ML_Intro

August 12, 2021

[3]:
```
# import the necessary libraries.
from mpl_toolkits import mplot3d
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
import seaborn as sea
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
from matplotlib.ticker import LinearLocator, FormatStrFormatter
import scipy as scipy
get_ipython().run_line_magic("matplotlib", "inline")
############################ Attributions #########################
# https://jakevdp.github.io/PythonDataScienceHandbook/04.
↪12-three-dimensional-plotting.html
# https://pythonprogramming.net/3d-graphing-pandas-matplotlib/
# https://stackoverflow.com/questions/36589521/
↪how-to-surface-plot-3d-plot-from-dataframe
# https://www.python-course.eu/matplotlib_contour_plot.php
# http://www.adeveloperdiary.com/data-science/
↪how-to-visualize-gradient-descent-using-contour-plot-in-python/
# https://www.coursera.org/learn/machine-learning
#################################################
```

[4]:
```
# import the Stata dataset with the housing data modified to also contain␣
↪grades and rand.
df = pd.read_stata("house.dta")
```

[5]:
```
# Look at the first five observations.
df.head(10)
```

[5]:

	age	price	area	agesq	grades	rand
0	48	60000.0	1660	2304.0	94	78.665031
1	83	40000.0	2612	6889.0	94	61.152706
2	58	34000.0	1144	3364.0	93	91.091667
3	11	63900.0	1136	121.0	92	82.843269
4	48	44000.0	1868	2304.0	91	86.785881

5	78	46000.0	1780	6084.0	90	59.637508
6	22	56000.0	1700	484.0	90	89.959587
7	78	38500.0	1556	6084.0	89	88.415070
8	42	60500.0	1642	1764.0	89	75.754196
9	41	55000.0	1443	1681.0	89	82.901779

[6]:
```
# Slide 29 shows observations that start with a house with a price of $59,000.
# Identify these below by identifying all
# all rows with a price that is equal to $59,000.
df[df["price"] == 59000]
```

[6]:

	age	price	area	agesq	grades	rand
17	70	59000.0	2458	4900.0	86	99.513565
23	15	59000.0	1215	225.0	81	66.654587
24	7	59000.0	2128	49.0	81	79.447250

[7]:
```
# The data on slide 29  either starts with observation number 17, 23, or 24␣
↪above.
df.iloc[17:27]
```

[7]:

	age	price	area	agesq	grades	rand
17	70	59000.0	2458	4900.0	86	99.513565
18	26	42000.0	750	676.0	85	79.647575
19	21	71500.0	2106	441.0	85	80.273384
20	24	43000.0	1000	576.0	84	82.347198
21	33	48000.0	1410	1089.0	83	75.582008
22	128	37500.0	2346	16384.0	81	86.530724
23	15	59000.0	1215	225.0	81	66.654587
24	7	59000.0	2128	49.0	81	79.447250
25	0	94000.0	2290	0.0	81	89.132675
26	0	95920.0	2464	0.0	80	90.347794

[8]:
```
# The above output does not line up with slide 29. Next, we look at the 10␣
↪observations
# starting with row the 24th index.
df.iloc[24:34]
```

[8]:

	age	price	area	agesq	grades	rand
24	7	59000.0	2128	49.0	81	79.447250
25	0	94000.0	2290	0.0	81	89.132675
26	0	95920.0	2464	0.0	80	90.347794
27	0	95000.0	2240	0.0	79	78.040932
28	0	95900.0	2464	0.0	79	87.203377
29	1	91000.0	2240	1.0	78	83.496582
30	0	96900.0	2464	0.0	78	77.350021
31	104	47000.0	2576	10816.0	78	71.748764
32	18	68500.0	1377	324.0	77	83.898636

```
33  188  36000.0  2071  35344.0     77  74.499268
```

[9]:
```python
# Slide 31 contains summary statistics in Excel for price. Let's calculate
# these summary statistics in Python.
# Below is the mean, median, variance, and standard deviation of price.
df["price"].mean(), df["price"].median(), df["price"].var(), df["price"].std()
```

[9]:
```
(96100.65625, 85900.0, 1868290304.0, 43223.72265625)
```

[10]:
```python
# Slide 31, below is the covariance of price and area.
# The covariance is in the first row, second column.
df[["price", "area"]].cov()
```

[10]:

	price	area
price	1.868291e+09	1.938513e+07
area	1.938513e+07	4.829665e+05

[11]:
```python
# Slide 31, below is the correlation of price and area.
# The correlation is in the first row, second column.
df[["price", "area"]].corr()
```

[11]:

	price	area
price	1.000000	0.645339
area	0.645339	1.000000

[12]:
```python
# Slide 32
# Next, let's look at the summary statistics for the dataframe.
df.describe()
```

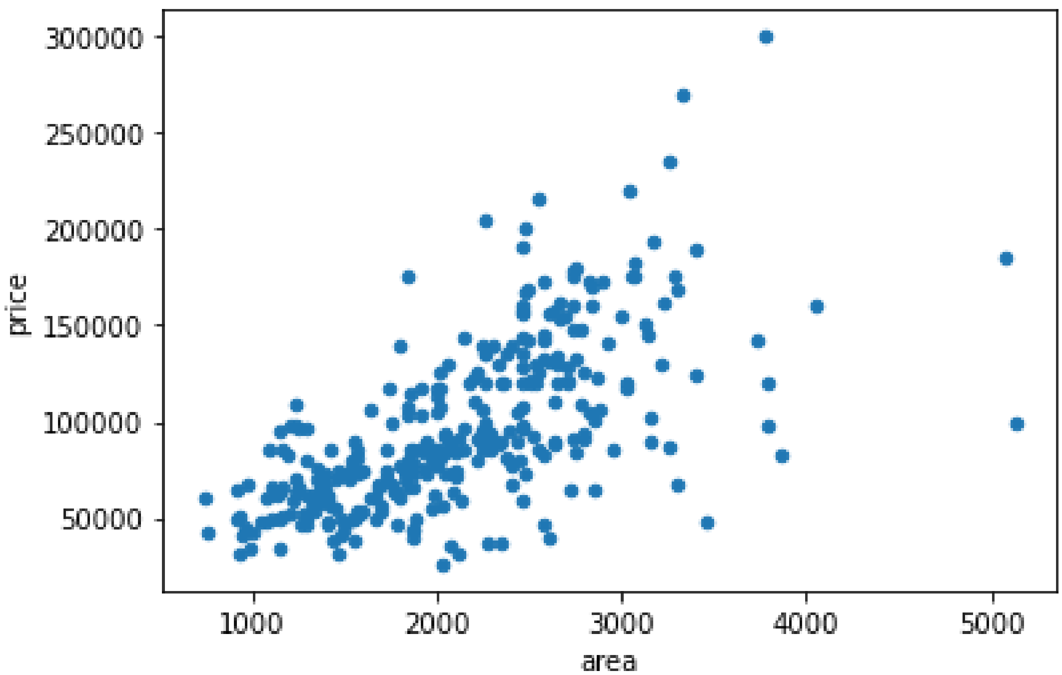
[12]:

	age	price	area	agesq	grades	\
count	321.000000	321.000000	321.000000	321.000000	321.000000	
mean	18.009346	96100.656250	2106.728972	1381.567017	80.461059	
std	32.565845	43223.722656	694.957902	4801.790039	11.644173	
min	0.000000	26000.000000	735.000000	0.000000	8.000000	
25%	0.000000	65000.000000	1560.000000	0.000000	76.000000	
50%	4.000000	85900.000000	2056.000000	16.000000	82.000000	
75%	22.000000	120000.000000	2544.000000	484.000000	87.000000	
max	189.000000	300000.000000	5136.000000	35721.000000	95.000000	

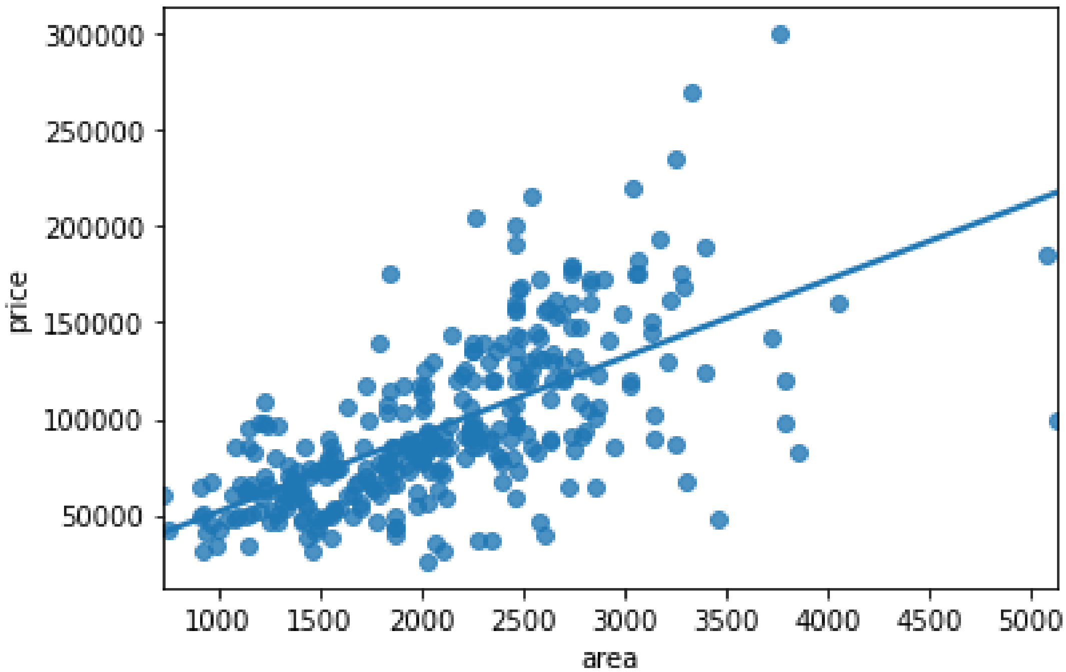
	rand
count	321.000000
mean	79.906532
std	11.066620
min	46.780296
25%	71.969383
50%	80.090462
75%	87.272881
max	106.812592

[13]:
```
# The format of the output above looks different than Slide 13, but the results␣
 ↪are the same.
# Slide 36 Next, create the scatterplot of price and area.
df.plot.scatter(y="price", x="area")
```

[13]: <matplotlib.axes._subplots.AxesSubplot at 0x22a7b69dca0>

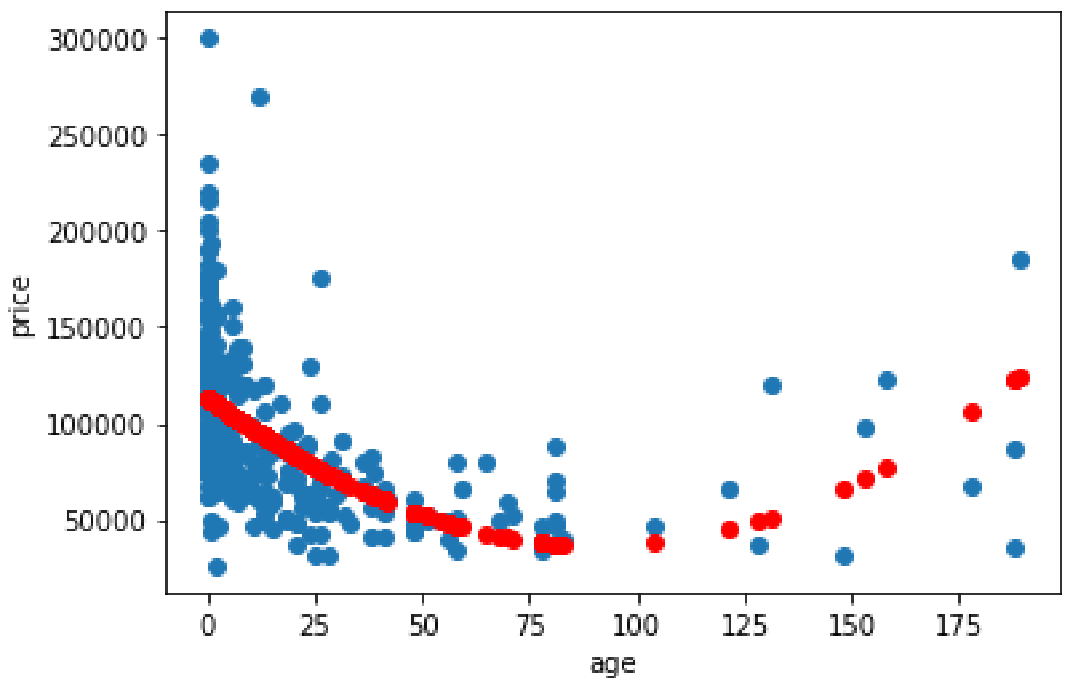


[14]:
```
# Slide 37
ax = sea.regplot(y=df["price"], x=df["area"], ci=None)
```



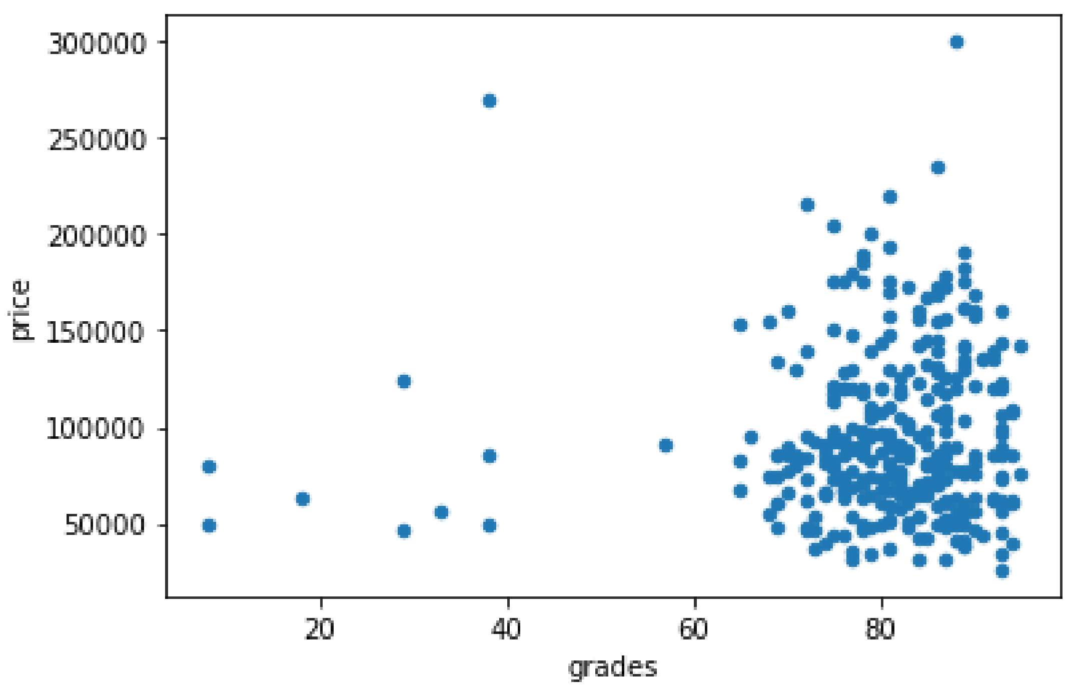
[15]:
```
# Silently create model with age and agesq to estimate the nonlinear␣
↪relationship
# and then add the predicted values to the dataframe.
X = df[["age", "agesq"]]
X = sm.add_constant(X)
Y = df["price"]
model = sm.OLS(Y, X).fit()
y_hat = model.predict(X)
df["y_hat"] = y_hat
```

[16]:
```
# Slide 42
fig, ax = plt.subplots()
ax.scatter(y=df["price"], x=df["age"])
ax.scatter(y=df["y_hat"], x=df["age"], color="red")
ax.set_xlabel("age")
ax.set_ylabel("price")
plt.show
del df["y_hat"]
```



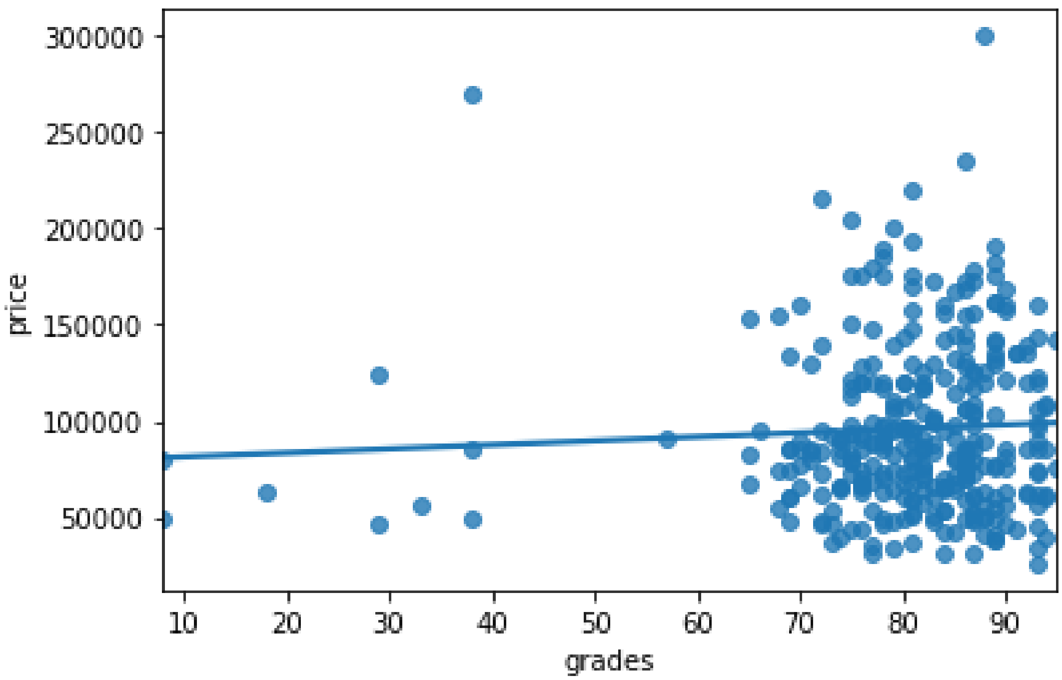
[17]:
```
# Slide 44
df.plot.scatter(y="price", x="grades")
```

[17]: <matplotlib.axes._subplots.AxesSubplot at 0x22a7d76bc10>



[18]:

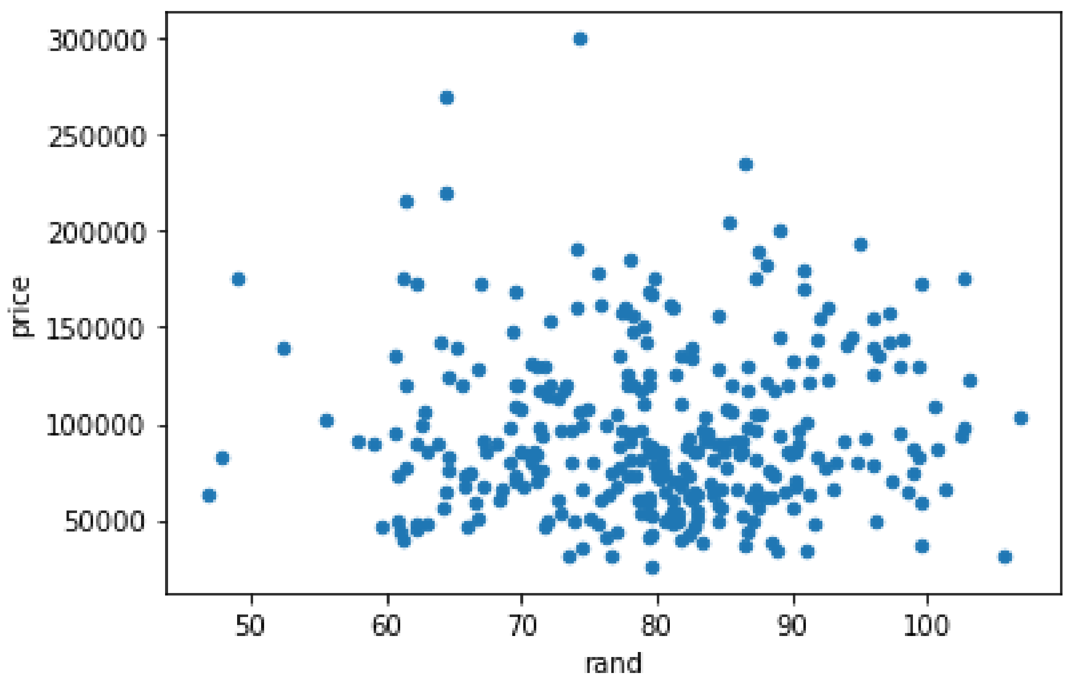
```
# Slide 45
ax = sea.regplot(y=df["price"], x=df["grades"], ci=None)
```



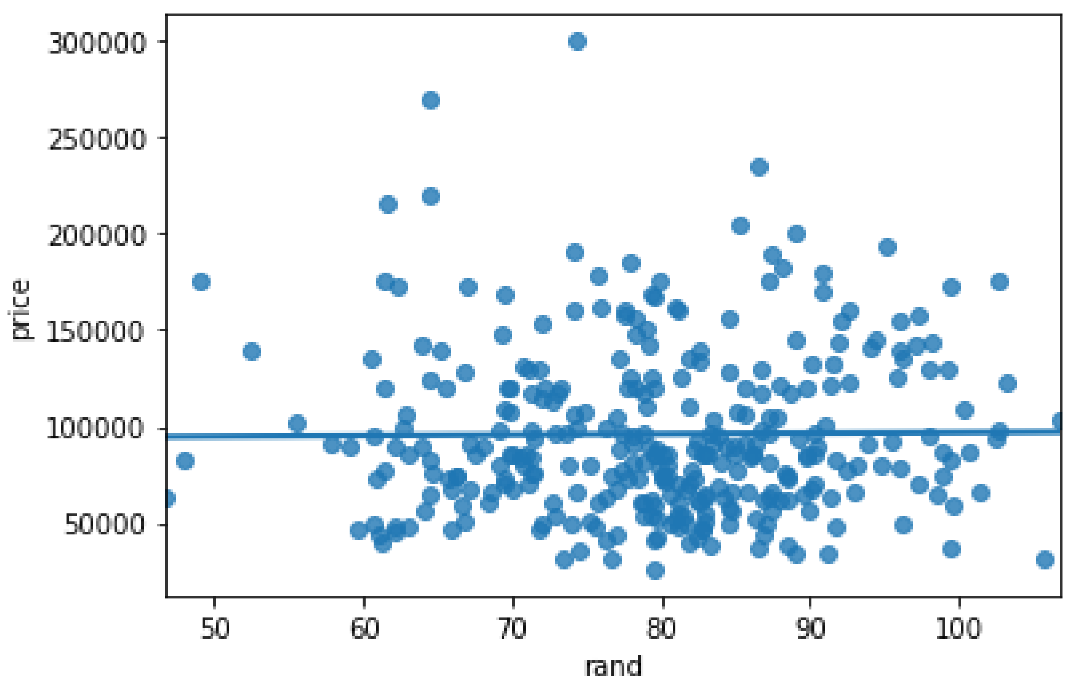
[19]:

```
# Slide 48
df.plot.scatter(y="price", x="rand")
```

[19]: <matplotlib.axes._subplots.AxesSubplot at 0x22a7daff160>



[20]:
```
# Slide 49
ax = sea.regplot(y=df["price"], x=df["rand"], ci=None)
```



[21]:
```
# Slide 52 - Area
X = df["area"]
X = sm.add_constant(X)
Y = df["price"]
model = sm.OLS(Y, X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.416
Model:                            OLS   Adj. R-squared:                  0.415
Method:                 Least Squares   F-statistic:                     227.7
Date:                Thu, 12 Aug 2021   Prob (F-statistic):           3.36e-39
Time:                        11:42:50   Log-Likelihood:                -3794.9
No. Observations:                 321   AIC:                             7594.
Df Residuals:                     319   BIC:                             7601.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       1.154e+04   5900.313      1.956      0.051     -66.910    2.31e+04
area          40.1376      2.660     15.089      0.000      34.904      45.371
==============================================================================
Omnibus:                       30.670   Durbin-Watson:                   1.173
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               72.802
Skew:                           0.462   Prob(JB):                     1.55e-16
Kurtosis:                       5.142   Cond. No.                     7.09e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 7.09e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[22]:
```
# Slide 52 - Grades
X = df["grades"]
X = sm.add_constant(X)
Y = df["price"]
model = sm.OLS(Y, X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.003
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                     1.008
```

```
Date:                    Thu, 12 Aug 2021   Prob (F-statistic):               0.316
Time:                            11:42:52   Log-Likelihood:                 -3880.9
No. Observations:                     321   AIC:                              7766.
Df Residuals:                         319   BIC:                              7773.
Df Model:                               1
Covariance Type:                nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       7.934e+04   1.69e+04      4.703      0.000    4.61e+04    1.13e+05
grades       208.3696    207.507      1.004      0.316    -199.886     616.625
==============================================================================
Omnibus:                       67.711   Durbin-Watson:                   0.886
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              121.817
Skew:                           1.156   Prob(JB):                     3.53e-27
Kurtosis:                       4.941   Cond. No.                         569.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[23]:

```python
# Slide 52 - Rand
X = df["rand"]
X = sm.add_constant(X)
Y = df["price"]
model = sm.OLS(Y, X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.003
Method:                 Least Squares   F-statistic:                   0.03881
Date:                Thu, 12 Aug 2021   Prob (F-statistic):              0.844
Time:                        11:42:53   Log-Likelihood:                -3881.4
No. Observations:                 321   AIC:                             7767.
Df Residuals:                     319   BIC:                             7774.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       9.266e+04   1.76e+04      5.253      0.000     5.8e+04    1.27e+05
rand          43.0775    218.668      0.197      0.844    -387.136     473.291
==============================================================================
Omnibus:                       65.576   Durbin-Watson:                   0.866
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              113.736
```

```
Skew:                           1.142   Prob(JB):                     2.01e-25
Kurtosis:                       4.812   Cond. No.                         589.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[24]:

```python
# Slides 63 and 68
X = df["area"]
X = sm.add_constant(X)
Y = df["price"]
model = sm.OLS(Y, X).fit()
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.416
Model:                            OLS   Adj. R-squared:                  0.415
Method:                 Least Squares   F-statistic:                     227.7
Date:                Thu, 12 Aug 2021   Prob (F-statistic):           3.36e-39
Time:                        11:42:53   Log-Likelihood:                -3794.9
No. Observations:                 321   AIC:                             7594.
Df Residuals:                     319   BIC:                             7601.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       1.154e+04   5900.313      1.956      0.051     -66.910    2.31e+04
area          40.1376      2.660     15.089      0.000      34.904      45.371
==============================================================================
Omnibus:                       30.670   Durbin-Watson:                   1.173
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               72.802
Skew:                           0.462   Prob(JB):                     1.55e-16
Kurtosis:                       5.142   Cond. No.                     7.09e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 7.09e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[24]:

```python

```

[25]:

```python
# Slidse 77 and 78.
# Below creates a new variable "scaled" which takes each value of
```

```
# area, subtracts the mean of area, and then divides by the standard deviation␣
↪of area.
df = df.iloc[0:25].copy()
df["scaled"] = (df["area"] - df["area"].mean()) / df["area"].std()
df.head(25)
```

[25]:

	age	price	area	agesq	grades	rand	scaled
0	48	60000.0	1660	2304.0	94	78.665031	0.106096
1	83	40000.0	2612	6889.0	94	61.152706	2.191225
2	58	34000.0	1144	3364.0	93	91.091667	-1.024079
3	11	63900.0	1136	121.0	92	82.843269	-1.041601
4	48	44000.0	1868	2304.0	91	86.785881	0.561671
5	78	46000.0	1780	6084.0	90	59.637508	0.368928
6	22	56000.0	1700	484.0	90	89.959587	0.193707
7	78	38500.0	1556	6084.0	89	88.415070	-0.121691
8	42	60500.0	1642	1764.0	89	75.754196	0.066672
9	41	55000.0	1443	1681.0	89	82.901779	-0.369191
10	78	39000.0	1439	6084.0	89	83.356407	-0.377952
11	38	41000.0	1482	1444.0	89	79.460243	-0.283770
12	18	50900.0	1290	324.0	88	75.122826	-0.704301
13	32	52000.0	1274	1024.0	87	79.652115	-0.739345
14	18	49000.0	1476	324.0	87	96.205551	-0.296912
15	58	80000.0	1838	3364.0	87	70.685295	0.495963
16	56	50000.0	1536	3136.0	86	86.993675	-0.165496
17	70	59000.0	2458	4900.0	86	99.513565	1.853925
18	26	42000.0	750	676.0	85	79.647575	-1.887042
19	21	71500.0	2106	441.0	85	80.273384	1.082953
20	24	43000.0	1000	576.0	84	82.347198	-1.339476
21	33	48000.0	1410	1089.0	83	75.582008	-0.441469
22	128	37500.0	2346	16384.0	81	86.530724	1.608616
23	15	59000.0	1215	225.0	81	66.654587	-0.868570
24	7	59000.0	2128	49.0	81	79.447250	1.131139

[26]:

```
# Slides 79 until the end.
# Initial guesses for B0 and B1 are zero.
coeff_guess = np.asarray([[0], [0]], dtype="int64")
# Set the learning rate paramater.
alpha = 0.01 / len(df)
results = []
# The code below does not use linear algebra on purpose.
# This talk was given to accounting and finance folks so I wanted to
# make the equations below more user friendly for that audience.
# Gradient descent is run with 1,000 iterations below.
for i in range(0, 1000):
one = (coeff_guess[0, 0] + (coeff_guess[1, 0] * df["scaled"]) - df["price"]).
↪sum()
two = (
```

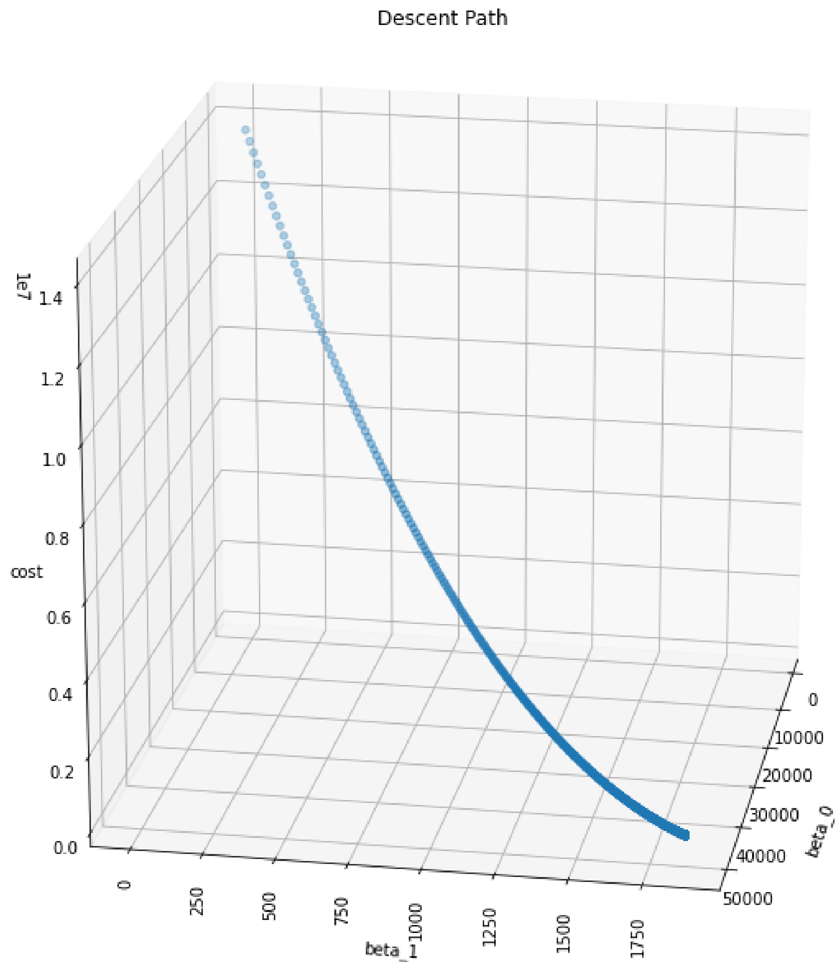
```
((coeff_guess[0, 0] + coeff_guess[1, 0] * df["scaled"]) - df["price"])
* df["scaled"]
).sum()
derivatives = np.asarray([[one], [two]], dtype="int64")
# Initial guess.
if i == 0:
coeff_guess = np.asarray([[0], [0]], dtype="int64")
cost = (
alpha
* (1 / 2)
* (
(coeff_guess[0, 0] + (coeff_guess[1, 0] * df["scaled"]) - df["price"])
** 2
).sum()
)
results.append([i, coeff_guess[0, 0], coeff_guess[1, 0], cost])
# Guesses for all iterations other than the first one.
else:
coeff_guess = coeff_guess - (alpha * derivatives)
cost = (
alpha
* (1 / 2)
* (
(coeff_guess[0, 0] + (coeff_guess[1, 0] * df["scaled"]) - df["price"])
** 2
).sum()
)
results.append([i, coeff_guess[0, 0], coeff_guess[1, 0], cost])
results
# Turn the results list into a dataframe.
df2 = pd.DataFrame(results, columns=["iteration", "beta_0", "beta_1", "cost"])
df2["iteration"] = df2["iteration"] + 1
# Below output is comparable to house_gradient.xlsx results.
# You can see that, within rounding errors, the paramter estimates
# match.
df2.head(3)
df2.tail(3)
# Create X matrix with a constant. Below lines transform df columns into
# vectors or matrices and then estimates and OLS model to compare to the
# gradient descent estimates later.
X_model = sm.add_constant(df["scaled"])
Y = df["price"]
model = sm.OLS(Y, X_model)
output = model.fit()
output.params
print(output.summary())
df2.to_excel("first_three_last_three.xlsx", index=False)
```

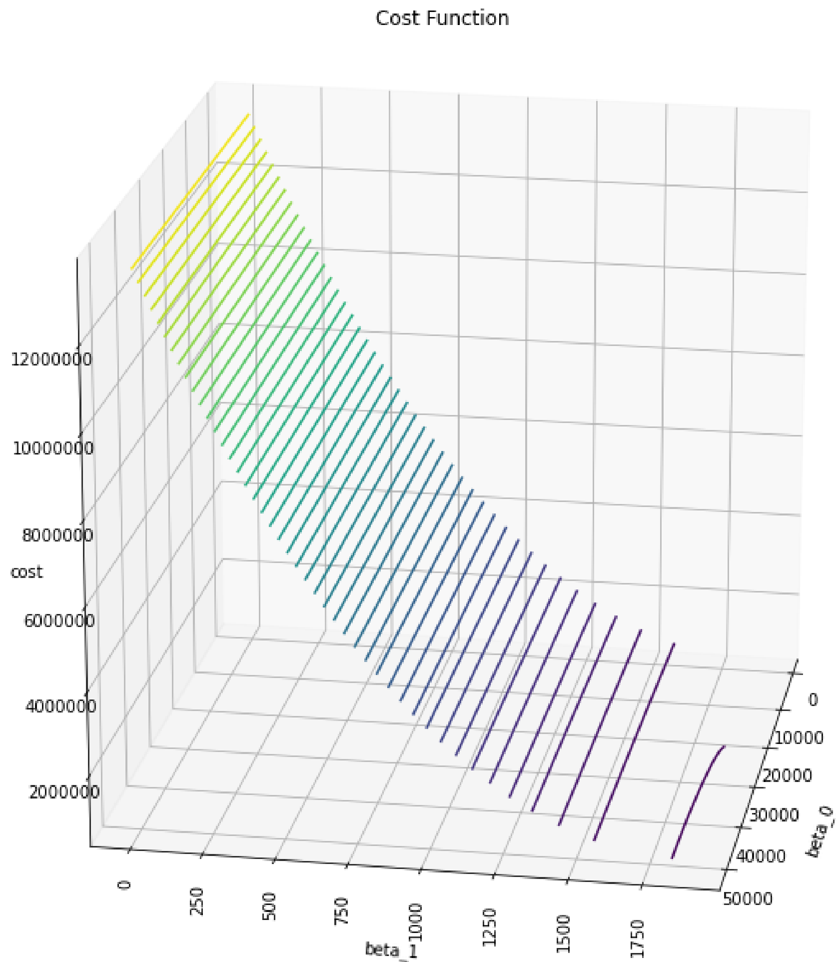
```
# Below creates a graph of the gradient descent path based on each B0 and B1
# guess and the cost function with that parameter combination.
fig = plt.figure(figsize=(12, 12))
gd = plt.axes(projection="3d")
plt.yticks(rotation=90)
gd.view_init(20, 10)
gd.set_title("Descent Path")
ax.ticklabel_format(style="plain")
gd.scatter(df2["beta_0"], df2["beta_1"], df2["cost"])
gd.set_xlabel("beta_0", labelpad=13)
gd.set_ylabel("beta_1", labelpad=13)
gd.set_zlabel("cost")
fig.savefig("descent.pdf")
# The graphs make it appear that the constant is less than $50,000.
# This appears strange, so I am double checking the cost function when
# B0 is around  $45K- $49K and seeing where the minimum of the cost function is.
# No exceptions noted. p/f/r.
df2[df2["cost"] == df2["cost"].min()]
df2.iloc[200:300]
# Above we have each combination of B0 and B1, but we don't have the cost
# function for all combinations of B0 and B1.Next, we write a loop, using
# linear algebra this time, to "sweep out" the cost function for all␣
↪combinations
# of each B0 and B1. The graph created from this will show more than just the
# gradient descent path because it will show, for example, for each B0 the cost
# with ALL of the other values of B1 in the range of B1 estimates.
# You can see this with the nested loops.
SST_Mat = np.zeros((1000, 1000))
X_model = np.asarray(X_model)
Y = np.asarray(Y)
Y = Y.reshape((25, 1))
G, H = np.meshgrid(df2["beta_0"], df2["beta_1"])
for i, j in enumerate(df2["beta_0"]):
for e, f in enumerate(df2["beta_1"]):
beta_hat = np.asarray([[j], [f]], dtype="int64")
XB = np.matmul(X_model, beta_hat)
Epsilon = np.subtract(XB, Y)
SST = alpha * (1 / 2) * np.matmul(Epsilon.T, Epsilon)
SST_Mat[i, e] = SST
fig = plt.figure(figsize=(12, 12))
ax = plt.axes(projection="3d")
plt.yticks(rotation=90)
ax.contour3D(G, H, SST_Mat, 50, cmap="viridis")
ax.set_title("Cost Function")
ax.ticklabel_format(style="plain")
ax.set_xlabel("beta_0", labelpad=13)
ax.set_ylabel("beta_1", labelpad=13)
```

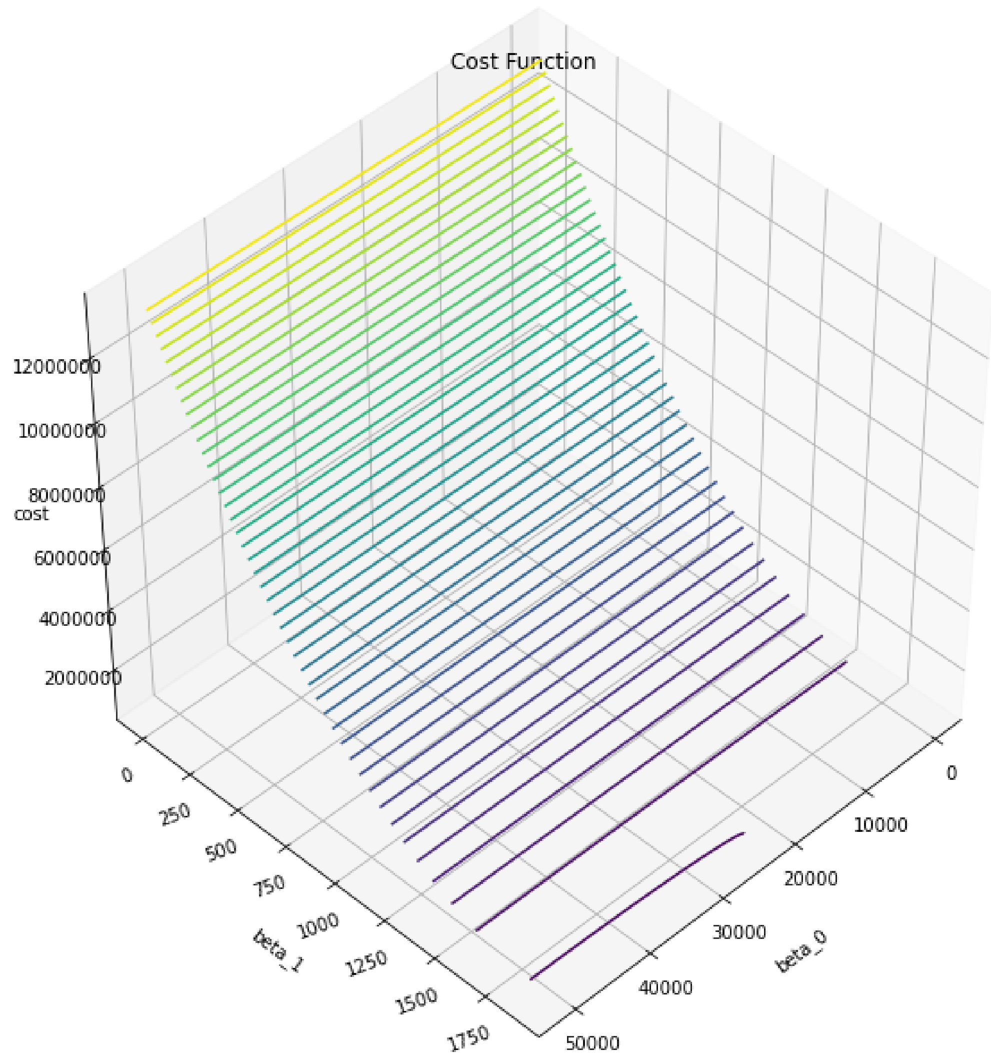
```python
ax.set_ylabel("beta_1")
ax.set_zlabel("cost")
ax.view_init(20, 10)
fig.savefig("plane.pdf")
# Rg: Note that it may appear that the value of B0 is actually less than
# $50,000 but that is just because of the rotation of the graph.
# See below.
fig = plt.figure(figsize=(11, 11))
ax = plt.axes(projection="3d")
plt.yticks(rotation=20)
ax.contour3D(G, H, SST_Mat, 50, cmap="viridis")
ax.set_title("Cost Function")
ax.ticklabel_format(style="plain")
ax.set_xlabel("beta_0", labelpad=13)
ax.set_ylabel("beta_1", labelpad=13)
ax.set_ylabel("beta_1")
ax.set_zlabel("cost")
ax.view_init(45, 45)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.028
Model:                            OLS   Adj. R-squared:                 -0.015
Method:                 Least Squares   F-statistic:                    0.6521
Date:                Thu, 12 Aug 2021   Prob (F-statistic):              0.428
Time:                        11:43:08   Log-Likelihood:                -267.90
No. Observations:                  25   AIC:                             539.8
Df Residuals:                      23   BIC:                             542.2
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       5.115e+04   2274.066     22.494      0.000    4.64e+04    5.59e+04
scaled      1874.1770   2320.959      0.808      0.428   -2927.092    6675.446
==============================================================================
Omnibus:                        1.687   Durbin-Watson:                   2.407
Prob(Omnibus):                  0.430   Jarque-Bera (JB):                1.110
Skew:                           0.515   Prob(JB):                        0.574
Kurtosis:                       2.919   Cond. No.                         1.02
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```


Descent Path
1e7
1.4
1.2
1.0
0.8
cost
0.6
0.4
0.2
0.0
0
250
500
750
1000
1250
1500
1750
beta_1
0
10000
20000
30000
40000
50000
beta_0


Cost Function
cost
12000000
10000000
8000000
6000000
4000000
2000000
beta_1
0
250
500
750
1000
1250
1500
1750
beta_0
0
10000
20000
30000
40000
50000



Cost Function
12000000
10000000
8000000
cost
6000000
4000000
2000000
0
250
500
750
1000
1250
1500
1750
beta_1
0
10000
20000
30000
40000
50000
beta_0