

# Problem 10

Give an example of a family of intervals  $A_n, n = 1, 2, \dots$  such that  $A_{n+1} \subset A_n$  for all  $n$  and  $\bigcap_{n=1}^{\infty} A_n$  consists of a single real number. Prove that your example has the stated property.

**PROOF** In order to give an example, I will use the same as the last problem:

$$\left(0, \frac{1}{2^{n+1}}\right) \subseteq \left(0, \frac{1}{2^n}\right)$$

$\frac{1}{2^{n+1}} < \frac{1}{2^n}$ , so the subset  $\left(0, \frac{1}{2^{n+1}}\right)$  is in the set  $\left(0, \frac{1}{2^n}\right)$ . So, if there is an arbitrary number  $x$  such that  $0 < x < \frac{1}{2^{n+1}}$ , then  $0 < x < \frac{1}{2^n}$ .