Chapter 1

A formal learning model

1.1 Probably Approximated Correct (PAC) learning

Definition 1.1.1 (PAC learnability)**.** A hypothesis class $\mathcal{H}$ is PAC learnable if there exist a function $m_{\mathcal{H}} : (0,1)^2 \to \mathbb{N}$ and a learning algorithm with the following property: $\forall \varepsilon, \delta \in (0,1)$, for every distribution $\mathcal{D}$ over $\mathcal{X}$, and for every labeling function $f : \mathcal{X} \to \{0,1\}$, if the realizable assumption holds with respect to $\mathcal{H}, \mathcal{D}, f$, then when running the learning algorithm on $m \geq m_{\mathcal{H}}(\varepsilon, \delta)$ i.i.d. examples generated by $\mathcal{D}$ and labeled by f, the algorithm returns a hypotesis h such that, with probability of at least $1 - \delta$ (over the choice of the examples), $L_{(\mathcal{D},f)}(h) \leq \varepsilon$.

The function $m_{\mathcal{H}} : (0,1)^2 \to \mathbb{N}$ determines the sample complexity of learning $\mathcal{H}$. So it determines how many samples are required to guarantee a probably approximately correct solution.

Corollary 1.1.0.1. *Every finite class is PAC learnable with sample complexity:*

$$m_{\mathcal{H}}(\varepsilon, \delta) \leq \left\lceil \frac{1}{\varepsilon} \log \left(\frac{|\mathcal{H}|}{\delta} \right) \right\rceil \tag{1.1}$$