Why Teach History of Mathematics?

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Why teach history of mathematics ?

TORKIL HEIEDE

There are questions which have only personal answers. One such question is the one in the title; another, which is intricately bound up with it, is: Why teach mathematics? On the following pages you will find my personal answers to both questions; a selection of other people's answers to one or both are listed at the end. Much longer lists could be made, and very long ones if one included also papers in other languages than English. As you will see, my list contains papers from different parts of the world and from different times in this century; it seems that many people agree, for different reasons, that one really should teach history of mathematics, but that it has to be said again and again.

Let me now begin with my answer to the second of the two questions. We teach mathematics–and I am thinking of all levels of teaching from primary school to university– for precisely the same reason that we teach any other subject: because it is there. It is part of a common heritage and must not be withheld from new generations. I do not teach mathematics in order to save the world or improve mankind or produce better engineers or make people more able to manage their daily lives, or to amuse them or to reach some general purpose common to all subjects – but because mathematics should be passed on and because I happen to like to do it. A teacher of mathematics is for me a person who brims over with mathematics, who is so full of this subject that he or she must tell other people about it. This is the one thing needful in the teaching of mathematics – and of course from that much of the rest will follow (except perhaps the saving of the world), but not the other way round.

Of course there are a lot of pertinent and difficult – but subordinate – questions to be answered here: which mathematics should be taught, to whom, by whom, how much, when, in which order, in which way and so on – all this constitutes the didactics of mathematics, and good answers are necessary for good mathematics teaching, but by no means sufficient.

Now where does history come into this? In the same way in which history comes in everywhere else. Man is a historical creature; it is that which makes the difference between man and every other creature on the earth. The eagle and the mouse have no history, or if they have, then no single eagle or mouse is aware of it, and it is imposed on them by man – just as he (in a more tangible way maybe) has imposed history on the horse. Every man and woman has a personal history, and also a family history – and it is a pity if one does not know or know about one's parents, grandparents, great-grandparents and if possible further back than that. A nation becomes a nation by knowing about its past – good

things and bad things, especially good acts and bad acts done in the name of that nation; and in a way it is the same with the whole of mankind: man has a history – it is only another name for having a memory. Of course we have specialists to take special care of this memory: the historians, who dig in it, think of it, discuss it, write on it and so on, but we ought to be aware of the past, all of us – if we are not it ceases to exist, and we too cease to exist, in a rather profound way maybe, but a historical creature is not alive if the past is not alive in him or her.

Everything man touches has a history – the tree, the axe, the house, the town, architecture, every trade, every art, every belief, every subject (even history itself), and therefore also mathematics. If you teach mathematics, you must also teach history of mathematics, for the history of a subject is part of the subject. If you are not aware that mathematics has a history, then you have not been taught mathematics – because you have then been cheated of an indispensible part of it. Again we have specialists, the historians of mathematics, who dig, think, discuss and write, but you are not a mathematics teacher if you do not teach also the history of mathematics when you teach mathematics.

An example: You have not been taught about logarithms if you have not heard about Napier: *Seeing there is nothing (right well beloved students in the Mathematics) that is so troublesome to Mathematicall practice, nor that doth more molest and hinder Calculators, than the Multiplications, Divisions, square and cubical Extractions of great numbers, which besides the tedious expence of time, are for the most part subject to many slippery errors. I began therefore to consider in my minde, by what certaine and ready Art I might remove those hindrances* – and how he then considered in his mind for twenty years and then brought forth his admirable logarithms. And more than that: you have really not heard of logarithms if you have not heard of prosthaphaeresis and Tycho Brahe – who was not the first to invent the method, but a lot of people from many countries learnt to use it at his two observatories Uranienborg and Stjerneborg on the (then Danish) island of Hven in the Sound near Copenhagen – and why logarithms were better than prosthaphaeresis, because with them one could handle more than two factors at a time, and also fractions with many factors both in the numerator and in the denominator, and even powers and roots. And how logarithms were an eminently important tool in calculations for more than three centuries, but then suddenly lost this significance, how they in the meantime had acquired another and much deeper theoretical importance in that branch of mathematics which for quite different reasons is called calculus – and how this importance is not lost and will never be lost. By the way: it is quite interesting that logarithms are now introduced later in school than they were before, because they are now no

longer necessary at the more elementary level – this is quite the reverse of the usual course of events.

In short: you have not learnt about logarithms if you have not become aware that even logarithms have their fate.

Maybe I have now brimmed over sufficiently; let me just say that obviously I do not agree with people who wish to teach history of mathematics in order to make mathematics more amusing, or more easy, or more human (or even to sugar the pill! If one thinks of mathematics as a bitter pill needing to be sugared one should not be a mathematics teacher at all). Again: if you teach history of mathematics as a part of mathematics, all these things will follow.

A consequence of all this is (at least for me) that teachers of mathematics should – in their initial training – have learnt about the history of mathematics, either as part of the mathematics they have studied, or maybe preferably also in special courses in the history of mathematics, just as they may possibly have studied other parts of mathematics in special courses.

This is of course not precisely the way things are done just now in teacher training colleges or universities or wherever future teachers are educated in the different countries of this imperfect world, and to bring about a change is a major task in itself. An even bigger task is to do something for all those who are already teachers of mathematics – and I am still thinking of all levels of teaching. Most of these teachers have not heard much about the history of mathematics, either in their own school-days or in their education as prospective mathematics teachers.

The problem is especially big with regard to the many teachers on the primary and lower secondary level, where the children of today receive their first and maybe often lasting impression of what mathematics is. Many of these teachers have not been taught very much mathematics and most often no history of mathematics at all. What should and what can be done for them?

This question is a professional and a personal one for me since I work at the Royal Danish School of Educational Studies, whose main purposes are research on education, and further education of teachers. Every year many hundreds of mathematics teachers attend shorter or longer courses at the department of mathematics of the School's main branch in Copenhagen or at its eight provincial branches. On a smaller scale there are part-time studies for a master's degree in mathematical education, and also every year a one-week course for the teachers of mathematics at the teacher training colleges.

It is not possible here to describe completely to which extent the history of mathematics figures in all this; let me as an example give a short description of one of the courses with which I have in later years been

involved at the Copenhagen branch.

Fifty teachers meet every Tuesday from 9am to 3pm through 33 weeks to participate in a course called Mathematics in Classes 7 to 10. Most of the time they are divided into two classes and work with several mathematical subjects under the guidance of five or six different lecturers; it depends to a high degree on these lecturers how much the history of these subjects will come out. On the Tuesdays in January and February the fifty teachers will split from 9am to noon into three or four smaller groups devoted to subjects chosen beforehand by the participants themselves; every year one of these subjects has been the history of mathematics.

In seven mornings the group then moves from Egyptian calculations and Babylonian equations through Greek geometry and Arabian algebra all the way to the emergence of non-Euclidean geometry, when mathematics once again defined its own domain in a new way. Of course one cannot cover the whole of the history of mathematics in any depth in so short a time, but the purpose is also rather to show that here is something of interest, something to return to, something to go on with. And of course it is not intended that what is presented in these somewhat packed mornings should or could pass unadapted into the participants' own teaching, but it is hoped that it in many ways might colour and maybe improve what goes on in their classrooms.

Many of last year's participants in this historical part of the course found it so important that they thought it should in future courses be given to all and not only be offered as a choice. That might be a good idea; it would be still better if all our courses had a historical part, even on as modest a scale as in this course. Luckily we have also had a few (somewhat shorter) courses devoted entirely to the history of mathematics.

Let me in conclusion mention another thing that has some bearing on the history of mathematics: the spirit of the time. We are all performing each of us our own personal dance to the music of time, either in complete rhythm and harmony or in some disagreement. But maybe we cannot always sense everything which is in the air – precisely because we breathe the air all the time. Professor Howard Levi of the City University of New York tried in his own way in the last years before his retirement both to learn something of this changing atmosphere at different moments in history and at the same time to make his students aware of this aspect of life. He offered a course to students whose main subject was not mathematics but who were interested in learning some mathematics and something about mathematics. He talked to them on some chosen mathematical subjects in their historical settings and asked the students if they could find in their main subjects, be it literature,

political science, biology or whatever, something from the same or nearly the same period which in some way had the same flavour as the mathematical topic in question or in some way reminded them of it. He was not asking for specific influences either from mathematics on the other subjects or vice versa, but rather wished they could find something which was in the air at that time, in philosophy, in politics, in science, in literature, in the view of what history itself might be. One of his own examples concerned the American Declaration of Independence of 1776; as everybody knows its second paragraph begins in this way: *We hold these truths to be self-evident... .* If it had been written fifty or sixty years later, might it then have begun in this more non-Euclidean way: *We wish to build our society on the following principles...?*

Nothing very concrete materialized from these courses, but they were very interesting both for Howard Levi and for his students. And of course there are no definite answers to the questions how we define the spirit of the time and how we sense it, and how we become aware of its changes – in the past and in the present. But it is more important to pose such questions than to answer them, and they should have a place in any engagement with history and also with the history of mathematics.

Another important question without an answer is: What is mathematics? And only the realization that mathematics has not always been what it is right now, and that in the future it will be something different, in other words that mathematics has a history, gives this question its real perspective. But that is maybe another story.

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