Assignment

Homework HL2

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Musical Acoustics

POLITECNICO
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Introduction

In this study, we're going to study the behaviour of an Helmholtz resonator, by simulations in COMSOL environment. We'll analyze its behaviour when changing some of his parameters and study its interaction with another resonator.

Part 1: simple resonator

Design of the Resonator

To define our resonator, we first need to calculate the size of the diameter D of its cavity. Having the following data:

- Resonance frequency: $f_0 = 300$ Hz.
- Opening diameter: $d_1 = 4$ cm.
- Opening lenght : $l_1 = 1$ cm.

we can derive the value of the diameter of the Sphere of the resonator inverting the formula:

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S}{V \cdot L}}$$

Considering the expression of the volume of a generic sphere:

$$V = \frac{4}{3}\pi r^3$$

we obtain D as:

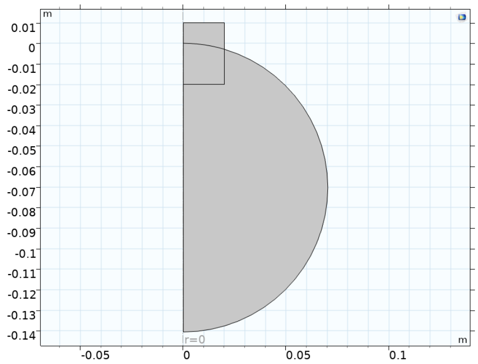
$$D = 2 \cdot r = 2 \cdot \sqrt[3]{\frac{c^2}{4\pi^2}\frac{S}{\frac{4}{3}\pi f^2 L}}$$

In order to have a correct simulation in the comsol environment, since we're using an analytical approach that works under precise hypotesis, we need to add a correction factor to the length of the pipe. In an empirical way, the new length of our pipe becomes $L = l_1 + 0.93 \cdot \frac{d_1}{2} = 2.86$ cm. And so, we find the diameter as:

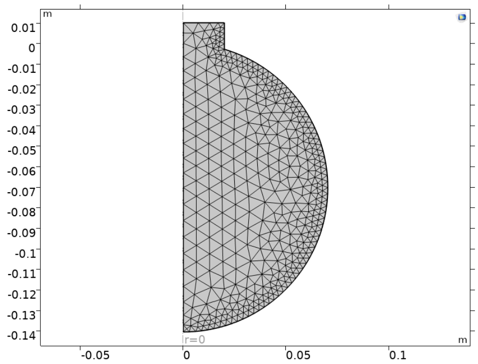
$$D = 2 \cdot r = 14.06 \text{ cm}$$

Building the model in COMSOL environment

We now proceed to define the geometry in COMSOL environment. To do so, we use a 2D axial symmetric environment, using the viscous gas condition in the *"pressure acoustics, frequency domain"* physics. Having defined the parameters from the data obtained before, we proceed to draw a half circle representing the sphere and a rectangle above it to model the opening. We then join the two components together to create a simple structure (The interior boundaries have been left here for better understanding of the component).



(a) 2D Model



(b) Mesh

To define the mesh for our study, we first created an edge mesh with extra fine size, in order to refine the mesh on its borders. We then finish it with a free triangular mesh considering the whole element.

We now define two probes at the opening of the resonator, in order to measure the pressure and the velocity at the entrance: this will be needed to calculate the input impedance of the model. Setting an input pressure of 1 kPa on the opening, and the material of the component as air, we're now ready to perform our analysis.

Frequency sweep analysis: 4cm diameter

We first perform a Frequency Sweep Analysis, considering the model described before, in a range that goes from 10Hz to 10kHz, with a step of 10Hz. The data from the probes has then been elaborated with a matlab script, obtaining the impedance as $Z_{in}(\omega) = -\frac{P(\omega)}{U(\omega)}$. We can see the obtained result below:

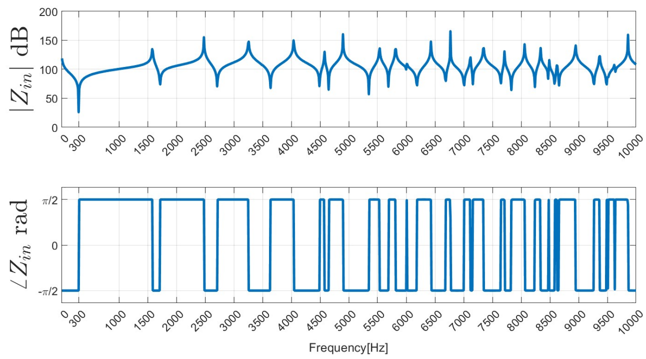


Figure 1: Frequency sweep analysis: 4cm diameter

As we can see from the graph, we notice a resonance peak at 300 Hz as expected. The graph represents the classical behaviour of two simple structures interacting with eachother. In particular, after about 4kHz, we can notice how the modes of the tube start interacting with the ones of the sphere, creating an uneven behaviour.

Frequency sweep analysis: changing the diameter

We now add a parameter sweep to the analysis, varying the diameter of the opening of the resonator, keeping the length of the opening and the diameter of the sphere fixed. We're going to analyze the following cases:

- d_1 = 1cm
- d_1 = 3cm
- d_1 = 8cm

We now plot the results, still elaborated with a matlab script:

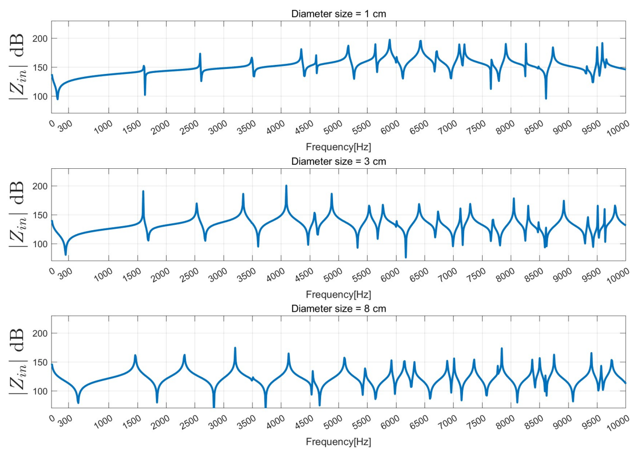
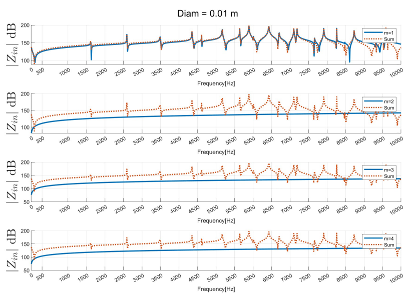


Figure 2: Frequency sweep analysis: 1cm, 3cm and 8cm diameter

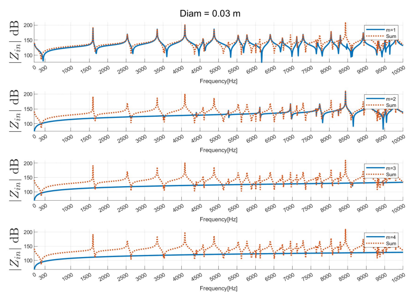
We can notice how the resonance frequency varies as the diameter changes: it goes way below 300Hz for the 1 cm case, while going above 300Hz for the 8cm case. As in the previous paragraph, we can notice the combination of the modes happening above 4kHz.

Modal Analysis

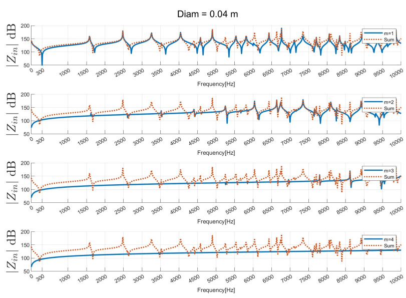
We now proceed to perform a modal analysis on the resonator. In particular, we're going to study the behaviour of the modes 0,1,2,3 at the various diameters of the opening seen before. This is done by performing a double parametric sweep in the comsol environment. Here are the results obtained from the probes (in red, the summation of the 4 modes):



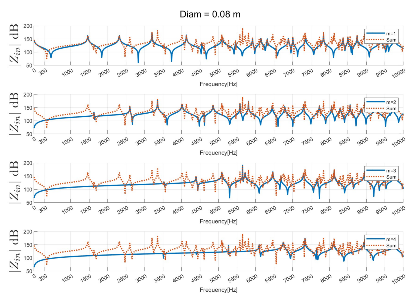
(a) Modal analysis, 1cm



(b) Modal analysis, 3cm



(c) Modal analysis, 4cm



(d) Modal analysis, 8cm

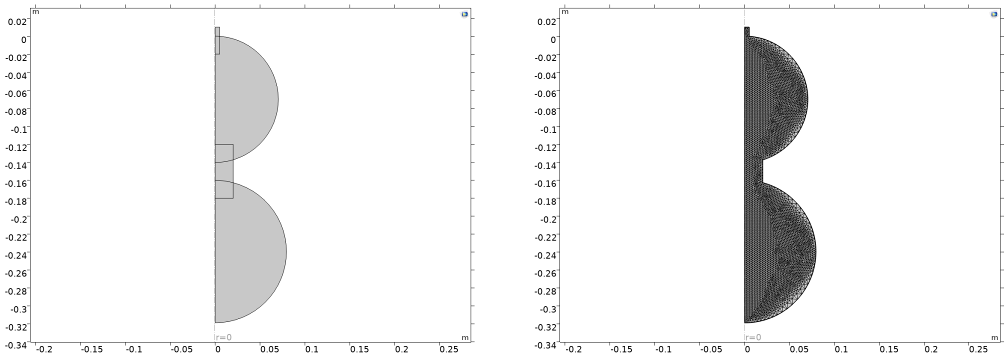
From the graphs, we can notice how the resonator with the diameter of 1 cm has less resonances in the range of interest. Therefore, we're going to use this condition four our next steps.

Part 2: Interaction with another Resonator

We're now going to study the behvaior of a system composed of two resonators, one connected to the other at the closed end.

Design and build of the system

We build a new model in comsol, still considering the pressure acoustics physics (in the viscous model condition) and using air as the material. We rebuild the first resonator using the diameter found in the previous paragraph. We now add another resonator, with opening diameter $d_2 = 4$ cm, opening length $l_2 = 2 \cdot l_1$ and sphere diameter $D_2 = 15.86$ cm. In the end we connect this last resonator to the bottom of the previous one, paying attention that the length of the tube between the two respects the value of l_2. We then build the mesh as previously, defining a finer mesh for the boundary edges and filling the rest of the component with a free triangular mesh. The result can be seen in the following figures:



(e) 2D Model

(f) Mesh

Adding as before a 1kPa pressure at the top opening, and using the same probes for the previous case, we're now ready to perform our study.

Frequency sweep analysis: double resonator

We now perform the same frequency sweep analysis executed before, in a range from 10Hz to 10kHz, with a step of 10 Hz. Elaborating the results of the probes on matlab, we obtain the following graph for the input impedance:

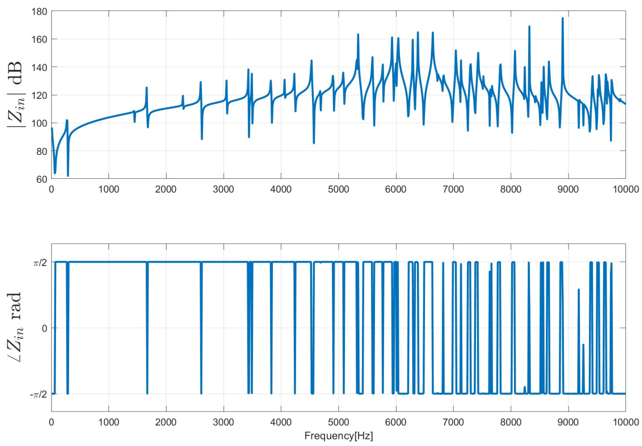


Figure 3: Frequency sweep, double resonator

We can see how the graph shows the interaction between the two components. In particular, we notice two minima in the lower frequencies range, representing the two first resonance frequencies of the system: the first at about 60-70 Hz, the second one at 290Hz. Between the two, there's an anti-resonance at a value of about 280Hz. We can see this behaviour repeating at the higher frequencies as well.

Electric analog: frequency response

We're now asked to set the electric analog of the system in Simscape environment. In order to do so, we need to take in consideration the equivalent component of the physical model as so:

- The opening pipe becomes an inductance: $L = \frac{\rho l}{S}$
- The cavity of the resonator becomes a capacitor: $C = \frac{V}{\rho c^2}$
- The pressure applied at the entrance of the resonator becomes the voltage generator
- The velocity of the air through the opening is equivalent to the current running through the circuit

where $\rho = 1.2kg/m^3$ is the density of air at 273 K and 101.325 kPa, and $c = 343m/s$ is the speed of sound in air at the same conditions. So, we now calculate the equivalent values for our model:

- $L_1 = {}^{\rho\cdot(l_1+\alpha_1)}/_{S_1} = {}^{1.2\cdot(0.01+0.93\cdot0.005)}/_{\pi\cdot0.005^2} \approx 223.84$ kg/m^4
- $L_2 = {}^{\rho\cdot(l_2+\alpha_2)}/_{S_2} = {}^{1.2\cdot(0.02+1.8\cdot0.02)}/_{\pi\cdot0.02^2} \approx 53.47$ kg/m^4
- $C_1 = {}^{\rho V_1}/_{c^2} = \frac{4}{3}\cdot{}^{\pi\cdot0.005^3}/_{1.2\cdot343^2} \approx 0.0015$ m^5/N
- $C_2 = {}^{\rho V_2}/_{c^2} = \frac{4}{3}\cdot{}^{\pi\cdot0.02^3}/_{1.2\cdot343^2} \approx 0.0021$ m^5/N

We now need to understand the position of the various elements on the line. Considering the structure of our element, we have the two resonators that appear to be in "Series" with each other, but placing all the component on a single line would give a single resonance frequency, considering that for an RLC circuit, the resonance frequency is found as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Analyzing better our model, we can see how the cavity of the first resonator directly interacts with the behaviour of the second one below, while its opening doesn't directly affect it. Therefore, we can place the equivalent capacitor of the first resonator in parallel with the components of the second resonator, obtaining an RLC circuit with two resonance frequencies as wanted.
To calculate the impedance of the circuit, we need to measure the output current, therefore we place a current sensor after the voltage generator. In the end, to obtain the frequency response of the system, we stimulate the circuit with an impulse signal so that we can find the spectrum of the circuit. Using a sampling frequency of 20kHz, we can then correctly reconstruct the spectrum of the circuit in the range of 0-10kHz as we wanted due to Nyquist's

theorem. To reduce the noise of the result, we add a resistor R_1 = 5 kOhm in series with the first resonator equivalent. Here is the final circuit:

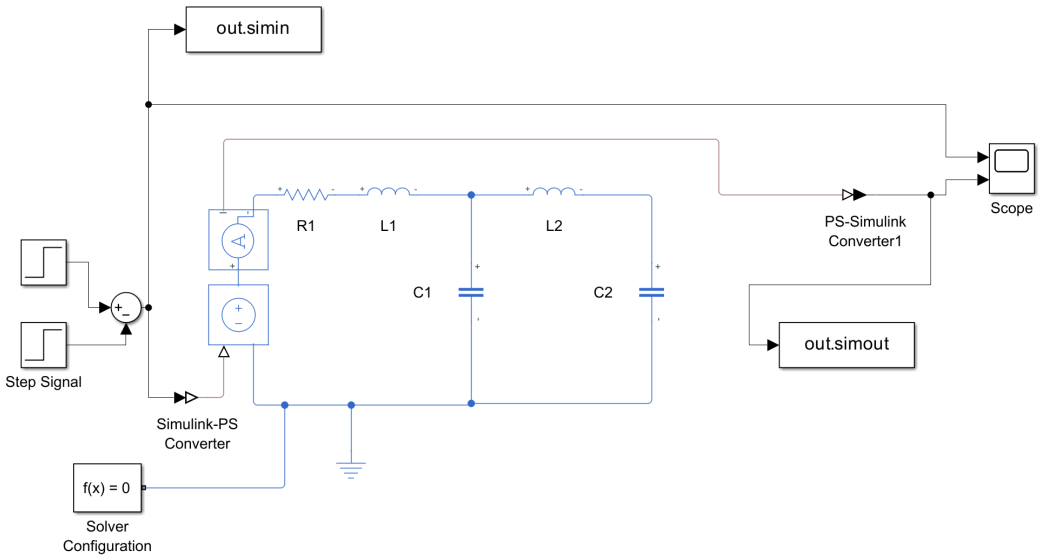


Figure 4: Equivalent circuit of the double resonator

Using the *"To workspace"* blocks, we can send the signal to a matlab script to elaborate the obtained signals. We now see the impulse response of the circuit and the graph obtained through this simulation (for the sake of semplicity, the second has been zoomed in the range 0-400Hz):

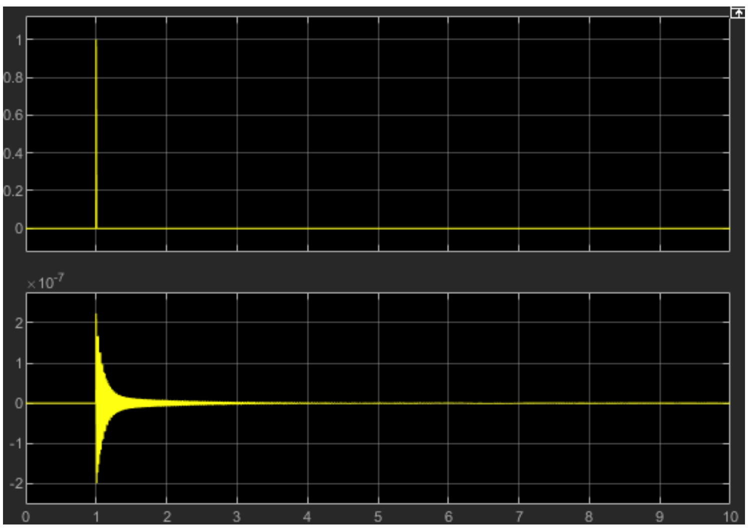


Figure 5: Impulse response of the electrical circuit

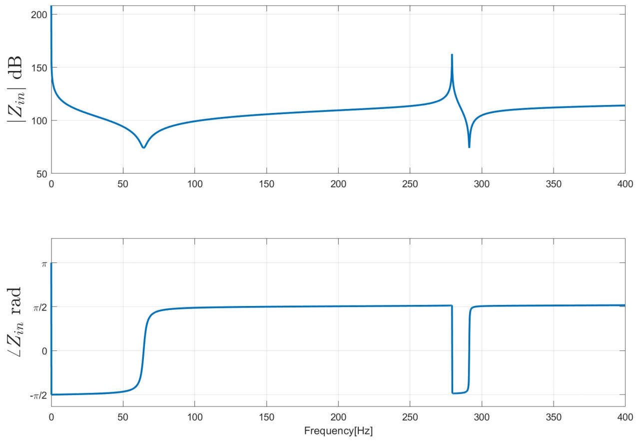


Figure 6: Frequency response of the electrical circuit

Given certain conditions, it becomes evident that the graphs derived from the electrical analogy closely align with those obtained through prior simulations, particularly at lower frequencies. Consequently, under appropriate assumptions, it can be affirmed that the electrical analogy serves as a suitable means of conceptualizing the Helmoltz resonator problem.

Conclusions

In conclusion, our investigation focused on the nuanced behavior of a Helmholtz resonator, with particular emphasis on the influence of varying tube width. Expanding our exploration, we systematically examined the characteristics of a dual resonator configuration through impedance analysis in the frequency domain, leveraging the framework of electrical analogy. The observed alterations in the resonator's response to changes in tube width underscore the sensitivity of Helmholtz resonators to geometric modifications. Additionally, the sequential arrangement of two resonators provided valuable insights into the combined system dynamics. The impedance-based approach, coupled with electrical analogy, not only facilitated a comprehensive understanding of the studied resonators but also highlighted the broader applicability of such analytical tools in unraveling the intricacies of acoustical systems.