Problem 8

Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.

PROOF I will prove the statement based on the formal definition of a limit:

$$a_n \to L \; as \; n \to \infty \iff (\forall \epsilon > 0)(\exists n \in \mathbb{N})(\forall m > n)[|a_m - L| < \epsilon]$$

By definition, we know that:

$$|a_m - L| < \epsilon$$

In order to prove the statement, I will multiply $|a_m - L| < \epsilon$ by M:

$$|M \cdot a_m - M \cdot L| < M \cdot \epsilon$$

So we get:

$$(\forall \epsilon > 0)(\exists n \in \mathbb{N})(\forall m > n)[|M \cdot a_m - M \cdot L| < \epsilon]$$

Hence, this proves that if the sequence $\{a_n\}_{n=1}^{\infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M > 0$, the sequence $\{Ma_n\}_{n=1}^{\infty}$ tends to the limit ML.