Problem 1

Say whether the following is true or false and support your answer by a proof.
$(\exists m \in N)(\exists n \in N)(3m + 5n = 12)$

ANSWER It is false.

PROOF We know that the equation has a solution in the integers because of the Extended Euclidean Algorithm, since the $gcd(3, 5) = 1$ and $1|12$. However, it does not prove that there are natural numbers that satisfy this equation, and I am going to prove that the statement is false.

To do this, I will solve the equation in terms of n:

$$5n = 12 - 3m$$

Since we can only use natural numbers, there are three possible values for m: 1, 2 and 3.

$$5n = 12 - 3 \cdot 1 = 9$$
$$or$$
$$5n = 12 - 3 \cdot 2 = 6$$
$$or$$
$$5n = 12 - 3 \cdot 3 = 3$$

There is no natural number n that satisfies this equation, because 9, 6 and 3 are not multiple of 5. Therefore, the statement $(\exists m \in N)(\exists n \in N)(3m + 5n = 12)$ is false.