Lecture 3.
Tuesday 17th March, 2020.
Compiled: Saturday 21st March, 2020.
Prof. Lucchesi

0.0.1 Light mesons

Now we can go back to the π mesons and other relatively light hadrons. πs are the strongly interacting particles and there are three π mesons: π^0, π^+ and π^-. Their history is the beginning of modern particle physics and they were discovered in 1947, when Lattes, Occhialini and Powell demonstrated the existence of $\pi^\pm$ through $\pi^\pm \longrightarrow \mu^\pm + \nu$.
By detailed study of their interactions, it was determined that the π mesons also had $J^P = 0^-$. The π^0 decays to 2 photons, so it is $C = +1$. All of this is consistent with the interpretation of the pions as spin-$\frac{1}{2}$ fermion-antifermion bound states.
There are 9 relatively light 0^- hadrons, also known as **pseudoscalar mesons**, and 9 somewhat heavier 1^- hadrons, called the **vector mesons**, presented in Figure 1.

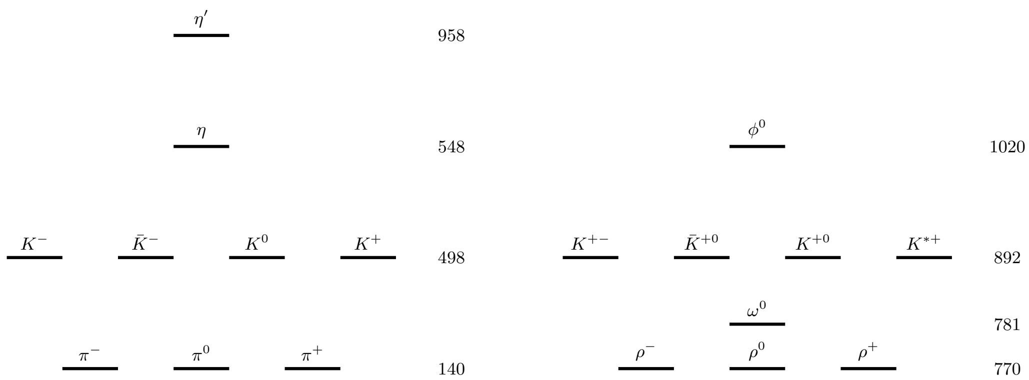

Figure 1: Light mesons summary. On the left there are the pseudoscalar mesons, on the right the vector mesons. The numbers given are the masses of the particles in MeV.

The K and K^* states are not produced singly in strong interactions. They are only produced together with one another, or with special excited states of the proton. For example, we see the reactions:

$$\pi^- p \longrightarrow nK^+K^-$$
$$\pi^- p \longrightarrow \Lambda^0 K^0$$

where Λ^0 is a heavy excited state of the proton, but we don't see the reaction:

$$\pi^- p \longrightarrow nK^0$$

For this reason, the K mesons and the Λ^0 baryon became known as the strange particles.
As a consequence of this discovery, a new quantum number, the **strangeness**, was introduced to describe the production and decay processes. It was found that the rules for K and K^* production can be expressed simply by saying that the strong interaction preserves the strangeness, with K^0, K^+, K^{*0} and K^{*+} having strangeness $S = -1$, their antiparticles having $S = +1$, and the Λ^0 having $S = +1$. Moreover, with the introduction of strangeness, a new kind of quark was introduced in the theories, namely the strange quark s. States with strangeness $+1$ will be assigned one s quark, and states with strangeness -1 will have one $\bar{s}$ antiquark.

0.1 Leptons

The leptons are fundamental particles, divided in several classes. We have:

- **Electron** e.
  It was discovered by J.J. Thomson in 1897 while studying the properties of cathode rays.

- **Muon** μ.
  It was discovered by Carl D. Anderson and Seth Neddermeyer in 1936 as component of the cosmic rays. At the beginning it was thought to be the Yukawa particle, the mediator of the strong force. Then Conversi, Pancini and Piccioni gave a proof that it does not interact strongly.

- **Tauon** τ.
  It was discovered by a group led by Martin Perl at Stanford Linear Accelerator Center. They used e^+e^- collisions with final states events $e\mu$.

- **Neutrino** ν.
  Neutrino hypothesis was formulated by Pauli to explain the β-decay. It was discovered by Clyde Cowan and Fred Reines in the 1953. We don't know if mass is given to neutrinos through the same mechanism (Higgs mechanism) for the other particles or if there is something that does it that we still don't know.

Chapter 1

Tools for calculations

1.1 Cross Section

The oservables we want to use in our case are:
A: unstable particle.

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = -\frac{P}{\tau_A} \Longrightarrow P(t) = P_0 e^{-\frac{t}{\tau_A}} \tag{1.1}$$

$$\tau_A = \frac{1}{\Gamma_A} \qquad \Gamma_A = \text{Total width of the state } A \tag{1.2}$$

$\Gamma(A \longrightarrow f)$ is the partial width.

$$\Gamma_A = \sum_f \Gamma(A \longrightarrow f) \tag{1.3}$$

$$\frac{\Gamma(A \longrightarrow f)}{\Gamma_A} = \text{Branching ration} \tag{1.4}$$

Let's introduce the **cross section**. Imagine we have a fixed target experiment. We have a beam of particles A, with density n_A, velocity v_A, and a target B. In this case we measure the rate, which is:

$$\text{Rate} = \frac{\text{Number of events}}{\text{Time}} = n_A v_A \sigma_i \tag{1.5}$$

with σ_i the cross section of the process, which has the dimension of an area and it is measured in barn (10^{-28} m^2). Another important quantity is the **luminosity**, i.e.:

$$\mathcal{L} = \frac{R}{\sigma_i} \tag{1.6}$$

In beam collisions we have two beams. For the first beam we have n_A, v_A, for the second beam n_B, v_B. The idea is that the second beam is the target, so we consider $N_B = n_B l_B A_B$ in order to calculate the rate:

$$R = n_A n_B l_B A_B |v_A - v_B| \sigma_i \tag{1.7}$$

The beam is composed of bunches with gaussian distributuion:

$$\frac{\mathrm{d}N}{\mathrm{d}s} = \frac{N}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)} \tag{1.8}$$

The number of interations per bunch is $N_mathrmint = \sigma_{\text{int}} \frac{N_1 N_2}{4\pi\sigma_x\sigma_y}$. The bunch frequency is f. The rate is:

$$R_i = N_{\text{int}} f = \sigma_{\text{int}} \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} \tag{1.9}$$

1.2 Partial Width

The partial width and the cross section for a certain process can be calculated through **Fermi's Golden Rule** in a very practical way. By using the time evolution operator T:

$$\langle 1,2,\dots,n|\, T\, |A(p_A)\rangle = \underbrace{\mathcal{M}(A \longrightarrow 1,2,\dots,n)}_{\text{Invariant matrix element}} (2\pi)^4 \underbrace{\delta^4\left(p_A - \sum_{i=1}^{n} p_i\right)}_{E,\vec{\mathbf{p}}\text{ conservation}} \tag{1.10}$$

To evaluate the full cross section, we need to do very difficult integrals over the phase space:

$$\int \mathrm{d}\pi_u = \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3 2E_1} \cdots \frac{\mathrm{d}^3 p_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^4\left(p_A - \sum_{i=1}^{n} p_i\right) \tag{1.11}$$

However, we also need to normalize:

$$|A\rangle \longrightarrow \frac{1}{2E_A} \qquad \text{Initial state} \tag{1.12}$$

$$\Gamma(A \longrightarrow f) = \frac{1}{M_A} \int \mathrm{d}\pi_u |\mathcal{M}(A \longrightarrow f)|^2 \tag{1.13}$$

For the cross section:

$$\sigma(A + B \longrightarrow f) = \frac{1}{2E_A E_B (v_A - v_B)} \int \mathrm{d}\pi_u |\mathcal{M}(A + B \longrightarrow f)|^2 (??????) \tag{1.14}$$

We will procede with a couple of examples/exercises.

Example 1: Phase space of 2 particles

$$\int \mathrm{d}\pi_2 = \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3 2E_1} \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p - p_1 - p_2) \tag{1.15}$$

Let's work in the CM system. We have to integrate over $\vec{\mathbf{p}}_2$ and exploit the properties of δ function:

$$\int \mathrm{d}\pi_2 = \int \frac{p\,\mathrm{d}\Omega}{16\pi^2(E_1 + E_2)} \longrightarrow \frac{2p}{E_{\mathrm{CM}}} \frac{1}{8\pi} \int \frac{\mathrm{d}\Omega}{4\pi} \tag{1.16}$$

We can describe the resonance through the Breit-Wigner formula:

$$\mathcal{M} \sim \frac{1}{E - E_{\mathrm{R}} + \frac{i}{2}\Gamma} \tag{1.17}$$

where E_{R} is the energy of the resonance and Γ is the width. In the case of resonance in the invariant mass, we have to slightly modify it:

$$\mathcal{M} \sim \frac{1}{p^2 - m_{\mathrm{R}} + i m_{\mathrm{R}} \Gamma_{\mathrm{R}}} \tag{1.18}$$

Example 2: $\pi^+\pi^- \longrightarrow \rho^0 \longrightarrow \pi^+\pi^-$

The final distributions of the particles are not in agreement with what we expect from the phase space distributions for two particles. In this case we can do the calculation in an easy way by studying:

1. $\pi^+\pi^- \longrightarrow \rho^0$ and treat it as a stable particle
2. Use Feynman diagrams:

$$\sigma(\pi^+\pi^- \longrightarrow \rho^0) = \frac{1}{2E_A 2E_B |v_A - v_B|} \int \frac{\mathrm{d}^3 p_C}{(2\pi)^3 2E_C} |\mathcal{M}|^2 (2\pi)^4 \delta^4 (p_C - p_A - p_B) \tag{1.19}$$

where $A = \pi^+$, $B = \pi^-$ and $C = \rho^0$.

$$\Gamma_\rho = \frac{1}{2m_\rho} \int \mathrm{d}\pi_2 |\mathcal{M}|^2 \tag{1.20}$$

$$\sigma(\pi^+\pi^- \longrightarrow \rho^0 \longrightarrow \pi^+\pi^-) = \frac{1}{2m_\rho}\frac{1}{8\pi}\frac{2p}{m_\rho} \int \frac{\mathrm{d}\Omega}{4\pi} \frac{1}{(E_{\mathrm{CM}}^2 - m_p^2)^2 - m_p^2 \Gamma_\rho^2} |k|^2 \tag{1.21}$$

where k is a part related to the spin of ρ.
We see a resonance and we are able to fit the data, so we can get the quantities we want to know as fit results parameters.