sjr

March 3, 2021

[1]:
```
%reset -f
import pandas as pd
import matplotlib
import matplotlib.pyplot as plt
import statsmodels.api as sm
import matplotlib.dates as mdates

"""
############  Attributions ##################
# https://matplotlib.org/stable/gallery/text_labels_and_annotations/date.html
# https://stackoverflow.com/questions/9750330/
 ↪how-to-convert-integer-into-date-object-python/37674465
# https://stackoverflow.com/questions/2623156/
 ↪how-to-convert-the-integer-date-format-into-yyyymmdd
# https://stackoverflow.com/questions/40511476/
 ↪how-to-properly-use-funcformatterfunc
# https://stackoverflow.com/questions/58881360/
 ↪python-plot-shows-numbers-instead-of-dates-on-x-axis

# Yves Hilpisch "Python for Finance"
# Theodore Petrou "Pandas Cookbook"
# Joel Grus "Data Science from Scratch"
# Daniel Chen "Pandas for Everyone"
# Wes McKinney "Python for Data Analysis"
# Jake VanderPlas "Python Data Science Handbook"
#############################################
"""
pd.set_option("display.max_columns", None)
pd.set_option("display.max_rows", None)
pd.options.display.float_format = "{:,}".format
```

[2]:
```
df = pd.read_excel("sjr.xlsx")
df = df.iloc[:16]
df.head()
```

[2]:
```
        date  tot_rev_mils  wline_rev  wline_cust_tot  wline_con_cust_tot  \
0 2020-11-30       1,370.0    1,053.0     5,156,262.0         4,510,873.0
```

```
1 2020-08-31       1,349.0    1,055.0     5,257,169.0         4,617,725.0
2 2020-05-31       1,312.0    1,060.0     5,328,412.0         4,697,228.0
3 2020-02-29       1,363.0    1,061.0     5,383,705.0         4,744,693.0
4 2019-11-30       1,383.0    1,065.0     5,434,210.0         4,794,689.0

   wline_biz_cust_tot  wless_rev  wless_cust_tot  wline_con_cab  \
0           645,389.0      317.0     1,922,543.0    1,356,083.0
1           639,444.0      294.0     1,821,514.0    1,390,520.0
2           631,184.0      252.0     1,761,690.0    1,423,509.0
3           639,012.0      302.0     1,767,155.0    1,445,113.0
4           639,521.0      318.0     1,716,096.0    1,464,423.0

   wline_con_sat  wline_con_int  wline_con_ph  wline_biz_cab  wline_biz_sat  \
0      617,140.0    1,888,800.0     648,850.0       37,479.0       38,367.0
1      650,727.0    1,903,868.0     672,610.0       37,512.0       36,002.0
2      658,027.0    1,918,320.0     697,372.0       35,832.0       34,253.0
3      658,137.0    1,923,423.0     718,020.0       40,686.0       39,088.0
4      671,348.0    1,917,351.0     741,567.0       43,465.0       37,989.0

   wline_biz_int  wline_biz_ph  wless_post  wless_pre  \
0      179,461.0     390,082.0 1,569,471.0  353,072.0
1      178,270.0     387,660.0 1,482,175.0  339,339.0
2      174,124.0     386,975.0 1,437,218.0  324,472.0
3      174,042.0     385,196.0 1,434,982.0  332,173.0
4      174,380.0     383,687.0 1,380,693.0  335,403.0

                                              source  \
0  https://www.sec.gov/Archives/edgar/data/932872…
1  https://www.sec.gov/Archives/edgar/data/932872…
2  https://www.sec.gov/Archives/edgar/data/932872…
3  https://www.sec.gov/Archives/edgar/data/932872…
4  https://www.sec.gov/Archives/edgar/data/932872…

          notes and variable definitions dictionary
0  Intersegment eliminations are netted against W…
1  {'tot_revenue_mils': 'total revenue in million…
2                                                NaN
3                                                NaN
4                                                NaN
```

[3]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 16 entries, 0 to 15
Data columns (total 20 columns):
 #   Column                                     Non-Null Count  Dtype
---  ------                                     --------------  -----
```

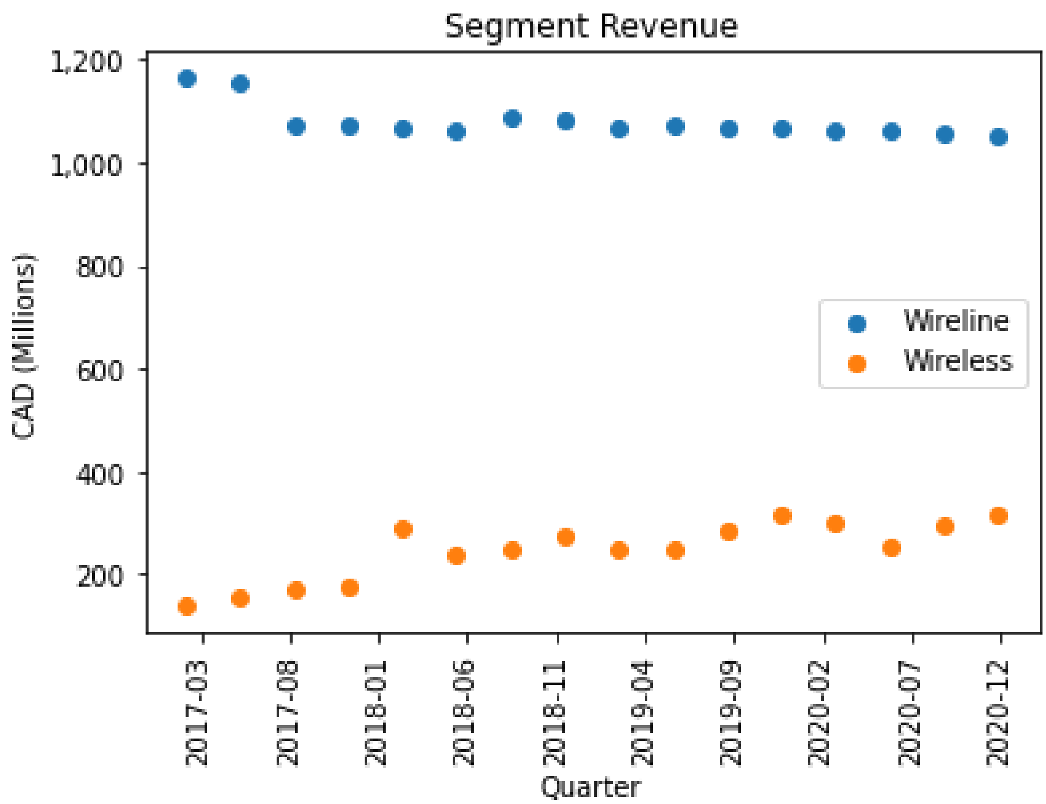
```
 0   date                                      16 non-null     datetime64[ns]
 1   tot_rev_mils                              16 non-null     float64
 2   wline_rev                                 16 non-null     float64
 3   wline_cust_tot                            16 non-null     float64
 4   wline_con_cust_tot                        16 non-null     float64
 5   wline_biz_cust_tot                        16 non-null     float64
 6   wless_rev                                 16 non-null     float64
 7   wless_cust_tot                            16 non-null     float64
 8   wline_con_cab                             16 non-null     float64
 9   wline_con_sat                             16 non-null     float64
 10  wline_con_int                             16 non-null     float64
 11  wline_con_ph                              16 non-null     float64
 12  wline_biz_cab                             16 non-null     float64
 13  wline_biz_sat                             16 non-null     float64
 14  wline_biz_int                             16 non-null     float64
 15  wline_biz_ph                              16 non-null     float64
 16  wless_post                                16 non-null     float64
 17  wless_pre                                 16 non-null     float64
 18  source                                    16 non-null     object
 19  notes and variable definitions dictionary  2 non-null      object
dtypes: datetime64[ns](1), float64(17), object(2)
memory usage: 2.6+ KB
```

[4]:

```
# Segment revenue graph- object oriented approach.
fig, ax = plt.subplots()
# Create first time series line with the appropriate label.
ax.scatter(df["date"], df["wline_rev"], label="Wireline")
# Create second time series line with the appropriate lable.
ax.scatter(df["date"], df["wless_rev"], label="Wireless")
# Title of graph.
ax.set_title("Segment Revenue")
# Labels for the x and y axes, respectively.
ax.set_xlabel("Quarter")
ax.set_ylabel("CAD (Millions)")
# Feed in the ticks for the x axis, rotate them, and display as dates.
ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
ax.xaxis.set_major_formatter(mdates.DateFormatter("%Y-%m"))
# Specify the maximum number of ticks. This option makes the ticks
# line up better and run through all of the time series. Without
# this option the last time tick is 2020-02.
ax.xaxis.set_major_locator(plt.MaxNLocator(12))
# Include the legend in the graph.
ax.legend()
# Format the y axis numbers to have a comma separater.
ax.yaxis.set_major_formatter(matplotlib.ticker.StrMethodFormatter("{x:,.0f}"))
# Save the figure and the "bbox_inches" option keeps the saved
# image from having the x axis labels cut off.
```

```
plt.savefig("Segment_Revenue.pdf", bbox_inches="tight")
```

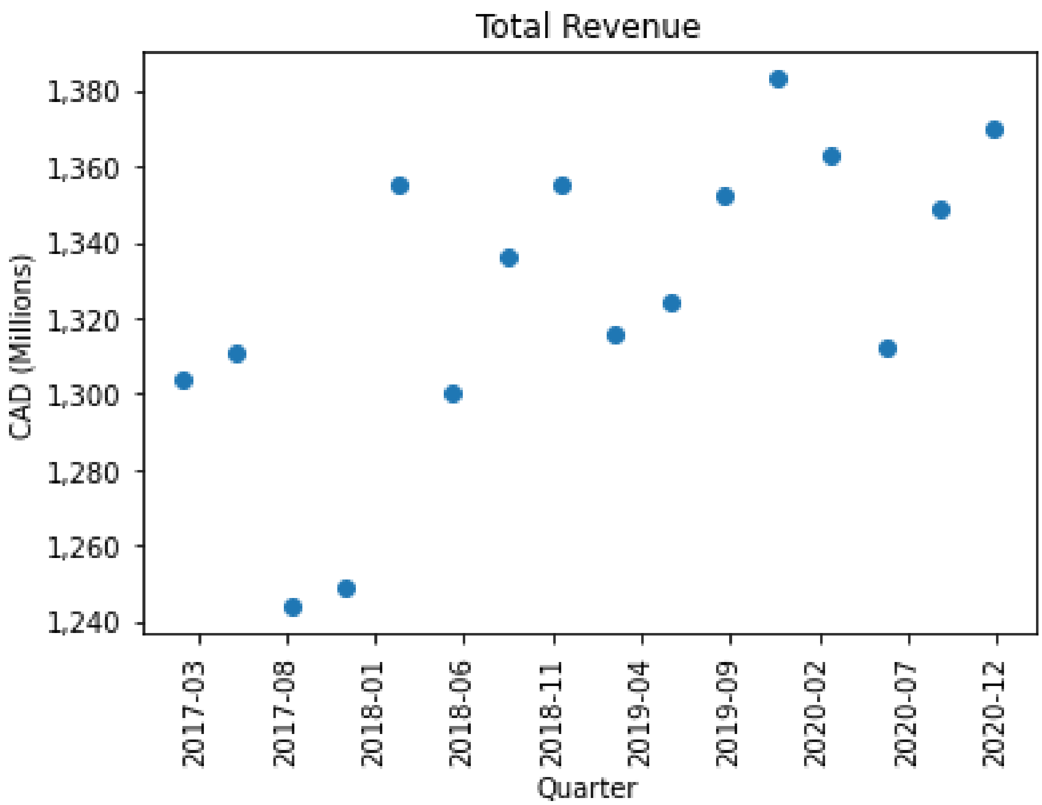
```
<ipython-input-4-88ce8b073a09>:13: UserWarning: FixedFormatter should only be
used together with FixedLocator
  ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
```



[5]:
```
# Total Revenue graph- object oriented approach.
fig, ax = plt.subplots()
# Create first time series line with the appropriate label.
ax.scatter(df["date"], df["tot_rev_mils"])
# Title of graph.
ax.set_title("Total Revenue")
# Labels for the x and y axes, respectively.
ax.set_xlabel("Quarter")
ax.set_ylabel("CAD (Millions)")
# Feed in the ticks for the x axis, rotate them, and display as dates.
ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
ax.xaxis.set_major_formatter(mdates.DateFormatter("%Y-%m"))
# Specify the maximum number of ticks. This option makes the ticks
# line up better and run through all of the time series. Without
# this option the last time tick is 2020-02.
```

```
ax.xaxis.set_major_locator(plt.MaxNLocator(12))
# Format the y axis numbers to have a comma separater.
ax.yaxis.set_major_formatter(matplotlib.ticker.StrMethodFormatter("{x:,.0f}"))
# Save the figure and the "bbox_inches" option keeps the saved
# image from having the x axis labels cut off.
plt.savefig("Total_Revenue.pdf", bbox_inches="tight")
```

```
<ipython-input-5-72f83cec529c>:11: UserWarning: FixedFormatter should only be
used together with FixedLocator
  ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
```



[6]:
```
# Individual Segment Models #
# Estimate Wireline average revenue per customer 2/28/17 through 11/30/19.
# This is the training period. The "test set" for "out of sample"
# model accuracy tests is 2/29/20 through 11/30/20.
y = df["wline_rev"].iloc[4:]
x = df["wline_cust_tot"].iloc[4:]
model = sm.OLS(y, x).fit()
model.summary()
model.params
wline_mfx = model.params[0] * 1000000
```

```
wline_mfx
```

```
C:\Users\Robso\anaconda3\envs\ml\lib\site-packages\scipy\stats\stats.py:1603:
UserWarning: kurtosistest only valid for n>=20 … continuing anyway, n=12
  warnings.warn("kurtosistest only valid for n>=20 … continuing "
```

[6]: 191.80795736374938

[7]:
```
# Individual Segment Model
# Estimate Wireless average revenue per customer for the same training period.
y = df["wless_rev"].iloc[4:]
x = df["wless_cust_tot"].iloc[4:]
model = sm.OLS(y, x).fit()
model.summary()
wless_mfx = model.params[0] * 1000000
wless_mfx
```

```
C:\Users\Robso\anaconda3\envs\ml\lib\site-packages\scipy\stats\stats.py:1603:
UserWarning: kurtosistest only valid for n>=20 … continuing anyway, n=12
  warnings.warn("kurtosistest only valid for n>=20 … continuing "
```

[7]: 170.93691137627562

[8]:
```
print(
    "The estimated difference in revenue per customer from\
    the segment models is",
    wline_mfx - wless_mfx,
)
```

```
The estimated difference in revenue per customer from    the segment models is
20.871045987473764
```

[9]:
```
# Total Revenue Model
# Estimate Wireline average revenue per customer 2/28/17 through 11/30/19.
# This is the training period. The "test set" period for "out of sample"
# model accuracy tests is 2/29/20 through 11/30/20.
y = df["tot_rev_mils"].iloc[4:]
x = df[["wline_cust_tot", "wless_cust_tot"]].iloc[4:]
model = sm.OLS(y, x).fit()
model.summary()
model.params
wline_nested_slope = model.params[0] * 1000000
wless_nested_slope = model.params[1] * 1000000
```

```
C:\Users\Robso\anaconda3\envs\ml\lib\site-packages\scipy\stats\stats.py:1603:
UserWarning: kurtosistest only valid for n>=20 … continuing anyway, n=12
  warnings.warn("kurtosistest only valid for n>=20 … continuing "
```

[10]:
```
print(wline_nested_slope)
print(wless_nested_slope)
print(
    "The estimated difference in revenue per customer from\
    the total revenue model is",
    wline_nested_slope - wless_nested_slope,
)

# The individual segment model estimates Wireline at $191 and Wireless at $170.
# The total revenue model estimates Wireline at $176 and Wireless at $232.
```

```
176.5643125475334
232.69388566005156
The estimated difference in revenue per customer from    the total revenue model
is -56.12957311251816
```

[11]:
```
# Averaging Approach
# Alternative approach to average revenue per customer using averages
df["wline_avg_per_cust"] = (
    df["wline_rev"].iloc[4:] / df["wline_cust_tot"].iloc[4:]
) * 1000000
df["wless_avg_per_cust"] = (
    df["wless_rev"].iloc[4:] / df["wless_cust_tot"].iloc[4:]
) * 1000000
```

[12]:
```
print(df["wline_avg_per_cust"].mean())
print(df["wless_avg_per_cust"].mean())

print(
    "The estimated differnce in revenue per customer from averaging is",
    df["wline_avg_per_cust"].mean() - df["wless_avg_per_cust"].mean(),
)
```

```
191.88509153745125
168.01274591330912
The estimated differnce in revenue per customer from averaging is
23.872345624142127
```

[13]:
```
# Comparison of total revenue predictions from 1) Individual Segment Models and
# 2) Total Revenue Model for the "test set" sample period of
# 2/29/20 through 11/30/20.
df["indiv_yhat"] = wline_mfx * df["wline_cust_tot"] \
                   + (wless_mfx * df["wless_cust_tot"])


df["indiv_yhat"] = (df["indiv_yhat"] / 1000000).round()
```

```python
df["indiv_wline_yhat"] = wline_mfx * df["wline_cust_tot"]
df["indiv_wless_yhat"] = wless_mfx * df["wless_cust_tot"]

df["indiv_wline_yhat"] = (df["indiv_wline_yhat"] / 1000000).round()
df["indiv_wless_yhat"] = (df["indiv_wless_yhat"] / 1000000).round()

df["total_yhat"] = wline_nested_slope * df["wline_cust_tot"] \
                   + (wless_nested_slope * df["wless_cust_tot"])

df["total_yhat"] = (df["total_yhat"] / 1000000).round()

df["total_wline_yhat"] = wline_nested_slope * df["wline_cust_tot"]

df["total_wless_yhat"] = wless_nested_slope * df["wless_cust_tot"]

df["total_wline_yhat"] = (df["total_wline_yhat"] / 1000000).round()
df["total_wless_yhat"] = (df["total_wless_yhat"] / 1000000).round()

df["indiv_sr"] = (df["indiv_yhat"].iloc[:4] - df["tot_rev_mils"].iloc[:4]) ** 2
df["total_sr"] = (df["total_yhat"].iloc[:4] - df["tot_rev_mils"].iloc[:4]) ** 2
```

[14]:
```python
df[["indiv_sr", "total_sr"]].iloc[:4]
```

[14]:

	indiv_sr	total_sr
0	2,704.0	144.0
1	841.0	9.0
2	121.0	1,521.0
3	784.0	1.0

[15]:
```python
df[["indiv_sr", "total_sr"]].sum()
```

[15]:
```
indiv_sr   4,450.0
total_sr   1,675.0
dtype: float64
```

[16]:
```python
df["indiv_sr"].sum() / df["total_sr"].sum()

# The sum of the squared difference between the predictions and
# actual values in the hold out sample period are approximately
# 2.7 times larger for the Individual Segment Models model compared
# to the Total Revenue Model.
```

[16]: 2.656716417910448

[17]:
```python
df[
    [
        "date",
```

```
        "tot_rev_mils",
        "indiv_yhat",
        "total_yhat",
        "wline_rev",
        "indiv_wline_yhat",
        "total_wline_yhat",
        "wless_rev",
        "indiv_wless_yhat",
        "total_wless_yhat",
        "indiv_sr",
        "total_sr"
    ]
].iloc[:4]

# Above shows that, while the Total Revenue Model estimates the
# Wireless segment revenue to be much higher than it is, the overall
# performance when predicting total revenue from all segments
# superior to Individual Segment Models.
```

[17]:

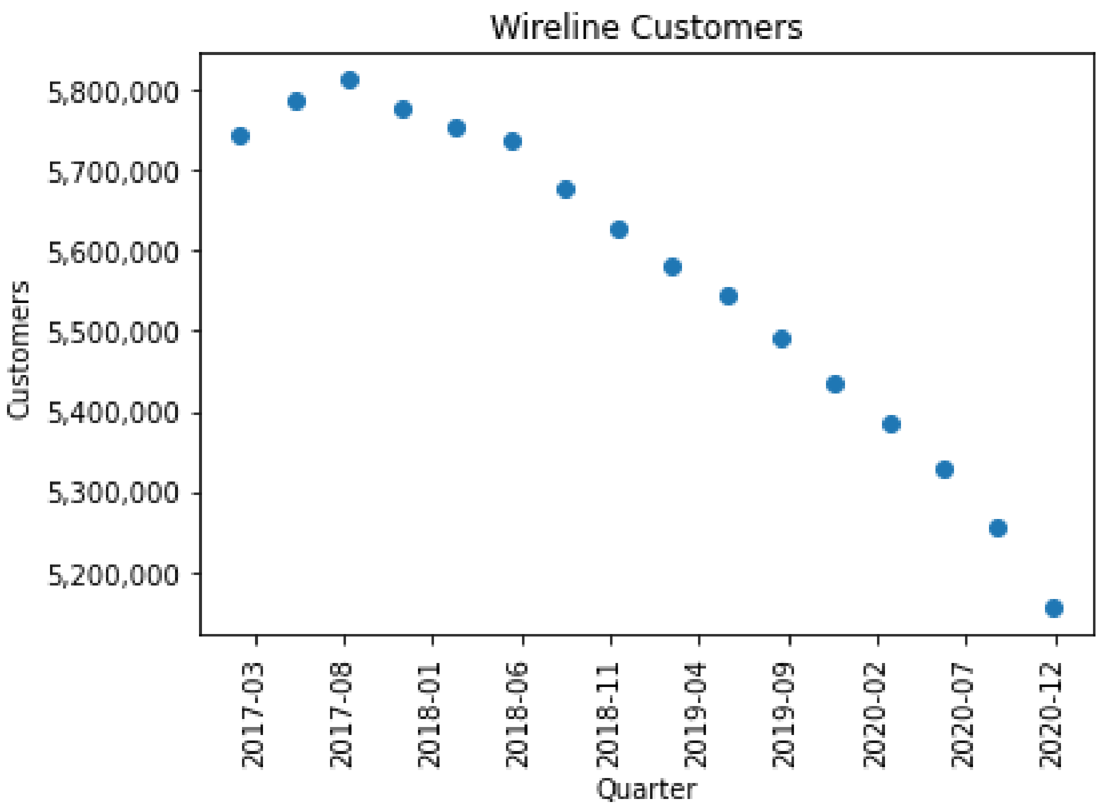
```
        date  tot_rev_mils  indiv_yhat  total_yhat  wline_rev  \
0 2020-11-30       1,370.0     1,318.0     1,358.0    1,053.0
1 2020-08-31       1,349.0     1,320.0     1,352.0    1,055.0
2 2020-05-31       1,312.0     1,323.0     1,351.0    1,060.0
3 2020-02-29       1,363.0     1,335.0     1,362.0    1,061.0

   indiv_wline_yhat  total_wline_yhat  wless_rev  indiv_wless_yhat  \
0             989.0             910.0      317.0             329.0
1           1,008.0             928.0      294.0             311.0
2           1,022.0             941.0      252.0             301.0
3           1,033.0             951.0      302.0             302.0

   total_wless_yhat  indiv_sr  total_sr
0             447.0   2,704.0     144.0
1             424.0     841.0       9.0
2             410.0     121.0   1,521.0
3             411.0     784.0       1.0
```

[18]:

```
# Scatterplot of Wireline customers- object oriented approach.
fig, ax = plt.subplots()
# Create first time series line with the appropriate label.
ax.scatter(df["date"], df["wline_cust_tot"])
# Title of graph.
ax.set_title("Wireline Customers")
# Labels for the x and y axes, respectively.
ax.set_xlabel("Quarter")
ax.set_ylabel("Customers")
# Feed in the ticks for the x axis, rotate them, and display as dates.
```

```
ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
ax.xaxis.set_major_formatter(mdates.DateFormatter("%Y-%m"))
# Specify the maximum number of ticks. This option makes the ticks line up
# better and run through all of the time series. Without this option the
# last time tick is 2020-02.
ax.xaxis.set_major_locator(plt.MaxNLocator(12))
# Format the y axis numbers to have a comma separater.
ax.yaxis.set_major_formatter(matplotlib.ticker.StrMethodFormatter("{x:,.0f}"))
# Save the figure and the "bbox_inches" option keeps the saved image
# from having the x axis labels cut off.
plt.savefig("Wireline_Customers.pdf", bbox_inches="tight")
```

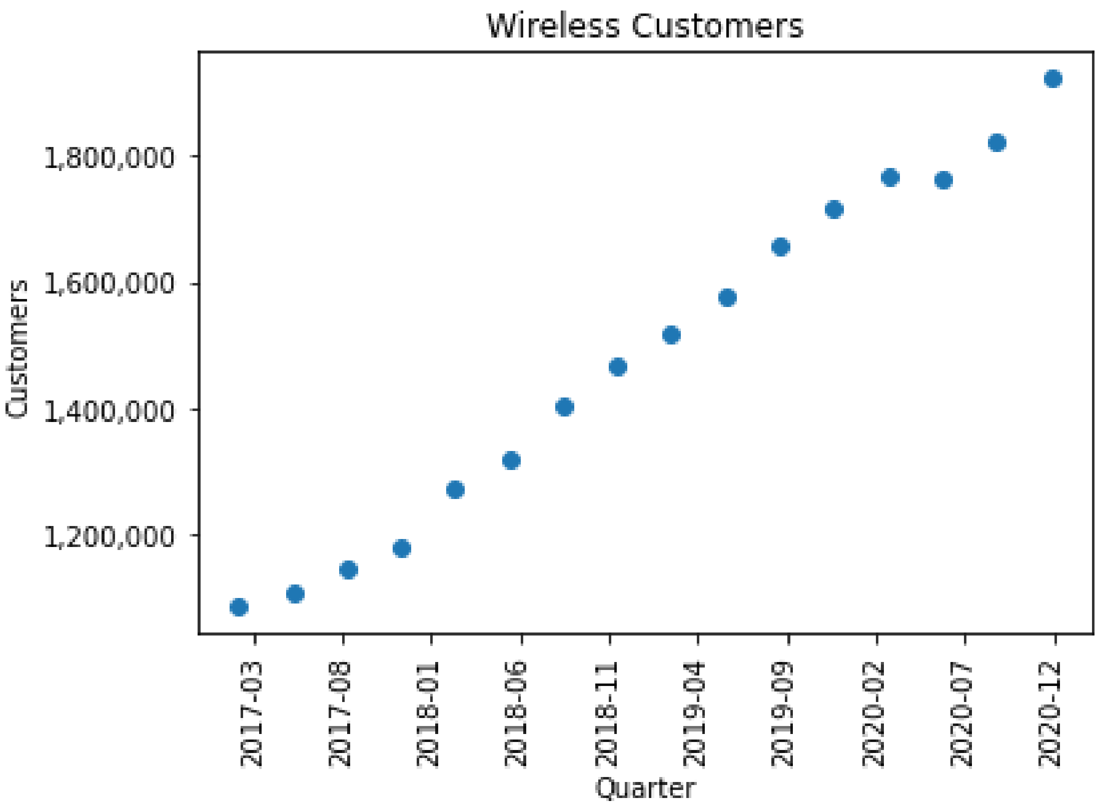
```
<ipython-input-18-e30deb90d946>:11: UserWarning: FixedFormatter should only be
used together with FixedLocator
  ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
```



[19]:
```
# Scatterplot of Wireless customers- object oriented approach.
fig, ax = plt.subplots()
# Create first time series line with the appropriate label.
ax.scatter(df["date"], df["wless_cust_tot"])
# Title of graph.
```

```
ax.set_title("Wireless Customers")
# Labels for the x and y axes, respectively.
ax.set_xlabel("Quarter")
ax.set_ylabel("Customers")
# Feed in the ticks for the x axis, rotate them, and display as dates.
ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
ax.xaxis.set_major_formatter(mdates.DateFormatter("%Y-%m"))
# Specify the maximum number of ticks. This option makes the
# ticks line up better and run through all of the time series.
# Without this option the last time tick is 2020-02.
ax.xaxis.set_major_locator(plt.MaxNLocator(12))
# Format the y axis numbers to have a comma separater.
ax.yaxis.set_major_formatter(matplotlib.ticker.StrMethodFormatter("{x:,.0f}"))
# Save the figure and the "bbox_inches" option keeps the saved image from
# having the x axis labels cut off.
plt.savefig("Wireless_Customers.pdf", bbox_inches="tight")
```

```
<ipython-input-19-00a81a456f0e>:11: UserWarning: FixedFormatter should only be
used together with FixedLocator
  ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
```



[20]:
```python
# Create a linear trend term. Note that the most recent obs is first
# in the dataset. Sort the data with oldest first before creating the trend.

df.sort_values(by=["date"], inplace=True)
df.reset_index(drop=True, inplace=True)

df["trend"] = df.index + 1
df["trend"]
```

[20]:
```
0      1
1      2
2      3
3      4
4      5
5      6
6      7
7      8
8      9
9     10
10    11
11    12
12    13
13    14
14    15
15    16
Name: trend, dtype: int64
```

[21]:
```python
# Models to forecast the number of customers for each segment.
y = df["wline_cust_tot"]
x = df["trend"]
x_model = sm.add_constant(x)
model = sm.OLS(y, x_model).fit()
model.summary()
model.params
wline_cons = model.params[0]
wline_slope = model.params[1]
```

```
C:\Users\Robso\anaconda3\envs\ml\lib\site-packages\scipy\stats\stats.py:1603:
UserWarning: kurtosistest only valid for n>=20 … continuing anyway, n=16
  warnings.warn("kurtosistest only valid for n>=20 … continuing "
```

[22]:
```python
wline_cons
```

[22]:
```
5923616.125000003
```

[23]:
```python
wline_slope
```

[23]: -41812.029411764786

[24]:
```
y = df["wless_cust_tot"]
x = df["trend"]
x_model = sm.add_constant(x)
model = sm.OLS(y, x_model).fit()
model.summary()
model.params
wless_cons = model.params[0]
wless_slope = model.params[1]
```

```
C:\Users\Robso\anaconda3\envs\ml\lib\site-packages\scipy\stats\stats.py:1603:
UserWarning: kurtosistest only valid for n>=20 … continuing anyway, n=16
  warnings.warn("kurtosistest only valid for n>=20 … continuing "
```

[25]:
```
wless_cons
```

[25]: 991871.1750000005

[26]:
```
wless_slope
```

[26]: 57744.22941176468

[27]:
```
# Create a dataframe with a one year ahead out of sample period, quarterly.
df_newdates = pd.DataFrame(
    ["20210228", "20210531", "20210831", "20211130"], columns=["date"]
)

# Format the dates as datetimes.
df_newdates["date"] = pd.to_datetime(df_newdates["date"], format="%Y%m%d")
```

[28]:
```
df_newdates
```

[28]:

	date
0	2021-02-28
1	2021-05-31
2	2021-08-31
3	2021-11-30

[29]:
```
df_newdates.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 4 entries, 0 to 3
Data columns (total 1 columns):
 #   Column  Non-Null Count  Dtype
---  ------  --------------  -----
 0   date    4 non-null      datetime64[ns]
```

```
dtypes: datetime64[ns](1)
memory usage: 160.0 bytes
```

[30]:
```python
# Append the out of sample quarters into the original dataframe.
df = df.append(df_newdates)
df.reset_index(drop=True, inplace=True)
df
```

[30]:

	date	tot_rev_mils	wline_rev	wline_cust_tot	wline_con_cust_tot
0	2017-02-28	1,304.0	1,164.0	5,742,051.0	5,171,048.0
1	2017-05-31	1,311.0	1,157.0	5,786,424.0	5,209,565.0
2	2017-08-31	1,244.0	1,072.0	5,811,776.0	5,231,359.0
3	2017-11-30	1,249.0	1,074.0	5,777,925.0	5,193,122.0
4	2018-02-28	1,355.0	1,065.0	5,752,290.0	5,161,740.0
5	2018-05-31	1,300.0	1,063.0	5,737,865.0	5,137,456.0
6	2018-08-31	1,336.0	1,086.0	5,678,634.0	5,066,426.0
7	2018-11-30	1,355.0	1,082.0	5,625,823.0	5,003,414.0
8	2019-02-28	1,316.0	1,069.0	5,581,193.0	4,955,023.0
9	2019-05-31	1,324.0	1,073.0	5,546,022.0	4,918,984.0
10	2019-08-31	1,352.0	1,069.0	5,491,661.0	4,861,042.0
11	2019-11-30	1,383.0	1,065.0	5,434,210.0	4,794,689.0
12	2020-02-29	1,363.0	1,061.0	5,383,705.0	4,744,693.0
13	2020-05-31	1,312.0	1,060.0	5,328,412.0	4,697,228.0
14	2020-08-31	1,349.0	1,055.0	5,257,169.0	4,617,725.0
15	2020-11-30	1,370.0	1,053.0	5,156,262.0	4,510,873.0
16	2021-02-28	NaN	NaN	NaN	NaN
17	2021-05-31	NaN	NaN	NaN	NaN
18	2021-08-31	NaN	NaN	NaN	NaN
19	2021-11-30	NaN	NaN	NaN	NaN

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	wline_biz_cust_tot	wless_rev	wless_cust_tot	wline_con_cab
0	571,003.0	140.0	1,086,185.0	1,650,789.0
1	576,859.0	154.0	1,106,159.0	1,663,710.0
2	580,417.0	172.0	1,147,173.0	1,671,277.0
3	584,803.0	175.0	1,181,483.0	1,653,269.0
4	590,550.0	290.0	1,271,185.0	1,635,554.0
5	600,409.0	237.0	1,317,844.0	1,619,222.0
6	612,208.0	250.0	1,402,858.0	1,585,232.0
7	622,409.0	273.0	1,468,473.0	1,561,464.0
8	626,170.0	247.0	1,516,256.0	1,532,511.0
9	627,038.0	251.0	1,578,355.0	1,508,208.0
10	630,619.0	283.0	1,658,185.0	1,478,371.0
11	639,521.0	318.0	1,716,096.0	1,464,423.0
12	639,012.0	302.0	1,767,155.0	1,445,113.0
13	631,184.0	252.0	1,761,690.0	1,423,509.0
14	639,444.0	294.0	1,821,514.0	1,390,520.0
15	645,389.0	317.0	1,922,543.0	1,356,083.0

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16	NaN	NaN	NaN	NaN
17	NaN	NaN	NaN	NaN
18	NaN	NaN	NaN	NaN
19	NaN	NaN	NaN	NaN

	wline_con_sat	wline_con_int	wline_con_ph	wline_biz_cab	wline_biz_sat
0	770,294.0	1,818,072.0	931,893.0	53,475.0	32,000.0
1	776,825.0	1,838,964.0	930,066.0	53,522.0	30,991.0
2	773,542.0	1,861,009.0	925,531.0	51,039.0	31,535.0
3	753,037.0	1,878,703.0	908,113.0	50,334.0	31,023.0
4	748,736.0	1,884,179.0	893,271.0	49,934.0	32,353.0
5	757,802.0	1,880,425.0	880,007.0	49,683.0	32,884.0
6	750,403.0	1,876,944.0	853,847.0	49,606.0	34,831.0
7	721,510.0	1,882,550.0	837,890.0	49,352.0	35,389.0
8	711,883.0	1,893,655.0	816,974.0	47,887.0	36,219.0
9	715,017.0	1,900,302.0	795,457.0	43,586.0	35,593.0
10	703,223.0	1,911,703.0	767,745.0	41,843.0	35,656.0
11	671,348.0	1,917,351.0	741,567.0	43,465.0	37,989.0
12	658,137.0	1,923,423.0	718,020.0	40,686.0	39,088.0
13	658,027.0	1,918,320.0	697,372.0	35,832.0	34,253.0
14	650,727.0	1,903,868.0	672,610.0	37,512.0	36,002.0
15	617,140.0	1,888,800.0	648,850.0	37,479.0	38,367.0
16	NaN	NaN	NaN	NaN	NaN
17	NaN	NaN	NaN	NaN	NaN
18	NaN	NaN	NaN	NaN	NaN
19	NaN	NaN	NaN	NaN	NaN

	wline_biz_int	wline_biz_ph	wless_post	wless_pre
0	173,144.0	312,384.0	714,917.0	371,268.0
1	172,709.0	319,637.0	735,002.0	371,157.0
2	170,644.0	327,199.0	764,091.0	383,082.0
3	170,150.0	333,296.0	797,141.0	384,342.0
4	170,312.0	337,951.0	890,649.0	380,536.0
5	171,125.0	346,717.0	944,838.0	373,006.0
6	172,859.0	354,912.0	1,029,720.0	373,138.0
7	174,107.0	363,561.0	1,115,787.0	352,686.0
8	172,667.0	369,397.0	1,180,457.0	335,799.0
9	173,094.0	374,765.0	1,241,736.0	336,619.0
10	173,686.0	379,434.0	1,313,828.0	344,357.0
11	174,380.0	383,687.0	1,380,693.0	335,403.0
12	174,042.0	385,196.0	1,434,982.0	332,173.0
13	174,124.0	386,975.0	1,437,218.0	324,472.0
14	178,270.0	387,660.0	1,482,175.0	339,339.0
15	179,461.0	390,082.0	1,569,471.0	353,072.0
16	NaN	NaN	NaN	NaN
17	NaN	NaN	NaN	NaN
18	NaN	NaN	NaN	NaN

```
19            NaN           NaN          NaN         NaN

                                                source  \
0   https://www.sec.gov/Archives/edgar/data/932872...
1   https://www.sec.gov/Archives/edgar/data/932872...
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3   https://www.sec.gov/Archives/edgar/data/932872...
4   https://www.sec.gov/Archives/edgar/data/932872...
5   https://www.sec.gov/Archives/edgar/data/932872...
6   https://www.sec.gov/Archives/edgar/data/932872...
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10  https://www.sec.gov/Archives/edgar/data/932872...
11  https://www.sec.gov/Archives/edgar/data/932872...
12  https://www.sec.gov/Archives/edgar/data/932872...
13  https://www.sec.gov/Archives/edgar/data/932872...
14  https://www.sec.gov/Archives/edgar/data/932872...
15  https://www.sec.gov/Archives/edgar/data/932872...
16                                                NaN
17                                                NaN
18                                                NaN
19                                                NaN

            notes and variable definitions dictionary  wline_avg_per_cust  \
0                                                 NaN  202.71502290732005
1                                                 NaN  199.95078134613019
2                                                 NaN  184.45308284421148
3                                                 NaN  185.87987902231336
4                                                 NaN   185.1436558309821
5                                                 NaN  185.26054551649437
6                                                 NaN  191.24317573557303
7                                                 NaN   192.3274159176355
8                                                 NaN  191.53611064874482
9                                                 NaN  193.47200570066258
10                                                NaN  194.65877445821948
11                                                NaN   195.9806485211282
12                                                NaN                 NaN
13                                                NaN                 NaN
14  {'tot_revenue_mils': 'total revenue in million...               NaN
15  Intersegment eliminations are netted against W...               NaN
16                                                NaN                 NaN
17                                                NaN                 NaN
18                                                NaN                 NaN
19                                                NaN                 NaN

    wless_avg_per_cust  indiv_yhat  indiv_wline_yhat  indiv_wless_yhat  \
```

0	128.89148717759863	1,287.0	1,101.0	186.0
1	139.22049180994776	1,299.0	1,110.0	189.0
2	149.9337937695535	1,311.0	1,115.0	196.0
3	148.11893188475838	1,310.0	1,108.0	202.0
4	228.13359188473748	1,321.0	1,103.0	217.0
5	179.83919189221183	1,326.0	1,101.0	225.0
6	178.2076304230364	1,329.0	1,089.0	240.0
7	185.9074017704105	1,330.0	1,079.0	251.0
8	162.90125150370386	1,330.0	1,071.0	259.0
9	159.0263280440712	1,334.0	1,064.0	270.0
10	170.66853216016307	1,337.0	1,053.0	283.0
11	185.30431863951668	1,336.0	1,042.0	293.0
12	NaN	1,335.0	1,033.0	302.0
13	NaN	1,323.0	1,022.0	301.0
14	NaN	1,320.0	1,008.0	311.0
15	NaN	1,318.0	989.0	329.0
16	NaN	NaN	NaN	NaN
17	NaN	NaN	NaN	NaN
18	NaN	NaN	NaN	NaN
19	NaN	NaN	NaN	NaN

	total_yhat	total_wline_yhat	total_wless_yhat	indiv_sr	total_sr	trend
0	1,267.0	1,014.0	253.0	NaN	NaN	1.0
1	1,279.0	1,022.0	257.0	NaN	NaN	2.0
2	1,293.0	1,026.0	267.0	NaN	NaN	3.0
3	1,295.0	1,020.0	275.0	NaN	NaN	4.0
4	1,311.0	1,016.0	296.0	NaN	NaN	5.0
5	1,320.0	1,013.0	307.0	NaN	NaN	6.0
6	1,329.0	1,003.0	326.0	NaN	NaN	7.0
7	1,335.0	993.0	342.0	NaN	NaN	8.0
8	1,338.0	985.0	353.0	NaN	NaN	9.0
9	1,347.0	979.0	367.0	NaN	NaN	10.0
10	1,355.0	970.0	386.0	NaN	NaN	11.0
11	1,359.0	959.0	399.0	NaN	NaN	12.0
12	1,362.0	951.0	411.0	784.0	1.0	13.0
13	1,351.0	941.0	410.0	121.0	1,521.0	14.0
14	1,352.0	928.0	424.0	841.0	9.0	15.0
15	1,358.0	910.0	447.0	2,704.0	144.0	16.0
16	NaN	NaN	NaN	NaN	NaN	NaN
17	NaN	NaN	NaN	NaN	NaN	NaN
18	NaN	NaN	NaN	NaN	NaN	NaN
19	NaN	NaN	NaN	NaN	NaN	NaN

[31]:
```
# Recalculate the time trend.
df["trend"] = df.index + 1

# Re-estimate the marginal effects of each Wireline and Wireless customer
```

```
# on total revenue using the Total Revenue Model on the entire sample period.
y = df["tot_rev_mils"].iloc[:16]
x = df[["wline_cust_tot", "wless_cust_tot"]].iloc[:16]
model = sm.OLS(y, x).fit()
model.summary()
model.params
wline_nested_slope = model.params[0] * 1000000
wless_nested_slope = model.params[1] * 1000000
```

```
C:\Users\Robso\anaconda3\envs\ml\lib\site-packages\scipy\stats\stats.py:1603:
UserWarning: kurtosistest only valid for n>=20 … continuing anyway, n=16
  warnings.warn("kurtosistest only valid for n>=20 … continuing "
```

[32]:
```
wline_nested_slope
```

[32]: 177.51793398014055

[33]:
```
wless_nested_slope
```

[33]: 227.91970800157858

[34]:
```
# Create one-year ahead predictions for customers by segment from the
# OLS estimates.
df["wline_cust_pred"] = (wline_cons + (wline_slope * df["trend"])).round()
df["wless_cust_pred"] = (wless_cons + (wless_slope * df["trend"])).round()

# Create one-year ahead forecasts based on Total Revenue Model for the entire
# sample period.
df["tot_rev_nested_pred"] = (
    (
        wline_nested_slope * df["wline_cust_pred"]
        + wless_nested_slope * df["wless_cust_pred"]
    )
    / 1000000
).round(2)

# Create one-year ahead forecasts based on Total Revenue Model for the training
# sample period. This is being done as a robustness check.
df["tot_rev_nested_pred_prior_betas"] = (
    (181.57 * df["wline_cust_pred"] + 215.14 * df["wless_cust_pred"]) / 1000000
).round(2)
```

[35]:
```
fig, ax = plt.subplots()
ax.scatter(
    df["date"], df["tot_rev_mils"], label="Actual Revenue", color="black"
)
ax.scatter(
```

```
    df["date"],
    df["tot_rev_nested_pred"],
    label="Predicted Revenue",
    color="orange",
)
ax.set_title("Actual vs. Predictions")
ax.set_xlabel("Quarter")
ax.set_ylabel("Revenue")
ax.legend()
ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
ax.xaxis.set_major_formatter(mdates.DateFormatter("%Y-%m"))
ax.yaxis.set_major_formatter(matplotlib.ticker.StrMethodFormatter("{x:,.0f}"))
plt.savefig("Actual_v_Pred.pdf", bbox_inches="tight")
```

```
<ipython-input-35-addea4c1444c>:15: UserWarning: FixedFormatter should only be
used together with FixedLocator
  ax.set_xticklabels(ax.get_xticks(), rotation="vertical")
```