Assignment

Homework HL4

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Musical Acoustics

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Introduction

In this study, we're going to implement the model of a trumpet in COMSOL Multiphysics environment, allowing us to simulate it's acoustic response. Through the *Finite Element Method* (FEM) simulation, we'll compute the input impedance of such trampet in frequency domain, its radiated sound pressure and directivity pattern.
We'll study three models progressively more complex and similar to the original instrument. In particular:

- The tube
- The tube + the bell
- The mouthpiece
- The tube + the bell + the mouthpiece

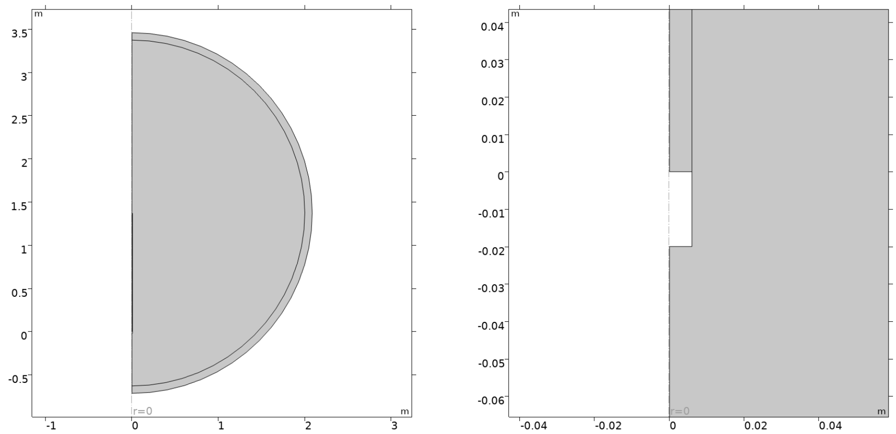
Part 1: The tube

In this section, we'll descrive the implementation of the first simple model of the trumpet: the tube. The tube of the trumpet is the place where sound waves are generated, and it can only produce "fixed - tones" (its eigenfrequencies).
Following, the data used to model the tube in comsol environment:

Name	Expression	Value	Description
rT	0.6[cm]	0.006 m	Tube radius
rS	2[m]	2 m	Air sphere radius
N	255	255	Number of frequencies
Lt	1.37[m]	1.37 m	Tube length
Lspace	20[mm]	0.02 m	Length empty space
lambdaMax	c0/fMax	0.08575 m	Wave length max frequency
fMin	50[Hz]	50 Hz	Min Frequency
fMax	4000[Hz]	4000 Hz	Max Frequency
c0	343[m/s]	343 m/s	wave speed

Figure 1: Simulation and Tube parameters

To build such model, we've taken advantage of the axisymmetric condition satisfied by the system to use a 2D geometry. To represent the tube, we'll use a thin and long rectangle.
We also need to take in consideration the free field condition. To do so, we first surrounded the tube with a sphere of air, represented by a semicircle that contains the rectangle. Then, we set a PML boundary condition between the tube and the air, by adding another semicircle made of totally absorbing material outside the previous one.
The complete model can be seen in the following figure:



(a) Tube model with air surrounding (b) Zoom of the opening

Figure 2: Tube Model

For the simulation, we've set up a really fine mesh that satisfies the " 5 points per wavelength condition" required.

We'll now plot the input impedance. This has been directly calculated by using the following formula:

$$Z(\omega) = \frac{P(\omega)}{U(\omega)}$$

where $P(\omega)$ is the sound pressure generated, while $U(\omega)$ is the velocity of the sound waves at the input boundary. The plot is shown here:

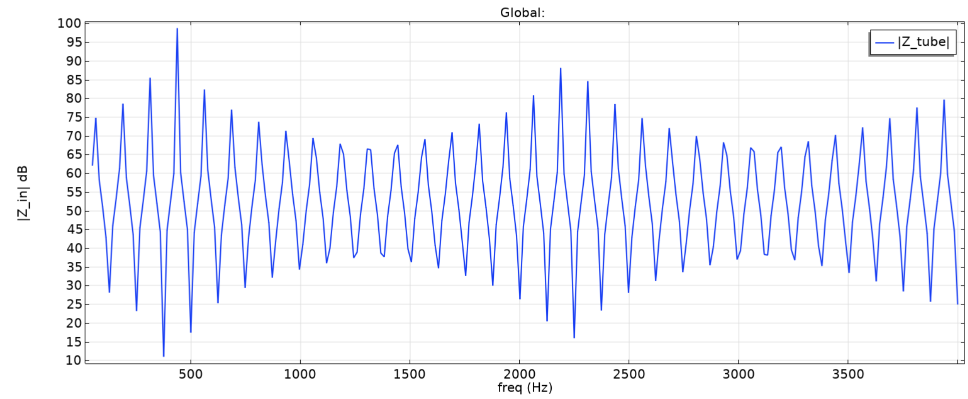


Figure 3: Input impedence of the pipe

We can notice how, for various frequencies, we have relative maxima and minima: the lowest values of the impedance represent the eigenfrequencies of our tube.

We then study the directivity pattern of such tube. To do so, we've traced a circumference of radius $rS = 2$ m around the pipe on the *Revolution 2D* dataset, generating a *Parameterized Curve 3D* dataset. This allowed us to calculate the SPL on the circumference and we can now plot it in a polar graph. The results are reported in the following figure. The sound pressure has been normalized in the graph.

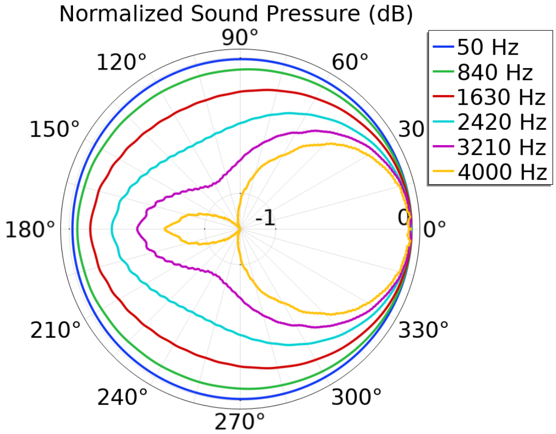


Figure 4: Directivity pattern - Tube

We can observe how the shape of the directivity varies as frequency grows, going from an omnidirectional circular shape to a more deformed one that loses components from 90° to 270°.

Part 2: Tube + Bell

We'll now continue our study by adding a bell to the end of the pipe. The bell has a particular effect on the trumpets sound, since it modifies the tube with a conical section that varies the overtones: this improves the radiation of higher frequencies, making the sound brighter and louder.
In particular, we've implemented an exponential bell whose radius obeys the following formula:

$$r_{BELL} = \sqrt{\frac{S_T}{\pi} e^{mx}}$$

where

- $S_T = 1.131 \times 10^{-4}$ m^2 is the tube's surface
- $m = 28$ is the bell exponent

The bell's length will be $L_H = 0.2$ m.
A zoom of such model can be seen in the following figure.

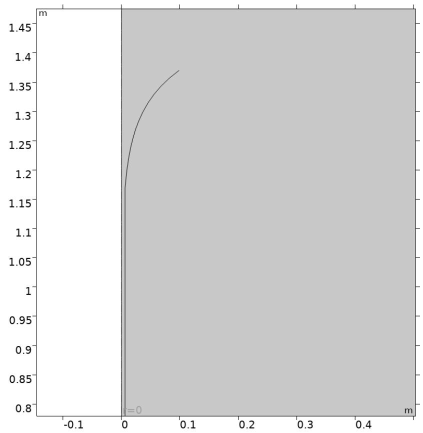


Figure 5: Tube and Bell Model Zoom

Once again, the mesh has been setup in order to satisfy the ”5 points for wavelength condition”. We’ll now show the obtained input impedance for such configuration.

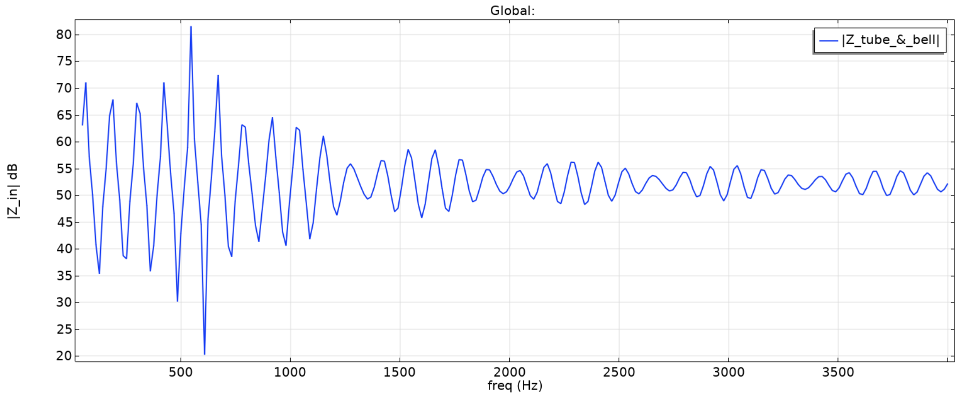


Figure 6: Tube + Bell Input Impedance

We can quickly notice the effect on the bell on the tube’s impedance. The magnitude of the response decreases as the frequency increases, and the resonances and antiresonances are shifted in value from above 500Hz up to the maxima of the simulation.
We’ll now consider the directivity pattern for such configuration, to study the possible effects of the bell.

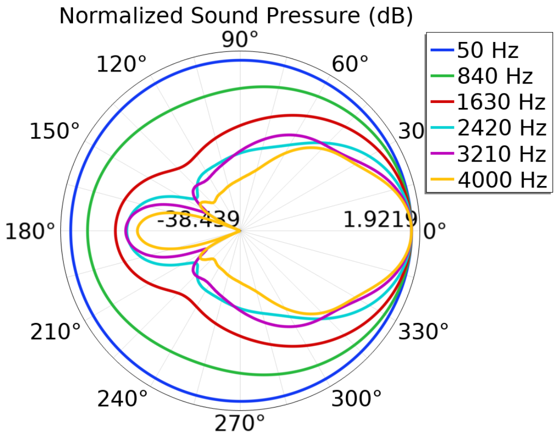


Figure 7: Directivity pattern - Tube & Bell

Once again, as the frequency grows, the directivity loses its omnidirectional characteristics. In particular, we can notice how here the frontlobe deforms more than in the previous configuration (Figure 4). Also, a change in the axis values can be noted, enhancing the forward directivity of such configuration.

Part 3: Mouthpiece

We'll now model the mouthpiece. The mouthpiece is composed of a cup and a narrow throat, where high acoustic pressure is generated. To recreate such structure, we've modelled the mouthpiece by combining together a circle a cone, having the short side equal to $\frac{1}{8}$ of the tube's radius. Following, the parameters for the mouthpiece model and a zoom on the generated geometry (For this study, the air domain radius has been reduced to $r = 1$ m).

- $r_M = 8 \times 10^{-3}$ m, Mouthpiece Radius
- $L_M = 10^{-1}$ m, Mouthpiece Length

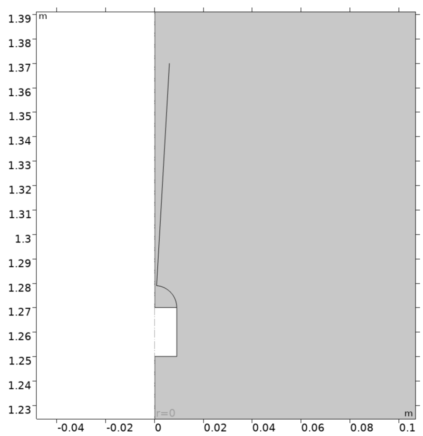


Figure 8: Mouthpiece Model

As for the previous sections, we're going to analyze the input impedance of the mouthpiece. The result is shown in the following plot.

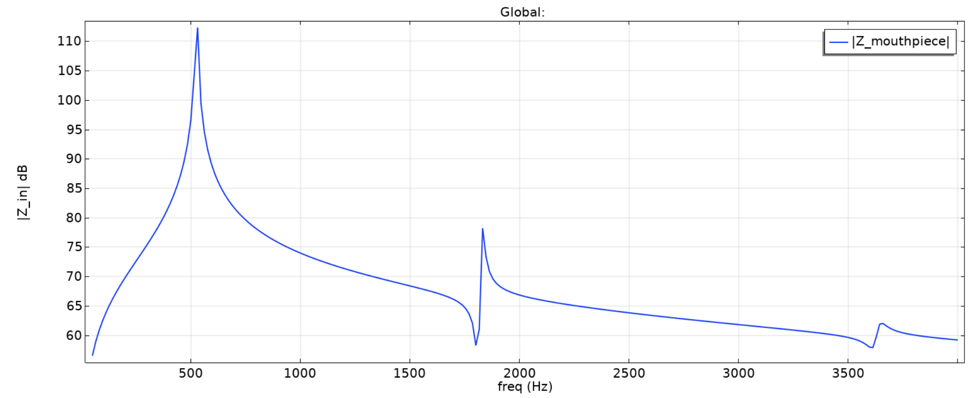
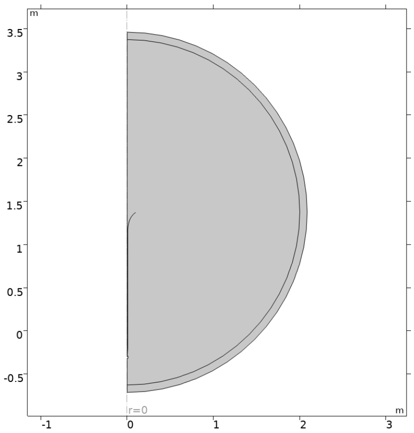


Figure 9: Mouthpiece Input Impedance

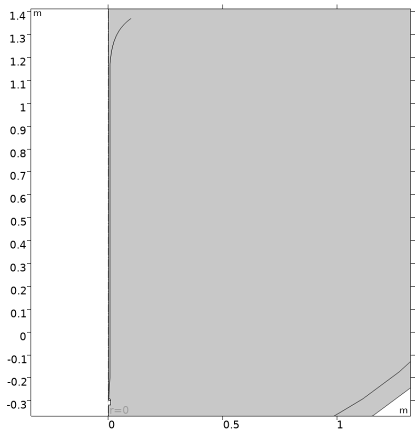
We notice three peaks of the impedance and two minima. In particular, the peak at $f = 530.2Hz$ has greater magnitude that the other anti-resonances found in the previous simulations. We can imagine how the mouthpiece will effect the overall input impedance of the trumpet.

Part 4: Complete Trumpet Model

In the last part of this study, we're going to combine together all the models we've studied up to here, by combining the tube with the bell to the mouthpiece and analyze the overall behavior of those components. The complete model is shown in the following figure.



(a) Complete Trumpet Model



(b) Zoom on the trumpet model

Figure 10: Complete Trumpet Model

Once again, we study the input impedance of the system.

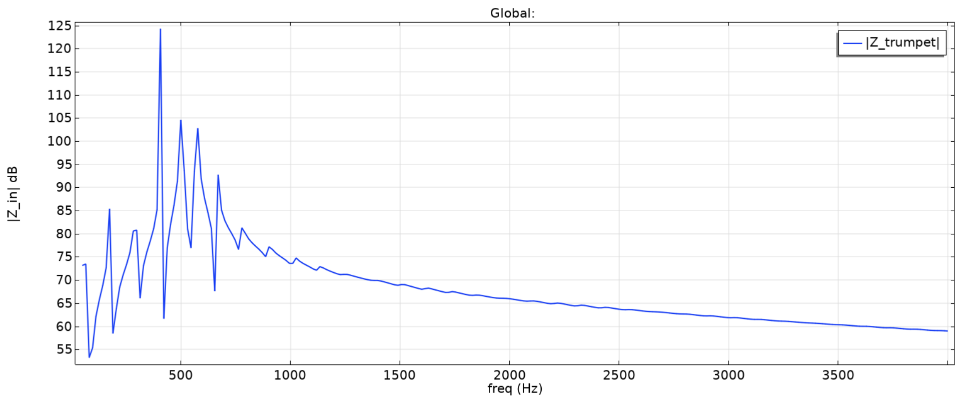


Figure 11: Trumpet Input Impedance

We can immediately notice the effect of the mouthpiece on the tube's impedance: the peaks and resonance of such system are accentuated around the mouthpiece's impedance peak. This result is more evident when comparing directly all the impedances, as show in the following plot:

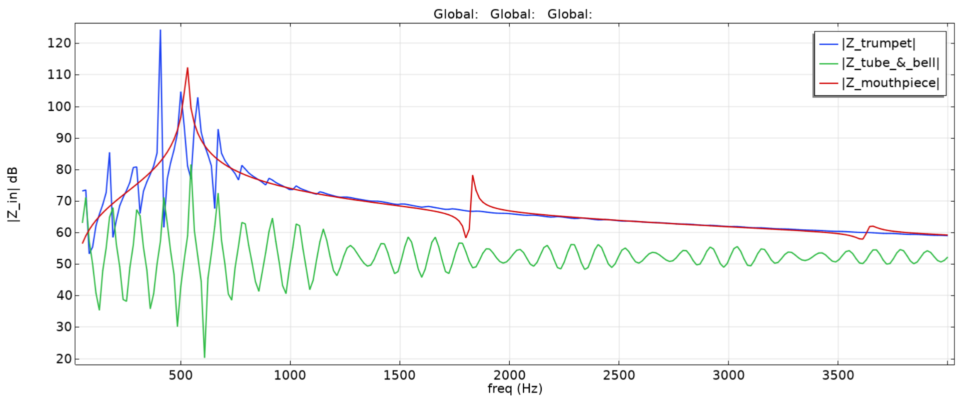


Figure 12: Input Impedance Comparison

As written before, we can notice how the overall impedance mainly follows the mouthpiece behavior. The tube and bell behavior is noticeable up to about 600 Hz, its contribution fades away for higher frequencies.
We're now going to analyze the directivity pattern for the complete model.

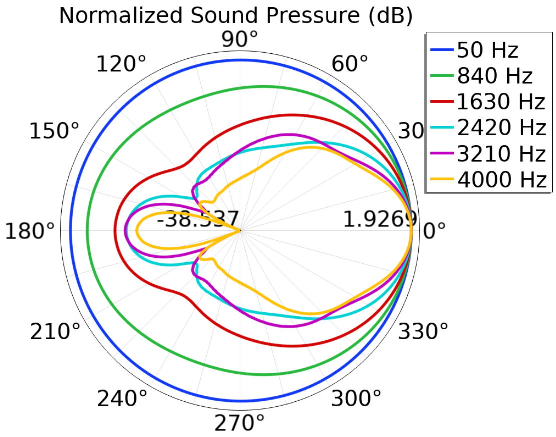


Figure 13: Directivity pattern - Trumpet

We can quickly notice how the addition of the mouthpiece didn't change much the directivity of the instrument (Figure 7. This shows how little the mouthpiece influences the directivity on the instrument, having a bigger impact on the impedance instead.

Conclusions

In summary, our exploration into simulating the behavior of a trumpet using the COMSOL simulation environment has yielded valuable insights. Divided into four distinct parts focusing on the tube, tube with bell, mouthpiece, and the complete trumpet assembly, our study delved into the intricacies of each component. By scrutinizing the input impedance for individual parts and examining their cumulative effects in the complete trumpet model, we gained a nuanced understanding of how these elements contribute to the overall acoustic performance. Additionally, our investigation into the directivity patterns further enriched our comprehension of the trumpet's sound production. This study not only deepens our understanding of the instrument's behavior but also offers practical insights for musicians, instrument designers, and acousticians. The simulation approach employed herein emerges as a valuable tool, enabling a more informed and comprehensive exploration of trumpet acoustics.