Lecture 8.
Wednesday 1st April, 2020.
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0.1 Crossing symmetry

To compute the cross section, we need to evaluate the matrix elements for electron-quark scattering, which can be described by the Feynman diagram in Figure ??. This diagram is similar to the one for $e^-e^+ \longrightarrow \mu^-\mu^+$, so the matrix element will have the same structure:

$$\mathcal{M}(e^- q_f \to e^- q_f) = (-e) \left\langle e^- \right| j^\mu \left| e^- \right\rangle \frac{1}{q^2} (Q_f e) \left\langle q_f \right| j_\mu \left| q_f \right\rangle \tag{1}$$

To evaluate this matrix element, we need the concept of **crossing symmetry**, applyed in a phenomenological way. To begin, we compare thee diagram in Figure ?? with the following one for $e^+e^- \longrightarrow q\bar{q}$ in Figure ??.

$$\sum_{\text{spin}} |\mathcal{M}_{\text{scat}}|^2 \longrightarrow \sum_{\text{spin}} |\mathcal{M}_{\text{pair}}|^2 \tag{2}$$

The two Feynamn diagrams actually show the same process, laid out in different ways in space-time. The situations with a final quark and an initial antiquark are strongly related, because the same quantum field that creates the electron destroys the positron, and similarly for a quark and antiquark. So, the matrix elements have the same functional form with appropriate identification of the external momenta.
Crossing symmetry is a theorem from QFT and it states that processes related by this kind of symmetry are described by the same function of external momenta. It is useful to continue to introduce a rigorous and standard notation for the kinematic invariants of 2-body scattering process. So, we want to study:

$$1(p_1) + 2(p_2) \longrightarrow 3(p_3) + 4(p_4) \tag{3}$$

The Mandelstam invariants reads (with $p_1, p_2 < 0$):

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \tag{4a}$$
$$t = (p_1 + p_3)^2 = (p_2 + p_4)^2 \tag{4b}$$
$$u = (p_1 + p_4)^2 = (p_2 + p_3)^2 \tag{4c}$$

where:

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2 \tag{5}$$

Kinematics:

$$p_1 = (-E, 0, 0, -E) \tag{6a}$$
$$p_2 = (-E, 0, 0, E) \tag{6b}$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta) \tag{6c}$$
$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta) \tag{6d}$$

and:

$$s = (2E)^2 = E_{\text{CM}}^2 \tag{7}$$

In the last chapter, we stated that we could represent an intermediate state in a Feynman diagram with a Breit-Wigner denominator:

$$\frac{1}{(p_1 + p_2)^2 - m_R^2 + i m_R \Gamma_R} \tag{8}$$

When the intermediate state separates the initial and the final state, the denominator depends on $(p_1 + p_2)^2 = s$:

$$\frac{1}{s - m_R^2 + i m_R \Gamma_R} \tag{9}$$

This type of reation is called an **s-channel process**. If the amplitude depends on t, we have a **t-channel process**, if it depends on u, we have an **u-channel process**.

0.2 Cross section for electron-quark scattering

We get:

$$\left|\mathcal{M}(e_R^- e_L^+ \to q_R \bar{q}_L)\right|^2 = \left|\mathcal{M}(e_L^- e_R^+ \to q_L \bar{q}_R)\right|^2 = 4 Q_f^2 e^4 \frac{u^2}{s^2} \tag{10}$$

$$\left|\mathcal{M}(e_R^- e_L^+ \to q_L \bar{q}_R)\right|^2 = \left|\mathcal{M}(e_L^- e_R^+ \to q_R \bar{q}_L)\right|^2 = 4 Q_f^2 e^4 \frac{t^2}{s^2} \tag{11}$$

These expressions are correct in any frame and they yield the expressions for the crossed amplitudes after an appropriate permutation of variables. For example, consider the crossing symmetry;
The eq scattering diagram on the right is obtained by moving the final antiquark $\bar{q}_L$ to the initial state, where it becomes the quark q_R, and moving the initial positron e_L^+ to the final state, where it becomes the electron e_R^-. Note that helicity is conserved. The interchange of momenta is:

$$\begin{aligned} p_1 &\longrightarrow p_1 \\ p_2 &\longrightarrow p_3 \\ p_3 &\longrightarrow p_4 \\ p_4 &\longrightarrow p_2 \end{aligned} \tag{12}$$

If we do this exchange, the matrix element for $e_R^- q_R \longrightarrow e_R^- q_R$ is given by:

$$\left|\mathcal{M}(e_R^- q_R \to e_R^- q_R)\right|^2 = 4 Q_f^2 e^4 \frac{s^2}{t^2} \tag{13}$$

Similarly:

$$\left|\mathcal{M}(e_R^- q_L \to e_R^- q_L)\right|^2 = 4 Q_f^2 e^4 \frac{u^2}{t^2} \tag{14}$$

There are amplitudes that do not contribute to the final state since the related processes violate helicity conservation. So, backward scattering is forbidden.