THE USE OF MATHEMATICAL HISTORY IN TEACHING *

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For several centuries subject matter was considered the only thing of importance in a course in mathematics. Then came a demand for the teaching of the applications of mathematics. Consequently there was written into our textbooks various types of applied problems. In the old order of things it was considered sufficient for the pupil to prove as an exercise-theorem that if the diagonals of a parallelogram are equal the figure is a rectangle. Now we suggest in connection with this exercise that a boy may test his accuracy in laying out a rectangular tennis court by measuring the diagonals. In more recent years we have also come to the realization that there is educational value in knowing the history of the subject; consequently we tell the pupil how and when the human race first discovered and proved this important fact about parallelograms with equal diagonals—or any other important and useful part of mathematics. So, in the last quarter of a century, we find the history of mathematics creeping into our high school textbooks. Let us consider then what value may be found in the history of mathematics, either in high school or college teaching, and how we may make use of mathematical history in teaching.

A knowledge of the history of the development of the mathematical processes he is learning will kindle the pupil's interest in the subject matter.

We hear much about human interest, the personal touch, and similar sentiments. Any newspaper publisher knows that matters pertaining to people have a greater news value than matters pertaining to things. He increases the circulation of his paper by printing what he knows the people like to read, and most of the people like best to read about what other folks have done or are doing. A reporter's orders are to get the names and facts about the people involved in a story, with pictures if possible.

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A plain story about an automobile wreck would attract only momentary interest if told without the names of the people concerned in it. Whether we know the victims of the accident or not, there is always some item concerning their lives or personalities that gives human interest to the news. So we pause in our reading because of the human interest and the incident is indelibly stamped upon our memory. I am sure that we all agree that the touch of human personality is the greatest factor in making news interesting.

We all know that efficient learning goes hand in hand with interest in the subject. Therefore, in seeking schemes to motivate the learning of fractions, for example, why not remove some of the abstractness from the processes by connecting them with human beings and let the pupil see how folks have struggled through greater difficulties than his to develop and simplify the rules he is learning? Would it not add to a seventh grade pupil's interest in fractions to be told something like the following?

Long, long ago when people could count only with the help of their fingers and toes, they had no need for fractions. Their herds could be counted in whole numbers. Their war clubs could be counted in whole numbers. Such an idea as half a tent or half a tree did not occur to them. As they became more civilized they tilled the fields, raised grain, and in various ways dealt with smaller things. Then the idea of measuring things gradually entered their minds. Corn and wheat came to be measured by bushels (or some similar measure) and soon they had half bushels and other fractional parts. It was thus that the idea of a fraction probably originated. The invention and use of fractions marked a very important step forward in civilization. Because of their invention in an arbitrary manner they are sometimes called artificial numbers, the whole numbers being called natural numbers.

Let us imagine a schoolboy of ancient Babylonia, in southern Asia, as he studied arithmetic more than 6000 years ago. With a hard, blunt-pointed instrument he made peculiar-looking wedge-shaped symbols on a soft, smooth clay surface. He could erase them with a flat board by patting the surface smooth again. His textbook, if he had any, had been written the same way on a sort of clay brick and had been baked hard so that the symbols could be erased. You wonder whether he had to learn fractions? He did, and to you his fractions may seem clumsy. All his fractions were 60ths, 3600ths, 216000ths, and so on, all the denominators being multiples of 60, just as our decimals are based on 10ths, 100ths, and 1000ths, and so on, all the denominators being multiples of 10. He wrote them much as you write decimals, omitting the denominators.

About the same time that the Babylonian youth was dealing with his 60ths, some schoolboy down in Egypt was struggling to express all his fractions with 1's for numerators. No one there could conceive of a fractional part other than the unit fraction. The Egyptian lad was taught to write 2/3 as $(1/2 + 1/6)$, 7/8 as $(1/2 + 1/4 + 1/8)$.

A few thousand years later we find a Roman schoolboy basing his fractions on 12ths, or with multiples of 12 as denominators, as the Babylonian used 60ths.

Out of the centuries of effort has grown our present system by means of which we deal easily with problems in business and common life that would have been very difficult, or quite impossible, for even the most learned scholars of those ancient times.

Suppose a pupil is trying to master the rules of operation with what seems to him to be very abstruse algebraic symbols. It ought to awaken his interest to some degree to be told, for example, that the symbols as he is using them were scarcely known and used by very few when Columbus sailed for America; that an Alexandrian Greek, Diophantus, living in the latter part of the third century, was the first person known to use any systematic form of abbreviation in mathematics; that his abbreviations were very crude and limited; that he used a symbol not unlike an inverted *h*, **h**, for the unknown in an equation; that Vieta, a Frenchman living in the sixteenth century, was the first to use a complete system of symbols in algebra as we do; that he used the vowels, A, E, I, O, U for unknowns; that he was a rich man that could pay to have his work published, thereby causing the spread of his methods; that Descartes, the man who introduced coordinates into mathematics, a generation later originated the custom of using x, y, and z for unknowns; that Robert Recorde, an Englishman (1557), first used the sign, =, for "is equal to" and explained that he did it to save time, adding that no two things could be more equal than a pair of parallels; that *p* and *m* were used for a long time for plus and minus; that students in American universities less than 200 years ago wrote *aaaaa* instead of a^5 though Descartes had used the exponential symbol a century before; that several methods were used to point off decimals before our present one became popular, people once writing $7\,\underline{|\,465}$ to mean 7.465; that we have no internationally uniform method of pointing off decimals yet, the English using the decimal point higher than we do (7·465) and some Europeans the comma (7,465).

Besides seeing the personal touch of human beings in the subject matter he studies, the pupil should react in a wholesome manner to the mere information about the origin of the things he is trying to learn. Thus they lose some of their abstractness. They become more concrete and real.

A knowledge of the history of mathematics on the part of the teacher gives him a source upon which he may draw to enrich and enliven his teaching.

There should be intervals of recreation in all labor. In periods of sustained and concentrated mental effort it is restful to have a recess in the nature of a change of thought. The successful public speaker realizes this and his speech is punctuated with jokes or stories. He does not allow his audience to grow tired or dull by too much concentration on a weighty subject.

In a mathematics class there is usually a sufficient variety of material in problem solving, but problem solving day after day grows monotonous. Some teachers enrich their teaching by illustrations of the various uses of mathematics. In addition to this every teacher can and should have stored in his mind ready for use the stories of the great mathematicians. In a few sentences the class can be told when studying similar triangles how Thales went down into Egypt and astonished the king by measuring the height of the pyramids without touching them. Many interesting stories of Pythagoras can easily be found. When a geometry class first learns to construct a perpendicular to a line they can be told that Oenopides, a Greek astronomer, was the first to do it, and he called it a gnomon-wise line after the gnomon that he and other astronomers would set up in a vertical position to cast a shadow for the purpose of determining the altitude of the sun. He *needed* to know how to set up a perpendicular and out of the necessity grew the solution of the problem.

What better impression could be made on a beginning class in geometry, wondering what it is all about, than to tell them something like this at the first meeting?

> The sources of geometry have been in the world from the beginning. Little by little, through the ages, man has discovered geometric relations as he has needed them. When the earth was formed its shape was made nearly like a sphere. Men were forced to study the properties of a spherical surface in order to sail a very long distance away from the sight of land. With the first leaves there grew a symmetry that can be expressed only in geometric terms. When the bees needed a type of cell that would give them the greatest room for honey with the least amount of cell walls and no waste space, they chose a geometric form (hexagonal) for their

cells. They had been storing honey in hexagonal cells for untold centuries before the mathematicians proved that hexagons of the same size with sides and angles equal may be fitted together on a plane in such a manner as to completely cover it. As civilization developed men began to discover more and more of the geometric relations we know so well. You have learned formulas for the measure of the areas and volumes of plane and solid figures. Has it ever occurred to you that there was a time when men did not know these formulas?

From this introduction it is easy to lead up to the Rhind Papyrus and some of its erroneous rules for areas, which should be compared with our formulas for the same. The Egyptian value of π $(16/9)^2$, the Hebrew value, 3, the Chinese value $\sqrt{10}$ and the Hindu value 142/45 should all be compared with the value we use.

The question arises at once, "Which method is the correct one?" or rather, "How do we know that our formulas are correct?" The study of geometry will convince us that we are right within the limit of errors of measurement. Its conclusions represent the result of centuries of study to improve and extend existing knowledge. Fact after fact was discovered and error after error corrected. The ancient Egyptians were satisfied when they had learned enough to resurvey their fields after each annual overflow of the Nile, which wiped out their boundaries. Because of this use of geometry the Greeks gave it its name which means literally "the measure of the earth". But the Greeks were not satisfied to accept the Egyptian mensuration formulas and other practical rules and go no farther. They tried to and did establish new facts and relations about triangles, squares, rectangles, circles, and other geometric figures. During the six centuries just preceding the Christian era they were very active. They studied the subject for its own sake rather than for the practical use of it. Some of the theory they developed has found applications in the centuries that followed. The things they did are the heritage of the centuries and on them is based a great part of our practical mathematics, modern architecture, and engineering, as well as much of our art.

Such an introduction will not only stimulate interest in the subject but will help the teacher to win the approbation of the class and enlist the hearty cooperation of the pupils in the coming adventure.

When the time comes in the course to begin proving theorems by the assumption of postulates and axioms the class should be told of Euclid and the *Elements.*

A story of the development of trigonometry can be given which is just as interesting to the class in trigonometry and a few words as to the life and works of Descartes will be interesting news to the class in analytic geometry.

A knowledge of the history of mathematics gives both pupil and teacher an appreciation of the value of the subject and its inseparable and vital connection with the development of civilization.

Our number system and the operations on numbers, our fractions, our algebraic symbolism, our geometry, our calculus, all were developed as the need for them arose in the progress of civilization. It is only fair to the pupil to let him see this, whether he be a high school or college student. As teachers we should certainly be fully informed on this subject. If the student is taught the history of mathematical processes he will see that they were not handed down from a mountain of inspiration but are the fruits of laborious thinking in the long and tedious journey up from barbarism to civilization and were developed for the most part as necessity demanded. He will see the worth while contribution that geometry has made to scientific progress from the time the ancient "rope stretchers" of King Sesostris laid out the right angles of their land surveys along the banks of the turbulent Nile to the present day when Einstein, forming a theory to explain the mysteries of the universe, assumes that figures may *not* be moved about in space without change of shape or size. He will see its contributing influence on art and culture to such an extent that he should feel that it is as much worth while to know about it as to know about many other subjects in the curriculum that do not contribute to his ability to earn a greater wage immediately on leaving school. He will see what a heritage is his.

The Bureau of Publications of Teachers College, 525 West 120th Street, New York City, still have copies of the Second and Third Yearbooks of the National Council on hand. These will soon be exhausted and many teachers to say nothing of libraries will be unable to secure them. Mr. C. M. Austin has taken it upon himself to dispose of 50 copies of the Fourth Yearbook as well as the others. Let us have a number of our members of the Council follow Mr. Austin's example.