Problem 9

Given an infinite collection $A_n, n = 1, 2, \ldots$ of intervals of the real line, their *intersection* is defined to be $\bigcap_{n=1}^{\infty} A_n = \{x \mid (\forall n)(x \in A_n)\}$ Give an example of a family of intervals

$A_n, n = 1, 2, \ldots$ such that $A_{n+1} \subset A_n$ for all n and $\bigcap_{n=1}^{\infty} A_n = \emptyset$. Prove that your example has the stated property.

$$\left(0, \frac{1}{2^{n+1}}\right) \subseteq \left(0, \frac{1}{2^n}\right)$$

PROOF $\frac{1}{2^{n+1}} < \frac{1}{2^n}$, so the subset $\left(0, \frac{1}{2^{n+1}}\right)$ is in the set $\left(0, \frac{1}{2^n}\right)$. So, if there is an arbitrary number x such that $0 < x < \frac{1}{2^{n+1}}$, then $0 < x < \frac{1}{2^n}$.

The sequence $\left(\frac{1}{2^n}\right)$ is decreasing and has limit zero. That means $(t < 0)\left((\exists n)\left[\frac{1}{2^n} < t\right]\right)$. That last bit means that nothing positive can be in every a_n but $(\forall n)[0 \notin a_n)$ so the intersection is $\emptyset$.