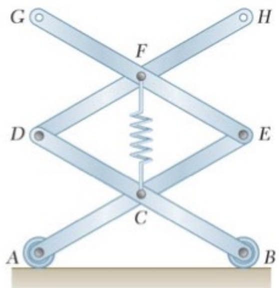
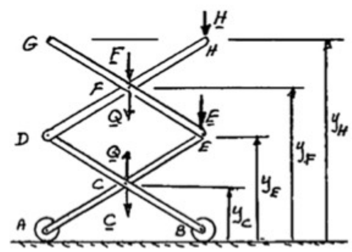



## PROBLEM 10.5

A spring of constant 15 kN/m connects Points $C$ and $F$ of the linkage shown. Neglecting the weight of the spring and linkage, determine the force in the spring and the vertical motion of Point $G$ when a vertical downward 120-N force is applied (*a*) at point $C$, (*b*) at Points $C$ and $H$.

## SOLUTION

$$y_G = 4y_C$$
$$y_H = 4y_C \quad \delta y_H = 4\delta y_C$$
$$y_F = 3y_C \quad \delta y_F = 3\delta y_C$$
$$y_E = 2y_C \quad \delta y_E = 2\delta y_C$$




For spring:

$$\Delta = y_F - y_C$$

**Q** = Force in spring (assumed in tension)

$$Q = +k\Delta = k(y_F - y_C) = k(3y_C - y_C) = 2ky_C \tag{1}$$

(*a*) $\mathbf{C} = 120\text{ N}, \quad \mathbf{E} = \mathbf{F} = \mathbf{H} = 0$

Virtual Work:

$$\delta U = 0: \quad -(120\text{ N})\delta y_C + Q\delta y_C - Q\delta y_F = 0$$
$$-120\delta y_C + Q\delta y_C - Q(3\delta y_C) = 0$$
$$Q = -60\text{ N} \qquad Q = 60.0\text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C, \quad -60\text{ N} = 2(15\text{ kN/m})y_C, \quad y_C = -2\text{ mm}$

At Point $G$: $y_G = 4y_C = 4(-2\text{ mm}) = -8\text{ mm} \qquad \mathbf{y}_G = 8.00\text{ mm}\downarrow \blacktriangleleft$

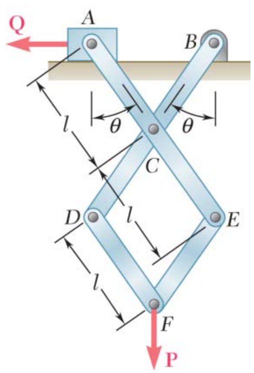
(*b*) $\mathbf{C} = \mathbf{H} = 120\text{ N}, \mathbf{E} = \mathbf{F} = 0$

Virtual Work:

$$\delta U = 0: \quad -(120\text{ N})\delta y_C - (120\text{ N})y_H + Q\delta y_C - Q\delta y_F = 0$$
$$-120\delta y_C - 120(4\delta y_C) + Q\delta y_C - Q(3\delta y_C) = 0$$
$$Q = -300\text{ N} \quad Q = 300\text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C \quad -300\text{ N} = 2(15\text{ kN/m})y_C, \; y_C = -10\text{ mm}$

At Point $G$: $y_G = 4y_C = 4(-10\text{ mm}) = -40\text{ mm} \qquad \mathbf{y}_G = 40.0\text{ mm}\downarrow \blacktriangleleft$



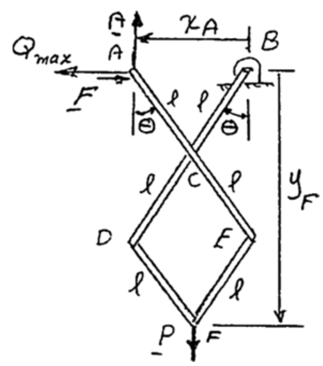

## PROBLEM 10.51

Denoting by $\mu_s$ the coefficient of static friction between the block attached to rod $ACE$ and the horizontal surface, derive expressions in terms of $P$, $\mu_s$, and $\theta$ for the largest and smallest magnitude of the force **Q** for which equilibrium is maintained.

## SOLUTION

For the linkage:

$$+\circlearrowleft \Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \quad \text{or} \quad \mathbf{A} = \frac{P}{2}\uparrow$$




Then:

$$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2}\mu_s P$$

Now

$$x_A = 2l\sin\theta$$
$$\delta x_A = 2l\cos\theta\delta\theta$$

and

$$y_F = 3l\cos\theta$$
$$\delta y_F = -3l\sin\theta\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

$$\left(Q_{\max} - \frac{1}{2}\mu_s P\right)(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

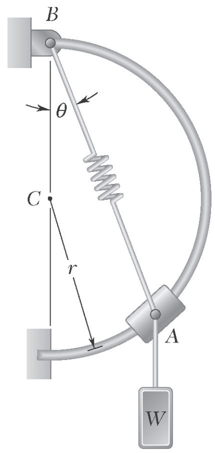
or

$$Q_{\max} = \frac{3}{2}P\tan\theta + \frac{1}{2}\mu_s P$$

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s) \blacktriangleleft$$

For $Q_{\min}$, motion of $A$ impends to the right and **F** acts to the left. We change $\mu_s$ to $-\mu_s$ and find

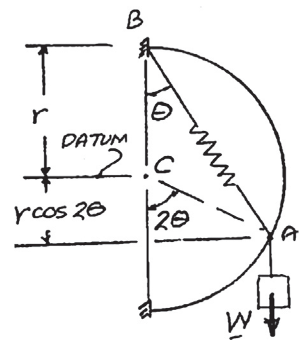
$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s) \blacktriangleleft$$




## PROBLEM 10.88

Collar $A$ can slide freely on the semicircular rod shown. Knowing that the constant of the spring is $k$ and that the unstretched length of the spring is equal to the radius $r$, determine the value of $\theta$ corresponding to equilibrium when $W = 200$ N, $r = 180$ mm, and $k = 3$ kN/m

## SOLUTION




Stretch of spring

$$s = AB - r = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

$$V = \frac{1}{2}ks^2 - Wr\cos 2\theta$$

$$= \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\cos 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta + 2Wr\sin 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta + Wr\sin 2\theta = 0$$

$$-kr^2(2\cos\theta - 1)\sin\theta + Wr(2\sin\theta\cos\theta) = 0$$

or

$$\frac{(2\cos\theta - 1)}{2\cos\theta} = \frac{W}{kr}$$

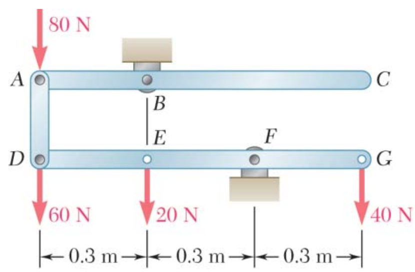
Now

$$\frac{W}{kr} = \frac{(200 \text{ N})}{(3000 \text{ N/m})(0.18 \text{ m})} = 0.37037$$

Then

$$\frac{2\cos\theta - 1}{2\cos\theta} = 0.37037$$

Solving $\theta = 37.4°$ ◀

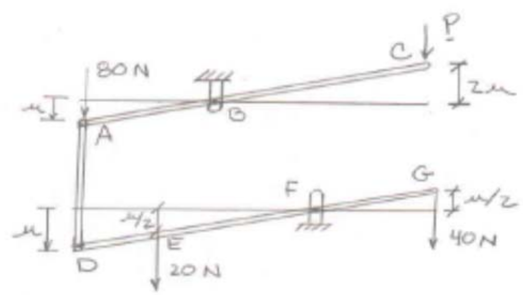



## PROBLEM 10.67

Show that equilibrium is neutral in Problem 10.1.

**PROBLEM 10.1** Determine the vertical force **P** that must be applied at *C* to maintain the equilibrium of the linkage.

## SOLUTION




Designate vertical distance between link *AC* and *DG* as *b*.

We have $$y_A = b - u, \quad y_C = b + 2u, \quad y_D = -u, \quad y_E = -\frac{1}{2}u, \quad y_G = +\frac{1}{2}u$$

$$V = (80\text{ N})y_A + P(y_C) + (60\text{ N})y_D + (20\text{ N})y_E + (40\text{ N})y_G$$

$$V = 80(\text{b} - u) + P(b + 2u) + 60(-u) + 20\left(-\frac{1}{2}u\right) + 40\left(\frac{1}{2}u\right)$$

$$\frac{dV}{du} = -80 + 2P - 60 - 10 + 20 = 0$$

$$P = 65\text{ N}$$

Substituting $P = 270\text{ N}$ in the expression for *V*, we have

$$V = (80 + 65)b + (-80 + 130 - 60 - 10 + 20)u$$

$$V = 145b$$

Thus *V* is constant

and equilibrium is neutral ◀