Now let us consider a few examples. The verification of one of the distributive laws, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, follows immediately since the indicator function for each side can be written, using Theorem 1, as $I_A + I_B I_C - I_A I_B I_C$. Verification of one of the DeMorgan Laws, $(A \cup B)' = A' \cap B'$, can be done as follows:

$$I_{(A \cup B)'} = 1 - I_{A \cup B} = 1 - (I_A + I_B - I_A I_B) = (1 - I_A)(1 - I_B) = I_{A'} I_{B'} = I_{A' \cap B'}.$$

As another illustration one can show (without having to recall any previously demonstrated statements) that

$$(A \cap B') \cap (C' \cap A) = A \cap (B \cup C)'$$

from the argument

$$I_{(A \cap B') \cap (C' \cap A)} = I_{(A \cap B')} I_{(C' \cap A)} = I_A I_{B'} I_{C'} I_A = I_A^2 (1 - I_B)(1 - I_C) = I_A (1 - I_B - I_C + I_B I_C)$$
$$= I_A (1 - I_{B \cup C}) = I_A I_{(B \cup C)'} = I_{A \cap (B \cup C)'}.$$

The method is not entirely a "mindless" one as the reader can discover by looking at how indicator functions can be used to consider such questions as: What is the relationship between $(A \cup B) \cap C$ and $A \cup (B \cap C)$? Are they equal for all A, B and C in U? Are they ever equal? Under what conditions, if any, is one a subset of the other?

In conclusion it is apparent that there is nothing astounding in what has been said above, but it is surprising that in an examination of some thirty to forty finite mathematics textbooks which included material on both functions and sets none were found which even suggested this idea. It may be that the argument against the use of the indicator function here is that it is mere "symbol manipulation," and that in part may be true. It may also be argued that this function and its arithmetic are more sophisticated than the elementary set theory to which it is applied. However, if the instructor chooses to take the time to introduce this idea, it could serve as a vehicle to provide the student with a greater appreciation and understanding of the concept of function itself (e.g., not every function is "formula" written as $y = f(x)$ which is defined on some set of numbers). In addition, it might also be good preparation for the functional concepts of probability measure and random variable. In any case, the use of indicator functions does provide a reasonably accessible method of verifying set-theoretic statements and might even give some students the pleasure of proving something in an elementary mathematics course.

WHAT EVER HAPPENED TO THE HISTORY OF MATHEMATICS?

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A recent survey [**1**] of the undergraduate mathematics program at 406 collegiate institutions revealed that less than 35% of them offered a course specifically on the history of mathematics. This appears to be a rather sad commentary on the state of undergraduate mathematics teaching in the United States. While many professions, e.g., law, medicine and architecture are presently moving towards requiring studies in the history of their major discipline, the teaching of mathematics is tending more and more to divorce itself from an historical perspective. Before World War II, the history of mathematics occupied an important place in the undergraduate mathematics curriculum, particularly that of teacher training institutions, but with the explosive growth of new areas of mathematical interest and applications (statistics, linear algebra, topology, operations research, computer science, etc.) it was relegated to a position of little importance. (It is interesting to note that this has not been the case in Europe, particularly the Soviet Union, where the history of mathematics still holds a position of prominence in the university curriculum and academic research.)

Many reasons can be given for teaching a course in the history of mathematics. Some of these would be:

1. to demonstrate the development of mathematics as a necessary human activity;
2. to show that mathematical ideas evolve over a period of time, are labored upon and subject to change;
3. to expose the interrelationship of mathematics with other disciplines: anthropology, sociology, economics, politics, music, arts, etc. [**2**];
4. to develop an appreciation of the structure of mathematics as viewed from an historical perspective, and
5. to expose the interrelationship of seemingly diverse areas of mathematics [**3**]; for example, a consideration of the development for the value of pi would reveal first the use of rough geometric approximation (1000 BC), then better geometric-analytical approximation by the exhaustion method of Archimedes (200 BC), analytic results of series approximation as employed by James Gregory (1671), Comte de Buffon's probabilistic approach (1771), and finally the iterative results obtained by modern computers (1950 +).

The history of mathematics can be taught informally in all mathematics courses or formally in a specific history of mathematics offering. Historical notes and justifications can easily be injected into lectures. Anecdotes concerning problem situations from which mathematical investigations and theories evolved would seem appropriate for use in most courses; for example, the expression we now know as a Fourier series emerged from Fourier's attempts to understand heat transfer.

Since the subject contains a huge body of information, the specific topics to be studied and their sequencing must be carefully planned [**4**]. It is a mistake to try to teach the mathematical accomplishments of 5000 years in one course. A course in the history of mathematics can most beneficially be given during a student's undergraduate career either in the sophomore or senior year of study. A sophomore level offering lays before the student a panoramic view of the territory to be explored; it can serve as an introduction to the content, techniques and trends of mathematical thought. Works by Struik [**5**] and Kline [**6**] lend themselves for such purposes. A senior level course also presents an overview of mathematics, but in addition serves to summarize the students' previous studies and give human meaning to much of the material they have learned. In such efforts issues can be put into perspective: "How did the rise of industrialization stimulate the development of the study of differential equations?"; "Why was the advent of modern mercantile capitalism in 15th century Italy accompanied by a period of mathematical growth and accomplishment?"; "What effects has warfare had on mathematics?", etc. Eves' book [**7**] with its excellent selection of problem studies nicely accommodates such a course. Projects and term papers can be usefully employed to supplement classroom discussions [**8**].

Ultimately, the history of mathematics and its meaningful success as a student learning experience depends on the instructors. Many of us have had no formal exposure to this discipline. However, the history of mathematics boasts a rich literature that encourages self-study [**9**]. While the references already cited provide an introduction to the subject, more specialized treatments are available [**10**]. In particular several journals either regularly feature articles on the history of mathematics or are completely devoted to it, e.g., *Isis*, *Scripta Mathematica*, *Historia Mathematica*.

What ever happened to the history of mathematics? It's there, waiting for us to teach it.

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PROBLEMS AND SOLUTIONS

EDITED BY DAVID BORWEIN, J. L. BRENNER, AND VLADIMIR DROBOT

Send all ***proposed*** *problems, in duplicate if possible, to Professor Vladimir Drobot, Department of Mathematics, University of Santa Clara, Santa Clara, CA 95053. Please include solutions, relevant references, etc.*

An asterisk (*) indicates that neither the proposer nor the editors supplied a solution.

Solutions *should be sent to the addresses given at the head of each problem set.*

A publishable solution must, above all, be correct. Given correctness, elegance and conciseness are preferred. The answer to the problem should appear right at the beginning. If your method yields a more general result, so much the better. If you discover that a MONTHLY *problem has already been solved in the literature, you should of course tell the editors; include a copy of the solution if you can.*

ELEMENTARY PROBLEMS

Solutions of these Elementary Problems should be mailed in duplicate to Dr. J. L. Brenner, 10 Phillips Road, Palo Alto, CA 94303 (USA), by March 31, 1983. Please place the solvers's name and mailing address on each (double-spaced) sheet. Include a self-addressed card or label (for acknowledgment).

E 2968. *Proposed by George Tsintsifas, Thessaloniki, Greece.*

The points A_1', A_2', A_3' lie on the sides A_2A_3, A_3A_1, A_1A_2 of an acute angle triangle $A_1A_2A_3$, respectively. Show that

$$2\sum a_i' \cos A_i \geqslant \sum a_i \cos A_i$$

where a_1, a_2, a_3 are the sides of the triangle $A_1A_2A_3$ and a_1', a_2', a_3' are the sides of the triangle $A_1'A_2'A_3'$.

E 2969. *Proposed by John Brillhart, The University of Arizona, and Constantin Sevici, University of Michigan.*

Let $f(x) = a_0x^n + \cdots + pa_n$ be a polynomial with integer coefficients such that $a_0a_n \neq 0$, $(a_0, a_1, \ldots, pa_n) = 1$, and let p be a prime such that $p > \sum_{s=0}^{n-1} |a_s||a_n|^{n-1-s}$. Prove that $f(x)$ is irreducible in $\boldsymbol{Z}[x]$.