Problem 3

Say whether the following is true or false and support your answer by a proof:
For any integer n, the number $n^2 + n + 1$ is odd.

ANSWER The statement is true.

PROOF I will prove by analyzing what $n^2 + n$ means, and to do this, let's simplify as $n(n+1)$.

Let n be an even number:

If n is even, $n+1$ is odd, and every odd number mutiplied by an even number is even.

Let n be an odd number:

If n is odd, $n+1$ is even, and every even number multiplied by an odd number is even.

Let n be zero:

If n is zero, the result of the multiplication is equal to zero.

Now, let's analyze the complete equation $n^2 + n + 1$:

By now it is possible to realize that whether n is even or not, $n^2 + 1$ is even, and an even number plus 1 is equal to an odd number, so $n^2 + n + 1$ is odd for all odd and even integers. About the zero, $0 + 1$ is equal to 1, which is an odd number.

So this proves that , for any integer n, the number $n^2 + n + 1$ is odd.