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Residential relocation dynamics: A microeconomic model based on agents' socioeconomic change and learning

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ABSTRACT
This article develops a mathematical model for residential location choice in which previous experience is considered via a dynamic learning process and each agent evaluates location decisions according to their utility. Agents' idiosyncratic behavior is modeled, assuming a stochastic willingness to pay based on dwelling and neighborhood attributes and households' socioeconomics. Additionally, the willingness to pay is affected by the agent's experience of the urban context in previous periods. Numerical examples are given, and simulations are conducted using linear bid functions. Additionally, concepts of urban dynamics are used in the long-term, assuming quasi-equilibrium in the bids considering the externalities of urban configurations. The results are compared with a static outcome corresponding to the equilibrium model of land use. The comparison is performed using indices of urban segregation and both short- and long-term configurations. The studied dynamics show that static modeling does not explain all the features associated with such a configuration of the urban system (in both short and long term).

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1. Introduction

Research regarding urban dynamics processes, particularly in residential relocation decisions, considers the interaction of a variety of agents that change their characteristics and preferences over time. Some key elements associated with the location decision-making of households have not been explicitly considered in the literature about short- and long-term urban land use models, such as memory, learning, habit formation, generation of expectations, uncertainty about resource availability, and social and economic changes, which all contribute to establish a complex system with a structure similar to other social and natural systems (Gunderson & Holling, 2002).

Households have internal dynamics, i.e., life changes such as new children, divorce, job changes, changing levels of education, that affect residential location decisions. Moreover, there are variations in urban land use due to new real estate projects funded by both private and public investment. In addition to these changes prompting relocation, it is also argued that previous experiences induce important inertia in relocation decision-making (Florez, 1998). For example, households tend to remain in the neighborhood in which they have lived or to relocate to a similar housing type (e.g., apartments or houses). In turn, it is also likely for households to maintain the status quo in terms of their residential area because of the knowledge generated by learning about the area and apprehension about change, despite changes in family structure and the environment. Additionally, potential changes in a household's lifecycle belong to a set of possibilities that can be anticipated by households themselves and considered *ex ante* in their decisions (Li & Tu, 2011).

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Such relocation forces are what Huff and Clark (1978) and Schinnar and Stewman (1978) have called cumulative inertia (resistance to movement) and residential stress (impulse to move). In addition to this work, there are other analytical and empirical studies that describe residential relocation dynamics based on agents' previous experiences. For example, in Chen, Chen, and Timmermans (2009), the effect of past location decisions is studied by considering the spatial correlation between new and previous decisions. The results indicate that there are preferences formed over time in connection with lifecycle events; this process is modeled through different extreme value distributions. Habib and Miller (2009) propose a model of residential choice founded on discrete choice theory in which individuals decide to relocate based on a reference point given by their former location. They postulate a utility function, conditional on the amount of incremental gain or loss of each attribute or feature of the property—with respect to the former location—using concepts from risk decision theory (Sugden, 2003; Tversky & Kahneman, 1991). Chen and Lin (2011) develop a lifecycle econometric model for residential location and, using the bounded rationality concepts described in Camerer (1998), show that the marginal valuation of each property attribute is generated by a learning process that depends on past experiences. In addition, they analyze the influence of lifecycle changes (e.g., having children, aging, and martial decisions) on location decisions.

In different types of studies, the relocation decision is analyzed as a two-stage process, first as a relocation probability and then as a search for a new option. For example, Nijkamp, Van Wissen, and Rima (1993) obtain households' probabilities of moving (logit binomial) from their current location based on age changes, changes in the characteristics of goods, and whether the agent owns their housing. Altogether, the probability of moving into a new zone is obtained, and conditional on this probability, the probability of switching to a different type of housing within the area is determined. Other econometric works based on logistic regression or logit binomial and probabilities to move from a current location by means of the utility thresholds obtained using longitudinal data are described in Clark, Huang, and Withers (2003) and Sommers and Rowell (1992).

Lee and Waddell (2010) extend this type of work by including sampling in the second stage of the model that corresponds to the selection of the new housing and creating a set of bounded decision alternatives given the relocation probability. Eluru, Sener, Bhat, Pendyala, and Axhausen (2009) presented an econometric formulation using multinomial-type choices and a system of equations that defines the reason for moving and the new choice duration. They used a retrospective survey to estimate the model parameters.

An important concept found in some of the relocation models is that of transaction costs, which are defined as not only monetary costs but also costs associated with distances, such as social or psychological losses, which are referred to as the disutility of change by Miller and Haroun (2000) and Parker and Filatova (2008), and according to Rust (1994) and Kennan and Walker (2011), are costs that increase with the agent's age.

Moreover, some studies in psychology and sociology show that habit formation and learning, in addition to socioeconomic changes in the lifecycle, are important factors when a household decides about intra-urban mobility (Anderson & Milson, 1989; Aarts, Verplanken, & Knippenberg, 1998; Ritchey, 1976; Rossi, 1955; Verplanken and Van Knippenberg, 1998).

Similar to relocation decisions, there are also dynamic economic models that explain and analyze the consumption of continuous and discrete goods of the agents through learning, habit formation, or memory models (Milani, 2007).

There are also interesting developments regarding dynamic models based on agents that seek to describe economic and urban systems such as real estate and land use markets (e.g., Benenson, 1998; Ettema, 2011; Filatova, 2015). Some of the important features of these models are the capability to analyze disaggregated and heterogeneous data associated with each agent, capturing micro-level interactions, and impacts that agents have on the macro or aggregate level (for example, on the urban distribution).

For instance, Benenson (1998) developed a multi-agent model in which agents could change their residential behavior, depending on the properties of his/her neighborhood, residents, and the entire city.

The agent is characterized as having an economic and cultural identity; in this manner, the model considers the consequences of these features of the individual's interactions. Ettema (2011) developed a multi-agent model of residential relocation that describes the decision to search for another home, the process of negotiation between sellers and buyers, and the pricing result. Some relevant aspects of the approach are its incorporation of the effects of the household lifecycle, which leads to preferences dynamics, and modeling and updating the perception of the availability and prices of homes.

Finally, there are some micro simulation models that attempt to model these dynamics and their influence on household choice (for example, Raju et al., 1998; Irwin, 2010). An important hypothesis of these models is that the evolution of the urban systems must be analyzed because urban systems evolve in a manner dependent on the historical path or with a memory processes that may not be fully captured by conventional static equilibrium models (Miller, 1996). For instance, Miller and Haroun (2000), Miller, Hunt, Abraham, and Salvini (2004), and Salvini and Miller (2005) developed a model called ILUTE that simulates the behavior of individual agents in time and space. The overall purpose of ILUTE is to simulate the evolution of an entire urban region for an extended period of time while analyzing the effects of changes in the transport system, the real estate market, and urban policies.

Given the previous findings of empirical and analytical articles, it can be concluded that the dynamics of learning and household lifecycles are important factors in urban processes such as residential relocation. Due to the lack of literature in the field of urban economy associated with urban equilibrium that integrates household learning and memory process and habit formation, we propose a microeconomic model of residential location choice that incorporates some of these aspects. To be more exact, the originality of our work is the development, design, and formulation of an urban equilibrium problem of residential location that is able to capture the dynamic behavior described, including the learning process and agents' changes. In the next section, we propose a discrete and microeconomic model of residential location choice that incorporates the utility gained from past experiences and the transition between periods; likewise, the households' willingness to pay is formulated for this case. In the fourth section, numerical examples are developed to illustrate the effects over both the short and long term in urban configurations. Finally, conclusions and a final discussion are presented.

2. Mathematical and microeconomic fundamentals

This section describes the basic urban economic guidelines needed to understand the memory and learning microeconomic model developed in Section 3. Urban economics proposes two main approaches to explain residential location decisions. The first methodology called "bid-auction," assumes an auction-type market in which agents bid for different locations, which are then assigned by the seller to the highest bidder (e.g., Alonso, 1964; Parker & Filatova, 2008). The second approach, called "choice," assumes that agents choose locations that maximize their utility level (Anas, 1982; Walker & Li, 2007; Dendoncker, Rounsevell, & Bogaert, 2007; Guo & Bhat, 2007; McFadden, 1978).

In the present section, the two approaches are briefly explained in a deterministic and stochastic context. Following Martinez (1992) and Martinez and Araya (2000), let us consider a household type $h \in H$ in period $t \in T$ choosing real estate $i \in D$ that maximizes its utility. Consumers are classified into socioeconomically homogeneous categories (index h), and the supply is described by location clusters (index i). The household values a property good, indexed by i in D, using a set of attributes denoted by vector Z_i, which includes dwelling characteristics, accessibility, and neighborhood quality (Guo & Bhat, 2007; Louviere & Timmermans, 1990). Neighborhood quality includes the location of residents and commercial activities that affect the utility of other locators in the neighborhood, called location externalities (Martínez & Henríquez, 2007). The following classical microeconomic formulation represents the household's location problem:

$$\max_i \max_x U_h^t(x, Z_i^t) \text{ subject to } p^t x + r_i^t \leq I_h^t. \tag{1}$$

where r_i^t is the rent for property (i), I_h^t is the exogenous income of household h, and p^t is the price vector associated with the set of market goods x. The indirect utility function associated with the solution of the problem (1) for real estate (i) is

$$V_{hi}^t \equiv V_h^t\left(I_h^t - r_i^t, Z_i^t, p^t\right). \tag{2}$$

In a stochastic environment under a multinomial logit decision protocol, if it is assumed that $V_{hi}^t = \hat{V}_{hi}^t + e_{hi}^t$, where e_{hi}^t is a random error and obeys a Gumbel distribution, then the probability that household h chooses an option i is given by (De Palma & Lefevre, 1983)

$$P_{i|h}^t = \frac{\exp\left(\mu V_{hi}^t\right)}{\sum_j \exp\left(\mu V_{hj}^t\right)}.$$

For a given utility level $\bar{U}_h$, if the inverse function of V_{hi}^t exists with respect to the rent variable, then we obtain

$$r_i^t = I_h^t - V_h^{-1}\left(\bar{U}_h, Z_i^t, p^t\right),$$

which represents the value that the consumer is willing to pay for location *(i)* to attain a utility level $\bar{U}_h$ (Ellickson, 1981), denoted by (bid problem)

$$B_{hi}^t = I_h^t - V_h^{-1}\left(\bar{U}_h, Z_i^t, p^t\right). \tag{3}$$

It is feasible to prove that if the utility function is quasilinear, then the indirect utility function and the willingness to pay are, respectively (see López-Ospina, Martínez, & Cortés, 2016),

$$V_{hi}^t\left(Z_i^t, I_h^t - r_i^t\right) = \lambda_h^t * \left(I_h^t - r_i^t\right) + \lambda_h^t b_{hi}^t\left(Z_i^t\right)$$

$$B_{hi}^t = I_h^t + b_{hi}^t\left(Z_i^t\right) - \frac{\bar{U}_h^t}{\lambda_h^t},$$

where λ_h^t is the marginal utility of income and $b_{hi}^t\left(Z_i^t\right)$ is a function that measures the household *h's* valuation of the property attributes.

Note that in the problem of choice of maximal utility, the term $\lambda_h^t * I_h^t$ does not affect the order of the utilities of the alternatives. Thus, the utility function $V_{hi}^t\left(Z_i^t, I_h^t - r_i^t\right)$can be truncated in the said term. Consequently, the expression generally used for econometrics applications of this type of model is

$$V_{hi}^t\left(Z_i^t, I_h^t - r_i^t\right) = -\lambda_h^t r_i^t + \lambda_h^t b_{hi}^t\left(Z_i^t\right). \tag{4}$$

Following Martínez and Henríquez (2007), we define B_{hi}^t as

$$B_{hi}^t = a_h^t + b_{hi}^t\left(Z_i^t\right), \tag{5}$$

where $a_h^t = I_h^t - \frac{\bar{U}_h^t}{\lambda_h^t}$, and the utility level is obtained from the market equilibrium defined by the condition that every agent should be located in the city. Additionally, assuming stochastic bids, $\hat{B}_{hi}^t = B_{hi}^t + \varepsilon_{hi}^t$, with ε_{hi}^t being identically and independently Gumbel distributed with dispersion parameter μ and B_{hi}^t being the deterministic bid. Under these assumptions, the probability that household h will be the highest bidder for property i in period t is

$$Q_{h|i}^t = \frac{\exp\left(\mu B_{hi}^t\right)}{\sum_{g \in C} \exp\left(\mu B_{gi}^t\right)}. \tag{6}$$

For household clusters of different sizes given by H_h^t, McFadden (1978) proposes a size correction that yields

$$Q^t_{h|i} = \frac{H^t_h \exp\left(\mu B^t_{hi}\right)}{\sum_{g \in C} H^t_g \exp\left(\mu B^t_{gi}\right)} (7).$$

.
In this manner, the number of type h households allocated to type i locations in period t is $H^t_{hi} = S^t_i Q^t_{h|i}$, where S^t_i is the exogenous supply, i.e., the number of real estate units of type i in period t.

This resulting bid-auction probability was first proposed by Ellickson (1981) and extended by Martinez (1992).

The rent at each location is obtained endogenously at equilibrium as the expected value of the maximum bid, which under independent and indentically distributed (i.i.d.) Gumbel distributed bids is given by the log sum expression as follows:

$$r^t_i = \frac{1}{\mu}\left\{\ln\left(\sum_{h \in C} H^t_h exp\left(\mu B^t_{hi}\right) + \gamma\right)\right\}, \forall i \tag{8}$$

where γ is Euler's constant. Finally, the residential market equilibrium condition is that every household is located in the available housing, which is attained by imposing $\sum_i Q^t_{h|i} S^t_i = H^t_h$, where S^t_i is quantity of exogenous supply of real estate (i). For this condition to hold, households must adjust their utilities to comply with the following equation:

$$a^t_h = -\frac{1}{\mu} ln\left(\sum_{i \in I} S^t_i \exp\left(\mu\left(b^t_{hi} - r^t_i\right)\right)\right), \forall h, \tag{9}$$

which represents a fixed-point system of equations whose solution yields the maximum utility level attainable in equilibrium by each cluster of households.

3. Memory effect and learning

As described previously, this work is based on the core elements of the The Random Bidding and Supply Land Use Equilibrium Model (RB&SM) (Martínez & Henríquez, 2007) that, in turn, is urban equilibrium based on the stochastic logit decision protocol approach. The reasons for using the bid approach are that it allows us to analyze the impact of learning on the incomes of form processes endogenously, not only for the system in equilibrium.

We propose a microeconomic theoretical model of housing choice that incorporates learning processes. This proposed microeconomic and equilibrium formulation, although innovative, is based on empirical and econometric works that incorporate these dynamics as a key factor in urban relocation processes. First, we present a deterministic model of discrete choices. Additionally, we develop a stochastic version of this deterministic model.

3.1. Deterministic process

In this section, based on the empirical, econometric, and sociological literature described above, we propose a simple dynamic microeconomic model of endogenous learning for each household associated with the classical urban process of residential choice. We assume that memory (inertia process) of past experiences is represented in the present time as a perceived utility, which affects the current utility in a linear combination. This current-memory utility is maximized in the following discrete choice problem for an agent h that belongs to the socioeconomic cluster H for the property (i) in each period t:

$$\max_i \max_x \alpha_h U^t_h\left(x, Z^t_i\right) + (1 - \alpha_h) m^{t-1}_{hi} subject\ to\ p^t x + r^t_i = I^t_h, \tag{10}$$

where $U_h^t(x, Z_i^t)$ is the utility associated with the consumption goods *x and real estate* attributes for the property ***(i)*** denoted by vector Z_i^t in the period *t*, which includes dwelling characteristics, accessibility, and neighborhood quality (Louviere & Timmermans, 1990). Additionally, r_i^t is the rent for the property (*i*), I_h^t is the exogenous income of the household h, and p^t is the price vector associated with a set of market goods *x*. In particular, m_{hi}^{t-1} hi is the memory, inertia, learning accumulation factor from past experiences in the real estate (i). $\alpha_h$is a valuation factor between current utility and memory in the decision period *t*.

Note that the microeconomic problem (10) is equivalent to some formulations of consumption's inertia or memory in other fields (e.g., Cantillo, Ortúzar, & Williams, 2007).

This approach assumes that the utility builds up in a continuous learning process formed by the accumulation of information over a long period of time. For example, the set of activities performed in previous periods generates a learning process on attractiveness and accessibility measures (see, e.g., Geurs, Montis, & Reggiani, 2015; Martínez, 1995; Jara-Díaz & Martínez, 1999; Páez, Scott, & Morency, 2012). In the proposed formulation, depending on the positive memory valuation, it is possible to generate a slow process of relocation because the memory factor in (1) represents the inertia to move.

We note that despite the dynamic effect introduced by memory, problem (1) remains static. However, our contribution is based on a static microeconomic formulation of an equilibrium problem, solved as a numeric solution based on previous algorithms, such as RB&SM (Martínez & Henríquez, 2007) and its dynamic extension (Martínez & Hurtubia, 2006).

Based on different experimental and technical articles and works, the memory utility may be specified in different manners, e.g., as a short-term or myopic memory (mm_{hi}^{t-1}), with agents carrying memory only from the location experienced in the past period. Mathematically,

$$mm_{hi}^{t-1} = \mathbb{I}_{hi}^{t-1} V_{hi}^{t-1}\left(Z_i^{t-1}, r_i^{t-1}\right) = \mathbb{I}_{hi}^{t-1} \lambda_h^{t-1} * \left(b_{hi}\left(Z_i^{t-1} - r_i^{t-1}\right)\right) \tag{11}$$

where

$$\mathbb{I}_{hi}^{t-1} = \begin{cases} 1 \ \textit{if int} - 1 \ \textit{the agent h was located in i} \\ \qquad\qquad 0 \ \textit{otherwise} \end{cases},$$

and $\lambda_h^{t-1} * \left(b_{hi}\left(Z_i^{t-1} - r_i^{t-1}\right)\right)$ is the utility obtained in the period t-1 (see Eq. 4).

Thus, previous experience increases the current utility if the past experience was positive; otherwise, it decreases the utility. Although in formulation (11) it seems that the agent makes decisions based only on the present and immediate past periods, there is a clear dependence on all past periods because in t-1, his/her decision was based on the two preceding periods, and so on.

In a long-term memory, households retain memory from their whole history:

$$m_{hi}^{t-1} = \sum_{k \le t} (1 - \alpha_h)^{t-k-1} mm_{hi}^k = \sum_{k \le t} (1 - \alpha_h)^{t-k-1} \mathbb{I}_{hi}^k V_{hi}^k\left(Z_i^k, r_i^k\right) \tag{12}$$

where $\sum_{k \le t} (1 - \alpha_h)^{t-k-1} mm_{hi}^k$ is a weighted average over past experiences discounted at a rate $(1 - \alpha_h)$. Another option is the average of the previous utilities:

$$m_{hi}^{t-1} = \frac{1}{t-1} \sum_{k \le t} mm_{hi}^k = \frac{1}{t-1} \sum_{k \le t} \mathbb{I}_{hi}^k V_{hi}^k\left(Z_i^k, r_i^k\right) \tag{13}$$

We note that expression (13) becomes unrealistic for experiences too far in the past due to changes in urban land use over such a long period, such as in real estate supply, construction of public and private property, changes in transportation systems and infrastructure, and aging of durable infrastructure, all of which generate considerable variation with respect to previously known attributes. Still, expressions (12) and (13) enrich the utility formation with past experiences, thus inducing inertia ($mm_{hi}^k > 0$) or impulse ($mm_{hi}^k < 0$) to move.

Assuming that U_h^t is quasi-linear, then the functional form of indirect utility function with memory of problem (10) is

$$V_{hi}^t = \alpha_h \left\{ \lambda_h^t \left(I_h^t - r_i^t \right) + \lambda_h^t b_{hi}^t \left(Z_i^t \right) \right\} + (1 - \alpha_h) m_{hi}^t.$$

Noting that $(1 - \alpha_h) m_{hi}^t$ is a constant in the consumer's problem (10) at t and inverting in rents, we obtain the agent h's bid:

$$\bar{B}_{hi}^t = B_{hi}^t + \frac{1 - \alpha_h}{\alpha_h} \frac{m_{hi}^{t-1}}{\lambda_h^t} \tag{14}$$

where $m_{hi}^{t-1}$is the utility of property (i) in t-1 converted into a monetary value by the marginal utility of income level at current time λ_h^t. That is, in a process of memory valuation, each household updates the utilities of prior experiences to the household's current income.

It is worth noting that $B_{hi}^t = a_h^t + b_{hi}^t \left(Z_i^t \right)$, with $a_h^t = I_h^t - \frac{U_h^t}{\alpha_h \lambda_h^t}$, thus implying that in equilibrium, utility U_h^t will be conditional on the memory level, i.e., $U_h^t \left(m_{hi}^{t-1} \right)$, and therefore, each agent h will reach a different utility in equilibrium depending on their past experiences.

Thus, the proposed formulation provides a useful tool in micro-simulation models, in which each agent can be analyzed in detail with their carrying history (residential relocation's trajectory). To simplify our analysis, in what follows, we consider a short term or myopic memory.

Additionally, the memory effect has a differentiated impact on the assessment of households to urban projects and policies because two agents of an urban system with identical conditions (residential location and household socioeconomics) in period t can perceive the utility of a new public facility differently because of their different past experiences.

In this sense, a social planner should include within the analysis not only the perceived utility in the period but also how the location attributes improve or worsen with respect to the location in previous periods.

3.2. Dynamic transition of agents

In addition to memory, we now consider a second dynamic in the location process caused by the evolution of a household's characteristics, such as changing jobs or income or changes in family structure. This makes households potentially transit from one cluster to another between two modeling periods, which implies that their behavior changes, i.e., their utility function changes. These types of dynamics have been analyzed in empirical and social articles, as described previously.

Consider a household that changes clusters between period t-1 and t from cluster g in t-1 to cluster h in t, denoted by (h; g), then, considering memory, the consumer's problem and the associated willingness to pay for a location (i) is

$$\max_i \max_x \alpha_h U_h^t \left(x, Z_i^t \right) + (1 - \alpha_h) m_{gi}^{t-1} \textit{subject to } p^t x + r_i^t = I_h^t \tag{15}$$

$$B_{(hg)i}^t = a_{hg}^t + b_{hi}^t \left(Z_i^t \right) + \frac{1 - \alpha_h}{\alpha_h} \frac{m_{gi}^{t-1}}{\lambda_h^t} \tag{16}$$

where $a_h^t = I_h^t - \frac{U_{hg}^t}{\alpha_h \lambda_h^t}$ and m_{gi}^{t-1} represents the transition from cluster g to h. Therefore, as mentioned above, two agents having the same socioeconomic characteristics and located in equal real estates in period t can have different levels of utility because of their experience or their evolution in socio-economic characteristics in previous periods. To keep track of this evolution path of household characteristics, we define a cluster by the following triplet (h; g; j), which reads as follows: a household currently in cluster h belonged to cluster g in period t-1 and was found at residential location j.

For such households, the bid function for real estate (i) in a period t with myopic memory is

$$B_{(hgj)i}^t = a_{hgj}^t + b_{hi}^t \left(Z_i^t \right) + 1_{\{i=j\}} \frac{1 - \alpha_h}{\alpha_h} \frac{\lambda_{gi}^{t-1}}{\lambda_h^t} \left(b_{hi}^{t-1} - r_i^{t-1} \right), \tag{17}$$

where

$$1_{\{i=j\}} = \begin{cases} 1, if\ i = j \\ 0, if\ i \neq j \end{cases} \tag{18}$$

where now $\frac{\lambda_{gi}^{t-1}}{\lambda_h^t}$ updates willingness to pay for the temporal transition. Note that if income has decreased, which means $I_g^{t-1} > I_h^t$ and $\lambda_g^{t-1} < \lambda_h^t$, then the value of memory is reduced; it is the opposite if income has increased. Furthermore, note that $\lambda_{gi}^{t-1}\left(b_{hi}^{t-1} - r_i^{t-1}\right)$is the utility obtained at i in period t-1.

Note that formulations (16) to (18) introduce dynamic elements in the household and urban behavior caused by socioeconomic evolution and individual learning.

Furthermore, an agent in cluster h being located in j in t-1 will relocate in period t to real estate i, with $j \neq i$, if $\alpha_h V_{hj}^t + (1-\alpha_h) m_{hj}^{t-1} < \alpha_h V_{hi}^t$, which yields the following relocation condition:

$$V_{hi}^t - V_{hj}^t > \frac{1-\alpha_h}{\alpha_h} m_{hj}^{t-1} \equiv ac_{hj} \tag{19}$$

Thus, the value ac_{hj} could be interpreted as an adjustment cost or pull factor, or the lower bound of differential utilities to make the relocation feasible. It also defines a resistance factor to change or transaction cost, which provides a formal microeconomic interpretation of what Miller and Haroun (2000) and Ettema (2011) called adjustment to relocation costs, which was defined by them in terms of monetary, psychological, and social losses. Additionally, $\frac{1-\alpha_h}{\alpha_h}$ represents the valuation of the past, such that if $\frac{1-\alpha_h}{\alpha_h} \geq 1$ then$\alpha_h \leq 0.5$, thus indicating that past experiences are valued comparatively more than present utility. This can be interpreted as agent h's risk aversion. In addition, if $r_i^t > b_{hi}^t$, we have $m_{hj}^{t-1} < 0$, indicating a negative consumer surplus in location i caused either by increased rent or a change in the valuation of the site. Such a change in value often happens due to changes in land use because of the arrival of unwanted activities in the neighborhood (e.g., prisons or landfills).

3.3. Stochastic modeling

To define the stochastic model, we will assume that there is an exogenous transition matrix of all agents in the system, denoted by $P(h^t|g^{t-1})$, that defines the aggregate percentage of population that transit between clusters g and h in periods t-1 and t as a result of lifecycle changes, such as income, education, children, job changes, macro decision-making, and exogenous economic shocks to the household. This transition matrix is assumed exogenous in this model. Thus, the number of agents belonging to the cluster defined by *(h; g; j)* in period t is

$$H_{hgj}^t = P\left(h^t|g^{t-1}\right) H_{gj}^{t-1} \tag{20}$$

where H_{gj}^{t-1} is the distribution of residents in period t-1.

Now, we introduce the idiosyncratic variability of consumers' behavior, assuming that the willingness to pay $B_{(hgj)i}^t$ is a random variable that satisfies an identical and independent Gumbel distribution with scale parameterμ. Additionally, in each time period, the following budget constraint must hold for the household to participate in the auction of real estate i:

$$\bar{B}_{(hgj)i}^t \leq I_h^t \forall h, g, j, t \tag{21}$$

Constraints (21) are important insofar as the effect of memory may be very high in some properties, but willingness to pay cannot exceed the current income. For example, income may decrease because of an economic shock, such as a job loss or increased spending on consumer durables (e.g., car or education). In this case, even though memory prompts the household to relocate, the income constraint may prevent it from making that decision. These restrictions may be modeled by including the multinomial constrained logit approximation (Martínez, Aguila, & Hurtubia, 2009;

López-Ospina et al., 2016) with *"cut-off"* functions defined as the probability that a household of a cluster h complies with income constraint at each location i, denoted by $\phi^t_{hgj(i)}$ and given by the binomial formulae

$$\phi^t_{hgj(i)} = \frac{1}{1+\exp\left(w\left(\bar{B}^t_{(hgj)i} - I^t_h + \zeta\right)\right)} = \begin{cases} 1, if\bar{B}^t_{(hgj)i} - I^t_h \to -\infty \\ \eta, if\bar{B}^t_{(hgj)i} - I^t_h \to 0 \end{cases} \tag{22}$$

The value of w determines how fast the probability value approaches the extreme value zero as the bid approaches the income constraint. The parameter ζ represents a tolerance of violating the income restriction, such that when the bid equals income, then $\phi^t_{hgj(i)} = \eta$; to achieve this, ζ is defined by

$$\zeta = \frac{1}{w}\ln((1-\eta)/\eta). \tag{23}$$

It follows that the number of bidders in the location i's auction at period t meeting the feasibility condition is $P(h^t|g^{t-1})H^{t-1}_{gj}\phi^t_{hgj(i)}$ for agent type (h; g; j).

Considering heterogeneous household sizes (McFadden, 1978), the probability that household (h, g, j) will be the highest bidder in real estate (i) is

$$Q^t_{hgj(i)} = \frac{P(h^t|g^{t-1})H^{t-1}_{gj}\phi^t_{hgj(i)}\exp\left(\mu\bar{B}^t_{(hgj)i}\right)}{A^t_i}, \tag{24}$$

where A^t_i is

$$A_i = \sum_{h'g'j'} P(h'|g')H^{t-1}_{g'j'}\phi^t_{h'g'j'(i)}\exp\left(\mu\bar{B}^t_{(h'g'j')i}\right).$$

Thus, the aggregate probability that a household type h will be the highest bidder and locates on property (i) is

$$Q^t_{h(i)} = \sum_{gj} Q^t_{hgj(i)} \tag{25}$$

In addition, the probability of relocation for (h, g, j) in the period t is

$$Prob(move|hgj) = 1 - Q_{hgj(i)} \tag{26}$$

which captures both urban system changes and household lifecycle variations. Thus, *Prob(move|hgj)* integrates dissatisfaction with current property and the impulse to search for a new home.

Additionally, the aggregate population distribution in the period t is

$$H^t_{hi} = Q^t_{h(i)} * S^t_i \tag{27}$$

S^t_i is quantity of exogenous supply for real estate (i) in period t. Similar to (8), the rent in (i) is obtained as follows:

$$r^t_i = \frac{1}{\mu}\ln\left(A^t_i + \gamma\right) \tag{28}$$

where γ is Euler's constant. Note that r^t_i depends on r^{t-1}_i because A^t_i preserves intertemporal information about bids; this is because A^t_i depends on all $\bar{B}^t_{(hgj)i}$ and each $\bar{B}^t_{(hgj)i}$ is in turn a function of the rent r^{t-1}_i (see Eq. (16)).

In each time period t, we assume that the following equilibrium condition holds:

$$\sum_{gj} Q^t_{hgj(i)} * S^t_i = P\left(h^t|g^{t-1}\right)H^{t-1}_{gj}, \forall(h,g,j) \tag{29}$$

which, solving for a^t_{hgj}(Eq. (17)), yield expressions for the utility level attainable in period t by an agent (h, g, j):

$$a^t_{hgj} - \frac{1}{\mu}\ln\left(\sum_i S^t_i \phi^t_{hgj(i)} \exp\left(\mu\left\{b^t_{hi} + \frac{1-\alpha_h}{\alpha_h}\frac{m^{t-1}_{g,j=i}}{\lambda^t_h} - r^t_i\right\}\right)\right), \forall(hgj) \tag{30}$$

Because $\phi^t_{hgj(i)}$ depends on a^t_{hgj}, and rent r^t_i depends on all residents' utilities$a^t_{h'g'j'}$, it follows that (34) defines a nonlinear fixed point system of equations in each period t. This equation system is the same problem of the static equilibrium problem in a Random Bidding and Supply Land Use Equilibrium Model (RB&SM) and is solved with the same algorithm.

To conclude this section, we note that the deterministic and stochastic formulations of this learning model capture important elements associated with the relocation process modeled as a dependence on past decision utilities. These past and present assessments of real estate properties generate analytical results regarding the intertemporal interaction between rents, population distributions, equilibrium utility levels, and relocation probabilities. Additionally, we note that the memory of one household influences its current relocation choice, which in turn induces neighbor changes at that location and therefore through location externalities influences all other households' locations choices. Thus, the memory factor disseminates across the population such that each household's memory eventually influences all other households' choices.

4. Numerical examples: Study of residential segregation

The numerical example of the proposed model is specified with bid functions based on the theory of dynamic models of residential segregation (see Fossett, 2006, 2011; Willms & Paterson, 1995; Zhang, 2004; Zhang, 2011; Grauwin et al., 2012). These types of works seek to analyze policies that reduce spatial segregation or promote inclusive cities in urban areas through simple numerical examples in which the agents' utility level depends on their neighbors' in a given period, with relocation rules given that utility (e.g., Feitosa et al., 2011; Wang, 2015). In O'Sullivan (2009), triangular bid functions (asymmetric preferences for integration) are used in deterministic and disaggregated approaches. Here, we analyze the urban system dynamics (long-term and short-term configuration) with the inclusion of learning processes using the logit model presented above. Specifically, we study the following question: if the agents change clusters over time (lifecycle evolution) and learn from their location experiences, does this process induce inertia affecting (reducing) the spatial segregation in the short and long run compared with the no-memory case?

We assume two equally sized zones and the only differentiating factor among the dwellings is who inhabits each zone. In this case, every agent evaluates each zone *i = (1; 2)* according to the proportion of each household type in the zone. There are two agent types with the same income but different preferences associated with their neighborhood. Note that their willingness to pay satisfies the budget constraint (it is not necessary to use the cutoff probabilities).

In this case, we study a bid function with lagged externalities given by $B^t_{hi} = a^t_h + b^t_{hi}\left(H^{t-1}_{gi}, \forall g\right)$, where H^{t-1}_{gi} is the population distribution in t-1.

The bid function with myopic memory is

$$If\ i = j \Rightarrow B^t_{ghj(i)} = a^t_{hgj} + b_{hi}\left(H^{t-1}_{fi}, \forall f\right) + \frac{1-\alpha_h}{\alpha_h}\frac{\lambda^{t-1}_g}{\lambda^t_h}\left(b_{gi}\left(H^{t-2}_{fi}, \forall f\right) - r^{t-1}_i\right) \tag{31}$$

$$If\ i \neq j \Rightarrow B^t_{ghj(i)} = a^t_{hgj} + b_{hi}\left(H^{t-1}_{fi}, \forall f\right) \tag{32}$$

In this example, bids have only one attribute, a location externality defined by the set neighbors as the following linear bid function:

$$b^t_{hi} = \sum_f \beta_{hf}\frac{H^{t-1}_{fi}}{\sum_{h'} H^{t-1}_{h'i}} \tag{33}$$

where β_{hf} is the valuation that has an agent type h of the percentage of agents type f in the zone. Thus, bids in a period t ($B^t_{ghj(i)}$) depend on the problem solution in t-1, through the lagged externality, and on t-2 through the myopic memory term.

Initially, the following values for β_{hf} are assumed to simulate attraction among peers:

$$\beta_{hf} = \begin{cases} 20 if \ h = f \\ 0 if \ h \neq f \end{cases} \tag{34}$$

We compute the first urban configuration (see Eq. (35)) with this setting and populations sizes *H = [100; 100]*, is the static equilibrium solution, or the with the no-memory case, obtaining perfect segregation:

	Zone 1	Zone 2
Household 1	100	0
Household 2	0	100

(35)

We now compare this result with the case with memory (31) and (32) and the same population sizes. For this, we initialize the population distributions for t = 0 and t = 1, which are necessary for calculating the urban configuration in the period 2:

$$H^0_{hi} = \begin{array}{c|c|c} & Zone\ 1 & Zone\ 2 \\ \hline Household\ 1 & 50 & 50 \\ \hline Household\ 2 & 50 & 50 \end{array} \ ; \ H^1_{hi} = \begin{array}{c|c|c} & Zone\ 1 & Zone\ 2 \\ \hline Household\ 1 & 51 & 49 \\ \hline Household\ 2 & 49 & 51 \end{array} \tag{36}$$

Moreover, we assume the following transition matrix:

$$P = \begin{array}{c|c|c} & Household\ 1 & Household\ 2 \\ \hline Household\ 1 & 0.6 & 0.4 \\ \hline Household\ 2 & 0.4 & 0.6 \end{array} \tag{37}$$

The long-term solution is attained when population distributions are numerically equal for at least three consecutive periods. If this condition does not hold, it makes no sense to speak about a long-term solution because the sizes of household clusters are changing. Finally, we compute the following index of segregation in each period:

$$dis^t = \frac{\sum_{h,i} \left| H^t_{hi} - H^0_{hi} \right|}{\sum_h H_h}, \tag{38}$$

with $dis^t \in [0, 1]$, which close to zero indicates a segregated system and close to 1 is fully integrated.

Figure 1 represents the dynamics of dis^t (% of integration) in several decision periods, with different memory levels, assuming the initial full integrationH^0_{hi}. We conclude that for low memory factor (αh > 0.5) the system eventually converges to full segregation, but for high memory factor (αh < 0.5) the long-term distribution exhibits an integration level greater than 2%. This means that the expected long-term outcome with memory is different from the static segregated solution. Furthermore, the dynamic process of segregation is slower when agents value memory. For example, with a memory valuation of 0.1 in 6 periods, the segregated solution is obtained, whereas with a valuation of 0.5, it takes 9 periods. When αh ≥ 0.5, the effect of valuating a segregated location strategy (that is, living close to other agents

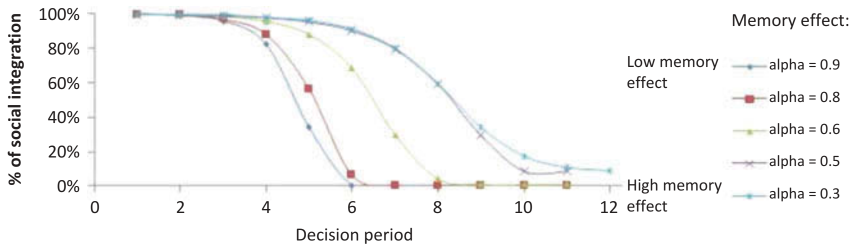

Figure 1. Segregation dynamic for different values of memory (α_h).

who belong to the same cluster and far from those belonging to a different one) is stronger than the effect of a dynamic transition of agents between clusters.

The results shown in Figure 1 were obtained with a fixed structure of the transition matrix P. We now perform a sensitivity analysis using the following symmetric matrix:

$$P = \begin{array}{c|c|c} & \textit{Household 1} & \textit{Household 2} \\ \hline \textit{Household 1} & 1-p & p \\ \hline \textit{Household 2} & p & 1-p \end{array} \tag{40}$$

Figure 2 shows that when $p = 0$, i.e., with no transition of agents, in all cases, the fully segregated solution is reached in the long term, but integration increases with p. In the case of $p = 0.1$, a level of integration of 8% is achieved for values of αh = 0.2. If the between-cluster probability (p) is high, then integration rates are also high, up to 20 or 30%, except in the case that αh = 0.2 and $p = 0.5$, for which we obtain full integration.

In Figure 3, we set αh = 0.5 to observe the number of periods needed to reach the long-term solution for different transition probabilities p. In all cases, the segregated solution is reached but

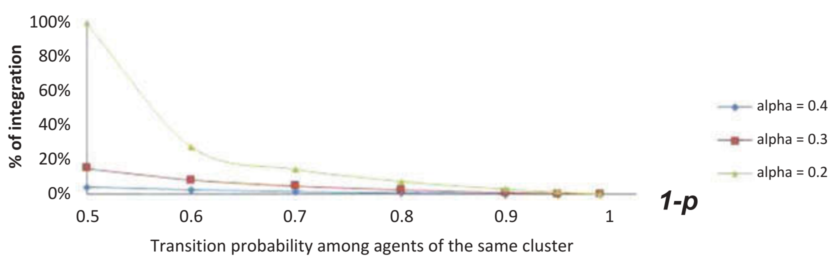

Figure 2. Integration levels for different transition matrices.

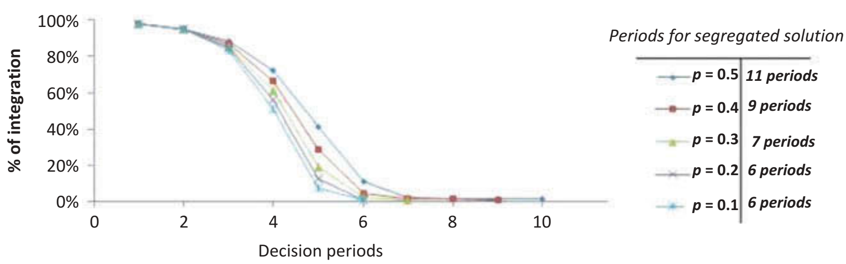

Figure 3. Level of integration in several periods if there are variations in *p*.

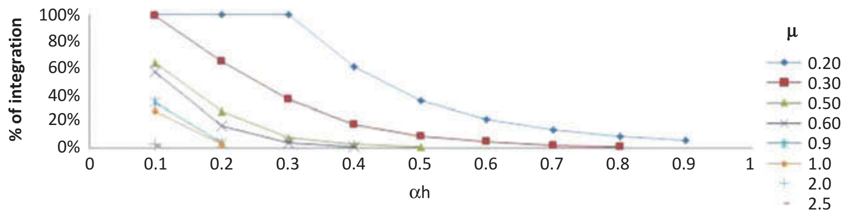

Figure 4. Level of integration in the long term under variations in α and μ.

requires a different number periods, such that when p decreases (low transition among clusters), the segregation rate increases over time due to the low probability of any agent to change its current cluster associated with the valuation of belonging to the other cluster.

All these simulations assume the same logit scale parameter $\mu = 0.5$ (see equations 26–27 and 34–35), which is the inverse of the variance of bids. Figure 4 shows that the results for the scale factor $\mu = 0.2$ are always different from the segregated case. This means that independent of the memory value (αh), the scale is so small, i.e., the behavior variance is so large, that full segregation is impeded. Several simulations lead us to conclude that variance in bidders' behavior reduces the level of segregation, at least to avoid full segregation outcomes.

In summary, the numerical examples with lagged externalities show the possibilities of memory to reduce segregation levels over both short and long term, owing to the inclusion of high memory valuation ($\alpha h < 0.5$). In this manner, we reach the conclusion that experiences or opportunities to interact with other types of households induce some levels of integration, depending on the internal dynamics of households $1\text{-}p > 0$ and their valuation of previous experiences $m_{t-1} > 0$. This conclusion could generate possible guidelines for strategies to induce less segregated cities because of endogenous agents' individual learning.

As a general remark about the simulations of models with memory, we remark that over both short and long term, non-segregated solutions are obtained when a memory effect exists in the agents' decision-making process. Thus, we conclude that myopic learning induces an integrating effect in society. We remark that by using linear bid functions and memory, we obtain results similar to those of other authors regarding segregation effects, even though they use triangular bid or utility functions but without memory valuation and other rules of relocation (e.g., see O'Sullivan, 2009; Zhang, 2011).

5. Conclusions and final discussion

This article presents a microeconomic formulation of a household choice model regarding residential location that incorporates the effects of past experiences and the dynamics of the population's socioeconomics. A static microeconomic model with endogenous learning is formulated. In the model, households assign higher utility to real estate with previous positive experiences and less value to a property in which they had negative experiences. Thus, memory of real estate experiences is modeled by the utility gained in former periods.

The process of utility maximization in decision-making with endogenous learning leads to the willingness to pay for real estate options at equilibrium conditions, including the effect of the household's transition between socioeconomic clusters. This induces different behaviors among households based on their history, and each household history affects the others. The first direct impact of memory is on household relocation choice, but location externalities make this effect produce interactions among households that induce further effects on other households' location choices. The stochastic formulation assuming logit distributions of behavior allows solving the equilibrium conditions at each period to obtain urban distributions based on myopic learning for the cases of lagged and simultaneous externalities.

Finally, numerical simulations were used to compare previous static equilibrium models of residential segregation with our model with memory and socioeconomic transition of households. For this, several scenarios were considered, including one with one-period lagged externalities. Starting with a fully integrated configuration, for the simulated conditions, the economic forces in the static model yields a fully segregated configuration. Conversely, in all cases simulated with our model, it was shown that the effects of memory and transition of agents yield partially integrated outcomes, achieving solutions with some levels of integration in both short and long term.

In our model, agents learn from previous experiences, having had the opportunity to interact with other types of households and establish valuable social networks. This process generates microeconomic forces that, depending on their strength or the household's value of memory, play against the classical segregation force associated with the will to live among peers. This conclusion could generate a possible strategy for public control of the urban segregation process by inducing a higher value of social ties.

Additionally, the microeconomic modeling of residential location with memory is an interesting contribution to the social assessment of private or public projects. Indeed, two agents of an urban system with identical conditions (residential location and household type) in a given period can perceive the utility of their current residential location differently because of their past experiences. In this sense, a social planner should consider not only the perceived utility of households in the current period, but also how location characteristics improve or worsen with respect to the location of those households in previous periods. Moreover, our formulation increases the amount of household categories because it includes the location background within the characteristics of the cluster, thereby generating a great variety of attributes within the agent's classification. Thus, each household will be described by its current socioeconomic characteristics and the characteristics in previous periods that define the perceived utilities in its anterior locations. The proposed formulation, therefore, provides a useful tool in micro-simulation models, in which each agent is analyzed in more detail. In short, this work develops a theoretical analysis of the effects of agents' learning on the urban configuration and individual choices regarding residential relocation, which previous econometric studies have examined and evidenced.

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