LECTURE NOTES
OF
SUBNUCLEAR PHYSICS

Comment.

Edited By

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Contents

Course structure and program

Lecture 1.
Tuesday 10^{th} March, 2020.
Compiled: Sunday 22^{nd} March, 2020.
Prof. Lucchesi

Informations

Suggested books:

- *Concepts of Elementary Particle Physics*, Michael E. Peskin.
  It has a very good experimental approach, with theoretical concepts explained as well.
- Any other book where the same topics are presented is fine. For example, the book of Alessandro Bettini.

Exam modalities: the exam is slitted into two parts. These are:

- **Written exercises**.
  The idea is to prepare two partial tests: one will take place almost at the middle of the course, one at the end. For each chapter of the reference book there are several exercises that are useful for the comprehension of the topics of the course.
- **Oral discussion**.
  It will be focused on a single topic and it will take place after the written part.

The final evaluation will be a weighted mean of the two written exercises and of the oral discussion.
Remeber to subscribe to the Facebook group *Subnuclear Physics at DFA* for further informations and for infos on seminars of particle physics.

Course Program

- **Introduction and recap**
- **Tools for calculation**.
  In order to understand all the following topics, we need some mathematical tools (that we already have but the way we are going to use them is different from the use we did in theoretical physics course). They are needed to evaluate the physical phenomena we are going to discuss.
- **Detectors for particle physics experiments**.
  They are needed to perform measurements, so it is important to acquire a certain knowledge on them. For example, in order to choose why a detector is better than another one for a certain task and to set up a particle physics experiment. This part is not well described in the reference book, so we will use other books for this purpose.
- **Cross section of $e^+e^- \longrightarrow \mu^+\mu^-$ and $e^+e^- \longrightarrow hh$.**
  The former is a very simple process and it is important for the study of many other processes. The ladder will be important to understand the basis of QCD.

- **Strong interactions**:
  - ▷ **Deep inelastic scattering**
  - ▷ **Gluon**
  - ▷ **QCD**
  - ▷ **Partons and jets**
- **Electroweak interactions** (This part and the part on strong interactions sum up into the discussion on Standard Model):
  - ▷ **V-A Weak theory**.
    It is the theory at the base of electroweak interaction, which we will build up.
  - ▷ **Gauge theory and symmetry breaking**.
    This part will be discussed not so deeply since it was treated during the course of *Theoretical Physics of Fundamental Interactions*.
  - ▷ W **and** Z^0 **bosons**.
    The most important items and measurements will be presented.
  - ▷ **Cabibbo theory and CKM**.
    This part is needed in order to put the hadrons, in particular the quarks, into the electroweak theory. However, it will not be discussed deeply since it was presented during the bachelor course *Introduction to Nuclear and Subnuclear Physics*.
  - ▷ **CP violation, the B meson system**.
    It will be a more experimental discussion.
- **New Physics** (we will try to give an answer to how we can go beyond the description given by Standard Model, in fact there are phenomena that are still not explained by this theory):
  - ▷ **Neutrino and Standard Model**
  - ▷ **Higgs properties**

Chapter 1

Introduction and Recap

1.1 Basic knowledge

Relativistic wave equations

Relativistic quantum field theory is necessary to describe quantitatively elementary particle interactions. Its description is not part of this course, so we will use it in simple cases and only when necessary.
It is assumed the following knowledge:

- Klein-Gordon equation (for boson fields):

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)\psi(t,\vec{\mathbf{x}}) = 0 \tag{1.1}$$

- Dirac equation (Klein-Gordon can't give a description for fermion fields):

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - m\right)\psi(t,\vec{\mathbf{x}}) = 0 \tag{1.2}$$

  with $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$

- Basic concepts of fields and particles

- Basic concepts of Feynman diagrams

Natural Units

During the course we will use the natural units, therefore:

$$\hbar = c = 1 \tag{1.3}$$

Considering that:

$$\begin{aligned} 1\ \text{eV} &= 1.6 \cdot 10^{-19}\ \text{J} \\ c &= 3 \cdot 10^{8}\ \text{m/s} \end{aligned}$$

we have:

$$1\ \frac{\text{eV}}{c^2} = 1.78 \cdot 10^{-36}\ \text{Kg}$$

Since $E^2 = p^2c^2 + m^2c^4$, it is convenient to measure p in GeV/c and m in GeV/c^2. For example the electron mass $m_e = 0.91 \cdot 10^{-27}$ g corresponds to $m_e = 0.51$ MeV/c^2. It is also useful to remember that $\hbar c = 197$ MeVfm.
An interesting quantity to consider in natural units is the strength of the electric charge of the electron or proton. By taking into account the potential $V(r) = \frac{e^2}{4\pi\varepsilon_0 r}$, the radius r in natural units has a dimension of Energy^{-1}. By this way it forces the following relation:

$$\alpha \equiv \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137.036} \tag{1.4}$$

namely, the **fine structure constant**.

Symmetries

They are the corner stones of particle physics. The most important ones for our studies are the **space-time symmetries**, which can be classified into:

- Continuous symmetries:
  - ▷ Translation in time. The generator of the group of time translations is the operator H, namely the Hamiltonian, which is linked to the energy quantity.
  - ▷ Translation in space. The generator of the group of space translations is the operator $\vec{\mathbf{p}}$, namely the momentum.
  - ▷ Rotations. In this case, the generator of the group of this kind of transformations is the angular momentum $\vec{\mathbf{L}}$.

  If a system is invariant under one of these transformations, the corresponding generator, so H, $\vec{\mathbf{p}}$ or $\vec{\mathbf{L}}$, is conserved.

- Discrete symmetries:
  - ▷ Parity P:

    $$x^\mu = (x^0, \vec{\mathbf{x}}) \xrightarrow{P} (x^0, -\vec{\mathbf{x}}) \tag{1.5}$$

    Fermions have half-integer spin and angular momentum conservation requires their production in pairs. We can define therefore just relative parity. By convention, the proton p has parity equal to $+1$. The parity of the other fermions is given in relation to the parity of the proton.

    Parity of bosons can be defined without ambiguity since they are not necessarily producted in pairs.

    Parity of a fermion and its antiparticle (i.e. an antifermion) are opposite, while parity of a boson and its anti-boson are equal.

    Moreover, the parity of the positron is equal to -1. Quarks have parity equal to $+1$, leptons have parity equal to $+1$. Their antiparticles have parity equal to -1.

    Lastly, parity of a photon is equal to -1.
  - ▷ Time Reversal T:

    $$x^\mu = (x^0, \vec{\mathbf{x}}) \xrightarrow{T} (-x^0, \vec{\mathbf{x}}) \tag{1.6}$$

- ▷ Charge Conjugation C:

$$\text{Particle} \xleftrightarrow{C} \text{Antiparticle} \tag{1.7}$$

It is needed in order to restore a complete symmetry under the exchange of a particle with its antiparticle. A photon has -1 eigenvalue under C, which means: $C\,|\gamma\rangle = -\,|\gamma\rangle$.
Fermion-antifermion have opposite intrinsic parity and for non elementary particles the total angular momentum has to be considered, in fact the C parity goes like $(-1)^\ell$ or $(-1)^{\ell+1}$ (depending on the intrinsic parity).

Fundamental constituents of the matter

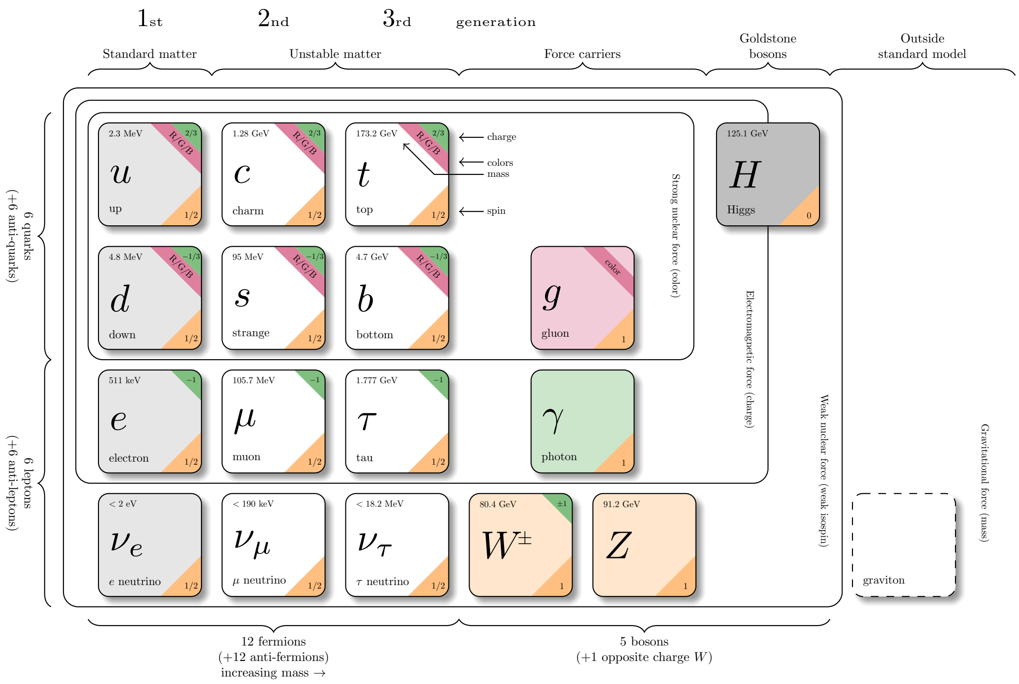

Figure 1.1: Standard Model particles.

Lecture 2. *Wednesday 11th March, 2020.* *Compiled: Sunday 22nd March, 2020.* *Prof. Lucchesi*

1.2 Hydrogen atom and Positronium

We are going to study the already known system of the hydrogen atom, and compare it to the system of positronium. More in detail, our goal is to understand the e^+e^- bound state and the possible application of this model to the description of other systems. Therefore, we start from the hydrogen atom since it has some characteristics in common with the positronium.
In QM Physics, this bound state is really similar to the hydrogen atom. The assumptions for this one in the non relativistic limit are that the mass of the proton is much bigger than the mass of the electron ($m_p >> n_e$) and the potential is given by:

$$V(r) = -\frac{e^2}{4\pi r} = -\frac{\alpha}{r} \tag{1.8}$$

From this potential, by solving the Schrödinger equation, we get the bound state energies:

$$E = -\frac{R_y}{n^2} \tag{1.9}$$

R_y is known as **Rydberg energy**, whose expression reads:

$$R_y = \frac{1}{2}\frac{me^4}{(4\pi)^2} = 12.6 \text{ eV} \tag{1.10}$$

$$R_y = \frac{1}{2}\alpha^2 m_p \qquad \text{In natural units} \tag{1.11}$$

The bound states of hydrogen are arranged in levels associated with integers $n = 1, 2, 3, \ldots$. Each level contains the orbital angular momentum states:

$$\begin{aligned} \ell &= 0, 1, \ldots, n-1 \\ m &= -\ell, \ldots, \ell \end{aligned} \tag{1.12}$$

The orbital wavefunctions are the spherical harmonics $Y_{\ell m}(\theta, \varphi)$, which are even under spatial reflection for even ℓ and odd for odd ℓ. Then, under P, these states transform as:

$$P\left|n\ell m\right\rangle = (-1)^\ell \left|n\ell m\right\rangle \tag{1.13}$$

However, with these assumptions, we are not considering that the real hydrogen atom has more structure. In fact, we are negleting that the electron is a particle with intrinsic spin and we have to take into account also this quantity. In a more technical way, we have to add the contribution of the spin-orbit interaction (fine splitting), which is proportional to the scalar product $\vec{\mathbf{L}} \cdot \vec{\mathbf{S}}$. Concerning the Hamiltonian of this contribution, it is given by:

$$\Delta H = \frac{g-1}{2}\frac{\alpha}{m^2 r^3}\vec{\mathbf{L}} \cdot \vec{\mathbf{S}} \tag{1.14}$$

The sign is such that the state with $\vec{\mathbf{L}}$ and $\vec{\mathbf{S}}$ opposite in sign has lower energy. Moreover, it may be useful to express the operator $\vec{\mathbf{L}} \cdot \vec{\mathbf{S}}$ in terms of J^2, L^2, S^2:

$$\vec{\mathbf{J}} = \vec{\mathbf{L}} + \vec{\mathbf{S}} \Longrightarrow \vec{\mathbf{L}} \cdot \vec{\mathbf{S}} = \frac{1}{2}\left(\left(\vec{\mathbf{L}} + \vec{\mathbf{S}}\right)^2 - L^2 - S^2\right) = \frac{1}{2}\left(J^2 - L^2 - S^2\right) \tag{1.15}$$

By this way it is straightforward to diagonalize the operator $\vec{\mathbf{L}} \cdot \vec{\mathbf{S}}$. At the end we get the order of magnitude of the spin-orbit interaction:

$$\left\langle \frac{\alpha}{m^2 r^3} \right\rangle \sim \frac{\alpha}{m^2 a_0^3} \sim \alpha^4 m \sim \alpha^2 R_y \tag{1.16}$$

Thus, this effect is a factor of 10^{-4} smaller than the splitting of the principal levels of hydrogen.
Another contribution that we have to add is the spin-spin interaction (hyperfine splitting) between electron and proton, which leads to the addition of another term into the total Hamiltonian. The magnetic moments of the proton and the electron interact, with the ground state favoring the configuration in which the two spins are opposite. Therefore:

$$\Delta H = C\vec{\mathbf{S}}_p \cdot \vec{\mathbf{S}}_e \tag{1.17}$$

where the C constant depends on the electron wavefunction.
Hence, we have several levels for the spin states. For example, the 1S state of hydrogen is split into two levels, corresponding to the total spin:

$$\vec{\mathbf{J}} = \vec{\mathbf{S}}_p + \vec{\mathbf{S}}_e \tag{1.18}$$

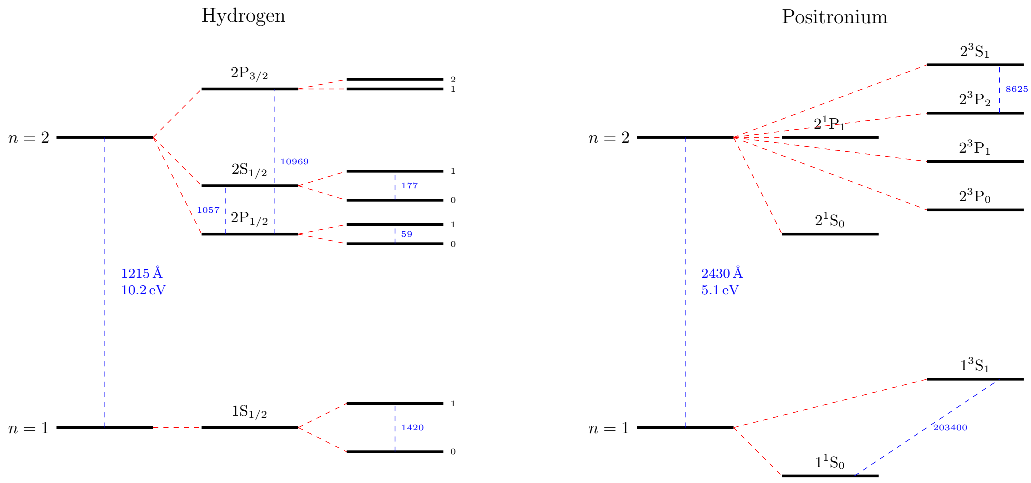

Figure 1.2: Comparison of the 1S, 2S, and 2P energy levels of hydrogen atom and positronium.

The possibilities we have are 2: $J = 0$ and $J = 1$, depending on how the two spin states of proton and electron combine. The projection on the z-axis gives 3 possibilities: $J_z = 1, 0, -1$ (corresponding to $|\uparrow\uparrow\rangle$, $\frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$, $|\downarrow\downarrow\rangle$).
Now the possibility that we have to evaluate is that e^+e^- forms bounded states. In fact, the same ideas can be applied to a particle-antiparticle system and the simplest case is the positronium.
It is relatively easy to make positronium. In colliders, when working with a beam of positrons which enter in the matter, they can pick up an electron and form a bounded state of positronium, so this the starting point of the idea. All the considerations applied to the case of hydrogen atom can be applied to the positronium case as well. All the calculations are omitted. The first consideration is that here we can't apply the approximation $m_p >> m_e$, in fact the two particles here have the same mass. The solution for this two-body problem is to use the reduced mass μ, namely:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_e}{2} \tag{1.19}$$

At the end of all the calculations we won't do, we get that the hyperfine splitting contribution is approximately of the same of order of magnitude of the fine splitting and both are of the order $\alpha^4 m_e$.
Now we have to classify the eigenstates under parity and charge conjugation of the positronium. Let's consider first P. The intrinsic parity of the electron is $P_{e^-} = +1$, of the positron $P_{e^+} = -1$. So the parity of a single particle goes like $P = (-1)^\ell$ and the overall parity goes like $P = (-1)^{\ell+1}$.
For C, we must account three effects:

- C converts the electron to the positron and the positron to the electron. The electron and positron are fermions, and so, when we put the electron and positron back into their original order in the wavefunction, we get a factor -1.

- Reversal of the coordinate in the orbital wavefunction gives a factor $(-1)^\ell$.

- Finally, the electron and positron spins are interchanged. The $S = 1$ state is symmetric in spin, but the $S = 0$ state is antisymmetric.

$$
\begin{aligned}
S = 0 &\longrightarrow \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \\
S = 1 &\longrightarrow |\uparrow\uparrow\rangle \qquad \frac{1}{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \qquad |\downarrow\uparrow\rangle
\end{aligned}
$$

and so gives another factor (-1).

In all, the positronium states have C:

$$
C = (-1)^{\ell+1} \cdot \begin{cases} 1 & S = 1 \\ -1 & S = 0 \end{cases} \tag{1.20}
$$

and what we get is the J^{PC} scheme. The low-lying states of the positronium spectrum then have the J^{PC} values as in Figure 1.3.

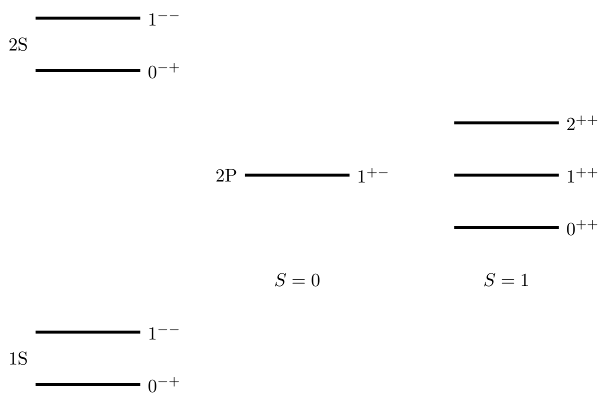

Figure 1.3: J^{PC} scheme. The 2P states 0^{++}, 1^{++} and 2^{++} arise from coupling the $L = 1$ orbital angular momentum to the $S = 1$ total spin angular momentum.

We know that electron and positron annhilate each other, so this state decays into something. The rules are E and $\vec{\mathbf{P}}$ conservation. It can't decay into a single photon since the momentum is not conserved. Recall that:

$$
C\,|\gamma\rangle = -1 \Longrightarrow C\,|n\gamma\rangle = (-1)^n \tag{1.21}
$$

If we are looking for the two photon decay (so positive conjugation) of the positronium, the only possible state is the one with $S = 0$. If we are looking for a three photon decay (so negative conjugation), the only possible state is the one with $S = 1$. This kind of decay has been verified experimentally.
Positronium with state $S = 0$ is also known as **para-positronium**. If the state is $S = 1$, it is also known as **ortho-positronium**. Their medium lifes are:

$$
\frac{1}{\tau_{\mathrm{p}}} = \frac{1}{2}\alpha^5 m \qquad\qquad \tau_{\mathrm{p}} = 1.2 \cdot 10^{-10}\ \mathrm{s} \tag{1.22}
$$

$$
\frac{1}{\tau_{\mathrm{o}}} = \frac{2}{9\pi}(\pi^2 - 9)\alpha^6 m \qquad\qquad \tau_{\mathrm{o}} = 1.4 \cdot 10^{-7}\ \mathrm{s} \tag{1.23}
$$

So, when we emit positrons into a gas, $\frac{1}{4}$ of the states decays quickly in τ_{p}, while $\frac{3}{4}$ of the states decays slower in τ_{o}. It is a strange result, but experiment verifi
es it (Berko and Pendleton, 1980).

1.3 Static Quark Model

A beautifully simple way to create any particle, together with its antiparticle, is to annihilate electrons and positrons at high energy. The annihilation results in a short-lived excited state of electromagnetic fi
elds. This state can then re-materialize into any particle-antiparticle pair that couples to electromagnetism and has a total mass less than the total energy of the annihilating e^+e^- system.

1.3.1 Light quarks: charm and beauty

By this way, the importance of the positronium state is clear. Moreover, it is linked to the discovery of quark charm and beauty.
Their discovery takes place in 1974 at SPEAR experiment, where by studying the process $e^+e^- \longrightarrow hh, \mu^+\mu^-,\ e^+e^-$, an enormous, very narrow, resonance at about 3.1 GeV was discovered. This resonance would correspond to a new strongly interacting particle.
When they announced this discovery, they learned that the group of Samuel Ting, working at Brookhaven National Laboratory in Upton, New York, had also observed this new particle. Ting's group had studied the reaction $pp \longrightarrow e^+e^- + X$, where the particles X are not observed.
This never observed particle is now called the J/ψ. A few weeks later, the SPEAR group discovered a second narrow resonance at 3686 MeV, the ψ'.
Another group of narrow resonances is found in e^+e^- annihilation at higher energy. The lightest state of this family, called Υ, has a mass of 9600 MeV. It was discovered by the group of Leon Lederman in the reaction $pp \longrightarrow \mu^+\mu^- + X$ at the Fermilab proton accelerator.
Concerning the J/ψ, this particle is given by a quark doublet $c\bar{c}$ called **charmonium**. If this state exists, we will see phenomena like the ones observed with positronium. In the process $e^+e^- \longrightarrow hh$, the highest rate reactions are those in which e^+e^- pair is annihilated by the electromagnetic current $\vec{\mathbf{j}} = \bar{\psi}\vec{\gamma}\psi$ through the matrix element:

$$\langle 0 | \vec{\mathbf{J}}(x) \left| e^+e^- \right\rangle \tag{1.24}$$

The current has spin 1, $P = -1$, and $C = -1$. These must also be properties of the annihilating e^+e^- state, and of the new state that is produced. So, all of the ψ and Υ states must have $J^{PC} = 1^{--}$.
The current creates or annihilates a particle and antiparticle at a point in space. So, if these particles are particle-antiparticle bound states, the wavefunctions in these bound states must be nonzero at the origin. Most probably, they would be the 1S, 2S, etc. bound states of a potential problem. If this guess is correct, the states with higher L must also exist. They might be produced in radiative decays of the ψ and Υ states. Indeed, there is an experimental evidence, with a pattern of states as in Figure 1.4.
Remarkably, this reproduces exactly the pattern of the lowest-energy states of positronium and makes even more clear that the analogy to positronium is precise. In the case of the ψ family, the fermion is called the charm quark (c); this quark has a mass of about 1.8 GeV. In the case of the Υ family, the fermion is called the bottom quark (b); this quark has a mass of about 5 GeV.

Lecture 3. *Tuesday 17th March, 2020. Compiled: Sunday 22nd March, 2020. Prof. Lucchesi*

1.3.2 Light mesons

Now we can go back to the π mesons and other relatively light hadrons. πs are the strongly interacting particles and there are three π mesons: π^0, π^+ and π^-.

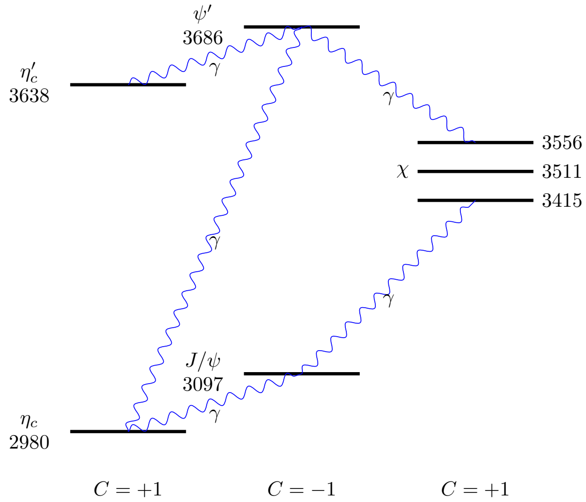

Figure 1.4: Pattern of states for the charmonium.

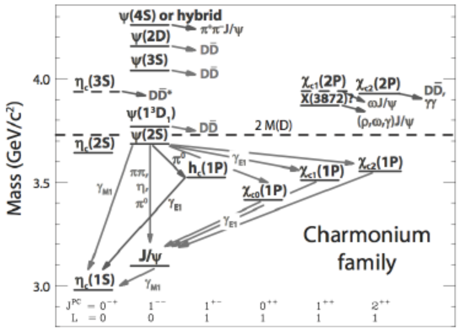

Figure 1.5: Observed states and transitions of the J/ψ system.

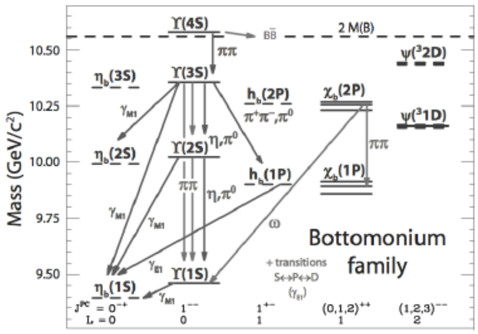

Figure 1.6: Observed states and transitions of the Υ system.

Their history is the beginning of modern particle physics and they were discovered in 1947, when Lattes, Occhialini and Powell demonstrated the existence of $\pi^{\pm}$ through $\pi^{\pm} \longrightarrow \mu^{\pm} + \nu$.
By detailed study of their interactions, it was determined that the π mesons also had $J^P = 0^-$. The π^0 decays to 2 photons, so it is $C = +1$. All of this is consistent with the interpretation of the pions as spin-$\frac{1}{2}$ fermion-antifermion bound states.
There are 9 relatively light 0^- hadrons, also known as **pseudoscalar mesons**, and 9 somewhat heavier 1^- hadrons, called the **vector mesons**, presented in Figure 1.7.
The K and K^* states are not produced singly in strong interactions. They are only produced together with one another, or with special excited states of the proton. For example, we see the reactions:

$$\pi^- p \longrightarrow n K^+ K^-$$
$$\pi^- p \longrightarrow \Lambda^0 K^0$$

where Λ^0 is a heavy excited state of the proton, but we don't see the reaction:

$$\pi^- p \longrightarrow n K^0$$

For this reason, the K mesons and the Λ^0 baryon became known as the strange particles.
As a consequence of this discovery, a new quantum number, the **strangeness**, was introduced to describe the production and decay processes. It was found that the

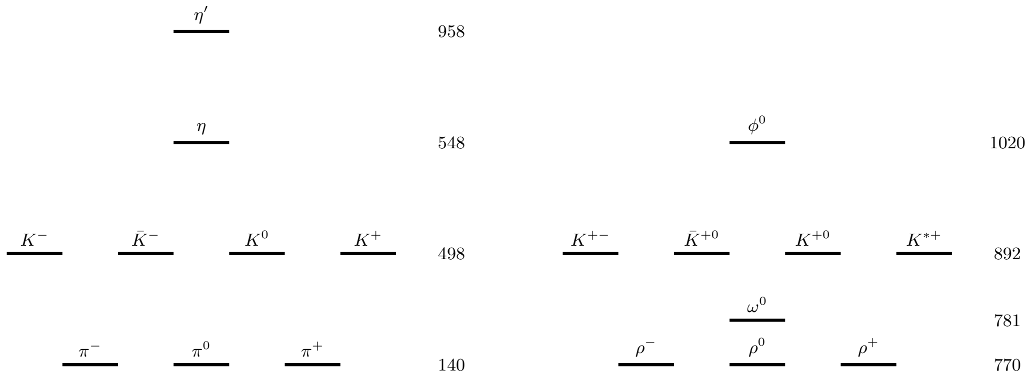

Figure 1.7: Light mesons summary. On the left there are the pseudoscalar mesons, on the right the vector mesons. The numbers given are the masses of the particles in MeV.

rules for K and K^* production can be expressed simply by saying that the strong interaction preserves the strangeness, with K^0, K^+, K^{*0} and K^{*+} having strangeness $S = -1$, their antiparticles having $S = +1$, and the Λ^0 having $S = +1$. Moreover, with the introduction of strangeness, a new kind of quark was introduced in the theories, namely the strange quark s. States with strangeness $+1$ will be assigned one s quark, and states with strangeness -1 will have one $\bar{s}$ antiquark.

1.4 Leptons

The leptons are fundamental particles, divided in several classes. We have:

- **Electron** e.
  It was discovered by J.J. Thomson in 1897 while studying the properties of cathode rays.

- **Muon** μ.
  It was discovered by Carl D. Anderson and Seth Neddermeyer in 1936 as component of the cosmic rays. At the beginning it was thought to be the Yukawa particle, the mediator of the strong force. Then Conversi, Pancini and Piccioni gave a proof that it does not interact strongly.

- **Tauon** τ.
  It was discovered by a group led by Martin Perl at Stanford Linear Accelerator Center. They used e^+e^- collisions with final states events $e\mu$.

- **Neutrino** ν.
  Neutrino hypothesis was formulated by Pauli to explain the β-decay. It was discovered by Clyde Cowan and Fred Reines in the 1953. We don't know if mass is given to neutrinos through the same mechanism (Higgs mechanism) for the other particles or if there is something that does it that we still don't know.

Chapter 2

Tools for calculations

To compare the results of elementary particle experiments to proposed theories of the fundamental forces, we must think carefully about what quantities we can compute and measure. We cannot directly measure the force that one elementary particle exerts on another. Most of our information about the subnuclear forces is obtained from scattering experiments or from observations of particle decay.
In scattering experiments, the basic measureable quantity is called the **differential cross section**. In particle decay, the basic measureable quantity is called the **partial width**.

2.1 Observables in experimental particle physics

The basic observable quantity associated with a decaying particle is the **rate of decay**. In quantum mechanics, an unstable particle A decays with the same probability in each unit of time. The probability of survival to time t then obeys the differential equation:

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = -\frac{P}{\tau_A} \overset{\text{solution}}{\Longrightarrow} P(t) = P_0 e^{-\frac{t}{\tau_A}} \tag{2.1}$$

The decay rate τ_A^{-1} is also called the **total width** Γ_A of the state A. Its dimension is 1/sec, equivalent to GeV up to factors of $\hbar$ and c.

$$\tau_A = \frac{1}{\Gamma_A} \qquad \Gamma_A = \text{Total width of the state } A \tag{2.2}$$

If there are multiple decay processes like $A \longrightarrow f$, each process has a rate $\Gamma(A \longrightarrow f)$, namely the **partial width**. Thus, the total decay rate is given by:

$$\Gamma_A = \sum_f \Gamma(A \longrightarrow f) \tag{2.3}$$

Another quantity called **branching ratio** can be defined by the definition of the previous ones:

$$\frac{\Gamma(A \longrightarrow f)}{\Gamma_A} = \text{Branching ratio} \tag{2.4}$$

We can now introduce the **cross section**. Let's imagine a fixed target experiment, where a beam of A particles of density n_A and velocity v_A, are shot at the fixed center B. What we can measure includes the rate R at which we see scatterings from the beam:

$$\mathrm{R} = \frac{\text{Number of events}}{\text{Time}} = n_A v_A \sigma_i \tag{2.5}$$

with σ_i the cross section of the process, which has the dimension of an area and it is measured in barn (10^{-28} m^2). It is the effective area that the target B presents to the beam. Another important quantity is the **luminosity**, i.e.:

$$\mathcal{L} = \frac{R}{\sigma_i} \tag{2.6}$$

Returning to the cross section, an alternative definition can be given. Imagine two bunches of particles A and B aimed at one another, namely a collision between two beams. The key idea is that the second beam is the target, so we consider $N_B = n_B l_B A_B$ in order to calculate the rate:

$$R = n_A n_B l_B A_B |v_A - v_B| \sigma_i \tag{2.7}$$

As pointed before, every beam is composed of bunches with the following gaussian distributuion:

$$\frac{\mathrm{d}N}{\mathrm{d}s} = \frac{N}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)} \tag{2.8}$$

The number of interations per bunch is $N_{\text{int}} = \sigma_{\text{int}} \frac{N_1 N_2}{4\pi\sigma_x\sigma_y}$ and the bunch frequency is f. Therefore, we can calculate the rate:

$$R_i = N_{\text{int}} f = \sigma_{\text{int}} \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} \tag{2.9}$$

2.2 Partial Width and Cross Section calculation

The partial width and the cross section for a certain process can be calculated through **Fermi's Golden Rule** in a very practical way. By using the time evolution operator T, we can write:

$$\langle 1, 2, \ldots, n | T | A(p_A) \rangle = \underbrace{\mathcal{M}(A \longrightarrow 1, 2, \ldots, n)}_{\text{Invariant matrix element}} \underbrace{(2\pi)^4 \delta^{(4)} \left(p_A - \sum_{i=1}^{n} p_i \right)}_{E,\vec{\mathbf{p}} \text{ conservation}} \tag{2.10}$$

It is useful to work out the dimension of $\mathcal{M}$. The operator T is dimensionless, and the states have total dimension GeV$^{-(\mathrm{n}+1)}$. The delta function has units GeV^{-4}. Then the invariant matrix element has the units:

$$\mathcal{M} \sim \text{GeV}^{3-\mathrm{n}} \tag{2.11}$$

Now, to find the total rate, we must integrate over all possible values of the final momenta. This integral is called **phase space** and for n final particles, the expression for the phase space integral is:

$$\int \mathrm{d}\Pi_n = \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3 2E_1} \cdots \frac{\mathrm{d}^3 p_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)} \left(p_A - \sum_{i=1}^{n} p_i \right) \tag{2.12}$$

However, we also need to normalize. So the initial state $|A\rangle$ will yield:

$$|A\rangle \longrightarrow \frac{1}{2E_A} \qquad \text{Initial state} \tag{2.13}$$

Finally, the Fermi Golden Rule formula for a partial width to an n-particle final state f is:

$$\Gamma(A \longrightarrow f) = \frac{1}{2M_A} \int \mathrm{d}\Pi_n |\mathcal{M}(A \longrightarrow f)|^2 \tag{2.14}$$

If the final state particles have spin, we need to sum over final spin states. The initial state A is in some state of definite spin. If we have not defined the spin of A carefully, an alternative is to average over all possible spin states of A. By rotational invariance, the decay rate of A can't depend on its spin orientation.
Concerning the cross section, a formula for this quantity is constructed in a similar way. We need the matrix element for a transition from the two initial particles A and B to the final particles through the interaction. So, it reads:

$$\sigma(A+B\longrightarrow f)=\frac{1}{2E_AE_B|v_A-v_B|}\int \mathrm{d}\Pi_n|\mathcal{M}(A+B\longrightarrow f)|^2 \tag{2.15}$$

2.3 Phase Space integral calculation

Phase space plays a very important role in particle physics. The default assumption is that final state particles are distributed according to phase space. This assumption is correct unless the transition matrix element has nontrivial structure. We will procede with a couple of examples/exercises in order to understand the way of working with this kind of computations.

Example 1: Phase space of 2 particles

Most of the reactions we will discuss will have two particles in the final state. So it's better to start with this example. We have to compute:

$$\int \mathrm{d}\Pi_2=\int \frac{\mathrm{d}^3p_1}{(2\pi)^3 2E_1}\frac{\mathrm{d}^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^{(4)}(p-p_1-p_2) \tag{2.16}$$

Let's work in the CM system, where $\vec{\mathbf{p}}_1+\vec{\mathbf{p}}_2=0$ and so $\vec{\mathbf{p}}_1=-\vec{\mathbf{p}}_2$. Hence:

$$P=(E_{\mathrm{CM}},\vec{\mathbf{0}}) \tag{2.17a}$$
$$p_1=(E_1,\vec{\mathbf{p}}) \tag{2.17b}$$
$$p_2=(E_2,-\vec{\mathbf{p}}) \tag{2.17c}$$

We have to integrate over $\vec{\mathbf{p}}_2$ and exploit the properties of δ function:

$$\begin{aligned}\int \mathrm{d}\Pi_2&=\int \frac{\mathrm{d}^3p}{(2\pi)^3)}\frac{1}{2E_1 2E_2}(2\pi)\delta(E_{\mathrm{CM}}-E_1-E_2)\\&=\int \frac{p^2\mathrm{d}\Omega}{16\pi^2E_1E_2)}\frac{E_1E_2}{pE_{\mathrm{CM}}}\\&=\frac{1}{8\pi}\left(\frac{2p}{E_{\mathrm{CM}}}\right)\int\frac{\mathrm{d}\Omega}{4\pi}\end{aligned} \tag{2.18}$$

Example 2: Phase space of 3 particles

It is also possible to reduce the expression for three-body space to a relatively simple formula. Let's work again in the center of mass frame where $\vec{\mathbf{p}}_1+\vec{\mathbf{p}}_2+\vec{\mathbf{p}}_3=0$ and let the total energy-momentum in this frame be $Q^0=E_{\mathrm{CM}}$. The three momentum vectors lie in the same plane, called **event plane**. Then the phase space integral can be written as an integral over the orientation of this plane and over the variables:

$$x_1=\frac{2E_1}{E_{\mathrm{CM}}}\qquad x_2=\frac{2E_2}{E_{\mathrm{CM}}}\qquad x_3=\frac{2E_3}{E_{\mathrm{CM}}}$$

which obey the constraint:

$$x_1 + x_2 + x_3 = 2$$

It can be shown that, after integrating over the orientation of the event plane, the integral over three-body phase space can be written as:

$$\int \mathrm{d}\Pi_3 = \frac{E_{\mathrm{CM}}^2}{128\pi^3} \int \mathrm{d}x_1 \mathrm{d}x_2 \tag{2.19}$$

It can be shown, further, that this integral can alternatively be written in terms of the invariant masses of pairs of the three vectors ($m_{12}^2 = (p_1 + p_2)^2$ and $m_{23}^2 = (p_2 + p_3)^2$):

$$\int \mathrm{d}\Pi_3 = \frac{1}{128\pi^3 E_{\mathrm{CM}}^2} \int \mathrm{d}m_{12}^2 \mathrm{d}m_{23}^2 \tag{2.20}$$

This formula leads to an important construction in hadron physics called the **Dalitz plot**.

Example 3: $\pi^+\pi^- \longrightarrow \rho^0 \longrightarrow \pi^+\pi^-$

One important type of structure that one finds in scattering amplitudes is a **resonance**. In ordinary quantum mechanics, a resonance is described by the **Breit-Wigner formula**:

$$\mathcal{M} \sim \frac{1}{E - E_{\mathrm{R}} + \frac{i}{2}\Gamma} \tag{2.21}$$

where E_{R} is the energy of the resonant state and Γ is its decay rate. The Fourier transform of Eq. 2.21 is:

$$\psi(t) = ie^{-iE_{\mathrm{R}}t}e^{-\Gamma\frac{t}{2}} \tag{2.22}$$

Then the probability of maintaining the resonance decays exponentially

$$|\psi(t)|^2 = e^{-\Gamma t} \tag{2.23}$$

corresponding to the lifetime:

$$\tau_{\mathrm{R}} = \frac{1}{\Gamma} \tag{2.24}$$

The final distributions of the particles are not in agreement with what we expect from the phase space distributions for two particles. In this case we can do the calculation in an easy way by studying:

1. $\pi^+\pi^- \longrightarrow \rho^0$ and treat it as a stable particle
2. Use Feynman diagrams:

$$\sigma(\pi^+\pi^- \longrightarrow \rho^0) = \frac{1}{2E_A 2E_B|v_A - v_B|} \int \frac{\mathrm{d}^3 p_C}{(2\pi)^3 2E_C} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_C - p_A - p_B) \tag{2.25}$$

where $A = \pi^+$, $B = \pi^-$ and $C = \rho^0$.

$$\Gamma_\rho = \frac{1}{2m_\rho} \int \mathrm{d}\pi_2 |\mathcal{M}|^2 \tag{2.26}$$

$$\sigma(\pi^+\pi^- \longrightarrow \rho^0 \longrightarrow \pi^+\pi^-) = \frac{1}{2m_\rho} \frac{1}{8\pi} \frac{2p}{m_\rho} \int \frac{\mathrm{d}\Omega}{4\pi} \frac{1}{(E_{\mathrm{CM}}^2 - m_p^2)^2 - m_p^2 \Gamma_\rho^2} |k|^2 \tag{2.27}$$

where k is a part related to the spin of ρ.
We see a resonance and we are able to fit the data, so we can get the quantities we want to know as fit results parameters.

We can describe the resonance through the Breit-Wigner formula:

$$\mathcal{M} \sim \frac{1}{E - E_{\mathrm{R}} + \frac{i}{2}\Gamma} \tag{2.28}$$

where E_{R} is the energy of the resonance and Γ is the width. In the case of resonance in the invariant mass, we have to slightly modify it:

$$\mathcal{M} \sim \frac{1}{p^2 - m_{\mathrm{R}} + i m_{\mathrm{R}} \Gamma_{\mathrm{R}}} \tag{2.29}$$

Chapter 3

Detectors for Particle Physics

Lecture 4. *Monday 18^{th} March, 2019. Compiled: Sunday 22^{nd} March, 2020.*

3.1 Recap: interaction of particles with matter

The way we identify particles is through their interaction with matter. So, we can detect:

- Charged particles based on ionization, breamsstrahlung, Cherenkov.
- γ-rays based on photo/compton effect, pair production.
- Neutrons based on strong interaction.
- Neutrinos based on weak interaction.

We will give only a phenomenological treatment since the goal is to be able to understand the implications for detector design.

3.1.1 Ionization

The equation that describe this interaction is the **Bethe-Bloch Equation**

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = K\rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\frac{1}{2}\log\frac{2m_e c^2\beta^2\gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{C}{z}\right] \tag{3.1}$$

3.1.2 Breamsstrahlung

$$-\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle = \frac{E}{X_0} \tag{3.2}$$

where X_0 is the radiation length in [g/cm^2] and its expression (approximation) is:

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \log \frac{183}{Z^{\frac{1}{3}}}} \tag{3.3}$$

After the passage of one X_0, electron has lost all but $(1/e)^{\text{th}}$ of its energy, namely 63%.
The critical energy E_C is the energy for which:

$$\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{ion}} = \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{rad}} \tag{3.4}$$

An approximation is:

$$E_C \approx \frac{600\ \text{MeV}}{Z} \tag{3.5}$$

3.1.3 Total energy loss for electrons

Ionization losses decrease logarithmically with E and increase linearly with Z. Bremsstrahlung increases approximately linearly with E and is the dominant process at high energies.

3.1.4 Interaction of photon with the matter

Photon lose energy by:

- Photoelectric effect on atoms at low energy.
- Compton effect important at intermediate range.
- Pair production.

In our case pair production is dominant:

$$\sigma_{\text{pair}} = \frac{7}{9}\frac{N_A}{A}\frac{1}{X_0} \tag{3.6}$$

3.2 Detectors: gaseous, scintillators and

3.2.1 Ionization in gas detectors

Primary ionization:

$$\text{Particle} + X \longrightarrow X^+ + e^- + \text{Particle} \tag{3.7}$$

Secondary ionization:

$$X + e^- \longrightarrow X^+ + e^- + e^- \tag{3.8}$$

The relevant parameters to evaluate the number of particles produced are the ionization energy E_i, the average energy/ion pair W_i and the average number of ion pairs (per cm) n_T. In particular:

$$\langle n_T \rangle = \frac{L\left\langle \frac{\mathrm{d}E}{\mathrm{d}x} \right\rangle}{W_i} \tag{3.9}$$

with L the thickness of the material. Typical values for E_i are
Concerning the diffusion, it is significantly modified by the presence of a magnetic field, with transverse and longitudinal orientation depending on it. By measuring the bending of the particle, we are able to infer the momentum of the particle itself. The electric field influences only the longitudinal diffusion and not the transverse diffusion.
The electrons can undergo to a multiplication process called townsend avalanche. Given the number of electrons at the position x, $n(x) = n_0 e^{\alpha x}$, we have the gain:

$$G = \frac{n(x)}{n_0} = e^{\alpha x} \tag{3.10}$$

where α can depend on x.
There are four regions of work:

- Geiger-Muller coun

3.2.2 Multiwire proportional chambers

Signal generations: electron drift to closest wire, gas amplification near wire that creates avalanche

3.2.3 Drift chambers

In this case we can have two dimensional informations through time measurments, namely drift time measurement. It starts by an external detector such as a scintillator counter. Electrons drift to the anode in the field created by anode and cathode. The electron arrival at the anode stops in the time measurement.

$$x = \int_0^{t_D} v_D \mathrm{d}t \tag{3.11}$$

We build the detector with a known drift velocity. We can introduce field wires

3.2.4 Semiconductor detectors

Semiconductor detectors have the following characteristics:

- High density (respect to gas detectors), so large energy loss in a short distance
- A small diffusion effect, so position resolution of less than 10 μm
- Low ionization energy, so it is easier to produce particles.

The materials employed for their construction are:

- Germanium, which needs to be operated at a very low temperature (77 K) due to small band gap.
- Silicon, which can operate at room temperature.
- Diamond, resistent to very hard radiations, low signal and high cost.

Silicon detectors are based on a p-n junction with reverse bias applied to enlarge the depletion region. The potential barrier becomes higher so that the diffusion across the junction is suppressed and the current across the junction is very small (“leakage current”).
Such a detector can be built in strips. By segmenting the implant we can reconstruct the position of hte transversing particle in one dimension. We have a higher field close to the collecting electrodes where most of the signal is induced. Strips can be read with dedicated electronics to minimize the noise. To have 2-dimensional measurements, double sided silicon detector are used. A type of silicon detector still in development is the pixel detector (for 3-dimensional measurements).
Noise contributions can be leakage current and electronics readout.
Position resolution is the spread of the reconstructed position minus the true position. For example:

$$\sigma = \frac{\text{pitch}}{\sqrt{12}} \qquad \text{One strip cluster} \tag{3.12}$$

$$\sigma = \frac{\text{pitch}}{1.5\frac{S}{N}} \qquad \text{Two strip cluster} \tag{3.13}$$

3.3 Track reconstruction

Track reconstruction is used to determine momentum of charged particles by measuring the bending of a particle trajectory in a magnetic field

Bibliography