The history of mathematics as a teaching tool

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The history of mathematics as a teaching tool

Raleigh Schorling once commented that for all that a knowledge of the history of mathematics had long been advocated by himself and others as a necessary part of a mathematics teacher's kit of tools, it still seemed to have had little effect on actual classroom practices. The purpose of this note is to suggest principles and procedures for making history a more useful and effective teaching tool. Additional principles relating to the use of historical materials in a classroom, concrete illustrations of these new principles or additional illustrations of those set forth in this article, pictures or descriptions of historically based or related projects, lists of historical materials and books which may be useful will all be welcomed by this department of THE MATHEMATICS TEACHER.

There are at least three broad categories of functions which may be performed by properly used historical materials. These are: (1) they may clarify meanings, give insights, and sharpen understandings of mathematics itself; (2) they may give students desirable "appreciations"; and (3) in addition to contributing directly to the achieving of such desired outcomes as mathematical understandings and appreciations, they may also serve as primarily a pedagogical device for improving instruction, that is as a methodological tool.

One of the more important understandings to be sought in teaching mathematics is an understanding of its structure, which involves in turn an understanding of the nature and role of undefined terms, definitions, axioms, and the logical deduction of theorems. For example, in geometry a presentation and discussion of Euclid's definitions of a *point* ("a point is that which has no part") and a *line* ("a line is breadthless length")[1] help to bring out the need for undefined terms, and illustrate that intuitive and physical concepts which are *not* mathematics have nevertheless often stimulated the thinking of mathematicians and influenced their choice of undefined terms and axioms. This sort of discussion also helps students understand the "ideal" nature of true points and lines as opposed to the physical representations of points and lines which they draw. Both the utility and the dangers of diagrams and thinking based on them are clearer for a discussion of early ideas and how they have changed.

History helps in an even more concrete way with many terms which youngsters must come to understand. For example, the stories of the words *sine, zero, root, radical,*[2] and many others will help students understand, remember, and recall them.

A student can hardly fail to understand and retain the meaning of *isosceles* after he has heard an analysis of the word *isosceles* into its Greek forbears *isos,* equal, and

[1] Sir T. L. Heath, *The Thirteen Books of Euclid's Elements* (Cambridge: The University Press, 1908), I, 154.

[2] Some of these are given and methods of finding others will be found in P. S. Jones, "Word Origins," THE MATHEMATICS TEACHER, XLVII (March 1954), 195 ff.

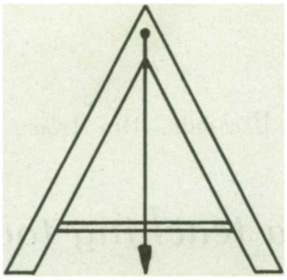

Figure 1

skelos, legs, and has participated in a listing of other "iso" words in English (isocline, isotherm, isobar, etc.), and has discovered that *skeleton* was the Greek word for a dried body. If the student is also shown a cardboard or wooden model of the Egyptian leveling device sketched in Figure 1, he will not only be further helped to remember what an isosceles triangle is, but he also can *discover* for himself (led by some questioning if necessary) such properties of the isosceles triangle as that its altitude bisects the base and that, conversely, the perpendicular bisector of its base passes through its vertex.

The story of the development of non-Euclidean geometry should be told, from early attempts to find a "better" axiom to replace Euclid's fifth or parallel postulate, to attempts to prove the postulate, to the essentially independent and simultaneous development of hyperbolic geometry by the Russian Lobachewsky and the Hungarian Bolyai. Not only does this story clarify the nature and role of axioms in modern mathematics, but it simultaneously teaches a number of things which we will list here but classify under our second heading, "appreciations."

These are:

1. The continuity of the growth of mathematics which often leads to simultaneous discoveries (the German Gauss also had essentially the ideas of Lobachewsky and Bolyai)

2. The internationalism of mathematics

3. The operation of intellectual curiosity and aesthetic sensitivity as motives and stimuli for developing mathematical theory (None of the developers anticipated any practical use for this geometry and much of the earlier dissatisfaction with Euclid's postulate was based on such aesthetic considerations as its complicatedness, its nonself-evident nature, and the fact that its converse is a theorem.)

4. Often totally unanticipated applications come after the development of a mathematical theory, as when a non-Euclidean geometry was found useful in the mathematical formulation of a relativity theory (which latter, in a sense, led to the development of atomic energy).

5. The rapid current growth of both pure and applied mathematics and the relationships between them. (These are shown by the fact that Riemann's elliptic non-Euclidean geometry was presented in 1854, about twenty years after Bolyai's and Lobachewsky's work, and that the tensor analysis by which the geometric ideas important in relativity are most easily developed was invented by the Italian, Ricci, prior to 1890 and has been extensively applied since 1920 in such areas as the theory of elasticity as well as in geometry and relativity.

Much of this material has to be presented by mere talk, it is true, but there are some expositions of non-Euclidean geometry which can be used with secondary school pupils.[3]

This should help to convince pupils that

[3] See Lillian R. Lieber, *Non-Euclidean Geometry; or, Three Moons in Mathesis* (Lancaster: The Science Press, 1940); or the appropriate chapter in Moses Richardson, *Fundamentals of Mathematics* (New York: Macmillan, 1941), which gives a sequence of simple theorems, or in H. R. Cooley, D. Gans, M. Kline, H. E. Wahlert, *Introduction to Mathematics* (Boston: Houghton Mifflin Co., 1949) or in Richard Courant and Herbert Robbins, *What is Mathematics?* (New York: Oxford, 1941). The recent reprint of Roberto Bonola, *Non-Euclidean Geometry* (New York: Dover Publications, 1955), contains translations of an historical account and of some of the original works, while Harold E. Wolfe, *Introduction to Non-Euclidean Geometry* (New York: The Dryden Press, 1945), approaches it in a manner similar to that of high school geometry.

rigorous proofs and careful criticisms of axioms have produced new and useful mathematics and will do so again. A further example of this is to be found in the development of such modern algebraic theories as quaternions and algebraic numbers. This followed Hamilton's recognition of the importance of the associative and other axioms of elementary algebra in his formulation of complex numbers as "number couples."[4]

Quaternions are not altogether beyond superior secondary school students, but so-called "modular arithmetics" and scales of notation are topics which may be more easily studied at the secondary level and which will also help students understand the motives of mathematicians, and the role of axioms and rigor in extending mathematics.

The history of the binary and sexagesimal systems shows the intertwining of intellectual curiosity and application as motives for advancement. It also shows the interrelationships between cultures as a whole and mathematics.

For example, the development of agriculture, division of labor, and trade by the Babylonians was made possible by fertile, watered soils in a warm climate. Agriculture produced an interest in both the calendar and astonomy. This grew along with the arithmetic and mensuration of commerce and construction. Babylonian sexagesimal fractions were later taken over by Greek astronomers and were passed by them to the succeeding Arabic civilization. In translations from Arabic into the scholarly Latin of medieval western Europe, sixtieths and sixtieths of sixtieths became the "pars minuta prima" and "pars minuta secunda" from which we got our "minutes" and "seconds."

Although more recently devised units of angular measure were defined entirely arbitrarily from a strictly logical viewpoint, they were consciously and deliberately devised for greater simplicity and convenience as opposed to the purely historical and almost random reasons behind our present system of degrees, minutes and seconds. In this particular case, *only* history can give the real reasons and a complete understanding of the system. The history of other angular units not only makes them more meaningful but also sheds light on the nature, source, and role of definitions in mathematics.[5]

In the course of presenting illustrations of my first contention that the history of mathematics may be used to teach some deeper understanding of the structure of mathematics, the roles of rigor and intuition, and of the mathematical motivation of some topics, I have incidentally given examples of my second point. This is that the history of mathematics may be used to teach several "appreciations." Among these are: (1) an appreciation of the abstract and logical nature of mathematics; (2) an appreciation of how the abstract systems of mathematics may be associated with physical systems and then used to solve "applied" problems; (3) a perception and appreciation of interrelationships within mathematics as well as with other areas of abstract thought such as philosophy, logic, and religion, and with the culture as a whole; (4) an insight into the motives which lead people to do mathematics; (5) an appreciation of the continuity and vigor of the growth of mathematics up to the present day; (6) a realization of the international nature of mathematical interests and productivity.

The pedagogical utility of the history of mathematics embraces all that has been claimed for it above. In addition, it may serve the more purely pedagogical purpose of: (1) creating interest of the type which

[4] P. S. Jones, "Complex Numbers: An Example of Recurring Themes in the Development of Mathematics, II," THE MATHEMATICS TEACHER, XLVII, (April 1954) 260. Reprints of three articles in this series may be purchased from the National Council of Teachers of Mathematics.

[5] P. S. Jones, "Angular Measure—Enough of Its History to Improve Its Teaching." THE MATHEMATICS TEACHER, XLVI (October 1953), 419 ff.

motivates; (2) providing enrichment materials which may be used for differentiated assignments, projects, and the construction of teaching aids; (3) suggesting devices for introducing topics and for aiding students to "discover" new concepts; (4) assisting both the teacher and the student to achieve and maintain a perspective on both the difficulty and the importance of comprehending the abstract nature of mathematics.

When a student hears how long it took for the concept of negative numbers to develop and become accepted, used, and understood, he does not feel quite so concerned that the concept didn't come to him easily. In fact these numbers which everyone today sees all around him on thermometers, stock market reports, antifreeze charts, etc., were called "fausse" or false by the great mathematician-philosopher René Descartes. But the fact that Descartes went ahead and used these numbers suggests to students and teachers that this concept was not an easy one to develop, that we have made really tremendous growth, and that what is today new, abstract, and difficult may in a not-too-distant future be relatively simple and common. This will happen, however, only as the concept is worked with, represented in various ways, and applied.

The word "imaginary" is a misnomer also used by Descartes which has unfortunately stayed with us to the present day. The story of the earlier beginning of imaginaries in Cardan's famous problem may be a good way to introduce the study of complex numbers. Are there two numbers whose sum is ten and whose product is forty? Cardan said that this problem was "obviously impossible." I don't know on what basis he called it impossible, but here is a good way to challenge youngsters by asking them how Cardan might have drawn this conclusion.

Students can easily be led to suggest, if they do not readily think of them for themselves, such approaches to Cardan's problem as numerical experimentations

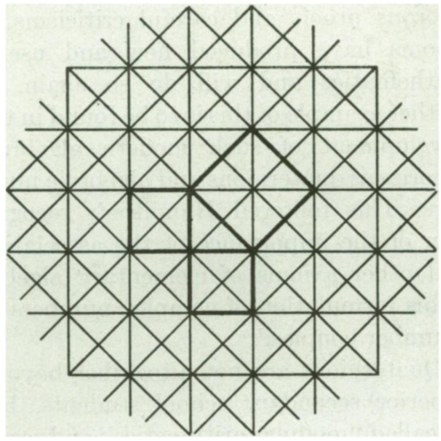

Figure 2

and graphs. Numerically, by taking such products as 1×9, 2×8, 3×7, 4×6, 5×5, 6×4, etc., one soon sees that 25 is the maximum product of two numbers whose sum is 10. This same result appears graphically if one writes and then plots the graph of the equation $y=x(10-x)$. Here, too, one can emphasize both the importance of the *if* in a mathematical statement and the roles of intellectual curiosity and analogy in mathematical discovery. Cardan's problem is impossible of solution *if* one limits oneself to the field of real numbers.

Cardan, motivated by a curiosity about what might happen in this "impossible" situation if he followed the same process which he used in possible cases, applied the method of completing the square to $x(10-x)=40$ and came up with $x=5 \pm\sqrt{-15}$. He then showed that if $\sqrt{-15}$ obeyed rules analogous to those followed by ordinary radicals, the equation was satisfied.[6]

Historical facts and speculations furnish many other suggestions for leading students to "discover" mathematical ideas. For example, some persons have thought that the Pythagorean theorem might have been first discovered in the case of the isosceles right triangle where it is geometrically quite obvious and might have been visualized in a tile floor or wall, as shown in Figure 2.

[6] P. S. Jones, "Complex Numbers," *op. cit.*

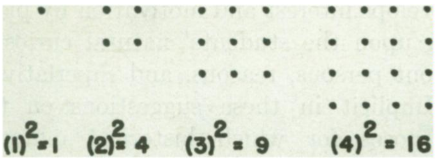

Figure 3

This is speculation, but it seems quite certain that the discovery of one formula for Pythagorean triples of numbers grew out of Greek interest in "figurate" numbers—numbers represented by geometric figures composed of dots. Thus Figure 3 shows the first four square numbers, and Figure 4 shows the first four triangular numbers. If we consider Figure 5 to represent the nth square number, we can then see that the next square number can be formed by adding n dots on each of two sides and filling in another dot at their intersection.

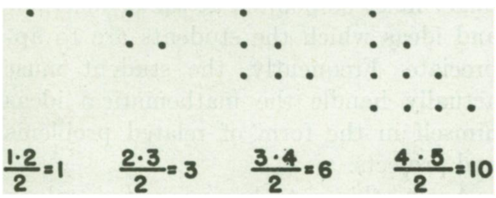

Figure 4

In other words $(2n+1)$ dots added to the nth square number makes it into the $(n+1)$th square number, or $n^2+(2n+1) = (n+1)^2$. This is almost a trivial identity when written this way, but students not only may be led to see in it a new interpretation of the square of the binomial $(n+1)$, but they may also be led to discover the numerical interpretation suggested by Table 1. The sum of the first n odd numbers is nth perfect square.

This ancient result can be made into a useful device for computing square roots on a modern desk calculator, and leads to the converse that if $y=f(x)$ is a quadratic function represented by a table in which the xs appear at equal intervals, then the second differences of the y values will be a constant. This is a special case of a fundamental theorem in the calculus of finite differences developed by Brook Taylor and Leonard Euler in the eighteenth century and useful to actuaries, statisticians, and engineers or to anyone who may wish to fit curves to data or do accurate interpolation or numerical integration.

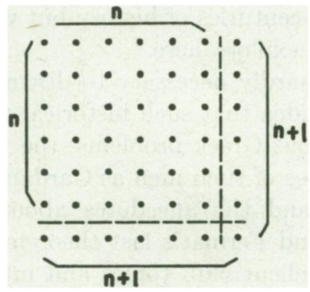

Figure 5

This one example not only shows chronologically the development and extension of a concept, but also displays interrelationships between geometry, number theory, algebra, analytic geometry, and both the finite and infinitesimal calculus as well as interrelationships between mathematics and its applications.

Further, a youngster who, perhaps, with the teacher's help, sees that $n^2 + (2n+1) = (n+1)^2$ is "almost" the Pythagorean theorem might then be led to try what would happen if $(2n+1)$ were a per-

TABLE 1

n	Odd Numbers	Sum of n Odd Numbers
1	1	1
2	3	4
3	5	9
4	7	16
5	9	25
.	.	.
.	.	.
.	.	.

fect square, say m^2. He should come out with

$$\left(\frac{m^2-1}{2}\right)^2+m^2=\left(\frac{m^2+1}{2}\right)^2,$$

which will manufacture a set of "Pythagorean triples" whenever m is an odd number. If he were then to try to get rid of this last restriction and further seek a formula which would give him *all* Pythagorean triples he would be discovering for himself a fascinating trail which has wandered through centuries of history but which we can not explore here.[7]

It is hardly necessary to discuss in detail the idea that such historical topics as the famous Greek problems; the fascinating stories of such men as Cardan, Galois, Gauss, and the anecdotes about Archimedes and Fermat's last theorem all furnish excellent club topics and interesting enrichment projects.

Many models, some of real value as teaching aids (e.g., the abacus, Napier's bones, trisectors, Gunter's scale) are also suggested by the history of mathematics as either student or teacher projects which make interesting demonstrations and bulletin board material.

Even a brief mention in class of who did it, why he did it, or what was the first or last, biggest or best solution to a problem develops interest and motivation by playing upon the students' natural curiosity about persons, reasons, and superlatives.

Implicit in these suggestions on the purposes for which historical material may be used in teaching have been suggestions as to *how* they may be used; e.g., to lead to discovery, to enrich the course and to provide topics for projects.

But if the history of mathematics is to be a real teaching tool a few other guides must be followed. History is not mere names and dates, but ideas developing for some reason. Names and dates will soon bore more than they interest. The mathematical ideas must be pointed out clearly and new ideas illustrated concretely, explored, and observed from several viewpoints. This means that the teacher must be sure that the interrelationships referred to so often above are actually perceived by the student, i.e., the teacher must see to it that a whole story is told with diagrams and worked examples. A figurative finger must be pointed at the connections and ideas which the students are to appreciate. Frequently, the student must actually handle the mathematical ideas himself in the form of related problems and projects.

Again, this note has suggested only a few of the materials, sources of ideas, and topics to be used in making the history of mathematics a teaching tool. We invite readers to add ideas and concrete examples to this discussion. Send them to the editor!

[7] P. S. Jones, "Pythagorean Numbers," The Mathematics Teacher, XLV (April 1952), 269 ff. gives a general formula and cites references wherein one may trace its development.

SIAM—the newest mathematical society

What is SIAM?

It is the Society for Industrial and Applied Mathematics, an organization of fairly recent origin, having been incorporated on April 30, 1952. Its aims and objectives are: (1) to further the application of mathematics to industry and science; (2) to provide basic research methods and techniques useful to industry and science; and (3) to provide mediums for the exchange of information and ideas between mathematicians and other scientific and technical personnel.

It seemed to those who conceived SIAM that a large gap existed in our professional structure

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