LECTURE NOTES
OF
SUBNUCLEAR PHYSICS

COMMENT.

EDITED BY

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Contents

Course structure and program

Lecture 1.
Tuesday 10^{th} *March, 2020.*
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Prof. Lucchesi

Informations

Suggested books:

- *Concepts of Elementary Particle Physics*, Michael E. Peskin.
  It has a very good experimental approach, with theoretical concepts explained as well.
- Any other book where the same topics are presented is fine. For example, the book of Alessandro Bettini.

Exam modalities: the exam is slitted into two parts. These are:

- **Written exercises**.
  The idea is to prepare two partial tests: one will take place almost at the middle of the course, one at the end. For each chapter of the reference book there are several exercises that are useful for the comprehension of the topics of the course.
- **Oral discussion**.
  It will be focused on a single topic and it will take place after the written part.

The final evaluation will be a weighted mean of the two written exercises and of the oral discussion.
Remeber to subscribe to the Facebook group *Subnuclear Physics at DFA* for further informations and for infos on seminars of particle physics.

Course Program

- **Introduction and recap**
- **Tools for calculation**.
  In order to understand all the following topics, we need some mathematical tools (that we already have but the way we are going to use them is different from the use we did in theoretical physics course). They are needed to evaluate the physical phenomena we are going to discuss.
- **Detectors for particle physics experiments**.
  They are needed to perform measurements, so it is important to acquire a certain knowledge on them. For example, in order to choose why a detector is better than another one for a certain task and to set up a particle physics experiment. This part is not well described in the reference book, so we will use other books for this purpose.
- **Cross section of** $e^+e^- \longrightarrow \mu^+\mu^-$ **and** $e^+e^- \longrightarrow hh$.
  The former is a very simple process and it is important for the study of many other processes. The ladder will be important to understand the basis of QCD.

- **Strong interactions**:
  - ▷ **Deep inelastic scattering**
  - ▷ **Gluon**
  - ▷ **QCD**
  - ▷ **Partons and jets**
- **Electroweak interactions** (This part and the part on strong interactions sum up into the discussion on Standard Model):
  - ▷ **V-A Weak theory**.
    It is the theory at the base of electroweak interaction, which we will build up.
  - ▷ **Gauge theory and symmetry breaking**.
    This part will be discussed not so deeply since it was treated during the course of *Theoretical Physics of Fundamental Interactions*.
  - ▷ W **and** Z^0 **bosons**.
    The most important items and measurements will be presented.
  - ▷ **Cabibbo theory and CKM**.
    This part is needed in order to put the hadrons, in particular the quarks, into the electroweak theory. However, it will not be discussed deeply since it was presented during the bachelor course *Introduction to Nuclear and Subnuclear Physics*.
  - ▷ **CP violation, the B meson system**.
    It will be a more experimental discussion.
- **New Physics** (we will try to give an answer to how we can go beyond the description given by Standard Model, in fact there are phenomena that are still not explained by this theory):
  - ▷ **Neutrino and Standard Model**
  - ▷ **Higgs properties**

Chapter 1

Introduction and Recap

1.1 Basic knowledge

Relativistic wave equations

Relativistic quantum field theory is necessary to describe quantitatively elementary particle interactions. Its description is not part of this course, so we will use it in simple cases and only when necessary.
It is assumed the following knowledge:

- Klein-Gordon equation (for boson fields):

$$\left(\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 + m^2\right)\psi(t,\vec{\mathbf{x}}) = 0 \tag{1.1}$$

- Dirac equation (Klein-Gordon can't give a description for fermion fields):

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - m\right)\psi(t,\vec{\mathbf{x}}) = 0 \tag{1.2}$$

  with $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$

- Basic concepts of fields and particles
- Basic concepts of Feynman diagrams

Natural Units

During the course we will use the natural units, therefore:

$$\hbar = c = 1 \tag{1.3}$$

Considering that:

$$\begin{aligned} 1\ \text{eV} &= 1.6 \cdot 10^{-19}\ \text{J} \\ c &= 3 \cdot 10^{8}\ \text{m/s} \end{aligned}$$

we have:

$$1\ \frac{\text{eV}}{c^2} = 1.78 \cdot 10^{-36}\ \text{Kg}$$

Since $E^2 = p^2c^2 + m^2c^4$, it is convenient to measure p in GeV/c and m in GeV/c^2. For example the electron mass $m_e = 0.91 \cdot 10^{-27}$ g corresponds to $m_e = 0.51$ MeV/c^2. It is also useful to remember that $\hbar c = 197$ MeVfm.
An interesting quantity to consider in natural units is the strength of the electric charge of the electron or proton. By taking into account the potential $V(r) = \frac{e^2}{4\pi\varepsilon_0 r}$, the radius r in natural units has a dimension of Energy^{-1}. By this way it forces the following relation:

$$\alpha \equiv \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137.036} \tag{1.4}$$

namely, the **fine structure constant**.

Symmetries

They are the corner stones of particle physics. The most important ones for our studies are the **space-time symmetries**, which can be classified into:

- Continuous symmetries:
  - ▷ Translation in time. The generator of the group of time translations is the operator H, namely the Hamiltonian, which is linked to the energy quantity.
  - ▷ Translation in space. The generator of the group of space translations is the operator $\vec{\mathbf{p}}$, namely the momentum.
  - ▷ Rotations. In this case, the generator of the group of this kind of transformations is the angular momentum $\vec{\mathbf{L}}$.

  If a system is invariant under one of these transformations, the corresponding generator, so H, $\vec{\mathbf{p}}$ or $\vec{\mathbf{L}}$, is conserved.

- Discrete symmetries:
  - ▷ Parity P:

    $$x^\mu = (x^0, \vec{\mathbf{x}}) \xrightarrow{P} (x^0, -\vec{\mathbf{x}}) \tag{1.5}$$

    Fermions have half-integer spin and angular momentum conservation requires their production in pairs. We can define therefore just relative parity. By convention, the proton p has parity equal to $+1$. The parity of the other fermions is given in relation to the parity of the proton.

    Parity of bosons can be defined without ambiguity since they are not necessarily producted in pairs.

    Parity of a fermion and its antiparticle (i.e. an antifermion) are opposite, while parity of a boson and its anti-boson are equal.

    Moreover, the parity of the positron is equal to -1. Quarks have parity equal to $+1$, leptons have parity equal to $+1$. Their antiparticles have parity equal to -1.

    Lastly, parity of a photon is equal to -1.
  - ▷ Time Reversal T:

    $$x^\mu = (x^0, \vec{\mathbf{x}}) \xrightarrow{T} (-x^0, \vec{\mathbf{x}}) \tag{1.6}$$

▷ Charge Conjugation C:

$$\text{Particle} \xleftrightarrow{C} \text{Antiparticle} \tag{1.7}$$

It is needed in order to restore a complete symmetry under the exchange of a particle with its antiparticle. A photon has -1 eigenvalue under C, which means: $C\,|\gamma\rangle = -\,|\gamma\rangle$.
Fermion-antifermion have opposite intrinsic parity and for non elementary particles the total angular momentum has to be considered, in fact the C parity goes like $(-1)^\ell$ or $(-1)^{\ell+1}$ (depending on the intrinsic parity).

Fundamental constituents of the matter

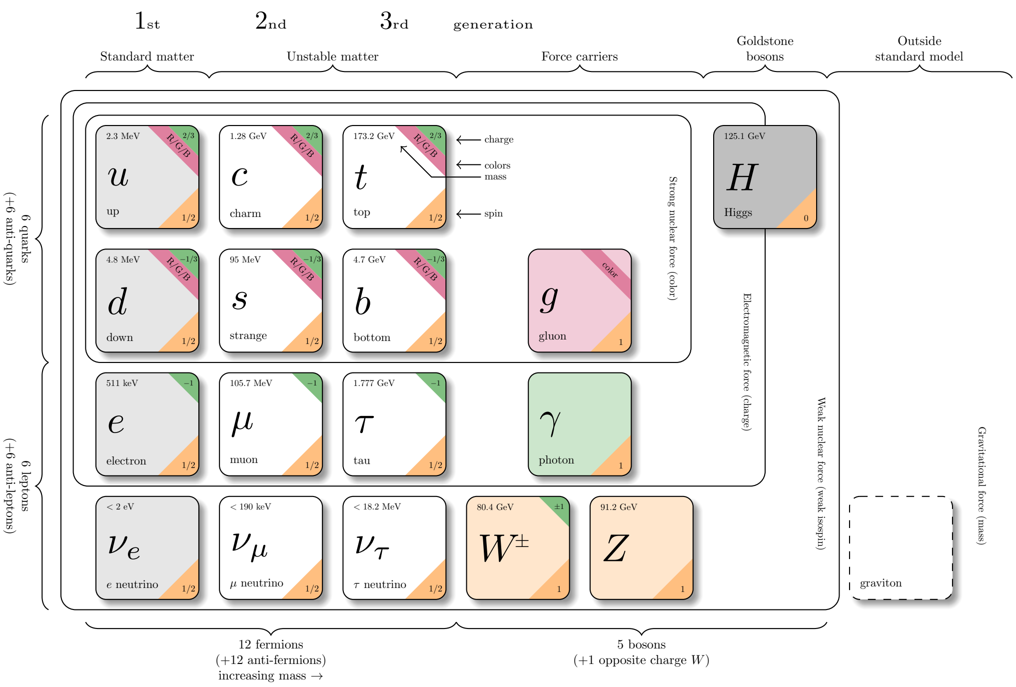

Figure 1.1: Standard Model particles.

1.2 Hydrogen atom and Positronium

Lecture 2. *Wednesday 11th March, 2020. Compiled: Tuesday 17th March, 2020. Prof. Lucchesi*

We are going to study the already known system of the hydrogen atom, and compare it to the system of positronium. More in detail, our goal is to understand the e^+e^- bound state and the possible application of this model to the description of other systems. Therefore, we start from the hydrogen atom since it has some characteristics in common with the positronium.
In QM Physics, this bound state is really similar to the hydrogen atom. The assumptions for this one in the non relativistic limit are that the mass of the proton is much bigger than the mass of the electron ($m_p >> n_e$) and the potential is given by:

$$V(r) = -\frac{e^2}{4\pi r} = -\frac{\alpha}{r} \tag{1.8}$$

From this potential, by solving the Schrödinger equation, we get the bound state energies:

$$E = -\frac{R_y}{n^2} \tag{1.9}$$

R_y is known as **Rydberg energy**, whose expression reads:

$$R_y = \frac{1}{2}\frac{me^4}{(4\pi)^2} = 12.6 \text{ eV} \tag{1.10}$$

$$R_y = \frac{1}{2}\alpha^2 m_p \qquad \text{In natural units} \tag{1.11}$$

The bound states of hydrogen are arranged in levels associated with integers $n = 1, 2, 3, \dots$. Each level contains the orbital angular momentum states:

$$\begin{aligned} \ell &= 0, 1, \dots, n-1 \\ m &= -\ell, \dots, \ell \end{aligned} \tag{1.12}$$

The orbital wavefunctions are the spherical harmonics $Y_{\ell m}(\theta, \varphi)$, which are even under spatial reflection for even ℓ and odd for odd ℓ. Then, under P, these states transform as:

$$P\,|n\ell m\rangle = (-1)^{\ell}\,|n\ell m\rangle \tag{1.13}$$

However, with these assumptions, we are not considering that the real hydrogen atom has more structure. In fact, we are negleting that the electron is a particle with intrinsic spin and we have to take into account also this quantity. In a more technical way, we have to add the contribution of the spin-orbit interaction (fine splitting), which is proportional to the scalar product $\vec{\mathbf{L}} \cdot \vec{\mathbf{S}}$. Concerning the Hamiltonian of this contribution, it is given by:

$$\Delta H = \frac{g-1}{2}\frac{\alpha}{m^2 r^3}\vec{\mathbf{L}} \cdot \vec{\mathbf{S}} \tag{1.14}$$

The sign is such that the state with $\vec{\mathbf{L}}$ and $\vec{\mathbf{S}}$ opposite in sign has lower energy. Moreover, it may be useful to express the operator $\vec{\mathbf{L}} \cdot \vec{\mathbf{S}}$ in terms of J^2, L^2, S^2:

$$\vec{\mathbf{J}} = \vec{\mathbf{L}} + \vec{\mathbf{S}} \Longrightarrow \vec{\mathbf{L}} \cdot \vec{\mathbf{S}} = \frac{1}{2}\left(\left(\vec{\mathbf{L}} + \vec{\mathbf{S}}\right)^2 - L^2 - S^2\right) = \frac{1}{2}(J^2 - L^2 - S^2) \tag{1.15}$$

By this way it is straightforward to diagonalize the operator $\vec{\mathbf{L}} \cdot \vec{\mathbf{S}}$. At the end we get the order of magnitude of the spin-orbit interaction:

$$\left\langle \frac{\alpha}{m^2 r^3} \right\rangle \sim \frac{\alpha}{m^2 a_0^3} \sim \alpha^4 m \sim \alpha^2 R_y \tag{1.16}$$

Thus, this effect is a factor of 10^{-4} smaller than the splitting of the principal levels of hydrogen.
Another contribution that we have to add is the spin-spin interaction (hyperfine splitting) between electron and proton, which leads to the addition of another term into the total Hamiltonian. The magnetic moments of the proton and the electron interact, with the ground state favoring the configuration in which the two spins are opposite. Therefore:

$$\Delta H = C\vec{\mathbf{S}}_p \cdot \vec{\mathbf{S}}_e \tag{1.17}$$

where the C constant depends on the electron wavefunction.
Hence, we have several levels for the spin states. For example, the 1S state of hydrogen is split into two levels, corresponding to the total spin:

$$\vec{\mathbf{J}} = \vec{\mathbf{S}}_p + \vec{\mathbf{S}}_e \tag{1.18}$$

The possibilities we have are 2: $J = 0$ and $J = 1$, depending on how the two spin states of proton and electron combine. The projection on the z-axis gives 3 possibilities: $J_z = 1, 0, -1$ (corresponding to $|\uparrow\uparrow\rangle$, $\frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$, $|\downarrow\downarrow\rangle$).

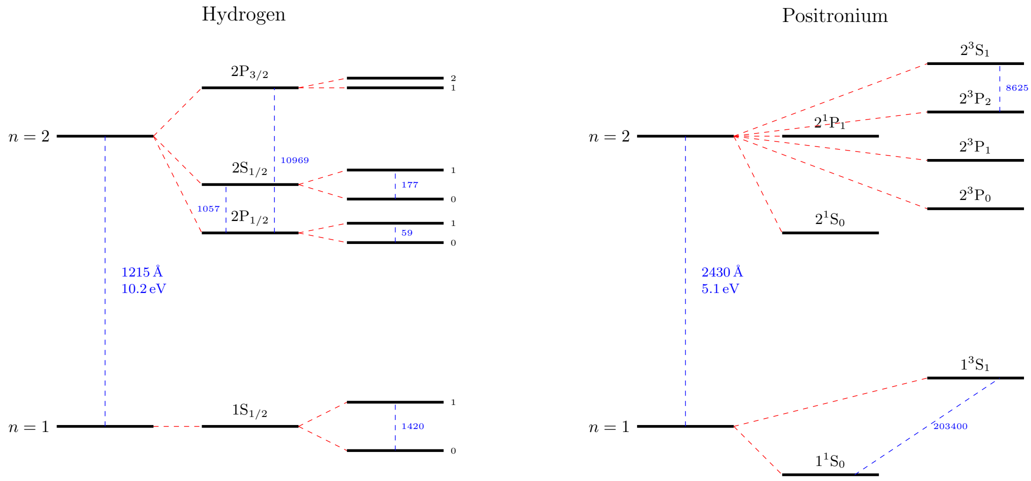

Figure 1.2: Comparison of the 1S, 2S, and 2P energy levels of hydrogen atom and positronium.

Now the possibility that we have to evaluate is that e^+e^- forms bounded states. In fact, the same ideas can be applied to a particle-antiparticle system and the simplest case is the positronium.
It is relatively easy to make positronium. In colliders, when working with a beam of positrons which enter in the matter, they can pick up an electron and form a bounded state of positronium, so this the starting point of the idea. All the considerations applied to the case of hydrogen atom can be applied to the positronium case as well. All the calculations are omitted. The first consideration is that here we can't apply the approximation $m_p >> m_e$, in fact the two particles here have the same mass. The solution for this two-body problem is to use the reduced mass μ, namely:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_e}{2} \tag{1.19}$$

At the end of all the calculations we won't do, we get that the hyperfine splitting contribution is approximately of the same of order of magnitude of the fine splitting and both are of the order $\alpha^4 m_e$.
Now we have to classify the eigenstates under parity and charge conjugation of the positronium. Let's consider first P. The intrinsic parity of the electron is $P_{e^-} = +1$, of the positron $P_{e^+} = -1$. So the parity of a single particle goes like $P = (-1)^\ell$ and the overall parity goes like $P = (-1)^{\ell+1}$.
For C, we must account three effects:

- C converts the electron to the positron and the positron to the electron. The electron and positron are fermions, and so, when we put the electron and positron back into their original order in the wavefunction, we get a factor -1.
- Reversal of the coordinate in the orbital wavefunction gives a factor $(-1)^\ell$.
- Finally, the electron and positron spins are interchanged. The $S = 1$ state is

symmetric in spin, but the $S = 0$ state is antisymmetric.

$$S = 0 \longrightarrow \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$
$$S = 1 \longrightarrow |\uparrow\uparrow\rangle \qquad \frac{1}{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \qquad |\downarrow\uparrow\rangle$$

and so gives another factor (-1).

In all, the positronium states have C:

$$C = (-1)^{\ell+1} \cdot \begin{cases} 1 & S = 1 \\ -1 & S = 0 \end{cases} \tag{1.20}$$

and what we get is the J^{PC} scheme. The low-lying states of the positronium spectrum then have the J^{PC} values:

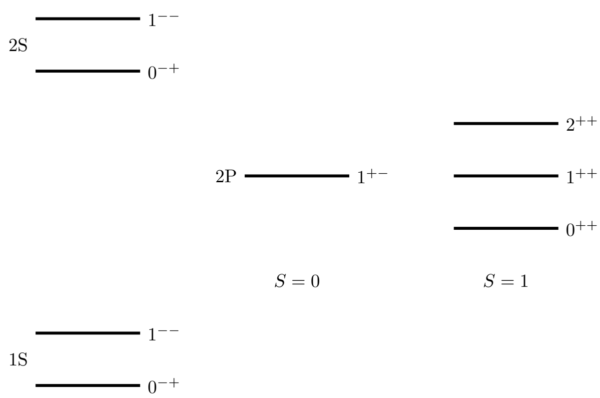

Figure 1.3: J^{PC} scheme. The 2P states 0^{++}, 1^{++} and 2^{++} arise from coupling the $L = 1$ orbital angular momentum to the $S = 1$ total spin angular momentum.

We know that electron and positron annhilate each other, so this state decays into something. The rules are E and $\vec{\mathbf{P}}$ conservation. It can't decay into a single photon since the momentum is not conserved. Recall that:

$$C\,|\gamma\rangle = -1 \Longrightarrow C\,|n\gamma\rangle = (-1)^n \tag{1.21}$$

If we are looking for the two photon decay (so positive conjugation) of the positronium, the only possible state is the one with $S = 0$. If we are looking for a three photon decay (so negative conjugation), the only possible state is the one with $S = 1$. This kind of decay has been verified experimentally.
Positronium with state $S = 0$ is also known as **para-positronium**. If the state is $S = 1$, it is also known as **ortho-positronium**. Their medium lifes are:

$$\frac{1}{\tau_{\mathrm{p}}} = \frac{1}{2}\alpha^5 m \qquad\qquad \tau_{\mathrm{p}} = 1.2 \cdot 10^{-10}\ \mathrm{s} \tag{1.22}$$
$$\frac{1}{\tau_{\mathrm{o}}} = \frac{2}{9\pi}(\pi^2 - 9)\alpha^6 m \qquad\qquad \tau_{\mathrm{o}} = 1.4 \cdot 10^{-7}\ \mathrm{s} \tag{1.23}$$

So, when we emit positrons into a gas, $\frac{1}{4}$ of the states decays quickly in τ_{p}, while $\frac{3}{4}$ of the states decays slower in τ_{o}. It is a strange result, but experiment verifi
es it (Berko and Pendleton, 1980).

1.3 Static Quark Model: Charm and Beauty

A beautifully simple way to create any particle, together with its antiparticle, is to annihilate electrons and positrons at high energy. The annihilation results in a short-lived excited state of electromagnetic fi
elds. This state can then re-materialize into any particle-antiparticle pair that couples to electromagnetism and has a total mass less than the total energy of the annihilating e^+e^- system.
By this way, the importance of the positronium state is clear. Moreover, it is linked to the discovery of quark charm and beauty.
Their discovery takes place in 1974 at SPEAR experiment, where by studying the process $e^+e^- \longrightarrow hh, \mu^+\mu^-,\ e^+e^-$, an enormous, very narrow, resonance at about 3.1 GeV was discovered. This resonance would correspond to a new strongly interacting particle.
When they announced this discovery, they learned that the group of Samuel Ting, working at Brookhaven National Laboratory in Upton, New York, had also observed this new particle. Ting's group had studied the reaction $pp \longrightarrow e^+e^- + X$, where the particles X are not observed.
This never observed particle is now called the J/ψ. A few weeks later, the SPEAR group discovered a second narrow resonance at 3686 MeV, the ψ'.
Another group of narrow resonances is found in e^+e^- annihilation at higher energy. The lightest state of this family, called Υ, has a mass of 9600 MeV. It was discovered by the group of Leon Lederman in the reaction $pp \longrightarrow \mu^+\mu^- + X$ at the Fermilab proton accelerator.
Concerning the J/ψ, this particle is given by a quark doublet $c\bar{c}$ called **charmonium**. If this state exists, we will see phenomena like the ones observed with positronium. In the process $e^+e^- \longrightarrow hh$, the highest rate reactions are those in which e^+e^- pair is annihilated by the electromagnetic current $\vec{\mathbf{j}} = \bar{\psi}\vec{\gamma}\psi$ through the matrix element:

$$\langle 0| \vec{\mathbf{J}}(x) \left|e^+e^-\right\rangle \tag{1.24}$$

The current has spin 1, $P = -1$, and $C = -1$. These must also be properties of the annihilating e^+e^- state, and of the new state that is produced. So, all of the ψ and Υ states must have $J^{PC} = 1^{--}$.
The current creates or annihilates a particle and antiparticle at a point in space. So, if these particles are particle-antiparticle bound states, the wavefunctions in these bound states must be nonzero at the origin. Most probably, they would be the 1S, 2S, etc. bound states of a potential problem. If this guess is correct, the states with higher L must also exist. They might be produced in radiative decays of the ψ and Υ states. Indeed, there is an experimental evidence, with the following pattern of states:

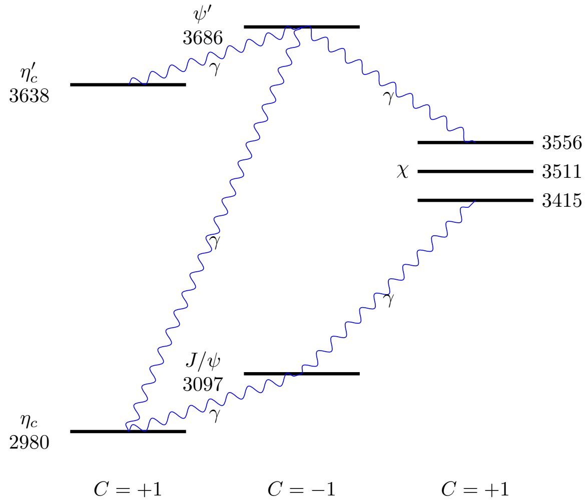

Figure 1.4: Pattern of states for the charmonium.

Remarkably, this reproduces exactly the pattern of the lowest-energy states of positronium and makes even more clear that the analogy to positronium is precise. In the case of the ψ family, the fermion is called the charm quark (c); this quark has a mass of about 1.8 GeV. In the case of the Υ family, the fermion is called the bottom quark (b); this quark has a mass of about 5 GeV.

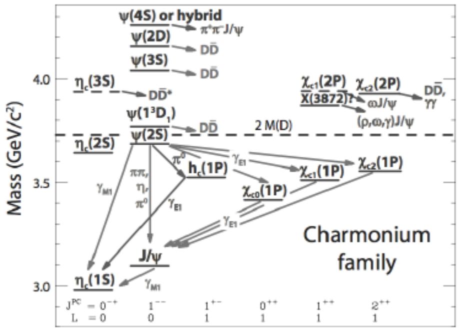

Figure 1.5: Observed states and transitions of the J/ψ system.

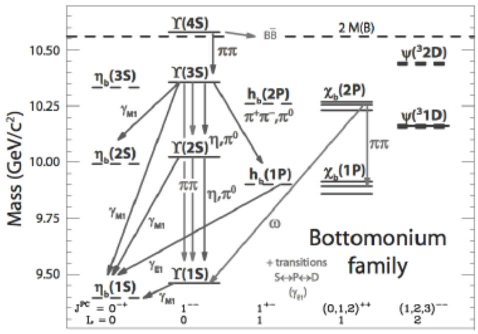

Figure 1.6: Observed states and transitions of the Υ system.

Bibliography