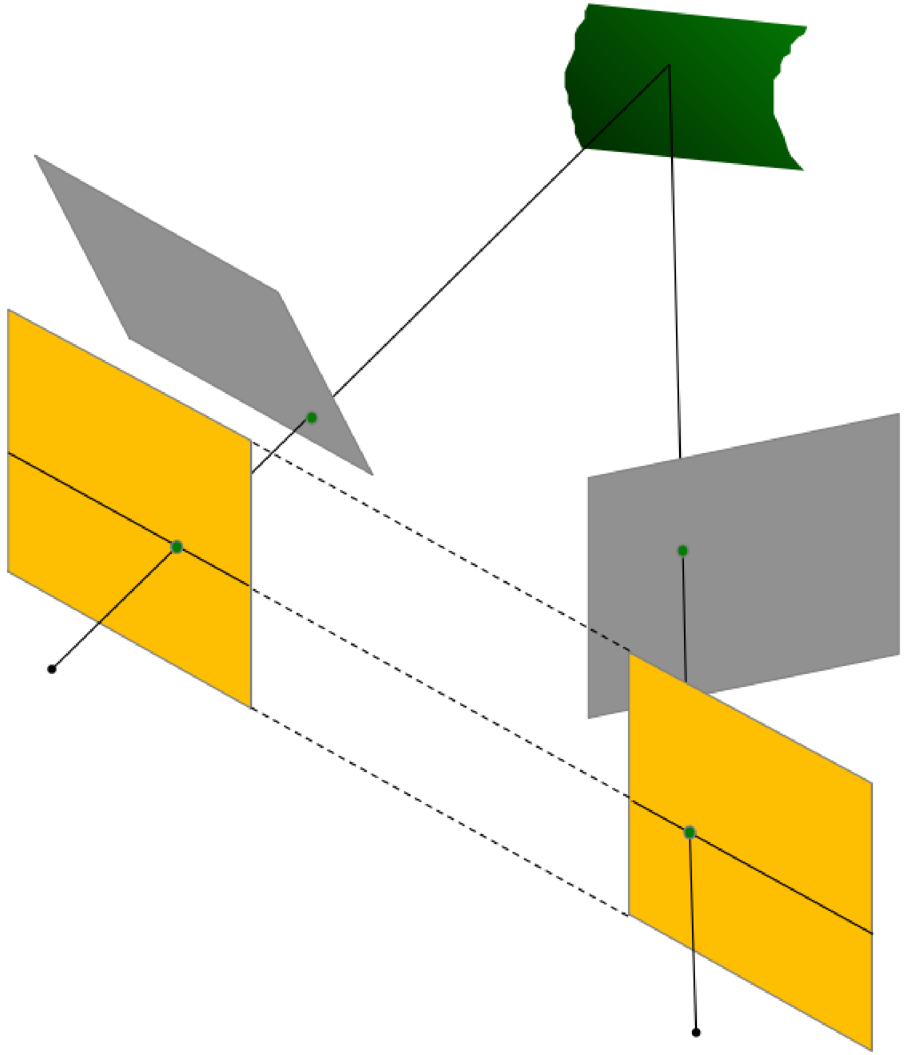


Need two homographies (3x3 transform), one for each input image reprojection

C. Loop and Z. Zhang. Computing Rectifying Homographies for Stereo Vision.Computer Vision and Pattern Recognition, 1999.

Stereo Rectification

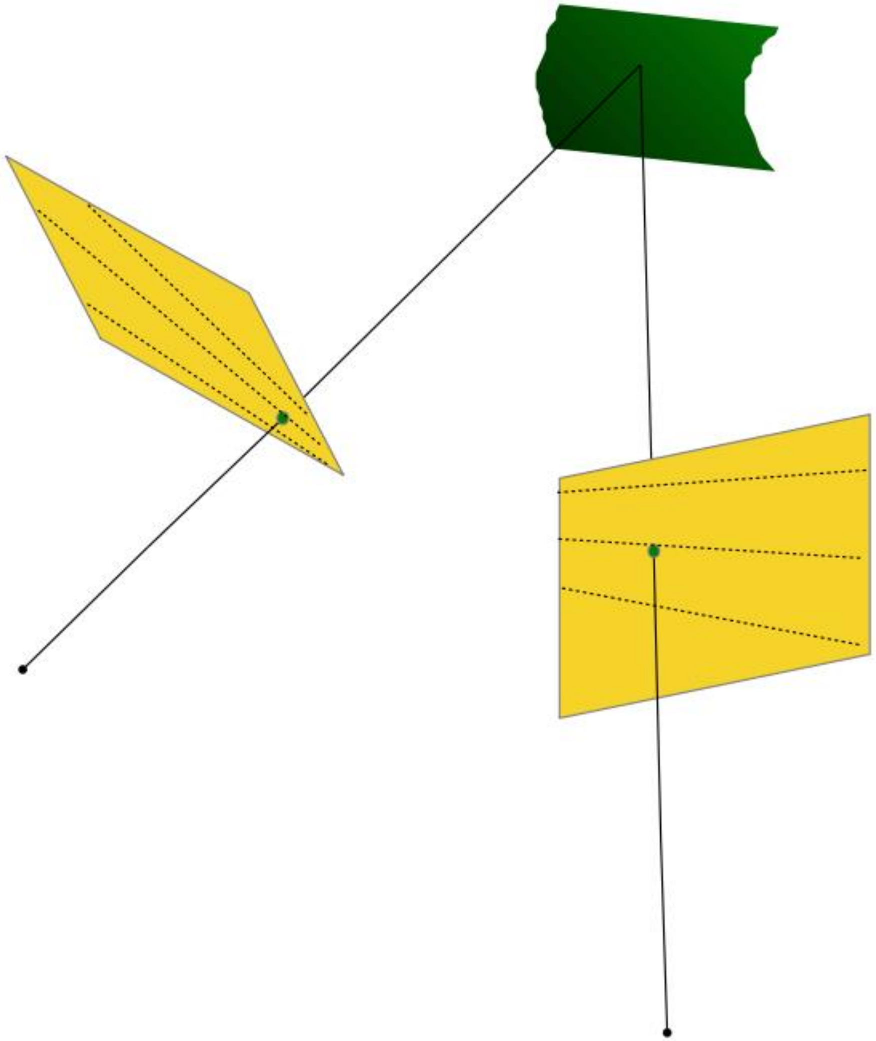
1. **Rotate** the right camera by **R**
   (aligns camera coordinate system orientation only)

2. Rotate (**rectify**) the left camera so that the epipole is at infinity

3. Rotate (**rectify**) the right camera so that the epipole is at infinity

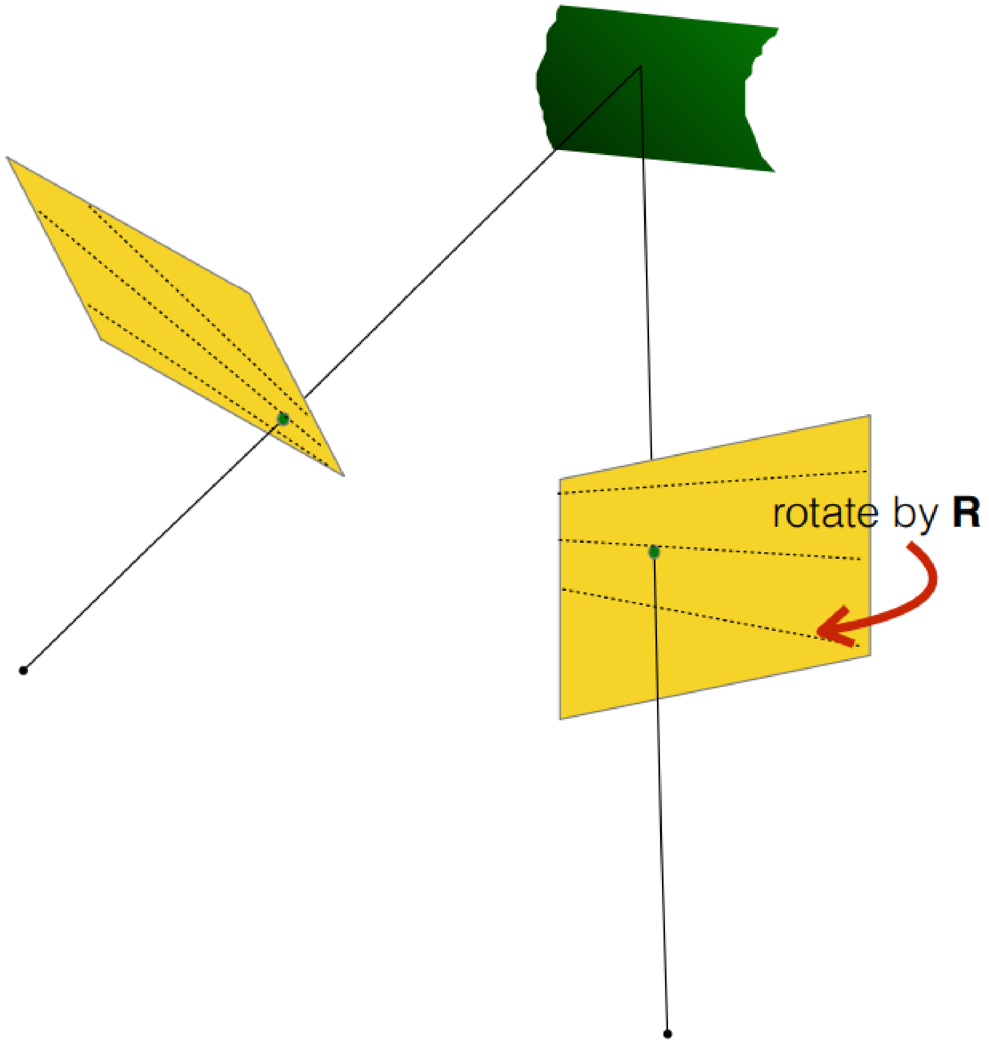
4. Adjust the **scale**

Stereo Rectification:



1. Compute **E** to get **R**
2. Rotate right image by **R**
3. Rotate both images by $\mathbf{R}_{\mathbf{rect}}$
4. Scale both images by **H**

Stereo Rectification:



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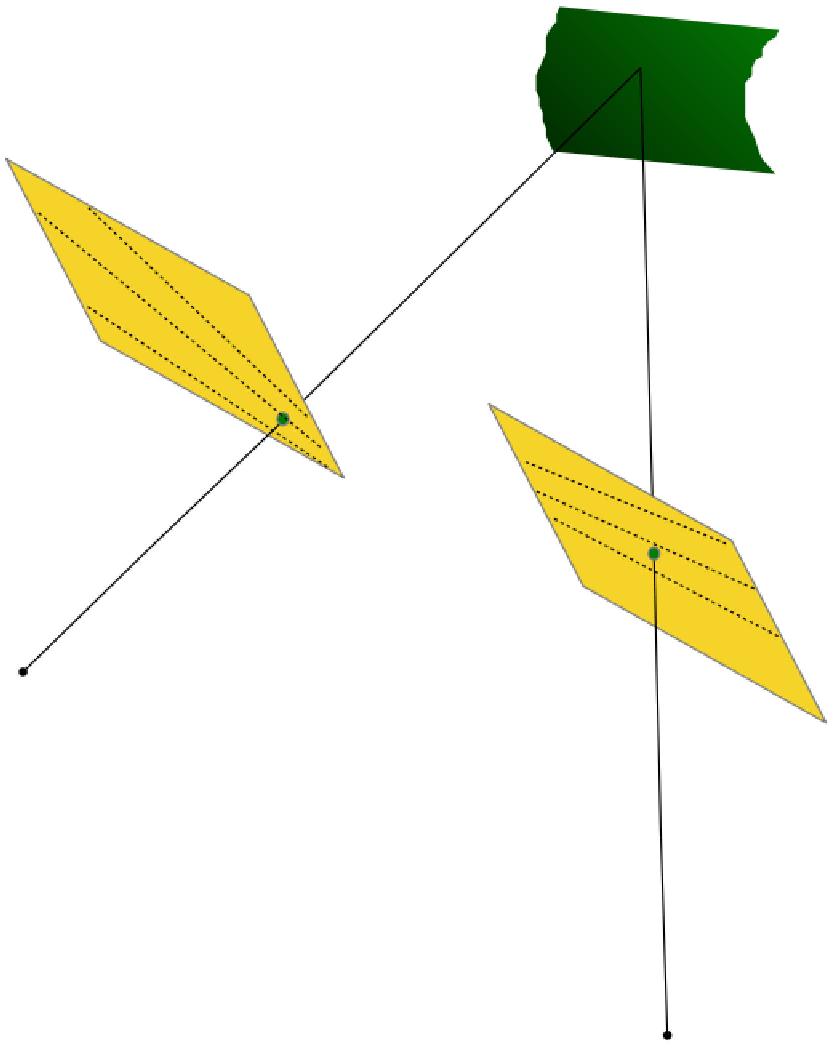

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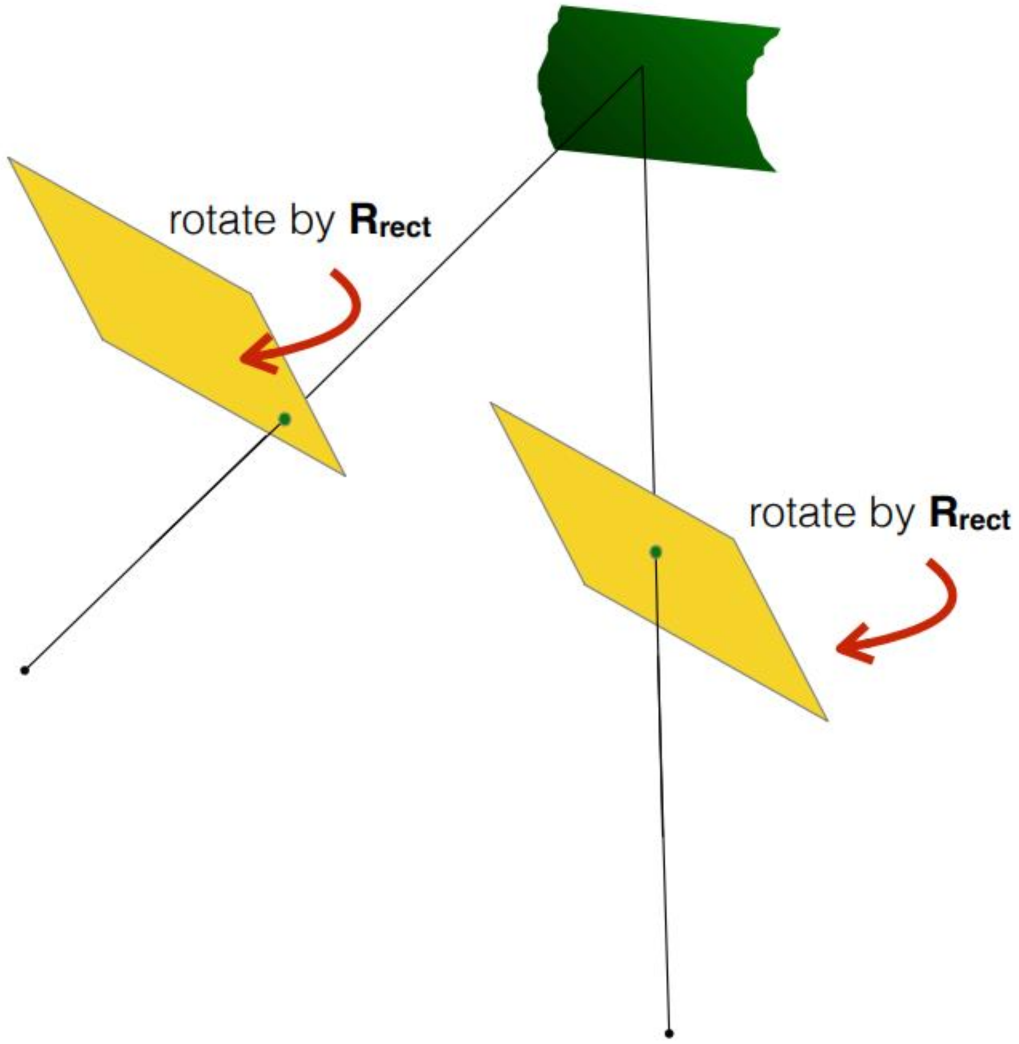

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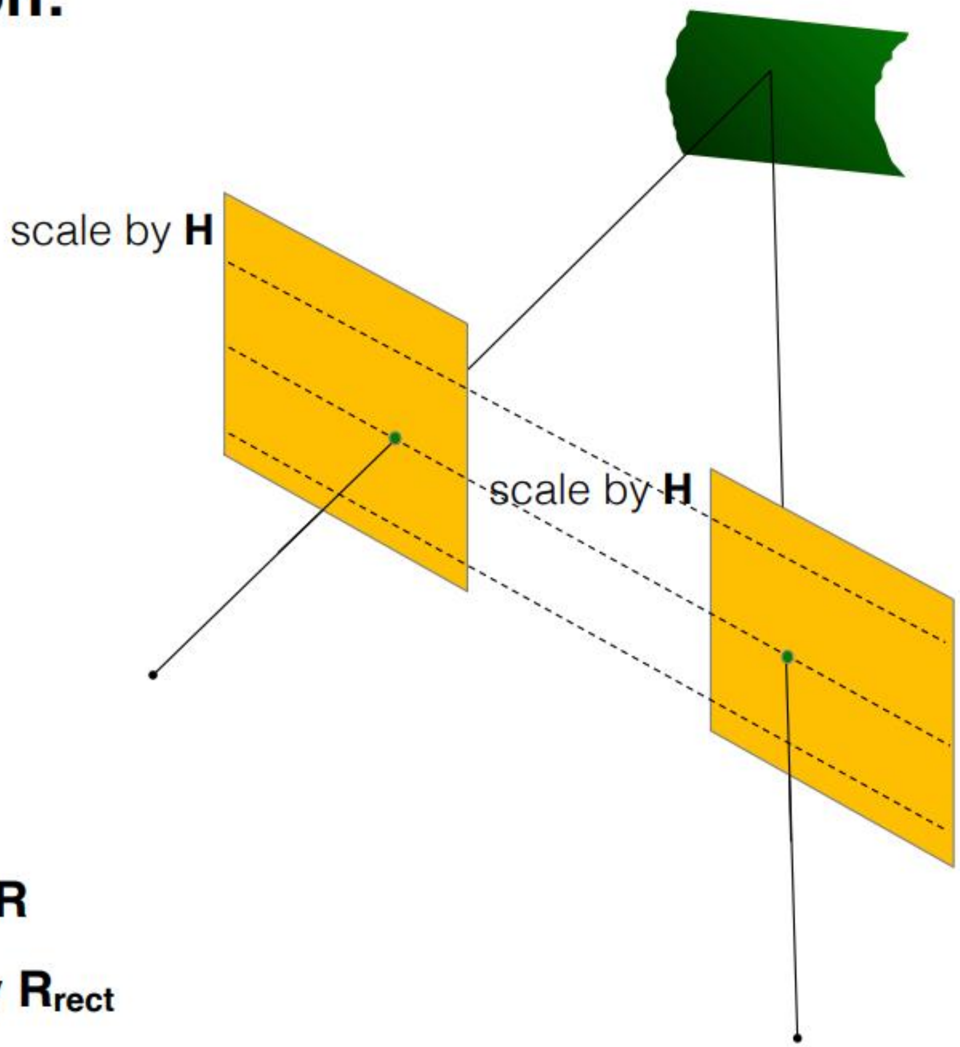

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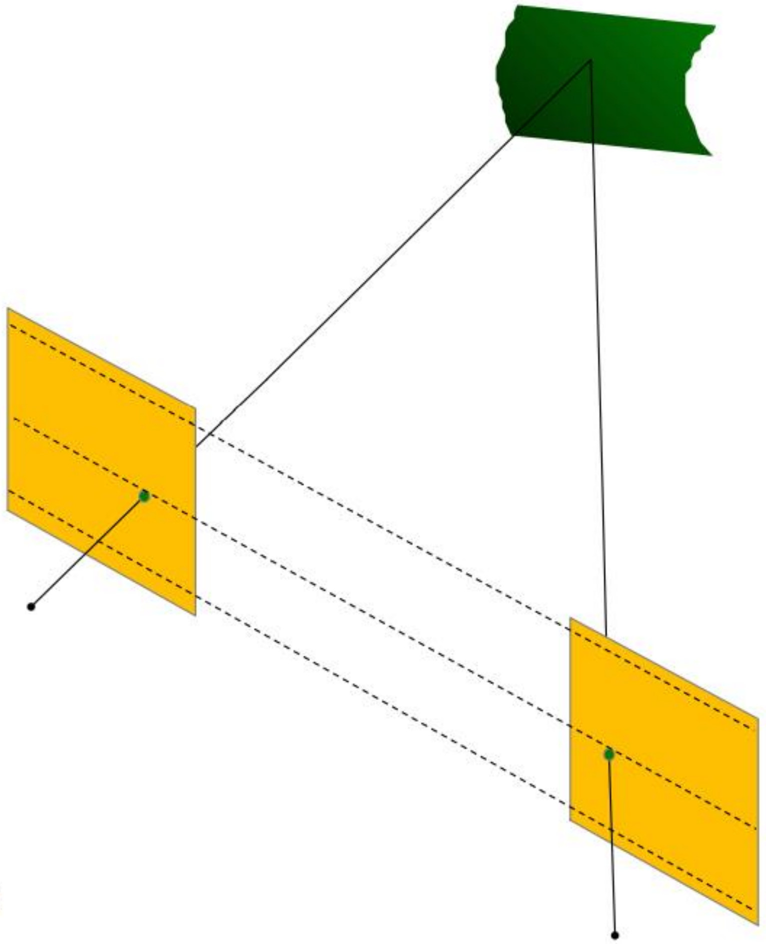

1. Compute **E** to get **R**
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4. Scale both images by **H**

We know the equation of the 1D search space

We match nxn patches between images

Drawbacks?

- Cameras need sufficient light
- Textureless regions in images (car door)
- We can use Textured light, infrared light
- Limited Range, Poor outdoor performance
- Relatively low range compared to LiDAR/RADAR

But wait… have I been lying?

- Portrait Mode – masking foreground
- How? Using depth!
- Stereo setup to extract depth - HTC One M8
- Pixel 2 with one lens able to achieve this bokeh

Enter Deep Learning! Ref: [Learning Single Camera Depth Estimation Using Dual-Pixels (thecvf.com)]