Problem 7

Prove that for any natural number n, $2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$.

PROOF I am going to prove the statement by induction. Let $n = 1$.

$$2^1 = 2^{1+1} - 2$$
$$2^1 = 2^2 - 2$$
$$2 = 2$$

The both sides of the equation are identical for $n = 1$, now let's prove if $A(n) \implies A(n+1)$.

To do this, I will take an arbitrary number n and add 1, so $n + 1$. Then I will replace the number n for $n + 1$ in the equation, and I will add 2^{n+1} to it.

$$2^{n+1} - 2 + 2^{n+1} = 2^{n+2} - 2$$
$$2 \cdot 2^{n+1} - 2 = 2^{n+2} - 2$$
$$2^{n+2} - 2 = 2^{n+2} - 2$$

Both sides of the equation are equal to $2^{n+2} - 2$. Hence, by induction, I have proved that for any natural number n, $2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$.