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Institución: UNAL Curso: Nota:

Parcial 1

a). Como la distancia media entre dos señales se expresa a partir de la potencia media de la diferencia entre ellas entonces tenemos:

$$d(x_1,x_2)=\bar{P}_{x_1-x_2}=\lim_{T\to\infty}\frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt$$

$$\frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt=\frac{1}{T}\left[\int_T |x_1(t)|^2dt-2\int_T x_1(t)x_2(t)dt+\int_T |x_2(t)|^2dt\right]$$

$$\frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt=\bar{P}_{x_1}-2\int_T x_1(t)x_2(t)dt+\bar{P}_{x_2}$$

Donde

$$\bar{P}_{x_1}=\frac{1}{T}\int_T |x_1(t)|^2dt=\frac{1}{T}\int_0^T |Ae^{jw_0t}|^2dt$$

$$=\frac{A^2}{T}\int_0^T (e^{jw_0t})(e^{-jw_0t})\,dt=\frac{A^2}{T}\int_0^T e^0\,dt$$

$$=\frac{A^2}{T}\int_0^T 1\,dt=\frac{A^2}{T}t\Big|_0^T=\frac{A^2}{\cancel{T}}(\cancel{T})=\boxed{A^2}$$

y donde

$$\bar{P}_{x_2}=\frac{1}{T}\int_T |x_2(t)|^2dt=\frac{1}{T}\int_0^T |Be^{jw_0t}|^2dt$$

$$=\frac{B^2}{T}\int_0^T (e^{jw_0t})(e^{-jw_0t})\,dt=\frac{B^2}{T}\int_0^T e^0\,dt$$

$$=\frac{B^2}{T}\int_0^T 1\,dt=\frac{B^2}{T}t\Big|_0^T=\frac{B^2}{\cancel{T}}(\cancel{T})=\boxed{B^2}$$

Ahora para

$$-\frac{2}{T}\int_T x_1(t)\,x_2(t)\,dt = -\frac{2}{T}\int_0^T \left(Ae^{j\omega_0 t}\right)\left(B\,e^{5j\omega_0 t}\right)dt$$

$$= -\frac{2}{T}\int_0^T Ae^{j\frac{2\pi}{T}t} \cdot Be^{j5\cdot\frac{2\pi}{T}t}\,dt = -\frac{2}{T}\int_0^T A.B.\,e^{j\frac{12\pi}{T}t}\,dt$$

$$= -\frac{2}{T}AB\int_0^T e^{j\frac{12\pi}{T}t}\,dt = -\frac{2}{T}A.B\left(\frac{T}{j12\pi}\,e^{j\frac{12\pi}{T}t}\right)\Big|_0^T$$

$$= -\frac{2}{T}AB\left(\frac{T}{j12\pi}\,e^{j\frac{12\pi}{T}T} - \frac{T}{j12\pi}\,e^{\cancel{j\frac{12\pi}{T}\cdot 0}}\right)$$

como $e^{j12\pi} = \cancel{\cos(12\pi)} + j\,\cancel{\operatorname{sen}(12\pi)} = 1$

$$= -\frac{2}{T}A\cdot B\left(\cancel{\frac{T}{j12\pi} - \frac{T}{j12\pi}}\right)$$

$$= -\frac{2}{T}AB(0) = \boxed{0}$$

Asi obtenemos:

$$\frac{1}{T}\int_T |x_1(t) - x_2(t)|^2\,dt = \bar{P}_{x_1} - 0 + \bar{P}_{x_2}$$

$$= \bar{P}_{x_1} + \bar{P}_{x_2} = A^2 + B^2$$

Finalmente

$$d^2(x_1, x_2) = \lim_{T\to\infty} A^2 + B^2$$

$$d(x_1, x_2) = \sqrt{A^2 + B^2}$$

b). tenemos:

$$x(t) = 3\cos(1000\pi t) + 5\,sen(2000\pi t) + 10\cos(11000\pi t)$$

$5\,KHz \rightarrow$ frecuencia de muestreo

$w_1 = 1000\pi$ $\quad T_1 = \frac{2\pi}{w_1} = \frac{1}{500}$ $\quad F_1 = 500\,Hz$

$w_2 = 2000\pi$ $\quad T_2 = \frac{2\pi}{w_2} = \frac{1}{1000}$ $\quad F_2 = 1000\,Hz$

$w_3 = 11000\pi$ $\quad T_3 = \frac{2\pi}{w_3} = \frac{1}{5500}$ $\quad F_3 = 5500\,Hz$

Ahora

$\frac{w_1}{w_2} = \frac{1000}{2000} = \frac{1}{2} \rightarrow \in \mathbb{Q}$ $\qquad \frac{w_2}{w_3} = \frac{2000}{11000} = \frac{2}{11} \rightarrow \in \mathbb{Q}$

$\frac{w_1}{w_3} = \frac{1000}{11000} = \frac{1}{11} \rightarrow \in \mathbb{Q}$ $\qquad$ La señal $x(t)$ es cuasiperiodica

$T = KT_1 = lT_2 = rT_3 \qquad k, l, r \in \mathbb{Z}$

$$T = \frac{K}{500} = \frac{l}{1000} = \frac{r}{5500}$$

$$11000\,T = 22K = 11l = 2r$$

Como MCM entre 2, 11 y 22 es 22

$$11000\,T = 22$$

$$T = \frac{22}{11000} = \boxed{\frac{1}{500}}$$

Por Nyquist $\quad F_s \geq 2\,F_{max}$

$F_s \geq 2(5500)\,Hz$

$F_s \geq 11000\,Hz$

$5000 \geq 11000$ X no cumple

Ahora obtenemos las velocidades angulare discretas:

$$t = \frac{n}{Fs}$$

$$x_1[n] = 3\cos\left[1000\pi \frac{n}{5000}\right]$$

$$x_1[n] = 3\cos\left[\frac{\pi}{5} n\right] \qquad \boxed{\Omega_1 = \frac{\pi}{5}}$$

$$x_2[n] = 5\,sen\left[2000\pi \frac{n}{5000}\right]$$

$$x_2[n] = 5\,sen\left[\frac{2\pi}{5} n\right] \qquad \boxed{\Omega_2 = \frac{2\pi}{5}}$$

$$x_3[n] = 10\cos\left[11000\pi \frac{n}{5000}\right]$$

$$x_3[n] = 10\cos\left[\frac{11\pi}{5} n\right] \qquad \boxed{\Omega_3 = \frac{11\pi}{5}} \rightarrow \text{es una copia}$$

Como Ω_3 es una copia entonces

$$\Omega_{OR3} = \Omega_3 - 2\pi = \boxed{\frac{\pi}{5}}$$

Así tenemos:

$$x[n] = \underbrace{3\cos\left[\frac{\pi}{5} n\right]} + 5\,sen\left[\frac{2\pi}{5} n\right] + \underbrace{10\cos\left[\frac{\pi}{5} n\right]}$$

$$x[n] = 13\cos\left[\frac{\pi}{5} n\right] + 5\,sen\left[\frac{2\pi}{5} n\right]$$