Relocation choice for different homophily preferences: hybrid scenarios for Schelling Model title to define

Rocco Paolillo, Andreas Flache

June 2020

Abstract

[shorten to 200 wrds] Schelling's model of residential segregation famously showed how high levels of residential segregation can emerge as unintended outcome of the interplay of individual relocations of actors who hold relatively mild ethnic preferences. Most of the work building on this model neglected two forms of heterogeneity which seem to become increasingly important empirically in contemporary societies. First, there is considerable heterogeneity of residential preferences not only between but also within ethnic groups, with especially younger, higher educated and more wealthy individuals having less strong preferences for ethnic homophily. Second, most of the research following Schelling focuses on ethnic similarity as relevant to residential preferences. However, recent theoretical and empirical research on spatial sorting emphasizes multidimensionality, as individuals prefer similar others not only regarding ethnicity, but also for social distinctions as shared values or shared status. Extending recent work (Paolillo Lorenz, 2018), we explore the interplay of heterogeneity in both forms of homophily preferences for ethnicity and shared values. Using a discrete choice version of Schelling's model, in which agents differ in their relative weights for ethnic and value similarity in relocation moves, we explore the consequences of deterministic or random relocation choice of agents, in addition to structural conditions of relative group sizes of ethnic and value groups. We find in particular that hybrid segregation patterns can emerge in which ethnically mixed but value homogeneous neighborhoods arise alongside ethnically segregated neighborhoods populated by agents driven more by ethnic homophily. Importantly and contrary to Schelling's model, we show how partial ethnic mixing can arise even if everyone has a preference for more co-ethnics in her neighborhood, all other things being equal.

Keywords (3-5): super-diversity, discrete choice, spatial sorting, Schelling

Introduction

Ethnic or racial residential segregation appears to be one of the most pervasive features of multi-ethnic cities all over the world (REFERENCE). Many possible and interconnected explanations for segregation have been proposed, such as discrimination by landlords, the sorting mechanisms built into housing markets, or income inequality in combination with features of urban geography (REFS here for example Clark Fosset 2008; Fosset 2006). Prominently, Schelling Schelling (1969, 1971) showed with a formal computational model that none of these processes would be strictly necessary to give rise to high levels of segregation.

Instead, segregation could be the unintended outcome of "preference dynamics" (Clark Fosset 2008), starting from the empirically plausible assumption that even if individuals are content with living in a mixed neighborhood, they typically also want at least a certain minimal fraction of co-ethnics nearby. Then, as the argument goes, as soon as some individuals feel outnumbered too much, they leave and settle elsewhere. In the process, movers unintendedly make it more likely that other members of their group follow suit, because after they left, their group becomes even more outnumbered in their prior neighborhood. Multiple modeling studies have since shown how this simple micro-level process can induce a self-reinforcing cascade in which more and more relocations occur until neighborhoods end up being highly segregated (for an overview, see Hegselmann, 2017). The model was widely applied as theoretical framework to explain empirical studies of residential segregation (Mägi et al., 2016; Müller et al., 2018). One key insight from a large number of formal modelling studies exploring variations and extensions of the model of preference dynamics is the astonishing robustness of its main result. High levels of segregation have been shown to arise as macro-level outcome if individual residents hold "mild" or even "integrationist" (Zhang, 2004) preferences also when other factors are included, such as randomness in residential choices (Bruch Mare 2006; Van de Rijt et al 2009), multiple groups of different sizes (Fosset, 2006), housing prices and income differences (Fosset, 2006; Zhang 2004), or empirically realistic spatial structures of real cities (Benenson ea 2009).

Preference dynamics have also been recognized as important element in understanding empirically observed segregation patterns (Clark, 2015). Clark and Fosset (2008), for example, characterize the empirically measured neighborhood preferences of different racial groups in a large-scale U.S. survey study such that "none of groups has a preference distribution that is compatible with that of any other group" (p. 4111) under the dynamics of a Schelling-type model. Their theoretical modelling work, incorporating empirically informed assumptions about ethnic preferences, evokes the impression that sharp ethnic residential segregation is almost inevitable in multi-ethnic cities as long as preference dynamics have a significant role to play in residential choices (Other fittings refs here?).

Yet, despite the strong theoretical evidence that preference dynamics should generate robust and high levels of ethnic segregation under empirically realistic conditions, empirical studies of recent trends in segregation suggest a somewhat different picture. Not only do U.S. studies point to declining levels of racial

segregation in recent decades (e.g. Glaeser and Vigdor 2012), but mixed neighborhoods increasingly start to arise in multi-ethnic cities (Clark, 2015; Lee et al 2012). In part, it has been suggested, this trend can be attributed to changing preference patterns. Goldman (2012), for example, finds evidence of reduced racial prejudice in society as a whole, a trend that seems to extend to residential ethnic preferences (Xie Zhou, 2012). This pattern is also reflected in studies from Europe (Blokland et al 2009). On the whole, it appears that younger and more highly educated citizens have increasingly more tolerant ethnic preferences when it comes to residential choice. A further important implication is that ethnic preferences in residential choices are highly heterogeneous, not only within but also between different ethnic groups. Clark, 1991 relocation histories of young adults: moving to more integrated neighborhoods, racial differences with Whites relocating to highest concentration neighborhoods than minorities. Differences between ethnicities, educational level of parents has more predictive power than income. Also educational level of individuals to relocate to better neighborhoods. Economic resources are not highly correlated with ethnicity anymore (see Crul "elite", Clark), some minorities have succeded in social mobility more than native counterpart, with transmission of educational and economic capital through generations. As a result, difference between cohorts are less than differences within cohorts. Then preferences can play a role higher than income itself -¿ see conclusions Clark 2019 Bruch2014 and VanGent2019: effect of mixed marriage, more internal and international migration, back to diversification of diversity in Vortec

Formal models of Schelling-type preferences dynamics in residential segregation have recently started to incorporate the insight that individuals differ in the degree of tolerance to local ethnic diversity. These models imposed heterogeneity in the proportion of co-ethnics that individuals find ideal in their neighborhoods, distinguishing between more and less tolerant individuals (Xie and Zhou (2012)Hatna and Benenson (2015)). Interestingly, these studies found that - similar to empirical patterns observed in modern multi-ethnic cities - preference dynamics could give rise to a division between ethnically mixed and segregated neighborhoods co-existing in the same city, together with a selection of more tolerant agents into the mixed neighborhoods. We call this pattern "hybrid segregation" in what follows.

The incorporation of heterogeneity in ethnic preferences has moved computational models of Schelling-type preference dynamics an important step closer towards mirroring complex empirical patterns of segregation. However, there is another important form of preference heterogeneity these models have not taken into account and which could profoundly affect dynamics of segregation. Shared values, defined as common beliefs, preferences or expectations on acceptable behavior induce perceptions of similarity across the boundaries of ethnicity (Wimmer, 2013). Recent empirical studies suggest that a preference for value-similar neighbors may sometimes even dominate preferences for ethnic similarity. For instance,van Gent et al. (2019) show how similarity with neighbors in sociocultural dispositions (i.e. gender balance in household tenure) is a better predictor to leave the neighborhood, compared to ethnic membership and

income similarity. In a similar vein, research on homophily in social networks recently has moved forward to recognize the importance of multidimensional similarity for the formation of social relationships (Block Grund, 2014; Hooijsma et al., 2020). This research implies that dissimilarity in ethnicity might not negatively affect the formation of relationships when compensated for by salient similarities individuals perceive in other categories.

While recent empirical studies seem to adopt the interplay of ethnicity with other social distinctions to explain hybrid segregation in diverse societies, this seems rarely the case in modeling literature. Yet, we argue that this work points to an intriguing new possibility for residential segregation dynamics. The seemingly unstoppable march towards segregation that Schelling-type preference dynamics induce may not only be stopped by higher levels of ethnic tolerance, as suggested by (Xie and Zhou (2012)Hatna and Benenson (2015)). It may also be stopped in a world where individuals still prefer being among co-ethnics, but at the same time hold an even stronger preference for having neighbors with similar values who also happen to be members of other ethnic groups. Given that such a predominance of value-orientation in residential preferences appears to some extent to be correlated with education, income and age, this possibility would offer a new explanation in the framework of Schelling-type preference dynamics of why well-off younger generations appear to increasingly move to more affluent and more ethnically mixed neighborhoods (Clark, 2002; Clark et al., 2018). It would also help to understand why low-income strata seem to become increasingly segregated through generations, meaning that their neighborhoods become progressively both ethnically and economically segregated (Clark, 2002). In line with this pattern, differences due to socio-economic status have increased in residential segregation, while ethnic segregation consistently has decreased from the 60's/70's, showing in particular different patterns of residential choices for members of the same ethnic group (Clark, 2015).

In this paper, we propose a formal computational model of Schelling-type preference dynamics that incorporates the possibility that individuals' residential choice are simultaneously driven by preferences for ethnic similarity as well as value similarity to their neighbors. Our study builds on and advances recent modelling work Paolillo and Lorenz (2018) which introduced the interaction between two types of agents present in each of two ethnic groups, agents who are only satisfied with neighborhoods with enough value-similar neighbors, and agents who are only satisfied with enough ethnically-similar neighbors. Following otherwise the classic Schelling framework, each type of agent in their model was given a threshold to indicate the desired concentration of members of same ethnicity for ethnic-oriented agents, and the desired concentration of members of the same shared values for value-oriented. Their results showed a general decrease in ethnic segregation compared to a world populated only be agents with ethnic preference. But they also pointed at more complex patterns, especially a spillover effect for ethnicity-oriented agents in the minority condition who found attractive ethnically mixed neighborhoods formed by value-oriented agents. Inflows of intolerant co-ethnics to ethnically mixed neighborhoods caused such neighborhoods to increase their ethnic homogeneity, until the presence of intol-

erant agents was not tolerated by tolerant agents, causing agents to leave and disruption of neighborhood.

The work of Paolillo and Lorenz (2018) is an important first step towards establishing that multidimensional homophily preferences can indeed induce hybrid segregation patterns in cities with a population heterogeneous in both ethnicity and value orientation. However, their model employs some highly unrealistic assumptions which potentially limit the empirical plausibility of the argument. In this paper, we advance their model in two central aspects. First, we relax the assumption that agents can only hold either a preference for value homogeneity or a preference for ethnic homogeneity and allow instead the residential choices are driven by a mix of both. Second, we replace the threshold decision model inherited from Schelling with a discrete choice model, following recent advances in agent-based modelling of residential choice dynamics (Bruch & Mare, 2006; Hess et al., 2018). This allows to incorporate that agents choose neighborhoods based on relative attractiveness given their preferences, with a stochastic component reflecting unobserved heterogeneity in residential preferences (Bruch Atwell 2015).

Our new model of preference dynamics with two-dimensional homophily preferences allows a systematic exploration of which mix of preferences in a population gives rise to hybrid segregation patterns, combining ethnically homogenous and ethnically heterogeneous neighborhoods with segregation between different value groups, or to 'ordinary' ethnic segregation in which the dividing line primarily ethnicity or a mix of both. We will further explore how this depends on one of the most important dimensions along which ethnically heterogeneous populations differ, the relative size of ethnic majority and minority groups.

In the next section we describe the components of our model, explaining modeling choices within the frame of our goal and previous literature. Section 3 presents results of our computational experiments. In section 4 we discuss and interpreting our results, offering avenues for future research and a tentative reflection on possible implication for desegregation policies in multi-ethnic cities.

¿¿¿¿ I suggest that this part below goes into the model section of the paper: This change allows to define a utility function to model the sensitivity of agents to neighborhood value and ethnic composition (Van de Rijt et al., 2009). Discrete choice let the inclusion of parameters quantifying the importance of either ethnic and value preferences in the relocation choice of agents (Hess et al., 2018; Manski, 1977). This last characteristic in particular can allow to define a hierarchy of preferences of agents and model heterogeneity of agents based on such preferences. These changes allow us to model how decisional processes of agents interact with structural conditions such as ethnic and value distribution of agents' population and change in neighborhood composition, with benefit to formalization of possible mechanisms underlying hybrid segregation

¿¿¿ Further material I could not integrate properly in this line argument. I propose to move this towards the discussion section of the paper. ¿¿¿ But perhaps you see way how to fold this into the intro above, Rocco? This condition results not only from a mix of increased diversity and demographic changes accumulated through years (Crul, 2016) and categories of self-identification elab-

orated within each domain (Vertovec, 2007), but also from inequalities in economic and cultural capital. On the last point, differences can emerge both between cohorts of the same ethnic group as result of integration policies adopted by countries (Crul et al., 2017), and between individuals due to transmission within families and success in social mobility (Esser, 2010). ... A common trait of most Western societies which have experienced decades of migration inflows and further generations of children of migrants is that of super-diversity (Vertovec, 2007). The concept refers not only to increased ethnic diversity, but also to the "diversification of diversity" (Martiniello, 2004, 3, cfr Hollinger 1995)(Hollinger, 1995, cfr.), i.e. how diversity spans along different domains in addition to ethnicity, and how members of the same ethnic group can experience different rates of integration along such domains. ... Although such complex segregation scenarios result from a mix of preferences, constraints and resources (Clark, 2015), in this paper we concentrate on the role of preferences because core to Schelling's dynamics to link individual behavior to change in segregation. In addition to ethnic preferences, we build on how people in diverse societies, belonging to different social distinctions, can adopt arbitrary criteria to define the limits of in-group inclusion, independent of ethnicity and attributing them different salience (Albeda et al., 2018).

Model Description

We developed our model in NetLogo 6.1.1 (Wilensky, 1999) upon Paolillo and Lorenz (2018). The model and its parameters are shortly described in Tab: 1. Agents represent households who relocate in a regular square grid of dimension 51 times 51 with periodic boundary conditions, i.e. a tours world. As in Paolillo and Lorenz (2018), agents are described by 2 static variables: ethnicity and value orientation. They are modeled respectively through color tag (blue for local population and orange for minority population) and shape tag (circle for tolerant value-oriented agents and square for intolerant ethnicity-oriented agents). Ethnicity is conceptualized as an ascribed status so as in the original Schelling (1969). Value orientation represents social distinctions based on shared values and opinions shared independently of ethnic membership and that can influence the relocation preference of agents. In the original Paolillo and Lorenz (2018), tolerant value-oriented would be satisfied with a neighborhood as long as the concentration of agents with same value category would fit their threshold preferences, independently of their ethnicity. On the other way, intolerant ethnicity-oriented would be satisfied with a neighborhood as long as the concentration of agents of same ethnicity would fit their desired threshold, independently of value orientation. In our current model, we introduce a certain degree of ethnic homophily preferences also for ethnic tolerant agents and value homophily preferences for ethnic conservative agents. Nevertheless, we keep a constant of our model that tolerant agents hold lower ethnic preferences of ethnicity-oriented agents. Agents are initially evenly distributed. At each step, an agent randomly selected makes a binary decision between its current

grid and an empty available one, deciding whether relocating or not.

Static agent variable	**Range**	
Ethnicity (*color*)	Blue, Orange	
Value orientation (*shape*)	Square, Circle	
Model Parameters	**Slider**	**Range**
Population density	density	$[0, 0.99]$
Ethnic group ratio	fraction_blue	$[0, 1]$
Value ratio circle/square blue	circle_blue	$[0, 1]$
Value ratio circle/square orange	circle_blue	$[0, 1]$
Determinism ethnic utility (β_e)	e	$[0, 25]$
Determinism value utility (β_v)	v	$[0, 25]$

Table 1: Model parameters

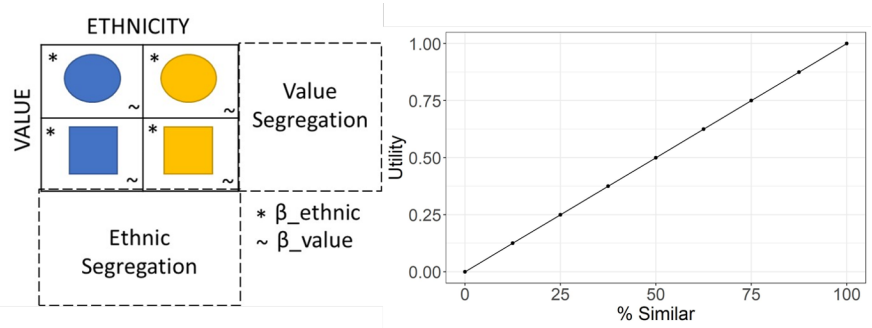

Figure 1: Agents type and utility function

Experiments and Results

Dominant and Secondary preferences

For JMS: We are excluding the case equal ethnic size, and different value distribution, we could leave in the appendix how the mechanism would change in that case, and how some distributions of 4 categories could be replicated in the majority/minority condition

Add justification for the 80% majority

We collect data for 1 Run and computed emerging segregation at steps 1000. The next observations span over the space model in Fig: 2.

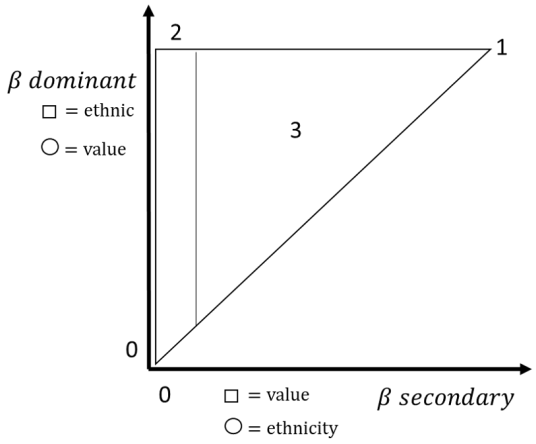

Figure 2: Space Model

Fig: 3 represents the baseline condition where each agent in Fig:1 represents the 25% of the overall population, i.e. the population is equally split into the two ethnic groups and each group (blue and orange) is equally split along the two value-orientation (circle and square). To explore the effect of parameter of determinism β, we impose $\beta_{dominant} = \beta_{secondary}$ for each agent. Thus, it represents the diagonal (1) of Fig:2 X-axis reports $\beta \in [0, 35]$ in each condition, on y-axis the value and ethnic segregation for each agent type. The figure shows how segregation emerges from initial random distribution at $\beta = 0$ linearly until $\beta = 10$, where full ethnic and value segregation is reached for all agents type.

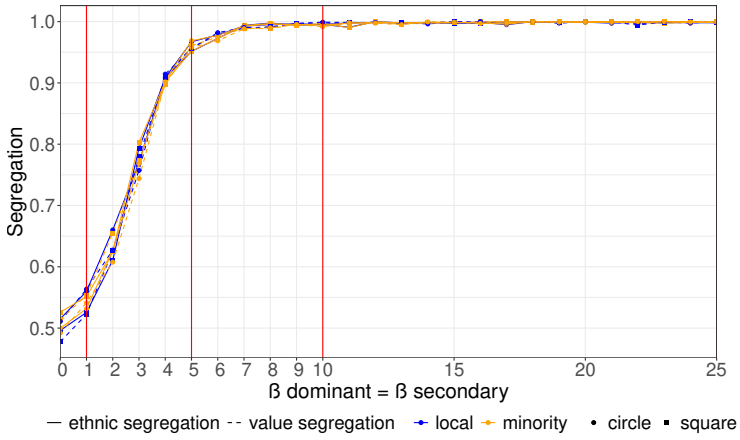

Figure 3: Baseline with $\beta_{dominant} = \beta_{secondary}$. Population is equally split into two ethnic groups: blue = 50%, orange = 50%. Each group is equally split into value orientation: circle = 50%, square = 50%

As Fig: 3 shows, each parameter of determinism β will increase segregation for the referred dimension with linear interpolation, until a stable pattern of full segregation is reached around $\beta \approx 10$. Since we are interested in the effect of different hierarchy of preferences of agents, in Fig: 4 we explore how the segregation dynamics would change due to secondary preferences of agents being totally random.

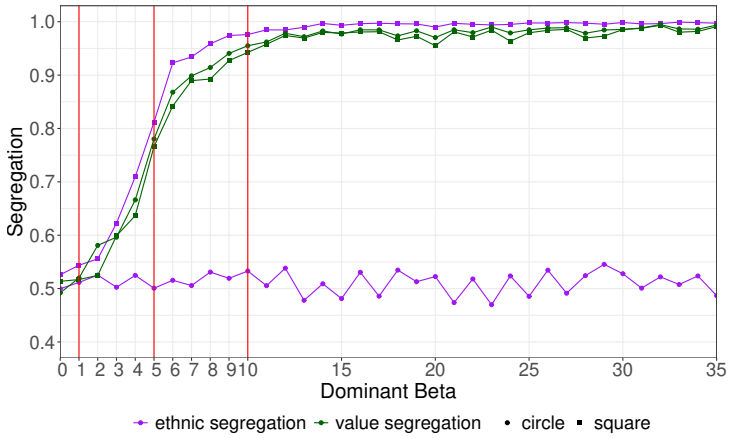

Figure 4: Baseline condition with $\beta_{secondary} = 0$

Fig: 5 shows the whole space of the model as in Fig: 2, and compares levels of dominant and secondary preference for circle and square. Here only results until $\beta = 15$ is reported for sake of reading. Ethnic and value integration for both types of agents is possible only under condition of random relocations. When both dominant and secondary preference reach a critical level of determinism ($\beta \approx 10$, the model replicates Schelling's results for both dimensions, i.e. not only segregation along ethnicity, but clustering of society in both ethnic and value segregation (4 groups segregated). The Paradox of high dominant preference low secondary preference is that only circle whose dominant preference is value become value segregated, and ethnically integrated if they don't care about ethnicity (secondary preference), while square whose dominant preference is ethnicity become both ethnically segregated (as expected) and value segregated, even if they don't care for value preference. While ethnic segregation of square is due to their preference, value segregation occurs because of isolation by circle agents due to their value, while potentially circle of their ethnicity can increase the utility of a neighborhood through ethnic homogeneity. Value preference can create a bridge between different ethnic groups (external integration), but create clashes within the same ethnic group when agents do not share the same value attribution (internal segregation).

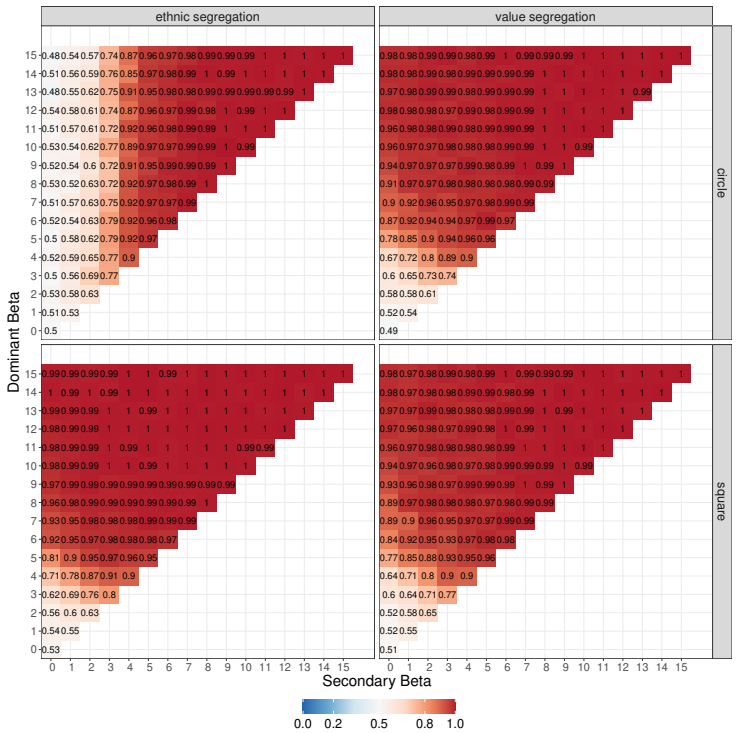

Figure 5: Heatmap baseline dominant and secondary preference

Effect of relative group size

These were results in baseline condition, now change the relative group size. Fig:6 replicates Fig: 4 for the condition of ethnic majority and ethnic minority. Both ethnic groups are equally split into circle value-oriented and square-ethnic oriented. Next graphs show the segregation patterns for type of agents in the condition of ethnic minority, which is the historical condition of segregation studies. Each graph is compared with the scenario where ethnic minority reaches same size of majority group.

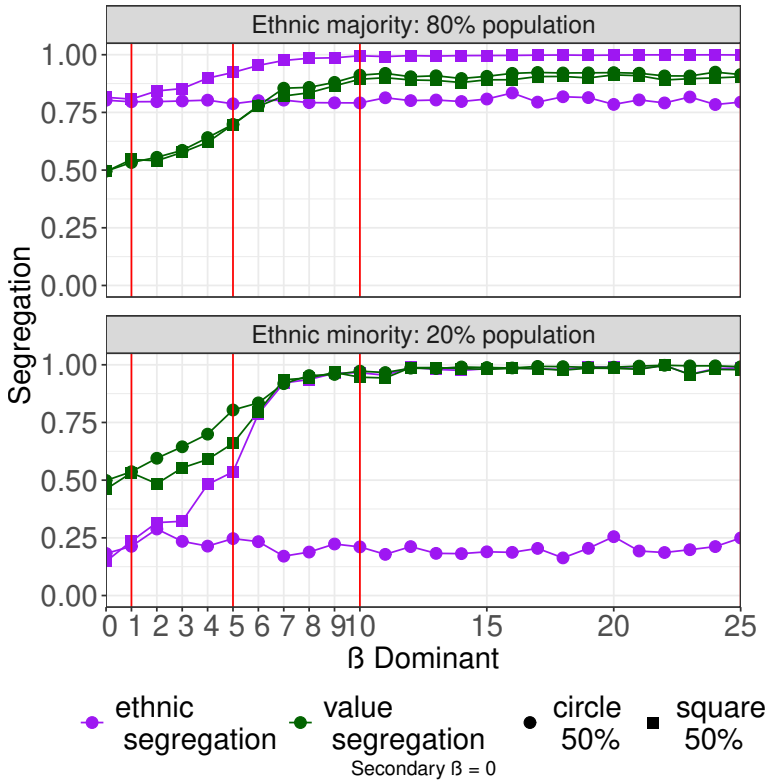

Figure 6: Secondary preference: 0 in asymmetric conditions. Ethnic majority = 80%, ethnic minority = 20%. Each group is equally split into value orientation: circle = 50%, square = 50%

Fig: 7 show behavior of circle agents leaving secondary preference $\beta_{ethnic} = 0$ in the condition of ethnic minority. Moving along the right diagonal, as circle are equally represented in both ethnic groups, for majority circle ethnic segregation is constant, because of numeric superiority. Minority circle are slightly more value segregated than majority counteparts, though this implies to be ethnically assimilated. The distance between value segregation of circle agents is higher in bottom left box, where circle minority the 0.05%. Bottom right box, where circle is the minority value in the majority ethnic population, and majority value in the minority ethnic: ethnic integration appears as of spillover to value segregation of local and minority circle. This only combination breaks the result of ethnic assimilation of minority circle when value segregating.

Equal ethnic size represents the condition of minority increasing in number through generations. As this occurs, ethnic integration occurs for both ethnic groups with circle maximizing value similarity and have same distribution of circle agents (along the right diagonal). This can be a case of how hybrid seg-

regation occurs due to change in ethnic composition of receiving societies. Top left box: value segregation still implies ethnic assimilation of minority orange (now numerically equal), when circle is minority value in the own ethnic group and majority in the other ethnic group. So this observation is symmetrical for local population in the bottom right box

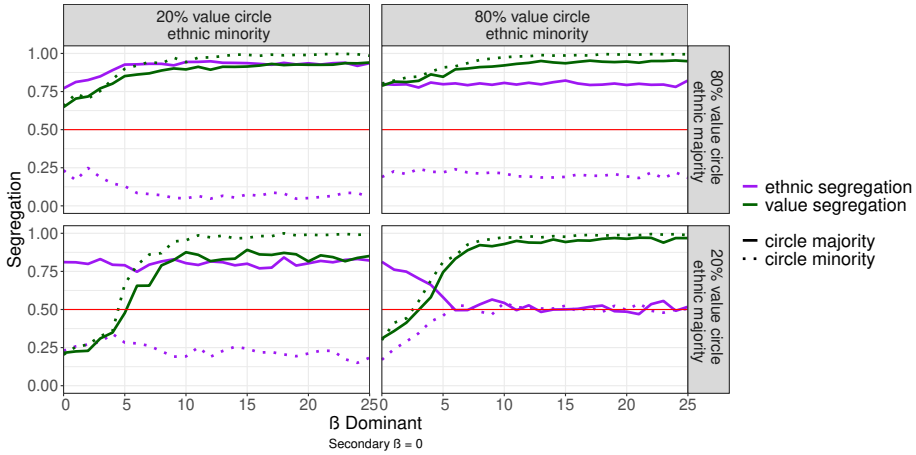

Figure 7: Comparison circle majority vs circle minority. Ethnic majority = 80% population, ethnic minority = 20% population. 2 conditions for each ethnic group: 20% value circle; 80% value circle

Fig: 8 show behavior of square agents leaving secondary preference $\beta_{value} = 0$ in the condition of ethnic minority. Bottom left box: square are majority value in both ethnic groups. For $\beta \leq 5$ assimilation of square minority occurs due to numeric inferiority; As there are less circle to refuse them, value segregation likely to occur here because of numeric superiority Top right box: square value is minority in both ethnic group. Double ethnic and value segregation of minority square confirms the mechanism, since they are refused by both circle (majority in overall population) and square of the other ethnic group. For square majority, the overlapping of ethnic and value segregation does not occur: both circle and square majority represent 40% of population, square have higher probability to maximize ethnic homophily, though relocating close co-ethnics of different value orientation. The most interesting result is for low-right box: square are majority value of minority ethnicity, and majority value in the majority local population. Here for limits of randomness $\beta \approx 5$, both ethnic and value segregation occurs, with value segregation decreasing from initial slight segregation 0.7 for $\beta = 0$. This is unexpected and coherent with preference of agents, and seems robust in simulations run. Still to think more why it happens in this specific combination.

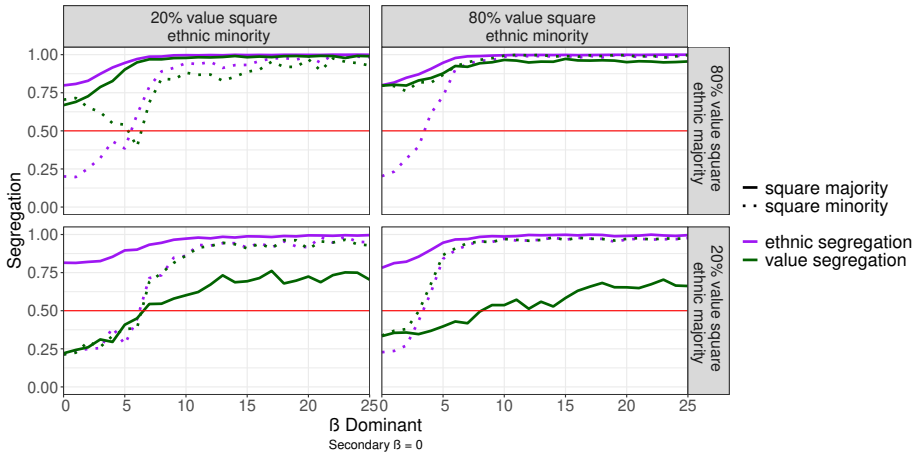

Figure 8: Comparison square majority vs square minority. Ethnic majority = 80% population, ethnic minority = 20% population. 2 conditions for each ethnic group: 20% value square; 80% value square

Conclusions and Discussions

* Recap * Future developmnets: value as dynamic variable and more opinion dynamics like * Comparison with discrete choice models * Role of social distance Clark and Fossett, 2008, we here focus only on aggregated consequences of heterogeneous preferences.

References

Albeda, Y., Tersteeg, A., Oosterlynck, S., & Verschraegen, G. (2018). Symbolic boundary making in super-diverse deprived neighbourhoods. *Tijdschrift voor economische en sociale geografie*, *109*(4), 470–484.

Bruch, E. E., & Mare, R. D. (2006). Neighborhood choice and neighborhood change. *American Journal of sociology*, *112*(3), 667–709.

Clark, W. A. (1991). Residential preferences and neighborhood racial segregation: A test of the schelling segregation model. *Demography*, *28*(1), 1–19.

Clark, W. A. (2002). Residential segregation trends. *Beyond the color line: New perspectives on race and ethnicity in America*, 83–94.

Clark, W. A. (2015). Residential segregation: Recent trends.

Clark, W. A., Andersson, E. K., & Malmberg, B. (2018). What can we learn about changing ethnic diversity from the distributions of mixed-race individuals? *Urban Geography*, *39*(2), 263–281.

Clark, W. A., & Fossett, M. (2008). Understanding the social context of the schelling segregation model. *Proceedings of the National Academy of Sciences*, *105*(11), 4109–4114.

Crul, M. (2016). Super-diversity vs. assimilation: Complex diversity in majority–minority cities challenges the assumptions of assimilation. *Journal of Ethnic and Migration Studies*, *42*(1), 54–68.

Crul, M., Keskiner, E., & Lelie, F. (2017). The upcoming new elite among children of immigrants: A cross-country and cross-sector comparison. Taylor & Francis.

Esser, H. (2010). Assimilation, ethnic stratification, or selective acculturation?. recent theories of the integration of immigrants and the model of intergenerational integration. *Sociologica*, *4*(1), 1–29.

Hatna, E., & Benenson, I. (2015). Combining segregation and integration: Schelling model dynamics for heterogeneous population. *Journal of Artificial Societies and Social Simulation*, *18*(4), 1–15.

Hess, S., Daly, A., & Batley, R. (2018). Revisiting consistency with random utility maximisation: Theory and implications for practical work. *Theory and Decision*, *84*(2), 181–204.

Hollinger, D. (1995). Beyond multiculturalism: Postethnic america. New York: Basic Books.

Mägi, K., Leetmaa, K., Tammaru, T., & van Ham, M. (2016). Types of spatial mobility and change in people's ethnic residential contexts. *Demographic Research*, *34*, 1161–1192.

Manski, C. F. (1977). The structure of random utility models. *Theory and decision*, *8*(3), 229.

Martiniello, M. (2004). How to combine integration and diversities: The challenge of an eu multicultural citizenship.

Müller, T. S., Grund, T. U., & Koskinen, J. H. (2018). Residential segregation and 'ethnic flight' vs. 'ethnic avoidance' in sweden. *European Sociological Review*, *34*(3), 268–285.

Paolillo, R., & Lorenz, J. (2018). How different homophily preferences mitigate and spur ethnic and value segregation: Schelling's model extended. *Advances in Complex Systems*, *21*(06n07), 1850026.

Schelling, T. C. (1969). Models of segregation. *The American Economic Review*, *59*(2), 488–493.

Schelling, T. C. (1971). Dynamic models of segregation. *Journal of mathematical sociology*, *1*(2), 143–186.

Van de Rijt, A., Siegel, D., & Macy, M. (2009). Neighborhood chance and neighborhood change: A comment on bruch and mare. *American Journal of Sociology*, *114*(4), 1166–1180.

van Gent, W., Das, M., & Musterd, S. (2019). Sociocultural, economic and ethnic homogeneity in residential mobility and spatial sorting among couples. *Environment and Planning A: Economy and Space*, *51*(4), 891–912.

Vertovec, S. (2007). Super-diversity and its implications. *Ethnic and Racial Studies*, *30*(6), 1024–1054.

Wilensky, U. (1999). *Netlogo. http://ccl.northwestern.edu/netlogo/.* Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.

Wimmer, A. (2013). *Ethnic boundary making: Institutions, power, networks.* Oxford University Press.

Xie, Y., & Zhou, X. (2012). Modeling individual-level heterogeneity in racial residential segregation. *Proceedings of the National Academy of Sciences*, *109*(29), 11646–11651.