Neighborhood Choice and Neighborhood Change[1]

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This article examines the relationships between the residential choices of individuals and aggregate segregation patterns. Analyses based on computational models show that high levels of segregation occur only when individuals' preferences follow a threshold function. If individuals make finer-grained distinctions among neighborhoods that vary in racial composition, preferences alone do not lead to segregation. Vignette data indicate that individuals respond in a continuous way to variations in the racial makeup of neighborhoods rather than to a threshold. Race preferences alone may be insufficient to account for the high levels of segregation observed in American cities.

INTRODUCTION

Sociologists have a longstanding interest in the relationship between individual behavior and collective outcomes. Explanations of social behavior are commonly considered more informative if they account for how

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the actions and motivations of individuals give rise to social organization, rather than assume that macrolevel phenomena are simple aggregates of individual characteristics and behavior (Coleman 1994, p. 197; Granovetter 1978, p. 1421). Tipping or threshold models (Schelling 1971, 1978; Granovetter 1978; Granovetter and Soong 1988) provide one useful framework for connecting the actions of individuals to population processes. The premise of these models is that human behavior is interdependent. On the one hand, people's actions may be influenced by the number (or proportion) of others who act in a given way or have a given characteristic. On the other, changes in individual behavior alter the makeup of the population. Thus, individuals' actions are both a response to some population statistic and contribute to that statistic. These models account for how collectivities "emerge" from the behavior of individuals and can also explain why the same individuals may experience a wide range of social outcomes, depending on the structure of their interaction.

Threshold, epidemic, and diffusion models make up a more general class of behavioral models that capture the feedback effects between micro- and macrolevel processes. These "interactions" models have been applied to a wide range of social phenomena, including, for example, outbreaks of crime or violence (LaFree 1999; Tolnay, Deane, and Beck 1996; Spilerman 1970; Pitcher, Hamblin, and Miller 1978); the adoption of technological innovation (Coleman, Katz, and Menzel 1966; Burt 1987; Ryan and Gross 1943; Hagerstrand 1967); neighborhood rates of teen sexual behavior and pregnancy (Rowe and Rodgers 1991; Crane 1991); the propagation of rumors and the persistence of urban legends (Noymer 2001); the spread of conventions, fads, and fashions (Young 1996; Lieberson, Dumais, and Baumann 2000); and the timing and occurrence of social movements and social protest (Tarrow 1998; McAdam and Rucht 1993).

These models of social interaction also have great potential for understanding the dynamics of residential mobility and residential segregation by race and ethnicity. In his pioneering work, Thomas Schelling (1971, 1972, 1978) laid the conceptual groundwork for understanding the relationship between individual preferences and behavior on the one hand and the evolution of neighborhoods on the other. Using rudimentary computational models applied to artificial agents, he showed how the preferences of individuals about where to live can give rise to (often unanticipated) aggregate patterns of residential segregation. These patterns, moreover, may be at odds with the majority of individuals' preferences. Schelling (1972, p. 157) adopted the term "tipping" to describe the point when a neighborhood reaches a race-ethnic composition that motivates one or more white residents to leave. The term implies that subsequent entrants who take the place of those who leave are predominantly of the

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minority group and that the process ultimately and irreversibly changes the composition of neighborhoods.[2] Schelling's ideas are consistent with threshold, epidemic, and diffusion models that are in widespread use in sociology. One important distinction, however, is that Schelling's model explicitly allows for the movement of individuals in and out of different neighborhood types. That is, people may reverse their choices. In contrast, many applications of threshold, diffusion, or social interaction models assume that, once an individual has made a particular transition, the process is irreversible.[3]

Schelling's ideas provide an account of neighborhood change that links notions of racial preference and prejudice, which have been documented in social survey data, to sociological research on patterns of residential segregation. Despite the significance of Schelling's contribution, it raises several issues that also arise in other applications of threshold, contagion, and diffusion models. One issue concerns the link between the underlying theoretical model and its empirical validation. That is, how well do the predictions of such models conform to known empirical regularities? A second, related issue is the robustness of the empirical predictions of such models at the macro level to alternative behavioral assumptions at the micro level. That is, do alternative assumptions about the behavior of individuals imply similar or dissimilar outcomes for populations?

In this article, we use an agent-based model to examine the implications of alternative assumptions about how individuals evaluate neighborhoods (based on their race-ethnic composition) for aggregate patterns of residential differentiation. We couple our agent-based model with survey data to determine what assumptions about individual preferences are most plausible. We find that relaxing seemingly innocuous assumptions about microlevel behavior can lead to vastly different macrolevel outcomes. While this article takes up the specific application of residential mobility and neighborhood change, our findings have implications for other research that relies on threshold, contagion, or diffusion models to explain collective behavior. Our analysis demonstrates that aggregate outcomes

[2] As Schelling (1972, p. 161) describes it: "We can foresee the possibility of a spiral or domino effect, or unraveling process. There will be some interdependence of decisions. Anyone who moves out reduces . . . the number of whites remaining. . . . Assuming some pressing black demand for housing, perhaps an increasing demand as the number of prospective black neighbors grows, and a diminishing white demand to move into a neighborhood as the black percentage rises, each white who reaches his tipping point and departs brings the remaining whites a little closer to their tipping points."

[3] Typically this assumption is implicit, and of course it depends on the application under consideration. One example of an irreversible choice is entry into first marriage (e.g., Hernes 1972). Once a person marries, he or she can never resume a never-married state. Other examples, such as the diffusion of a technical innovation, may or may not be reversible. We return to this issue in the conclusion of the article.

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may be quite sensitive to one or more assumptions at the micro level and underscores the importance of determining (empirically) the most appropriate model of individual behavior.

Static and Dynamic Approaches to Residential Segregation

Many studies have used census data to describe patterns of residential segregation in large American cities (e.g., Taeuber and Taeuber 1965; Duncan and Duncan 1957; Frey and Farley 1996; Massey and Denton 1993; Denton and Massey 1991; Jargowsky 1996). Sociologists have also used survey questions to investigate directly the willingness of whites, blacks, Asians, and Hispanics to live in neighborhoods of varying race-ethnic composition (e.g., Farley et al. 1993, 1994, 1978; Farley, Fielding, and Krysan 1997; Charles 2000; Bobo and Zubrinsky 1996). Researchers typically use data on individuals' preferences about neighborhood composition to explain observed patterns of residential differentiation. But whereas these two strands of literature assume a link between individuals' preferences about neighbors, mobility behavior, and aggregate patterns of segregation, this link is rarely modeled directly.

Although Schelling's tipping model is well known to students of residential mobility and segregation, it is seldom directly used to analyze neighborhood change in real populations. Any effort to use the tipping model for this purpose needs to address several issues. First, Schelling's results are derived from an extremely small population.[4] Second, the model is limited to only two race-ethnic groups. It is not clear whether the relationships Schelling observes hold in a world with multiple, discriminating race-ethnic groups. Finally, and the motivating force behind this article, while simple models are crucial for developing a theoretical understanding of the mechanisms that produce observed patterns of segregation, Schelling's model rests on strong assumptions about how individuals appraise neighborhoods and decide where to live. Specifically, Schelling's model assumes that highly nonlinear choice functions describe how individuals evaluate their neighborhoods. It is not clear how robust Schelling's findings are to alternative assumptions about individual behavior.

Our main goals here are to elucidate the conditions under which the race-ethnic preferences of individuals can produce high levels of segre-

[4] Schelling's city consisted of a 13×16 grid populated with 138 individuals. In results not shown here, we show that, when individuals evaluate neighborhoods according to smooth (continuous) preference functions, grid size affects segregation outcomes. However, when individuals behave according to Schelling's original function (or other step functions) the aggregate results are robust across grid size (results available from authors on request). See appendix A below for further discussion of lattice size effects.

gation and to use survey data to determine whether these conditions are met. In the first section, we explicate the behavioral assumptions underlying the Schelling choice function and discuss some alternative assumptions. We then simulate mobility using several of these behavioral models and show that aggregate patterns of segregation vary under alternative specifications of individual behavior. Very high segregation occurs only when individual behavior is governed by strict thresholds; that is, when individuals are indifferent about a subset of neighborhoods (e.g., all neighborhoods 0%–49% own-group are considered equally undesirable, and all neighborhoods at least 50% own group are equally desirable). Given that different models of residential choice produce different patterns of residential segregation, which model best reflects how people make choices? In the second section, we use survey data to examine the empirical shape of respondents' preference functions.[5] We then simulate mobility under the assumption that individuals follow the preferences of survey respondents, and we examine what segregation outcomes emerge. We find that, even though the survey data suggest that most people are unwilling to live in neighborhoods in which their own race-ethnic group is the minority, the level of segregation implied by these empirical preference functions is far lower than that predicted under the Schelling regime. If the survey-based choice functions are valid, it is unlikely that the preferences of persons to live among their own group will lead to the dramatic segregation outcomes predicted by the Schelling model. Our analysis suggests that for residential tipping to occur individuals must choose neighborhoods according to a threshold function. That is, a threshold at the individual level leads to a threshold at the aggregate level. In the final section, we examine the segregation outcomes under a variety of assumptions to show further how the form of individuals' responses to neighborhood characteristics affects aggregate outcomes.

This article focuses on the implications of alternative assumptions about individuals' preferences for neighborhoods of varying race-ethnic composition for residential mobility and segregation. Obviously, in any specific context, individuals may consider many other neighborhood factors beyond race and ethnicity (e.g., Harris 1999). Residential patterns are also determined by limits on individuals' ability to pay for housing and by institutional constraints, such as discriminatory behavior by realtors and lenders. Our goal is not a full explanation of residential segregation.

[5] The preference data analyzed here provide information only on attitudes, not actual mobility behavior. However, in other research, we have been working with both behavioral and attitudinal approaches to neighborhood preferences. Our results suggest that these two approaches yield similar conclusions about individuals' preferences and residential mobility.

Rather, we seek to evaluate the relevance of behavioral models and data used by other investigators to an explanation of neighborhood change. In this sense, our models provide a baseline for assessing the effects of individuals' race-related preferences on segregation.

RESIDENTIAL PREFERENCES AND RESIDENTIAL SEGREGATION

The Causes of Residential Segregation

Trends and causes of residential patterns continue to be a major social issue in the United States. The characteristics of neighborhoods in which individuals grow up may be important determinants of their lifetime socioeconomic success or failure and may be source of socioeconomic inequality (Brooks-Gunn, Duncan, and Aber 1997; Crane 1991; Garner and Raudenbush 1991; Herrnstein and Murray 1994; Reich 1991). Place of residence remains a barrier to upward social mobility and, for some groups, may be even more of a barrier today than in the past (Borjas 1999; Durlauf 1996). The causes of segregation, however, are not well understood. One enduring issue is the relative importance of racial preferences and prejudices in residential mobility decisions (Yinger 1995; Harris 1999; Clark 1991). Researchers have used vignette data to show that, while both blacks and whites are willing to tolerate some degree of integration, the majority of whites will not tolerate neighborhoods that are more than 20% black. In contrast, most blacks prefer a neighborhood that is at least 50% black. Thus, the neighborhood that most blacks prefer is the same neighborhood from which whites would move (Farley et al. 1978; Farley et al. 1993, 1997).[6] The low tolerance for integration with blacks expressed by whites (and also to a lesser degree, Asians and Hispanics) is taken to be an important source of persistent segregation (Bobo and Zubrinsky 1996; Clark 1986, 1991, 1992, 1996; Farley et al. 1978, p. 343; 1997, p. 766; Charles 2000, p. 193).[7] But without a framework for understanding *how* individuals' location choices influence neighborhood formation and change in the aggregate, this research cannot specify the extent to which preferences contribute to residential differentiation and what the expected consequences would be should the preferences of one or more groups change.

[6] However, Farley et al. (1997) provide evidence for some degree of overlap between the preferences of blacks and whites.

[7] While survey respondents demonstrate that the race composition of a neighborhood is correlated with its desirability, racial neighborhood preferences may be a proxy for other neighborhood characteristics (e.g., neighborhood poverty) (Harris 1999).

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Schelling's Model of Residential Tipping

Schelling noted that in Chicago all of the mixed neighborhoods (defined as neighborhoods 25%–75% nonwhite) in 1940 became entirely nonwhite over the next 10 years (Duncan and Duncan 1957, p. 11, cited in Schelling 1971, p. 181). To understand what mechanisms might have produced this phenomenon, Schelling constructed a simple spatial model in which two groups of people ("blacks" and "whites") are distributed in a stylized city in accordance with their preferences about the composition of their local areas. Each individual wishes to live where at least 50% of his neighbors are members of his own group. If individuals are initially distributed randomly but subsequently try to move whenever they are surrounded by a majority of the other color, then, when all feasible moves have been completed, the city is far more segregated than any individual alone prefers. Each individual chooses his or her own neighborhood, but no one chooses the high level of segregation that results from all of these moves.[8] This exemplifies models for the ways that aggregate features of the environment result from the behavior of individual actors (e.g., Krugman 1996; Axelrod 1997; Lieberson, Dumais, and Baumann 2000; Macy 1991; Granovetter 1978; Granovetter and Soong 1988; Hedström 1994; Bongaarts and Watkins 1996). These models are useful for understanding phenomena in which the characteristics of the environment and the behavior of the individuals who constitute that environment are dynamically interdependent (Durlauf 2001). For example, individuals who move out of a neighborhood because they cannot tolerate its racial composition simultaneously respond to and modify neighborhood racial composition.

Despite the elegance and power of such models, it is important to realize that they rest on specific behavioral assumptions. In the case of segregation, the Schelling model rests on only one out of a number of possible assumptions about how individuals choose their neighborhoods. In this article, we explicate the behavioral assumptions underlying different residential preference functions. In addition, we investigate what levels of neighborhood segregation result from alternative models and how well these models conform to the stated preferences of individuals.[9]

[8] Other behavioral functions also produce residential tipping. For example, Zhang (2004) finds that segregation emerges even if individuals have a strict preference for integrated (half-black, half-white) neighborhoods.

[9] Clark (1991) also uses survey data to argue that empirical preference functions look similar to Schelling's theoretical distributions of tolerance. However, he assumes that people behave according to threshold functions. In contrast, we treat this assumption as potentially problematic and examine its implications for residential mobility and segregation.

THE COMPUTATIONAL MODEL

We specify a model of residential mobility that incorporates individuals' preferences for the racial makeup of their neighborhoods and the race-ethnic composition of the city. This model is similar to Schelling's but is based on a city with a much larger number of dwelling units.[10] We use this model to compute the patterns of residential segregation that result from alternative assumptions about the model's parameters.

The City

Our computational model uses a two-dimensional 500×500 lattice; that is, a grid with 250,000 cells.[11] Each cell corresponds to a dwelling unit. This lattice is populated with a mixture of "agents" who belong to one of several ethnic groups. Agents are the hypothetical people who interact in our computational model. We present results for a population that is 50% white and 50% black.[12] Each agent can only occupy one cell on the lattice at a time but can move to any vacant cell. To allow agents to move relatively freely on the lattice, 15% of the cells on the lattice are vacant.[13] Agents respond only to the ethnic composition in their immediate neighborhood; they have no information about the overall level of segregation in the city. The size of the agents' neighborhoods is determined by a radius. Figure 1 illustrates the type of neighborhood (with a radius of 2 cells) that is used in the results reported here. The agent is located in the center of the neighborhood, and its white and black neighbors are shown in the cells labeled "W" and "B," respectively. Because agents evaluate neighborhood boundaries defined by their own position relative to others, each agent has a unique set of neighbors (albeit overlapping with the neighborhoods of nearby agents). Nonetheless, we compute a variety of measures of residential segregation both for individual-specific neighbor-

[10] Fossett (1999) also uses a computational approach to adjudicate between explanations of residential segregation. Benenson (2004, chap. 4) uses a realistic computational model based on GIS data to simulate residential dynamics in Yaffo, Israel. While informative, both of these studies only loosely rely on residential choice data. In contrast, we focus specifically on the micro-level assumptions that underpin the model, and use empirical data to assess the validity of these assumptions.

[11] This model is programmed in Java and utilizes REPAST software framework for agent-based simulation (http://repast.sourceforge.net/).

[12] In work not reported here we also simulate what segregation outcomes occur for a multiethnic metropolis with the race-ethnic composition of Los Angeles (31% white, 10% black, 14% Asian, 45% Hispanic). These simulations yield the same substantive conclusions as the two-group results reported here.

[13] By leaving 15% of the cells vacant, we assume that housing is relatively plentiful. Thus, our simulated people can usually find an available housing unit in a satisfactory neighborhood.

W	W			B
	B	B		W
B	W	Agent	W	
	B	W	W	
B			B	B

FIG. 1—Example of one agent and its neighborhood

hoods and for the fixed grid of neighborhoods typically assumed in segregation studies.

The Choice Function

Each agent uses a preference function to calculate whether or not it is happy in its current neighborhood. The Schelling function assumes that agents wish to move away from their current address if the proportion of own-race neighbors dips below 50%. All destinations with at least 50% own-group neighbors are considered equally satisfactory. Agents use the same preference function to evaluate the desirability of both staying in the current neighborhood and moving to another vacant cell on the lattice. This function, which is illustrated in figure 2 part a, is based on several assumptions.[14] First, residential preference is a pure step function in which people only distinguish between two types of neighborhoods. For example, neighborhoods that are 25% own group are equally desirable to those that are 45% own group. Second, all members of the same race-ethnic group have the same preferences for neighbors. Third, no agents have a taste for diversity that would make neighborhoods in which their own group was heavily overrepresented less attractive than any other type of integrated neighborhood. Finally, agents rate neighborhoods only on their

[14] Note that the scale of the *y*-axis is arbitrary because the probability of choosing any given neighborhood depends on the numbers and proportions of neighborhoods of each race-ethnic makeup. What matters for the purpose of the present argument is the shape of the preference function.

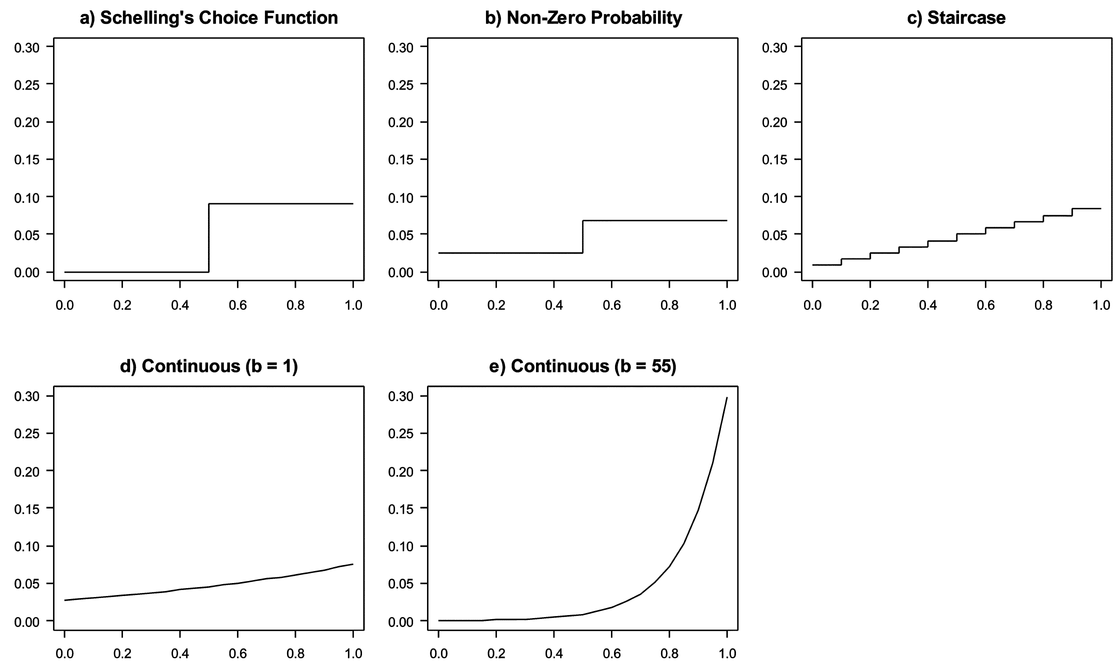

FIG. 2.—Alternative residential choice functions; for all four images, the vertical axis is "probability of residential choice" and the horizontal axis is "neighborhood proportion own group."

static conditions and ignore how neighborhoods have changed in the recent past.

There are a number of ways to conceptualize individual behavior and alternative assumptions may lead to different aggregate results. To better understand why the Schelling function produces high levels of segregation, we systematically vary the behavioral assumptions that underlie this function. The Schelling function can be written,

$$\text{prob (moving into the } j\text{th neighborhood at time } t \mid X_{kt}, k = 1,2,\ldots,K) = \frac{X_{jt}}{\sum_{k=1}^{K} X_{kt}}, \tag{1}$$

where $X_{kt} = 1$ when the kth neighborhood has at least 50% agents of like color at time t, and 0 otherwise, and k indexes all possible destination neighborhoods. A minor variation on Schelling's model is to allow for a small probability of moving into neighborhoods less than 50% own group, using a conditional logit formulation (McFadden 1973) which assumes a nonzero probability of choosing any given neighborhood. That is,

$$\text{prob (moving into the } j\text{th neighborhood at time } t \mid X_{kt}, k = 1,2,\ldots,K) = \frac{e^{X_{jt}}}{\sum_{k=1}^{K} e^{X_{kt}}}, \tag{2}$$

which is illustrated in figure 2, part b. This model, which differs in substance only slightly from Schelling's, provides a basis for comparing alternative models that vary in their substantive implications.[15]

In a second model, we assume a "staircase" function, in which agents experience a small increase in desirability for each k% increase in the proportion of own-group agents in the neighborhood (see fig. 2, part c). The model assumes that there are m types of neighborhoods, ranging from unattractive to attractive (e.g., 0%–10% to 90%–100% own group, with $m = 10$). Thus, agents distinguish between m types of neighborhoods but are indifferent to small changes in neighborhood composition within types (e.g., agents consider a neighborhood that is 11% own group to be as attractive as a neighborhood that is 17% own group). The model is

$$\text{prob (moving into the } j\text{th neighborhood at time } t \mid d_{kt}, k = 1,2,\ldots,K) = \frac{e^{d_{jt}}}{\sum_{k=1}^{K} e^{d_{kt}}} \tag{3}$$

[15] A more general formulation is to weight X in eq. (2) by a coefficient β, which can, in principle, be estimated.

where d_{jt} is a discrete variable that ranges from zero to one in m steps (e.g., 0, .1, .2, . . . , .9, 1.0). When $m = 2$, this model reduces to the single threshold model, or equation (2). When $m > 2$, agents distinguish among more than two levels of race-ethnic composition, although the attractiveness of a neighborhood remains a monotonic function of the proportion of neighbors who belong to the agent's own group.

In a final model, we relax the assumption that neighborhood choice is a step function and allow neighborhood attractiveness for each agent to vary smoothly with the proportion of neighbors who are in its own race-ethnic group. In contrast to the step functions, continuous functions allow agents to respond to even slight changes in neighborhood proportion own group, a response that may be linear or nonlinear and monotonic or nonmonotonic.[16] For example, agents may prefer to live where they are neither the overwhelming majority nor the minority. Under this model, the probability that an agent in the lth race group moves into the jth neighborhood at time t, conditional on q_{jt} and q_{kt}, is

$$\frac{e^{F_l(q_{jt})}}{\sum_{k=1}^{K} e^{F_l(q_{kt})}}, \tag{4}$$

where q_{jt} is the proportion own group in neighborhood j at time t, and F_l defines the shape of the response function for the lth race group. For example, if the log odds of choosing one neighborhood over another is proportional to the percentage of one's own group for the lth race, then $F_l(q_{jt}) = q_{jt}$. This function is illustrated in figure 2, part d. A variation on equation (4) is shown in figure 2, part e, where $F_l(q_{jt}) = 55 * q_{jt}$.

We use these preference functions to calculate the predicted probability (p) that an agent will move into a neighborhood or remain in its current neighborhood.[17] To translate these probabilities into decisions to move,

[16] Eq. (4) is a relatively minor modification of the step function used by Schelling. One can reparameterize the choice model using the logistic function $f(h,\lambda) = \frac{1}{1+e^{-\lambda h}}$, where λ is a parameter that controls the shape of the function and h is a linear transformation of proportion own-group ($h_{jt} = 2q_{jt} - 1$). With appropriate parameterization, this function can generate a threshold function (when $\lambda = \infty$) or a continuous function with varying shape ($0 < \lambda < \infty$). As λ approaches ∞, the function approximates a very steep (albeit continuous) sigmoid function. Nonlinear continuous functions with steep slopes can generate high levels of segregation on finite-sized lattices. This is because it is the steepness of the Schelling function, not the discontinuity at the critical value 0.5, that produces high levels of segregation. Note 25 below provides further explanation of this point.

[17] In particular, if p_{ijt} denotes the probability of choosing the jth neighborhood in the tth period by the ith individual, then the preference functions described above can be written as $p_{ijt}(U_{ijt}) = [\exp(U_{ijt})]/[\Sigma_{k\varepsilon C(i)}\exp(U_{ikt})]$, where U_{ijt} and U_{ikt} denote the expected relative desirability of neighborhoods j and k, and $C(i)$ denotes the set of potential destination neighborhoods for individual i.

we specify that the decision is a random draw from a multinomial distribution for the probabilities of moving into each possible neighborhood.

The models discussed in this section illustrate some alternative ways that the race-ethnic composition of a neighborhood may affect its attractiveness to potential movers. Our goal is to assess whether these alternative behavioral assumptions have implications for residential mobility and residential segregation.

Implementing the Computational Model

At the beginning of the simulation, all agents are evenly distributed on the lattice; that is, they are arranged with an index of dissimilarity of zero across a fixed grid of tracts.[18] Next, one agent is sampled from the population using simple random sampling with replacement. Using one of the preference functions described above, the selected agent calculates transition probabilities for its current neighborhood and the neighborhoods surrounding all available vacancies. Based on these probabilities, the agent moves into another neighborhood in the city or remains in its current residence. Any agent that moves leaves its previous cell vacant for another agent to move into. In the next time period, a second agent is randomly sampled, evaluates its options, and decides whether and where to move based on its vector of transition probabilities. In the third period, yet another agent is sampled, and the process continues. Obviously, the opportunity structure for each agent changes over subsequent moves. Thus, the race-ethnic composition of neighborhoods available to agents as they make their mobility decisions is a function of all previous moves by other agents.[19]

One point of uncertainty is for how many periods we should run the model in order to be confident that we have captured its essential properties. Typically, researchers working with computational models focus

[18] An alternative approach is to allow the initial distribution of agents be random (Schelling 1971). But with random placement, the initial values of the segregation scores are affected by the proportion minority in the city's population (Cortese, Falk, and Cohen 1976). In work not shown here, we experimented with other starting distributions (e.g., completely segregated), but these do not change the substantive conclusions of this article.

[19] Time in our simulation is approximately continuous; each period is so minute that agents evaluate their neighborhoods sequentially (not simultaneously). However, the time scale does not have a straightforward mapping into minutes, days, months, or years. Because we sample with replacement, it is possible for one agent to be sampled twice before another agent has an opportunity to move. This is a realistic aspect of mobility behavior. It is possible to calibrate the model to real time by linking the simulated mobility rate to annual mobility rates reported in survey or census data, although this link is unnecessary for the analyses reported in this article.

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on how a model behaves at equilibrium. Given the amount of time it takes to run each of these models, however, we have limited the simulations to 1 million iterations per model, which reveal the essential behavior of the models, albeit not at equilibrium. We nonetheless believe that our simulations are adequate to demonstrate the macrolevel implications of alternative microbehavioral assumptions.[20]

In the simulation results reported below, we measure segregation using the index of dissimilarity, based on 2,500 equally sized "tracts" that contain 100 dwellings. This index measures the departure of the observed race-ethnic spatial distribution from evenness across a city. Although this index has a number of well-known limitations, including its insensitivity to distances between the residences of members of racial groups and reliance on an arbitrary grid of neighborhoods, it is adequate for our purpose of showing the implications of alternative behavioral models. We have also summarized our simulations using alternative measures of segregation that are based on the race composition of each individual's unique neighborhood (see fig. 1). These measures, which to conserve space are not reported in this article, point to identical conclusions to those based on the index of dissimilarity.

SIMULATIONS OF SEGREGATION DYNAMICS

We simulate mobility using the four choice functions described above: the Schelling threshold function, the nonzero probability function, the staircase function, and the continuous function (where $F_l(q_{jt}) = q_{jt}$ in eq. [4]). As we show below, whether neighborhoods tip to a high level of segregation depends critically on whether individuals follow a simple threshold preference function.

Figure 3 contrasts the segregation outcomes implied by these four choice functions. Modifying Schelling's preference function to allow for nonzero probabilities of moving into all possible destinations produces similar high levels of residential segregation to those generated by Schelling's function. Assuming that individuals have some small, nonzero probability of moving into areas less than 50% own group has no discernable impact on the observed neighborhood outcomes. In contrast, preference functions that allow individuals to recognize variation among neighborhoods above or below the simple threshold produce much lower different levels of segregation from what is implied by the threshold function. When agents evaluate neighborhoods according to a continuous linear function of the

[20] In app. A, we present an aggregate interactive Markov chain version of our models and show that the equilibrium results obtained from these models are substantively equivalent to the results shown here after one million iterations.

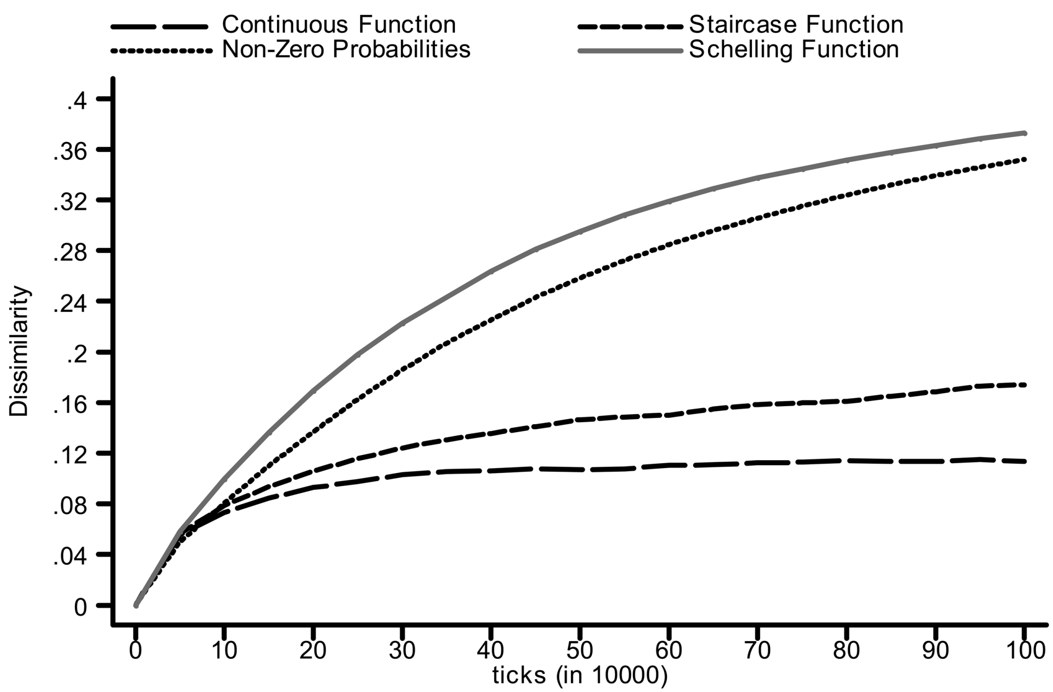

FIG. 3.—Index of dissimilarity, modified Schelling functions

percentage own-group, the index of dissimilarity increases initially and then flattens out very quickly around 0.1. Assuming agents respond to small changes in neighborhood composition, therefore, appears to eliminate tipping. Not surprisingly, the staircase function, which allows for a series of small threshold responses (in this case, 10 intervals of neighborhood proportion own group), generates an intermediate level of segregation. This is further evidence that the threshold form of Schelling's preference function may drive neighborhood tipping.

Assumptions about how individuals evaluate their neighborhoods have a large impact on macrolevel outcomes. In particular, the assumption that people are indifferent over intervals of neighborhood proportion own group (as with the threshold function, and to a lesser extent, the staircase function) leads to higher segregation, while the assumption that individuals are sensitive to even slight changes in neighborhood proportion own group leads to lower segregation. These simulations demonstrate the importance of studying the *shape* of individuals' responses to their neighborhood characteristics, not just the average level of tolerance in the population.

RESIDENTIAL PREFERENCES IN STATED PREFERENCE DATA

Given that different assumptions about the shape of individuals' response curves strongly affect residential segregation, it is important to assess which assumption or assumptions best approximate how people evaluate the desirability of neighborhoods. Stated residential preference data from the Los Angeles and Boston modules of the 1992–94 Multi-City Study of Urban Inequality (MCSUI), and the 1976 and 1992 Detroit Area Studies (DAS) can be used to estimate neighborhood preference functions for various race-ethnic groups. We compare these functions to the models outlined in the previous section to determine what assumptions about individual behavior seem most plausible.

Vignette Data from the 1976 and 1992 DAS

We use one measure of preferences from the MCSUI and one from the DAS. We have information collected in both the 1976 and 1992 DAS regarding the willingness of blacks to live among whites (and vice-versa), assuming a city with only two race groups. Schelling's ideas about tipping were formulated in response to neighborhood conditions in the early 1970s, and the DAS data allow us to explore the possibility that preference functions have changed over time. For example, the threshold function may represent individual choice behavior in the past, but the continuous

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function may better describe contemporary neighborhood choice. The vignette neighborhoods (described in detail by, e.g., Farley et al. 1997) consist of five cards depicting three rows of five houses, with the respondent's house situated in the middle. The remaining houses are populated by either the respondent's own race-ethnic group or another group. The respondent is told that he or she has found an attractive, affordable house in a neighborhood with the race composition shown on the card. The respondent is then asked if he or she would move into this area. Thus, we observe five responses for each individual (one for each card).

The top panel of table 1 shows that in 1976 most whites were willing to tolerate a small number of blacks in their neighborhoods. For example, almost 71% of the whites interviewed said they would move into an area that was 7% black, and half said they would move into a neighborhood that was 21% black. Whites did not want to live in an area with a sizable black presence. Approximately 26% of the whites interviewed said they would live in an area that was 36% black, and only 16% of whites said they would live in an area that was 57% black. Between 1976 and the early 1990s whites became somewhat more willing to tolerate a larger percentage of blacks in their neighborhoods. Over 40% of the Detroit white respondents interviewed in the 1992 DAS say they would move into an area that was 36% black, and approximately 28% of white respondents reported a willingness to move into neighborhoods that were 57% black. These estimates are consistent with those reported by Farley et al. (1993, p. 27) who conclude that there has been a "significant shift among whites toward more tolerant attitudes about residential integration."

Table 1 also shows the distribution of responses for black respondents. The evidence suggests that blacks over time became somewhat less willing to live in a neighborhood populated by a white majority and more likely to enter an all-black neighborhood. The proportion of blacks willing to move into an all-black neighborhood increased from 70% in 1976 to 75% in 1992. Similarly, in 1976 approximately 37% of the DAS's black respondents expresssed willingness to live in an all-white neighborhood, whereas in 1992 only 28% were willing to move into an all white area. Almost all blacks from both surveys, however, express a strong preference for neighborhoods where neither race held an overwhelming majority.

These vignette data have both advantages and disadvantages for evaluating the realism of alternative models of individual behavior. On the one hand, the hypothetical neighborhoods that survey respondents were asked about are approximately the same size and shape as the neighborhoods used in the computational model. The DAS data assume a world in which there are only blacks and whites, which is consistent with our two-group computational model. These data also provide a relatively pure

TABLE 1
DISTRIBUTION OF PREFERENCES TO MOVE INTO NEIGHBORHOODS BY RACE OF INDIVIDUAL AND RACE COMPOSITION OF NEIGHBORHOOD

NEIGHBORHOOD PROPORTION OTHER RACE	DETROIT AREA STUDY 1976	DETROIT AREA STUDY 1992
	% Whites Willing to Move to Neighborhood	
0	95.8	90.8
.07	70.5	82.6
.21	48.6	66.2
.36	25.5	40.9
.57	15.7	28.1
N	711	783
	% Blacks Willing to Move to Neighborhood	
0	69.4	75.3
.29	99.2	98.2
.50	99.5	97.6
.86	95.4	85.8
1.00	37.5	27.7
N	395	740

measure of preferences compared to observations of actual residential mobility, which are constrained by housing costs and availability of information.

On the other hand, the DAS data have several limitations. First they include a limited range of white responses (up to only 57% black) even though some whites may tolerate neighborhoods with greater black representation. Second, blacks and whites were not shown the same vignettes, making interracial comparison less precise. Third, because both black and white respondents were shown only five vignettes, the data provide less than optimal information for determining the functional form of individuals' residential choices. Fourth, these data do not reveal whether respondents evaluate the desirability of their current neighborhood differently from possible destination neighborhoods.[21] Finally, like most stated

[21] In related work (Mare and Bruch 2003), we estimate preference functions based on observed mobility data for Los Angeles County residents, which allow us to determine the extent to which people evaluate their current neighborhood differently from other possible destinations. Although individuals tend to prefer their own neighborhoods over other neighborhoods regardless of their race composition, patterns of racial preference are similar for both own and other neighborhoods.

choice data, the DAS data were collected under artificial conditions. Despite these shortcomings, data of this sort are commonly used to document preferences for neighborhoods that vary in racial makeup. In addition, as shown below, we can resolve some of these problems by supplementing the analysis of the DAS data with analysis of the MCSUI. On balance, the vignette data provide a reasonably solid basis for adjudicating among alternative simple models of individual residential preference.

The DAS survey responses are an incomplete ranking of neighborhoods as either first (the respondent would live in these areas) and or last (the respondent would not live in these areas) but more precise rankings are unobserved. We estimate preference functions for these data using an exploded logit model with ties (Allison and Christakis 1994) for the probability that a respondent selects a neighborhood as a function of the proportion of neighbors in the hypothetical neighborhood who are in the other race-ethnic group.[22] This model provides estimates of individuals' response curves. We estimate separate models for blacks and whites, and we examine whether the data support a continuous or threshold specification. Given the large number of data points (number of individuals × the number of vignettes) in our samples, which make all coefficients and model contrasts statistically significant, we do not to rely on traditional tests of model specification but rather examine the shape of the curves directly.

Table 2 shows the estimated coefficients of models for blacks and whites that do not impose any functional form assumptions on the data; they include a dummy variable for each measured category of race composition. If black preferences followed a threshold function, we would expect to see a sharp increase (or decrease) in the response function at the threshold point and constant betas above and below the threshold. Instead, for both the 1976 DAS and the 1992 DAS, the coefficients suggest that choice functions are nonlinear and approximately continuous. This suggests that we can simplify these models to a linear and a quadratic term; that is, $F_i(q_{jt}) = \beta_1 * q_{jt} + \beta_2 * q_{jt}^2$. Predicted probabilities from both the dummy variable and quadratic specification for 1976 are shown in figure 4. The estimated preference functions show that most blacks wish to live in neighborhoods that are dominated by neither whites nor blacks. Most important, the data provide no evidence for a threshold response.

If whites' preferences follow a threshold function, we would expect to see a sharp decrease in response at a certain level of neighborhood proportion black and constant levels of response above and below that point. Instead, while whites do not want to live among blacks, they are responsive to changes in neighborhood proportion black across the five

[22] These models are estimated in Stata using the rologit command.

TABLE 2
Coefficients for Effects of Neighborhood Proportion Other Group on Residential Preferences of Blacks and Whites

Neighborhood Proportion Other Group	1976 Detroit Area Study			1992 Detroit Area Study		
	β	$SE(\beta)$	$\lvert Z(\beta)\rvert$	β	$SE(\beta)$	$\lvert Z(\beta)\rvert$
	Preferences of Blacks					
.00						
.29	3.672	.502	7.32	2.272	.210	10.82
.50	4.258	.753	5.65	2.129	.193	11.02
.86	2.000	.245	8.16	.676	.117	5.77
1.00	−1.292	.163	7.95	−2.186	.182	12.04
Log likelihood	−172.3			−416.6		
N	1,975			3,700		
	Preferences of Whites					
.00						
.07	−2.478	.261	9.48	−.502	.149	3.35
.21	−5.422	.396	13.71	−1.842	.158	11.65
.36	−9.303	.556	16.72	−3.872	.212	18.24
.57	−13.104	.787	16.66	−5.744	.296	19.43
Log likelihood	−104.6			−362.3		
N	3,544			3,915		

intervals. This is consistent with a continuous nonlinear function. White predicted probabilities in figure 4 indicate that the quadratic model fits the data well.

These results suggest that a continuous, quadratic function, albeit different in shape for blacks and whites, is sufficient to describe preferences for neighborhoods with varying race composition. We find little support for a threshold formulation. However, the DAS data only show black and whites responses over five possible neighborhoods and, in the case of whites, a large part of the potential range of responses is not recorded. To offset these limitations of the DAS, we use the MCSUI preference data to explore more rigorously the possibility that individuals' race preferences for neighbors may follow a threshold function.

Ideal Neighborhood Data from the Multi-City Study of Urban Inequality

We estimate preference functions using the Los Angeles and Boston modules of the 1992–94 MCSUI in which respondents were given a card depicting a neighborhood with 15 empty houses. They were asked to use that card to illustrate the racial composition of their ideal neighborhood,

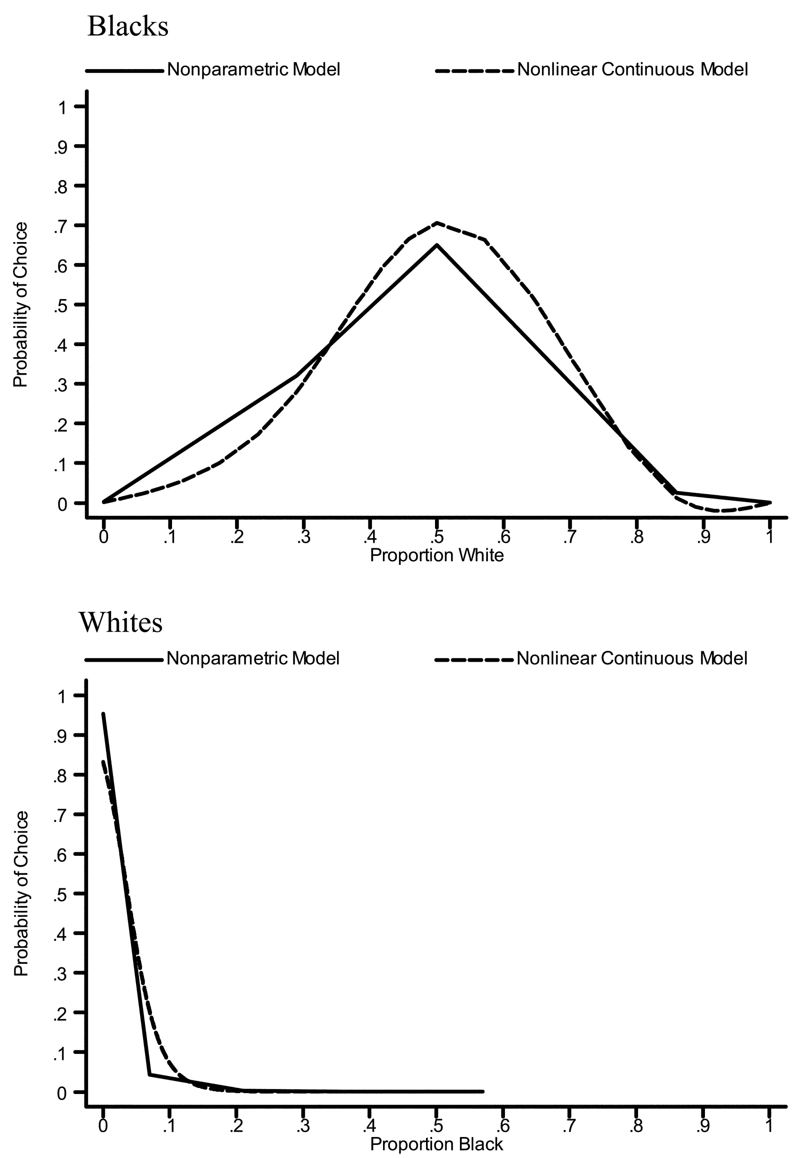

FIG. 4.—Predicted probabilities for 1976 DAS preference question, nonparametric models, and nonlinear continuous specification.

assuming that they lived in one house and could allocate neighbors from four race-ethnic groups (white, black, Asian, Hispanic) to the 14 remaining houses. Any configuration of the 15 houses was possible. These data allow respondents to select one of the 680 unique possible neighborhood compositions formed by arranging the four race-ethnic groups in the 14 empty houses.[23] Each respondent's card was coded by the proportion of black, Asian, Hispanic, and white households present in that person's ideal neighborhood. We use this information to estimate a conditional logit model (McFadden 1973) of the relationship between the proportion of own-group neighbors for blacks and whites and the probability that a black or white respondent selects this neighborhood. We estimate models that allow for the possibility of one or more thresholds in individuals' response curves. As in the case of the DAS data, we estimate a model that includes a dummy variable for each observed category of proportion own group and compare this categorical specification to a nonlinear continuous function. Compared to the DAS data, which distinguish among only five levels of proportion of the respondent's own group, the MCSUI data allow for up to 15 levels. The MCSUI data therefore provide a stronger basis for investigating possible threshold behavior. The nonlinear continuous model contains both a quadratic and a cubic term.

Figure 5 shows the probability that a respondent will select an ideal neighborhood by neighborhood proportion in his or her own group. Consistent with the DAS data, blacks prefer neighborhoods where they are neither the minority nor the majority. Blacks most prefer a neighborhood that is around 35% black, where neighbors are a mixture of whites, blacks, Asians, and Hispanics. The nonlinear continuous model fits the observed responses well. Moreover, there is no evidence of a threshold response for blacks. The predicted probabilities, however, are multimodal, with a large peak for integrated neighborhoods and a smaller peak for areas that are entirely black. This is suggestive of a mixture distribution, where there are two types of persons, one that prefers integrated areas and another that prefers to live entirely among blacks. The corresponding predicted probabilities for whites also suggest heterogeneity in which some persons most prefer an integrated neighborhood while others prefer to be completely surrounded by whites. Compared to blacks, however, whites are much more likely to prefer a neighborhood that is homogeneous in their own race. As in the case of blacks, there is no evidence that whites' preferences follow a threshold function.

In sum, neither the vignette data from the 1976 and 1992 DAS nor the 1992–1994 MCSUI ideal neighborhood data provide evidence for a thresh-

[23] That is, respondents choose from among $(14 + 4 - 1)!/(14!(4 - 1)!) = 680$ possible neighborhoods.

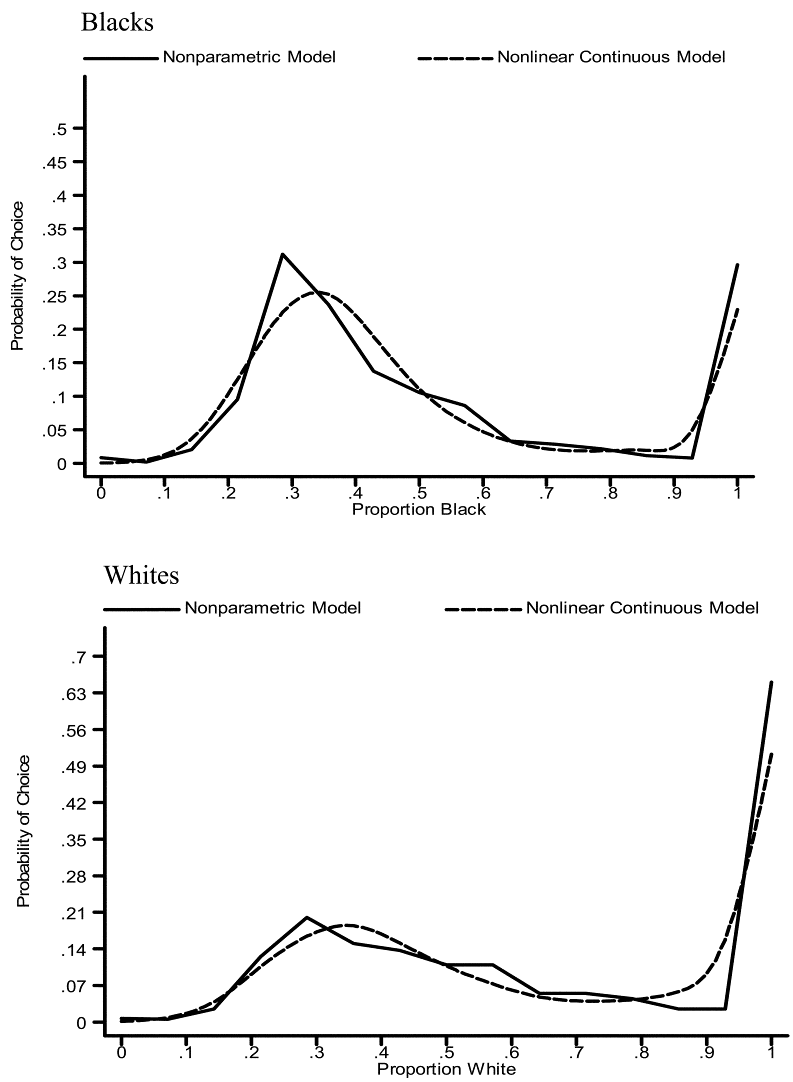

FIG. 5.—Predicted possibilities using the 1992–94 MCSUI ideal neighborhood question, nonparametric models, and nonlinear continuous specification.

old response function. Rather, our estimates are all more consistent with a nonlinear, continuous function. It remains to be seen what the patterns of preferences that we observe in these survey data imply for aggregate neighborhood change.[24]

Neighborhood Dynamics Implied by Empirical Choice Functions

Based on the data reported above, we assign our agents continuous, nonlinear MCSUI and DAS preference functions. Figure 6 shows the index of dissimilarity for black and white agents based on the 1976 and 1992 DAS and the 1992–1994 MCSUI data as well as the Schelling function. Despite the low tolerance expressed by DAS and MCSUI respondents for areas where their group is the minority, these preference functions generate very low levels of segregation. In fact, the segregation generated by all three empirical functions is almost identical to that generated by the continuous function results shown in figure 3. Once we relax the threshold assumption, a wide range of continuous functions (with varying slopes, and possible nonlinearities) generate approximately the same low level of segregation on the 500×500 lattice.[25]

CONTINUOUS VERSUS THRESHOLD FUNCTIONS

The results reported above suggest that the assumption that individuals have a threshold response to neighborhood composition generates high levels of segregation. All the choice functions that allow agents to respond to more finely grained changes in neighborhood proportion own group produce very low levels of segregation. This suggests that tipping may be a result of specifying a threshold functional form for residential choice. That is, the effect of the individual threshold translates into a threshold at the aggregate level. In this section, we examine the segregation that results from a range of continuous and threshold choice functions to try to better understand why populations of individuals whose preferences follow continuous functions have low levels of segregation, while popu-

[24] While these data suggest that the black and white populations are internally heterogeneous, in this article we present preference functions that assume homogeneous preferences within races. A model of heterogeneous responses within race groups (not shown here) provides evidence of heterogeneous choice functions for both blacks and whites, all of which are nonlinear and continuous in form. None follows a threshold.

[25] In work not shown, we simulated the segregation outcomes that occur for individuals who have the preferences reported in the 1992 and 1976 DAS, allowing for unobserved heterogeneity. Allowing for heterogeneity does not change our results. The segregation observed for heterogeneous DAS agents are identical to those observed for homogeneous DAS agents. These figures are available from the authors on request.

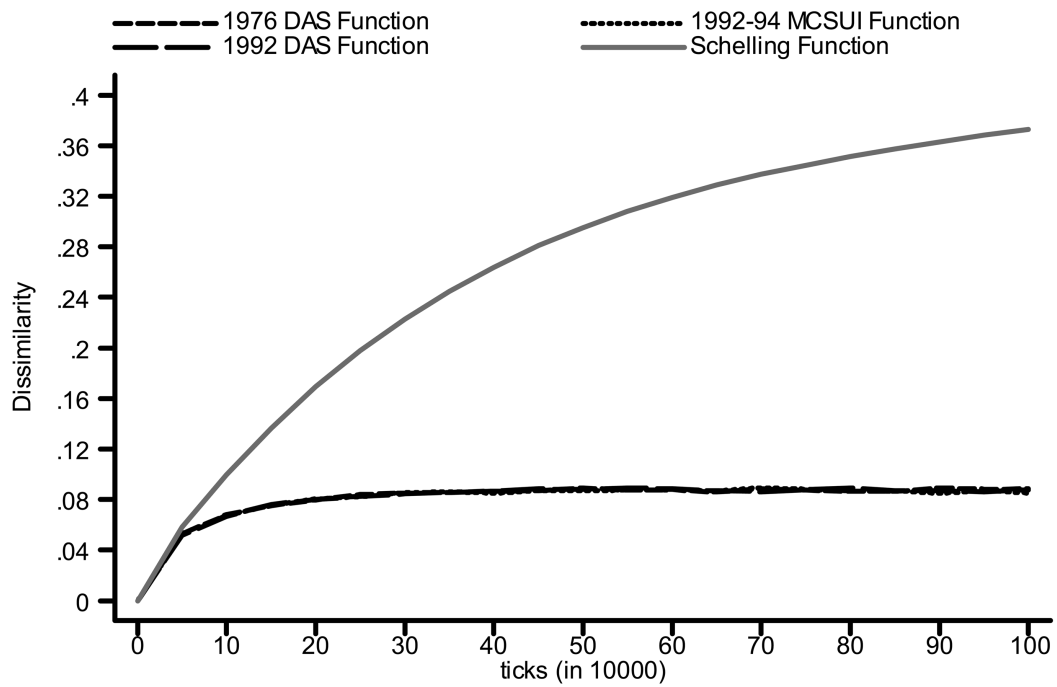

FIG. 6.—Index of dissimilarity, DAS/MCSUI functions

lations of individuals whose preferences follow threshold functions have higher levels of segregation.

The function used to simulate expected levels of segregation when individuals respond in a continuous way to variation in the proportion of a neighborhood made up of their own group can be expressed as a variation of equation (4), where $F_i(q_{ji}) = \beta * q_{ji}$ and $\beta = 1.0$. This function is illustrated in figure 2, part d. We considered the possibility that, even though this continuous function fails to yield high levels of segregation, a larger value of β, indicating a steeper response to variations in neighborhood racial composition, might lead to higher segregation levels. We simulated the segregation outcomes that occurred in a city where both blacks and whites had tolerance levels dictated by β's ranging from 5 to 55 (e.g., see fig. 2, part e, where $\beta = 55$). However, on our 500×500 lattice, all of these functions produce segregation levels similar to that produced by the continuous function shown in figure 2, part d.[26]

Another possibility is that individuals evaluate neighborhoods according to a threshold function but that this threshold is not 0.5 own group. They may be willing to remain in neighborhoods as long as the proportion own group in the area exceeds 0.3, 0.4, 0.5, or 0.6. Figure 7 shows the segregation that results if all agents are assigned one of these alternative thresholds. Segregation is highest when all the agents have a 0.6 threshold. However, predicted segregation for the agents with a threshold point at 0.4 is still higher than it is when choice functions are continuous. Thus, whereas the actual tipping point affects segregation outcomes, a wide range of tipping points produce greater segregation than any of the continuous functions considered here.

Our results indicate that whether neighborhood preference is a continuous or a threshold function of proportion own group matters more than

[26] The continuous function with a sufficiently steep slope will produce high levels of segregation on a sufficiently small (e.g., 10×10) lattice. As the slope of the continuous function increases, for larger intervals of neighborhood proportion own-group the change in neighborhood desirability associated with a change in neighborhood composition gets smaller (while for smaller intervals of neighborhood proportion own-group the change in neighborhood desirability gets larger). The continuous function leads to integration because the changing desirability of a neighborhood to a given race group offsets the proportion of that race group at risk of moving to that neighborhood. On a small lattice, changes in the size of the population at risk of entering a neighborhood occur in larger increments than do changes in the size of the population at risk of entering a neighborhood on a big lattice (because one agent moving in or out of a neighborhood changes the proportion of the population living outside that neighborhood by a smaller amount when the population is big than when it is small). Thus, for a sufficiently steep continuous function on a sufficiently small lattice, changes in the proportion of the population at risk of moving into a neighborhood no longer offset changes in neighborhood desirability and segregation will occur. See app. A for a more detailed discussion of this issue.

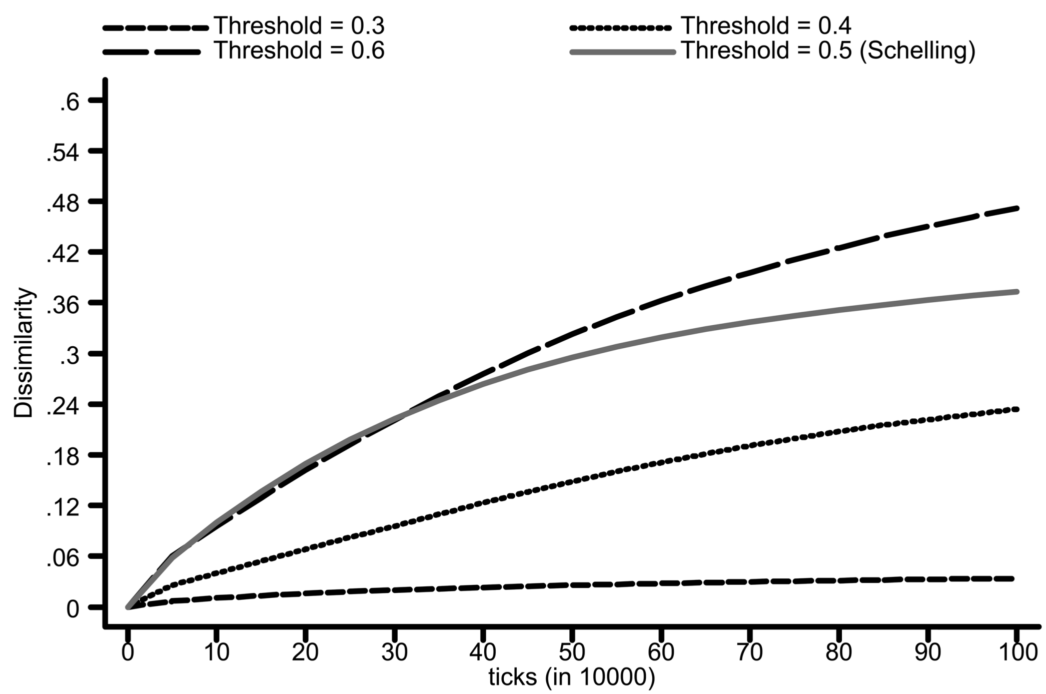

FIG. 7.—Index of dissimilarity, alternative threshold functions

the actual tipping point or average level of tolerance. It may seem counterintuitive that even an extremely steep continuous function, such as $\beta = 55$, implies lower segregation than a step function with a threshold at 0.4 own group. These results, however, should be thought of in terms of population flows rather than transition probabilities (Quillian 1999, p. 17). A low probability of moving into a less desirable neighborhood (i.e., an area with few own-race neighbors) can still lead to a net increase in own-race neighbors in that area if enough individuals are at risk of making this transition. Threshold functions have large intervals on percentage own group across which individuals are indifferent. In the threshold model, the inflow of whites (and outflow of blacks) to areas less than 50% white is not large enough to push the proportion of white agents above the 50% threshold. However, because the continuous function is responsive to the smallest change in percentage own group, even a small inflow of whites to areas less than 50% white generates a slightly larger expected flow of whites into this area in the next period, and this effect cumulates over time. In continuous models, individuals are always sensitive to small changes in race composition, creating a cascade toward integration. Neighborhoods change until the flow of agents into a particular neighborhood is offset by the flow of agents out of that neighborhood (thus, the relative desirability of an area is offset by the number of agents at risk of entering that area). Appendix A provides a more detailed discussion of this point.

We have presented simulations of mobility under several assumptions about how individuals evaluate neighborhoods in an effort to isolate the source of residential tipping. The tipping observed under the original Schelling preference function disappears when the model allows for a continuous response to own-group neighborhood proportion. Tipping may also be slowed or eliminated when thresholds are heterogeneous within race groups.[27] However, even heterogeneous thresholds produce higher segregation than a model that assumes a continuous response to own-group proportion. Thus tipping only occurs under the special circumstances when individuals follow a threshold preference function.

CONCLUSION

Alternative assumptions about individuals' behavior imply different aggregate patterns of residential segregation. In particular, whether neighborhood tipping results from the residential preferences of individuals

[27] Simulation results assuming heterogeneous thresholds within race groups are available from the authors by request.

depends critically on the form of these preferences. The threshold preference function that underlies Schelling's model predicts very different levels of segregation from models that allow for a continuous response to neighborhood composition. The same average level of tolerance but different response functions (i.e., threshold vs. continuous) give rise to different neighborhood formation patterns. Thus, researchers who wish to link individuals' neighborhood race preferences and the observed distribution of neighborhoods must be explicit about their assumptions about how individuals respond to neighborhood conditions.

Survey data suggest that people evaluate their neighborhoods according to a continuous rather than a threshold function. Thus, while the Schelling function seems compelling because it reproduces the high levels of segregation observed in actual cities, it may be misleading because the data do not support this model of individual behavior. However, the MCSUI and DAS functions, while empirically based, imply unrealistically low levels of segregation. This may be because these data fail to provide enough information to identify the threshold functional form of people's preferences or because the data may obscure other aspects of residential mobility that produce high levels of segregation. For example, recent changes in a neighborhood's race composition, rather than its current racial makeup, may govern individuals' preferences. Alternatively, it may be that it is not just the immediate neighborhood that affects residents, but a function of larger areas. Thus, mobility may create a ripple effect on seemingly unrelated parts of a city.

Residential Sorting by Race and Income

Another promising explanation for the high levels of segregation in American cities is that, even if race composition does not affect residential preferences through a threshold mechanism, race is correlated with other variables that may follow a threshold function. For example, income and wealth constraints make it impossible for poor people to live in certain neighborhoods, given the cost of to rent, high housing prices, and the unavailability of mortgages. Persons within an income stratum may share a price threshold that determines whether they can move into neighborhood. If neighborhood choices based on income follow a threshold function, and if income is correlated with race, this may imply high levels of race segregation. Redlining or racial disparity in information about available vacancies may also imply a threshold choice function because racial minorities may have no access to some types of neighborhoods.

Here we provide an alternative model of neighborhood formation in which income thresholds, coupled with income inequalities among race groups, drive observed patterns of race-ethnic segregation. Figure 8 de-

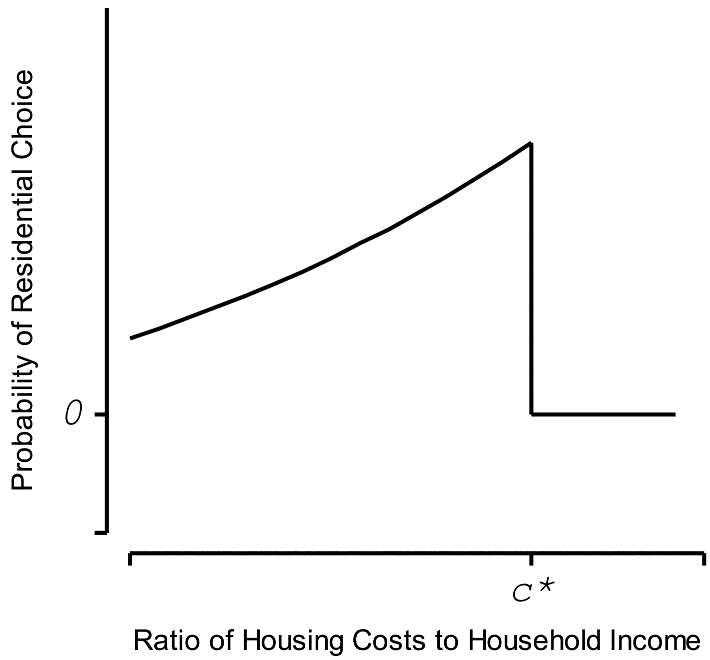

FIG. 8.—Hypothetical relationship between housing costs and household resources for an individual with a given race and economic status.

scribes one possible relationship between housing costs and the probability of choosing a housing unit, for an individual with a given level of income or wealth. For an individual with a given economic status, the probability of moving into a given housing unit increases with the unit price up to a threshold c*. People want to live in the best housing they can afford, but once housing becomes unaffordable, the probability of moving into a unit drops abruptly. Individuals distinguish among affordable units, but are indifferent over all unaffordable (unattainable) units. This choice function assumes that price is an indicator of quality and that individuals prefer the most desirable housing subject to their price constraints. Following the results presented in this paper, let us assume that (1) individuals respond to continuous variations in neighborhood racial composition; (2) blacks prefer integrated neighborhoods while whites prefer predominantly white areas; (3) whites are wealthier than blacks on average, but the wealthiest blacks are better off than the poorest whites; and that (4) higher income and higher cost areas are more desirable to both race groups. Under this model, as a few whites cluster together, this area both becomes more attractive to other whites, and also the incomes (and housing prices) in this area will be a bit higher on average than those in mixed or black areas. Thus, some blacks will not be able to afford to move into this area.[28] As more whites move in (and blacks move out), incomes and prices continue to increase, thereby barring a larger proportion of blacks from entering (due to price constraints). We can imagine how this effect may cumulate over time. A continuous preference function results in integration because a decrease in the probability of moving to a given neighborhood for a given race group is offset by an increase in the proportion of agents of that race group at risk of moving to that neighborhood. However, once we introduce income thresholds and income inequality among race groups, this is no longer true. It is not clear what level of race segregation a model that incorporates price thresholds and income inequalities among race groups would imply, but, based on the results of this paper, it seems plausible that residential sorting by both race and income may produce higher levels of racial segregation than either factor alone (Bruch 2005).

Reversible and Irreversible Choices

Our investigation of the robustness of Schelling's segregation model to alternative behavioral assumptions and of the empirical basis for these assumptions may hold lessons for other areas of sociological research that have relied on threshold, contagion, and diffusion models. One important condition, however, is that our results concerning the macrolevel implications of threshold and continuous individual behavior functions only apply to circumstances where the microlevel process is reversible. When the individual behavior under consideration is not reversible, a continuous function will create macrolevel social dynamics that closely resemble those generated by a threshold function.

Consider contrasting examples of reversible and irreversible processes. Hernes's (1972) applied diffusion models to entry into first marriage. His model assumes that the social pressure to marry is proportional to the number of those already married in the same cohort, and the rate of change into marriage is proportional to this pressure. Thus individuals' choice functions are continuous. However, unlike residential choice, the decision to enter into one's first marriage is, by definition, not reversible. Thus, if the decision to marry is a continuous function of the proportion of one's peers who have already married, the female population will still tip to a state in which all are married. When a few women are married, this raises the probability that others in their reference group will also

[28] The extent to which blacks are unable to afford to live in predominantly white areas depends on the level of income inequality between race groups.

marry. As more women marry, pressure mounts even further on those still single and the flow of women into marriage cannot be offset by a flow of women from marriage into a never-married state.[29]

In contrast, less permanent traits, such as the choice to drink Coke instead of Pepsi, to select a hair style, or to support a presidential candidate during the primary election, which may depend on how many others in the population behave, are clearly reversible decisions. For example, in U.S. presidential primaries, individuals' beliefs about a candidate are influenced by perceived public opinion (Bartels 1988, pp. 110–12). If voters wish to support the most electable candidate in their party during the presidential primary and they vote for a candidate if and only if at least, say, 50% of polled voters indicate that they support this person, as soon as a candidate reaches this level of support, his or her victory is virtually assured. Underdog candidates have no hope of reaching the threshold needed to attract voters. In contrast, if the decision to vote for a candidate is a continuous function of the proportion of polled voters who say they will vote for this person, the outcome is less clear. As a candidate gains popular support, she or he attracts a growing number of persons who may defect to a less favored contender whom individual voters may support with a lower probability.

Threshold, contagion, and diffusion models provide an explicit and fruitful link between individuals' choices and collective outcomes. Many applications of these models, however, have not been accompanied by efforts to examine the robustness of theoretical results to microbehavioral assumptions. Moreover, researchers often lack empirical data that would support one or another set of assumptions. The aesthetic and scientific appeal of these kinds of models notwithstanding, their future success will depend on further efforts to place their formal assumptions on a solid empirical footing.

APPENDIX A

Interactive Markov Chain Models

To explore further the result that continuous preference functions yield low levels of segregation and as a check on our results, we can reformulate the problem as a deterministic, nonlinear, discrete-time dynamic system. That is, if we assume that neighborhoods have fixed boundaries, we can view the mobility process as an interactive Markov chain (Conlisk 1976),

[29] This holds with equal force if the decision to marry follows a threshold rather than a continuous function.

in which the transition probabilities at time t depend on the population distribution at time t; that is,

$$m[t+1] = P(m[t])m[t], \tag{A1}$$

where the vector $m[t]$ denotes the expected distribution of the population across neighborhoods at time t. The number of rows in $m[t]$ is $S \times k$, where k is the number of neighborhoods, and S is the number of race-ethnic groups. For example, if $S = k = 2$, the population vector will take the form

$$m[t] = \begin{bmatrix} \text{blacks in neighborhood 1 at time } t \\ \text{blacks in neighborhood 2 at time } t \\ \text{whites in neighborhood 1 at time } t \\ \text{whites in neighborhood 2 at time } t \end{bmatrix}. \tag{A2}$$

The transition matrix P is an $Sk \times Sk$ matrix of mobility probabilities between all possible pairs of neighborhoods. In our model, we assume that the probability of entering state j at time $t + 1$ is the same for all states i (including $i = j$). Thus, all the rows of this matrix are identical. In this model, the transition matrix is a function of $m[t]$, which itself is a function of previous mobility. Conlisk derived stationary distributions for some interactive Markov chains, but no analytic equilibrium is known for our model. For any given initial transition probabilities and population composition, however, we can compute the equilibrium numerically.

To show how alternative preference functions affect segregation, we consider a highly simplified city with only two neighborhoods and a population that consists of 10 blacks and 10 whites. At time zero, the population is completely segregated; all blacks are in one state, and all whites are in the other. Thus, $m[0] = [1, 0, 0, 1]$. Next, we compute the population trajectory for whites and blacks using alternative preference functions.

For example, if people evaluate their neighborhoods according to the continuous preference function shown in figure 2, part d, $m[1]$ is

$$m[1] = P_0 \times m[0]$$

$$= \begin{bmatrix} \frac{e^{m[0][1]}}{e^{m[0][1]} + e^{m[0][2]}} & \frac{e^{m[0][1]}}{e^{m[0][1]} + e^{m[0][2]}} & 0 & 0 \\ \frac{e^{m[0][2]}}{e^{m[0][1]} + e^{m[0][2]}} & \frac{e^{m[0][2]}}{e^{m[0][1]} + e^{m[0][2]}} & 0 & 0 \\ 0 & 0 & \frac{e^{m[0][3]}}{e^{m[0][3]} + e^{m[0][4]}} & \frac{e^{m[0][3]}}{e^{m[0][3]} + e^{m[0][4]}} \\ 0 & 0 & \frac{e^{m[0][4]}}{e^{m[0][3]} + e^{m[0][4]}} & \frac{e^{m[0][4]}}{e^{m[0][3]} + e^{m[0][4]}} \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{e^1}{e^1 + e^0} & \frac{e^1}{e^1 + e^0} & 0 & 0 \\ \frac{e^0}{e^1 + e^0} & \frac{e^0}{e^1 + e^0} & 0 & 0 \\ 0 & 0 & \frac{e^0}{e^1 + e^0} & \frac{e^0}{e^1 + e^0} \\ 0 & 0 & \frac{e^1}{e^1 + e^0} & \frac{e^1}{e^1 + e^0} \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.731 \\ 0.269 \\ 0.269 \\ 0.731 \end{bmatrix}. \tag{A3}$$

Figure A1 shows the two-state discrete time Markov chain models for the threshold (eq. [1]), nonzero (eq. [2]), staircase (eq. [3]), and continuous (eq. [4], where $F(q_{jt}) = q_{jt}$) preference functions at selected time points. The continuous function equilibrates in a completely integrated state. In contrast, the threshold (Schelling) function remains completely segregated, and the nonzero and staircase functions equilibrate at a low level of integration.[30]

[30] Because agents in the computational model evaluate the 24 cells surrounding a vacancy, whereas the interactive Markov models assume that neighborhoods are a fixed grid, the level of segregation attained in the interactive Markov model exceeds that attained in the agent-based model. However, both methods lead to the same substantive conclusions. We examined what segregation outcomes emerge if agents evaluate neighborhoods according to the Schelling preference function for up to five million iterations. In these longer simulations, segregation reaches a stable plateau

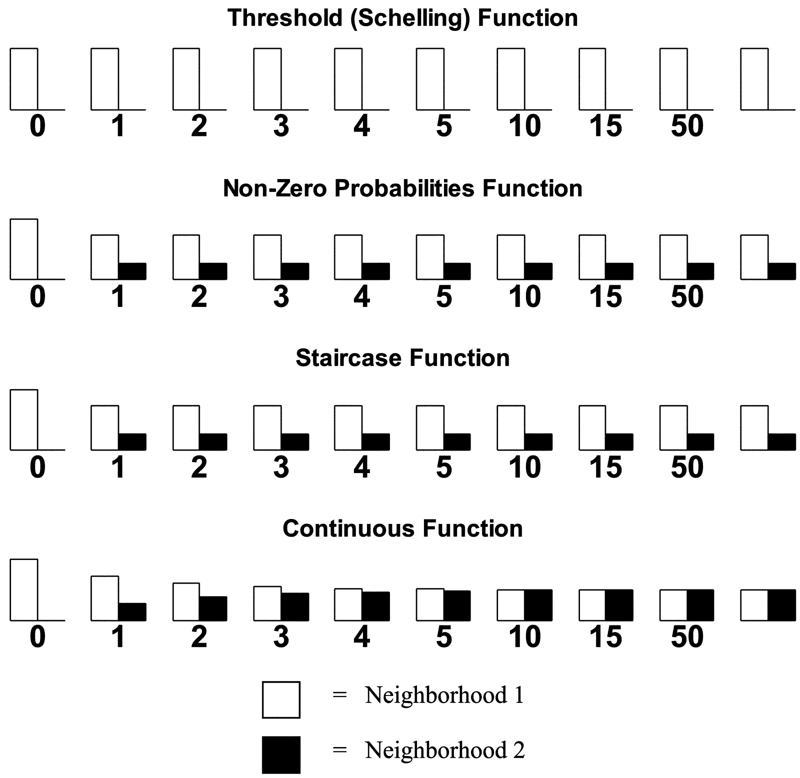

FIG. A1.—Proportion black in neighborhoods 1 and 2; for all four images, the vertical axis is "proportion black," and the horizontal axis is "time."

To see how the continuous function yields neighborhood integration, even though the transition probabilities imply that people prefer to live among their own group, it is helpful to examine a few steps of the Markov

after approximately one million iterations with a maximum index of dissimilarity of 0.42. In contrast, if agents with the Schelling choice function treat neighborhoods as a fixed grid of tracts, the final index of dissimilarity is 1.0.

chain. Equation (A3) shows the first step for the continuous, linear function. Below we compute the next two steps. In step 2,

$$
\begin{aligned}
m[2] &= P_1 \times m[1] \\
&= \begin{bmatrix}
\frac{e^{0.731}}{e^{0.731}+e^{0.269}} & \frac{e^{0.731}}{e^{0.731}+e^{0.269}} & 0 & 0 \\
\frac{e^{0.269}}{e^{0.731}+e^{0.269}} & \frac{e^{0.269}}{e^{0.731}+e^{0.269}} & 0 & 0 \\
0 & 0 & \frac{e^{0.269}}{e^{0.731}+e^{0.269}} & \frac{e^{0.269}}{e^{0.731}+e^{0.269}} \\
0 & 0 & \frac{e^{0.731}}{e^{0.731}+e^{0.269}} & \frac{e^{0.731}}{e^{0.731}+e^{0.269}}
\end{bmatrix} \times \begin{bmatrix} 0.731 \\ 0.269 \\ 0.269 \\ 0.731 \end{bmatrix} \\
&= \begin{bmatrix}
0.613 & 0.613 & 0 & 0 \\
0.387 & 0.387 & 0 & 0 \\
0 & 0 & 0.387 & 0.387 \\
0 & 0 & 0.613 & 0.631
\end{bmatrix} \times \begin{bmatrix} 0.731 \\ 0.269 \\ 0.269 \\ 0.731 \end{bmatrix} = \begin{bmatrix} 0.614 \\ 0.387 \\ 0.387 \\ 0.613 \end{bmatrix},
\end{aligned}
\tag{A4}
$$

and in step 3,

$$
\begin{aligned}
m[3] &= P_2 \times m[2] \\
&= \begin{bmatrix}
\frac{e^{0.614}}{e^{0.614}+e^{0.387}} & \frac{e^{0.614}}{e^{0.614}+e^{0.387}} & 0 & 0 \\
\frac{e^{0.387}}{e^{0.614}+e^{0.387}} & \frac{e^{0.387}}{e^{0.614}+e^{0.387}} & 0 & 0 \\
0 & 0 & \frac{e^{0.387}}{e^{0.614}+e^{0.387}} & \frac{e^{0.387}}{e^{0.614}+e^{0.387}} \\
0 & 0 & \frac{e^{0.614}}{e^{0.614}+e^{0.387}} & \frac{e^{0.614}}{e^{0.614}+e^{0.387}}
\end{bmatrix} \times \begin{bmatrix} 0.614 \\ 0.387 \\ 0.387 \\ 0.613 \end{bmatrix} \\
&= \begin{bmatrix}
0.557 & 0.557 & 0 & 0 \\
0.443 & 0.443 & 0 & 0 \\
0 & 0 & 0.443 & 0.443 \\
0 & 0 & 0.557 & 0.557
\end{bmatrix} \times \begin{bmatrix} 0.614 \\ 0.387 \\ 0.387 \\ 0.613 \end{bmatrix} = \begin{bmatrix} 0.557 \\ 0.443 \\ 0.443 \\ 0.557 \end{bmatrix},
\end{aligned}
\tag{A5}
$$

At time zero, a small number of whites move from neighborhood 2 into neighborhood 1. This slightly increases whites' preferences for neighborhood 1, and slightly decreases their preferences for neighborhood 2. Since there are more whites in neighborhood 2 than in neighborhood 1, even a small probability of moving into neighborhood 2 results in a net inflow of whites into that area. Meanwhile, even though whites are leaving neighborhood 1 and moving into neighborhood 2, the number of whites in neighborhood 1 is too small for even a large probability of entry into neighborhood 2 to offset the outflow of whites from neighborhood 2. This process continues until the inflows and the outflows are equal; that is, when the population is evenly distributed across the two neighborhoods. This happens with the continuous function, but not with others, because even the smallest change in neighborhood proportion own group results in a change in the attractiveness of that neighborhood. In contrast, with threshold models, individuals are equally attracted to neighborhoods across some interval of proportion own group, and a small change in proportion own group is not enough to increase a neighborhood's appeal.

Lattice Size, Continuous Preference Functions, and Integration

Threshold preference functions lead to residential tipping on both small and large lattices. However, for continuous preference functions of varying degrees of nonlinearity, the segregation outcome depends on the size of the lattice. Continuous functions that lead to integration on a large lattice can generate segregated neighborhoods on a small lazttice. The reason for this is as follows. When the effect of proportion own group on residential preference follows a continuous function but is small to moderate in size, a small change in neighborhood composition produces a corresponding change in neighborhood desirability. However, for highly nonlinear continuous functions, a small change in neighborhood composition produces almost no change in neighborhood desirability for most values of neighborhood composition. As discussed above, continuous functions tend to lead to integration because any change in the size of the population at risk of moving in (because an agent has entered or exited the neighborhood) is offset by a corresponding change in neighborhood desirability.

As the continuous function approaches a threshold function, the change in utility associated with a change in neighborhood composition gets smaller for a wider range of values of neighborhood proportion own-group. Thus, for continuous functions with steep slopes, the change in the size of the population at risk of moving into a neighborhood must be small enough to offset the diminished change in neighborhood desirability. On a small lattice, such as a 20×20 lattice populated by 80 black agents and 80 white agents, changes in the size of the black or white population at risk of moving into any given neighborhood occur in 1/80 increments. In contrast, on a 500×500 lattice populated by 106,250 black agents and 106,250 white agents, changes in the size of the black or white population at risk of moving into any given neighborhood occur in 1/106,250 increments. These changes are more fine grained. On a smaller lattice, when agents behave according to a continuous function with a steep slope, changes in the size of the population at risk may be too large to be offset by changes in neighborhood desirability. However, on a larger lattice, because changes in the size of the population at risk occur in smaller increments, these two may have offsetting effects thereby leading to integration.

Threshold functions are robust across lattice sizes because the desirability of neighborhoods is invariant except at threshold points. Even for an infinite population (with infinitesimal increments in the size of the population at risk), a change in the population at risk of entering the neighborhood is not offset by a change in neighborhood desirability except at the threshold point.

A Closer Look at the Agent-Based Model

In this section we provide further details about the implementation of the agent-based model. The model, which is programmed in Java, uses a 500 $\times$ 500 cell grid populated by interacting agents. Each agent lives in a single cell; no more than one agent can occupy any cell. Fifteen percent of the cells are vacant. Each agent has a race and a preference for neighborhood composition. This preference is given by a choice function, as shown above in equations (1)–(4).

When the model is initialized, the agents are evenly distributed across the grid and the index of dissimilarity is zero. Next one agent is chosen from the population using random sampling with replacement. That agent evaluates the ethnic makeup of the neighborhood surrounding its current cell as well as the neighborhoods surrounding all vacant cells on the grid. The agent chooses a new destination or stays put based on the relative desirability of its possible destination neighborhoods. Each sampled agent's mobility opportunity makes up one time step of the model. As time unfolds, the neighborhoods in the model change as a function of agents' mobility decisions.

Figure A2 illustrates the decision process for a single choice function and a small grid size. In particular we use the choice function shown in equation (4), where $F_l(q_{jt}) = q_{jt}$ and there are only four available neighborhoods. The figure is divided into four panels. For each sampled agent, we repeat the following process. In part A, an agent examines the neighborhoods where it might move. The jth neighborhood has a race-ethnic composition (q_{jt}) where t indexes the time step. The agent uses its choice function to evaluate the relative desirability of each available neighborhood. Thus, if $V(1)$ denotes the relative desirability of neighborhood $j = 1$ and q_{jt} is the proportion own-group in that neighborhood at time t, then $V(1) = q_{1t}$. This is illustrated in figure A2, part B. The result is a list of relative desirability scores associated with each neighborhood. The agent

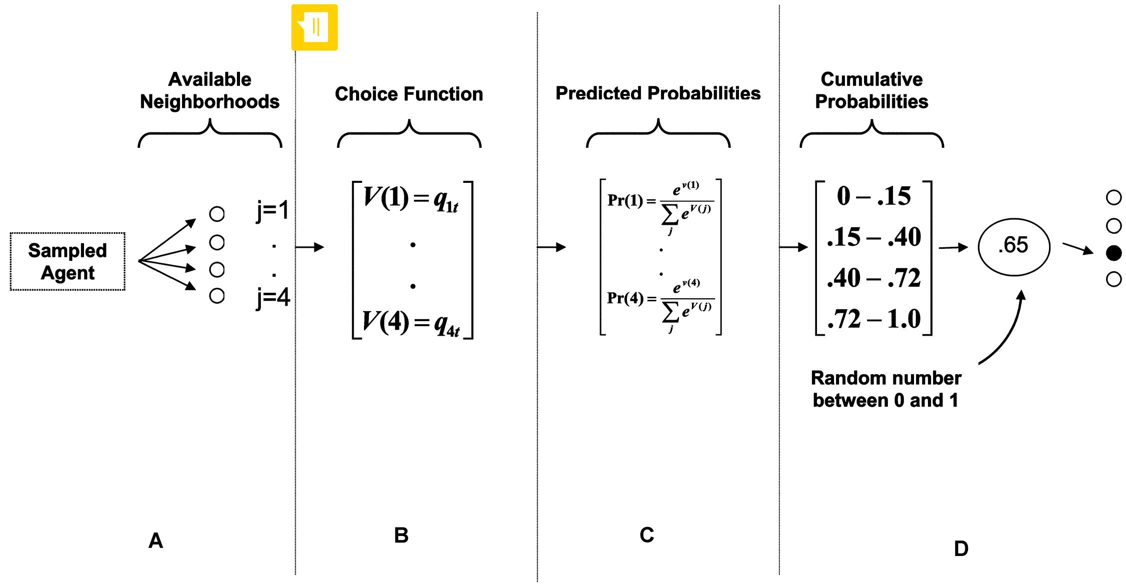

FIG. A2.—Residential mobility process in agent-based model (with four neighborhoods) then turns these relative desirability scores into predicted probabilities by dividing each score by the sum of the scores, as shown in part C. The list of predicted probabilities sums to one by construction. In part D the agent converts the predicted probabilities into cumulative probabilities, where, for example, the cumulative probability associated with neighborhood $j = 3$ is the sum of the probabilities associated with all neighborhoods where $j \leqslant 3$. This yields a list in which neighborhoods with higher choice probabilities have wider intervals. Finally, the agent samples a random number from a uniform (0,1) distribution and "picks" the neighborhood with the interval in panel D that contains the selected number. In the example in figure A2, the random number is 0.65, which falls into the interval associated with the $j = 3$ neighborhood. This neighborhood is shaded black in the final panel of the figure.

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