Using the history of mathematics in teaching on the secondary school level
Author(s): Herta T. Freitag and Arthur H. Freitag
Source: *The Mathematics Teacher*, Vol. 50, No. 3 (MARCH 1957), pp. 220-224
Published by: National Council of Teachers of Mathematics
Stable URL: https://www.jstor.org/stable/27955379
Accessed: 15-11-2024 02:34 UTC

Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

Using the history of mathematics in teaching on the secondary school level

by Herta T. Freitag, Hollins College, Hollins College, Virginia, and Arthur H. Freitag, Jefferson Senior High School, Roanoke, Virginia

More than two thousand years ago, Pythagoras lectured on the power of number, meaning only integers and fractions. He reiterated again and again, "Number rules the universe; the essence of all things is number." Then one day he evaluated the diagonal of a unit-sided square. Alas, there was no number as he understood number which gave an answer.

Legend has it that one of his grief-stricken students contrary to the rules of Pythagoras' esoteric school divulged this number fiasco to an outsider. The student was drowned. Students are no longer drowned for breaking the rules of a school. But to know the difference between rational and irrational numbers is as important today as it was then.

Any educator interested in improving the teaching of mathematics must consider a more intelligent use of historical material. In ancient times, man was a local creature. During the Middle Ages he grew in social awareness and became provincial in his outlook. At the onset of modern history, nationality became his important social interest. Today not only do nearly all our problems affect the entire world, but many of them can be understood only in the light of their history. When discussing the uses of historical material in teaching secondary school mathematics, pertinent questions naturally arise. How does the knowledge and proper use of historical material aid both the teacher and learner of secondary school mathematics? Which historical approach will be most beneficial? Should the historical material be presented as part of an integrated course, or should specific units be given separately? Should it be chronological facts, as well as the biographies of the great mathematicians? And above all of these questions, do we have any valid criteria to offer as a basis of selection from these many possible approaches?

The historically-oriented plan

The historical approach to the teaching of mathematics requires that mathematics be presented as an ever-evolving human endeavor. In this "evolutionary approach" plan, an occasional remark about a mathematician, a footnote of an historical nature are in no way adequate. Nor should the history of mathematics be given supplementary status or relegated to the mathematics club. Investigations for determining the value of the integration of this historical material in courses in mathematics should run concomitant with actual teaching under this "evolutionary plan." This latter implies that the complete evolution of an idea may be traced, and the life of one of its creators should be described.

The human interest factor as well as mathematical content seem the all-important bases upon which the selection of the historical material is to be efficiently integrated. The essence of this suggested method is *flexibility*. The over-all basic principle for the choice of material is a constant concern for the improvement of mathematical learning. That the history of mathematics is inextricably interwoven with the sum total of mathematical knowledge is the basic assumption. Furthermore, there is no one ideal method of teaching. If this approach is to be of educational value, the teacher's judgment must be final.

AN ANALYSIS OF THE POSSIBLE BENEFITS THAT MAY BE DERIVED FROM THE HISTORICAL APPROACH

A proper fusion of the history of mathematics with its teaching and learning can only produce desirable results. Some of the possible improvements are in deepening, fostering, and strengthening the outcomes of the traditional approach. Others lie in areas not so easily reached by the usual and ordinary methods. The methods and materials that ought to be used must in the very nature of the problem remain in the form of general suggestions.

The historical method is an asset to the attainment of mathematical objectives of both teaching and learning. It leads to a better understanding of theoretical mathematics and greater technical skill in its application. Mathematics' role in the rise of civilization is appreciatively recognized and this recognition is apt to influence favorably the whole personality of the student. The endeavor to make the history of mathematics an *integral* part of the teacher-learner rapport magnifies the instructor's power of presentation.

Mathematical meanings become more lifelike by portraying the ontogeny of the evolutionary structure of mathematics. One illustration of the ecological basis of this approach is shown in the development of decimal fractions and the invention of an adequate symbolism for its growth. Furthermore, here we see that the historical order is not necessarily the logical one, and the vital part that intuition plays in the realm of human thought. The importance of considering details is recognized through the correlative and cumulative nature of mathematics. It takes its rightful place as a progressively growing phase of man's search for knowledge.

The superiority of the language of mathematics in its terminology and symbolism are better comprehended through this historical approach. This superiority has brought mathematics to its present high order of abstraction. It may be a real challenge to the student to decipher some obsolete symbolism. Mathematical language then becomes understood as a growing part of the evolution of the field, and the interdependence of thought and symbolic communication is shown. The problem of mathematical skills (technical ability) is made up of two components. The first is the proficiency with which factual knowledge is applied. The other is an attitude or desire to perform the required operations. Viewing from the peaks of history the panorama of attempts to record numbers opens the infinite vistas of modern electrical computation.

The ever-changing and fragmentary character of knowledge and understanding is exposed by an historical approach. A continuity in mathematics, a possibility of international collaboration among mathematicians (individuals or groups), and the correlating and spirally-linking tendencies are portrayed.

Trigonometric relationships illustrate the cosmopolitan contributions to mathematics. The Greeks gave us geometric equivalents to $\sin^2\theta+\cos^2\theta=1$, the Law of Cosines, the cosine of the difference between two angles expressed as functions of the separate angles, and the relationship for sine of half an angle. The other Pythagorean relationships were bequeathed by the Arabs who also supplied the formulas for the sine of twice an angle. The Germans developed the reciprocal relationships between the cosine and the secant, the Law of Tangents, a one-half angle formula, and the area of a triangle equivalent to half the product of any two sides times the sine of the included angle. The country of Lafayette contributed the sine of a treble angle, the tangent and secant of any multiple angle, the nth power of the expression $\cos\theta + i\sin\theta$, and other half-angle formulas. Swiss research produced the formulas for the tangent and cotangent of a double angle. An expansion into an infinite series of the sine and cosine of a multiple angle came from the British.

The invention of the wheel by our remote ancestors illustrates the inception of the spirally-linking trend in mathematics. Much later this led to higher levels of abstraction with theorems about the circle. The Greeks had already analyzed conic sections. Descartes reached new levels of abstraction via the function concept in analytic geometry. Projective geometry continued the crystallization of the family relationships among conic sections.

Historically, pure and applied mathematics developed as mutual correlatives. A list of instances of the two-way influencing interaction between pure and applied mathematics would be impressive in extent and length. Frontier work done out of sheer joy in intellectual curiosity has much later been appropriated for far-reaching practical applications. The non-utilitarian ancient Greeks brought forth the conic sections more than a century before Christ. Almost two millenniums later, Kepler used these curves as the basis of his laws on celestial motion. Ballistics and navigation came next and drew forth from these same wellsprings. And now the orbits of the electrons in the hydrogen atoms are found to be conic section gyrations.

Faith in human dignity is encouraged by acquaintance with some of the creative pioneers in this discipline. Social sensitivity (especially in co-operation and tolerance) and a widened perspective are envisioned with those mathematical ideas underlying the beauteous correlation in nature and art. Realizing the relative insignificance of anyone's accomplishments, however great, humility—the very essence of life—is born.

At this point I can feel the sceptic breathing down my neck. What kind of divine enthusiasm could possibly justify such ideal attainments? But I still believe where there is no vision, a nation perisheth, and so without that divine spark does mathematics.

Some reasons for the possible benefits derived from the evolutionary approach

The contributions of an historical approach are either natural concomitants of the suggested method or of the psychological characteristics of the subject matter itself. Gestalt theory emphasizes the mental-emotional unity of the learner. From a vague integrated configurational response the student should be drawn to differentiating recognitions of particular properties and relationships. Then the final step of the master teacher, a clear and thorough grasp of the whole picture, becomes a work of art!

The suggested plan takes care of the mind as well as the feelings. Field theory considers the individual as part of a still greater whole. Interrogate the past, for the human race is more important than the individual. The warp and woof of mathematics, important as it is, can be observed as a part of the whole realm of human endeavors. Connectionism stresses the need for establishing associational bonds in the learning situation. The historical method associates mathematical ideas with their creators, mathematics with the history of human affairs (be it ever so inhuman). Here is the chance to take drudgery out of drill and review and train that memory.

Let little Gauss provide an illustration of our approach in arithmetic and arithmetical progressions. An annoyed teacher hands out a tough problem to a group of elementary school cherubs. Add up all the numbers from one through a hundred! That will keep them busy for a long time. The little boy, Gauss, beamed and put down his pencil, saying "My answer is 5050." The story does not say how well the teacher reacted to these words of wisdom from the mouth of an infant in arithmetic. But it is fair to surmise that Carl Friedrich Gauss or at least his method will be remembered when there is need for adding a series of numbers or an arithmetic progression sum.

Mathematics by its inherent nature is spiral and cumulative. The circular-ladder type of organizing areas in this field is based on sound psychological considerations. To enrich the review of familiar patterns of thought, the spiral procedure distributes drill, provides that much-needed variety of material, and facilitates the proper spacing of learning. Concept formation is a matter of slow and often arduous growth. The evil of rigid over-compartmentalization is automatically thwarted. The imitative urge of young people to be like their hero is adequately satisfied.

An intelligent use of the past can help us solve our problems more reasonably. Mankind's intellectual heritage unfolds as the common property of all. Scientific and humanistic ideologies are reconciled and unified. The universality of change, the fallibility of the mind, and the present uncertainty of all human knowledge (Gödel's theorem) are glimpsed from this perspective.

A final group of reasons explains why the historical method presents possibilities for setting the stage for new topics. It lends itself effectively to the unit presentation plan.

It now remains to show that this plan increases the efficiency of the teacher. By thoroughly absorbing a background that integrates the history of his field with the teacher-learner conjunction, the real meaning of education pervades this setting. A better and fuller knowledge of the psychological principles dawns upon the instructor. The ontogeny of learning recapitulates its phylogeny. The simple counting of objects to natural numbers, fractions, and then negative numbers is a human trait. Concrete experiences are first encountered before abstractions. Generalization is ordinarily the final step in an investigation. The increase of rigor in mathematics is beaten out from the primal rhythms of the Congo to the Music of the Spheres. As George Sarton remarks, only that individual with his "own inveterate and insatiable curiosity, his constant itching for intellectual adventure" may take this path.

Michael Faraday, an average pupil, was ridiculed by his pedantic and sadistic teacher for his speech defect. Steinmetz, after he had made sacrifices for a new suit, was not allowed to appear on the platform with his graduating class because he was a hunchback. Galileo's gift for independent thought was drowned out by the "magister dixit." Gregor Mendel was reminded by his examiners that he had not sufficiently mastered his subject. Henry Huxley's introduction to formal education left nothing but bitter memories. Madame Curie's life at school was made miserable by a fanatic of nationalism. And what was Henry Poincare's score on the Binet test? The same Poincare who has been called the living brain of rational sciences was rated an imbecile. And yet one more, who should be a favorite with all teachers of mathematics, Evariste Galois who first went to school as if to a prison. Reports on his work were continually mediocre, his conduct, argumentative and dissipated. Twice he failed the entrance examinations to the École Polytechnique. This blindness to recognize native intelligence is inherent in all educational systems. George Sarton puts the whole lesson of this tragic life in, "One can never be too kind to the young;

one can never be too tolerant of their faults, even of their intolerance."

What a challenge and a responsibility for a teacher! He must always be a student intellectually struggling for knowledge and perhaps wisdom. Ingenuity, imagination, and flexibility only can assure his wise timing of historical illuminations. Enthusiasm not to be "just a teacher of mathematics but a master teacher of mathematics" will exert that inspirational influence which may be his immortality. Let us close with Glaisher's conviction that, "I am sure that no study loses more than mathematics by any attempt to dissociate it from its history."

What's new?

BOOKS

College

Descriptive Geometry, Steve M. Slaby. New York, Barnes and Noble, 1956. Paper, xiii+353 pp., $2.25.

Elements of Mathematics (2nd Edition), Helen Murray Roberts and Doris Skillman Stockton. Reading, Massachusetts, Addison-Wesley Publishing Company, 1956. Cloth, x+308 pp., $3.50.

Intermediate Algebra (2nd Edition), Paul K. Rees and Fred W. Sparks. New York, McGraw-Hill Book Company, 1957. Cloth, x+306 pp., $3.90.

Structure of Rings, Nathan Jacobson. Colloquium Publications, XXXVII. Providence, Rhode Island, American Mathematical Society, 1956. Cloth, xii+263 pp., $7.70.

Miscellaneous

Calculus Refresher for Technical Men, A. Albert Klaf. New York, Dover Publications, Copyright 1944, Dover Edition, 1956. Paper, viii+431 pp., $1.95.

Mathematics Magic and Mystery, Martin Gardner. New York, Dover Publications, 1945. Paper, xii+176 pp., $1.00.

Theory of Approximation, N. I. Achieser (translated by Charles J. Hyman). New York, Frederick Ungar Publishing Company, 1956. Cloth, x+307 pp., $8.50.

Trigonometry Refresher for Technical Men, A. Albert Klaf. New York, Dover Publications, Copyright 1946, Dover Edition, 1956. Paper, x+629 pp., $1.95.

BOOKLETS

Money Management, Your Automobile Dollar, Money Management Institute of Household Finance Corporation, Prudential Plaza, Chicago 1, Illinois. 36-page illustrated booklet, 10¢ each.

DEVICES

Direct-O-Percenter, Educational Supply and Specialty Co., 2823 Gage Ave., Huntington Park, California, *or* A. C. Vroman, 367 S. Pasadena Ave., Pasadena, Calif. 11″×14″ cardboard printed with right triangle and scales for teaching percentage problems, 50¢ each, quantity discount.

EQUIPMENT

Math-Master Chalk Board, GAMCO Products, Box 305, Big Spring, Texas. 3½′×4′ framed green boards with either rectangular co-ordinates or polar co-ordinates embossed on surface, $32.50 each, or $29.95 each in lots of two or more.

MONOGRAPH

Providing for Outstanding Science and Mathematics Students, Education Monograph XVI, University of Southern California Press, University Park, Los Angeles 7, California. 111 pp., $4.50.