VeCBFPdFactor Mathematical Formulation

1 Core Equations

1.1 Distance and Velocity Terms

$$\boldsymbol{p}_{p0} = \text{pi.translation}() - \boldsymbol{obs} \tag{1}$$
$$d = \|\boldsymbol{p}_{p0}\| \tag{2}$$
$$\boldsymbol{v}_{\text{diff}} = \boldsymbol{v}_i - \boldsymbol{obs}_{\text{vel}} \tag{3}$$

1.2 Measurement Function (h_i)

$$h_i = \left(\frac{1}{d_{\text{safe}}} - \frac{1}{d}\right) + \beta \left(\frac{\boldsymbol{p}_{p0}^\top}{d}\right) \boldsymbol{v}_{\text{diff}} \tag{4}$$

1.3 Dynamics Terms

$$\boldsymbol{g} = \begin{bmatrix} 0 & 0 & 9.81 \end{bmatrix}^\top \tag{5}$$
$$\boldsymbol{a}_i = \begin{bmatrix} 0 & 0 & u_i(0) \end{bmatrix}^\top \tag{6}$$
$$\boldsymbol{Ra}_i = \text{pi.rotation}().rotate(\boldsymbol{a}_i) \tag{7}$$

2 Error Function

$$\text{err} = h_i + \alpha \left(H_{ti}(\boldsymbol{p}_i, \boldsymbol{v}_i)\dot{\boldsymbol{x}}_0 + H_{vi}(\boldsymbol{p}_i)\dot{\boldsymbol{x}}_2\right) \tag{8}$$

where:

$$\dot{\boldsymbol{x}}_0 = \boldsymbol{v}_i$$
$$\dot{\boldsymbol{x}}_2 = -\boldsymbol{g} + \boldsymbol{Ra}_i$$

3 Jacobian Calculations

3.1 Position Jacobian (H_{ti})

$$H_{ti} = \frac{\boldsymbol{p}_{p0}^\top}{d^3} + \beta \left(\frac{\boldsymbol{v}_{\text{diff}}^\top}{d} - \frac{(\boldsymbol{p}_{p0}^\top \cdot \boldsymbol{v}_{\text{diff}})\boldsymbol{p}_{p0}^\top}{d^3}\right) \tag{9}$$

3.2 Velocity Jacobian (H_{vi})

$$H_{vi} = \frac{\beta}{d} \boldsymbol{p}_{p0}^{\top} \tag{10}$$

3.3 Extended Position Jacobian ($H_{ti}^{\mathbf{err}}$)

$$H_{ti}^{\text{err}} = H_{ti} + \alpha \left[\beta \left(\frac{\boldsymbol{t}_3^{\top}}{d} - \frac{(\boldsymbol{p}_{p0}^{\top} \cdot \boldsymbol{t}_3) \boldsymbol{p}_{p0}^{\top}}{d^3} \right) \right] \tag{11}$$

where $\boldsymbol{t}_3 = -\boldsymbol{g} + \boldsymbol{R}\boldsymbol{a}_i$.

3.4 Extended Velocity Jacobian ($H_{vi}^{\mathbf{err}}$)

$$H_{vi}^{\text{err}} = H_{vi} + \alpha H_{ti} \tag{12}$$

4 Error Evaluation Logic

- If err > 0:
  - Set err $= 0$
  - Set all Jacobians to zero matrices
- If err ≤ 0:
  - Compute Jacobians:

$$\begin{aligned} H1 &= H_{ti}^{\text{err}} J_{ti} + \alpha H_{vi} J_{r,ri} J_{ri} \\ H2 &= H_{vi}^{\text{err}} \\ H3 &= \alpha H_{vi} J_{r,ai} \boldsymbol{e}_2 \quad \text{(with proper block assignment)} \end{aligned}$$

5 Key Parameters

- α: Weight for derivative terms
- β: Weight for velocity-dependent terms
- d_{safe}: Safety distance threshold
- $\boldsymbol{obs}$: Obstacle position
- $\boldsymbol{obs}_{\text{vel}}$: Obstacle velocity