Problem 4

Prove that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

PROOF I will prove the statement by induction and by looking at one property of the odd and even numbers: any odd or even number multiplied by an even number will be equal to an even number.

So $4n$ will be equal to an even number, whatever n is.

Since $4n$ is an even number, $4n + 1$ or $4n + 3$ will be equal to an odd number, because every even number plus an odd number is equal to an odd number.

So it is possible to realize that $4n + 1$ and $4n + 3$ will always be equal to an odd number, and now I am going to prove by induction that every odd natural number is one of the forms above.

Let $n = 0$.

$$4 \cdot 0 + 1 = 1$$
$$and$$
$$4 \cdot 0 + 3 = 3$$

The result is 1 and 3, which are the first two odd natural numbers.

Now let $n = n + 1$.

$$4(n + 1) + 1 = 4n + 5$$
$$and$$
$$4(n + 1) + 3 = 4n + 7$$

- This way, if $n = 0$, the result will be 5 and 7, which are the two next odd natural numbers.

- If we keep it going, using $n = n + 2$, $n = n + 3$ and so on and so forth, the result of the equations will always be the next odd natural number.
- They will also always produce two sequencial odd natural numbers, and by sequencial I mean with the difference of two.

Hence, by induction, I have proved that every odd natural number is of one of the forms $4n + 1$ or $4n + 3$, where n is an integer.