PH203: Optics

Lecture #5

15.11.2018

Interference

Interference of white light in a soap bubble

https://www.youtube.com/watch?v=CAe3lkYNKt8

$\Rightarrow$ whenever two waves superimpose they produce an intensity distribution having max and min

The intensity distribution is called <u>interference pattern</u>

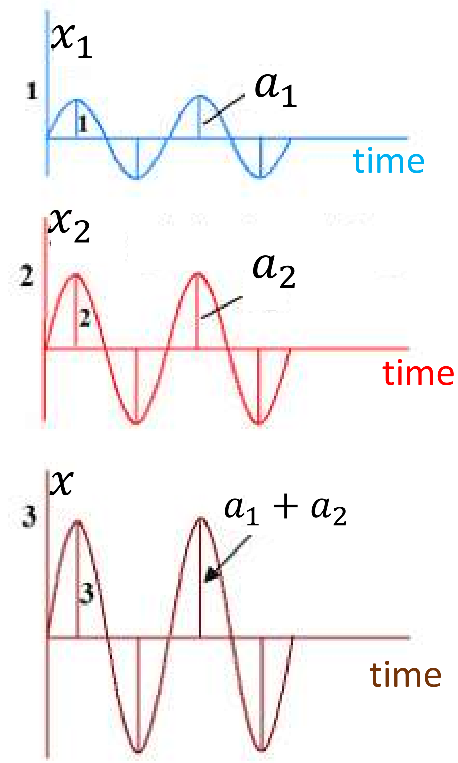


Fig.1 Constructive Interference

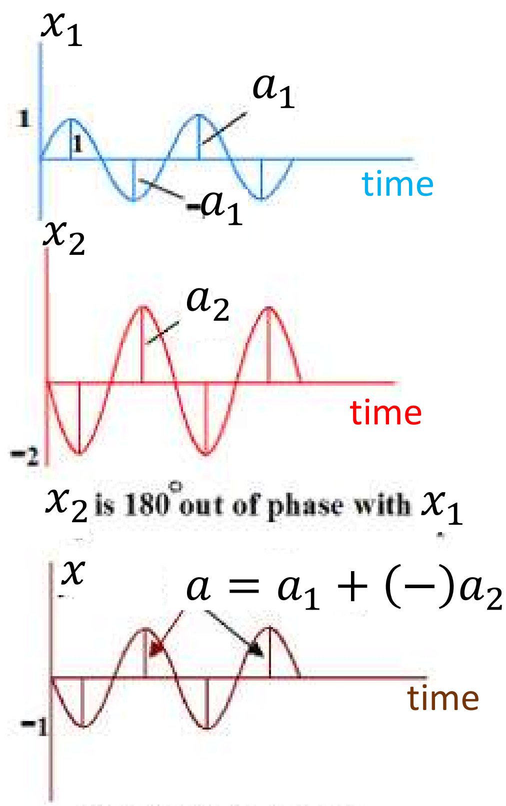


Fig.2 Destructive Interference

Interference

Consider superposition of two non-identical (amp & phase different) sinusoidal waves of same frequency:

$$x_1(t) = a_1 \cos(\omega t + \theta_1) = a_1 \cos \omega t \cos \theta_1 - a_1 \sin \omega t \sin \theta_1$$
$$x_2(t) = a_2 \cos(\omega t + \theta_2) = a_2 \cos \omega t \cos \theta_2 - a_2 \sin \omega t \sin \theta_2$$

Resulting displacement due to superposition of the two

$$x(t) = x_1(t) + x_2(t)$$
$$= \cos \omega t \underbrace{\left[a_1 \cos \theta_1 + a_2 \cos \theta_2\right]}_{= a \cos \theta} - \sin \omega t \left(\underbrace{\left[a_1 \sin \theta_1 + a_2 \sin \theta_2\right]}_{= a \sin \theta}\right)$$

$$\Rightarrow \; x(t) = a \cos(\omega t + \theta)$$

where $a \left(\cos^2 \theta + \sin^2 \theta\right)^{1/2} = \left[a_1^2 + a_2^2 + 2 a_1 a_2 \cos(\theta_1 - \theta_2)\right]^{1/2}$

We also get $\tan \theta = \dfrac{a_1 \sin \theta_1 + a_2 \sin \theta_2}{a_1 \cos \theta_1 + a_2 \cos \theta_2}$

From $$a = \left[a_1^2 + a_2^2 + 2a_1a_2 \cos\left(\theta_1 - \theta_2\right)\right]^{1/2}$$

For phase difference $\theta_1 - \theta_2 = 0, 2\pi, 4\pi, \cdots \quad \Rightarrow \quad \theta_1 - \theta_2 = 2m\pi; \;\; m = 0,1,2, \ldots.$

If we assume *a* to be always positive, from the above $a = [a_1^2 + a_2^2 + 2a_1a_2]^{1/2}$

$$\Rightarrow \;\; a = a_1 + a_2$$

$\Rightarrow$ If the two waves are in phase, two amplitudes add up to form the resultant amplitude:

called <u>constructive interference</u>

For $\theta_1 - \theta_2 = \pi, 3\pi, \cdots \quad \Rightarrow \quad \theta_1 - \theta_2 = (2m+1)\pi; \;\; m = 0,1,2, \ldots.$

$$a = a_1 - a_2$$

called <u>destructive interference</u>

Whenever/wherever constructive interference takes place, we have an intensity maximum

Similarly in case of destructive interference, we have an intensity minimum

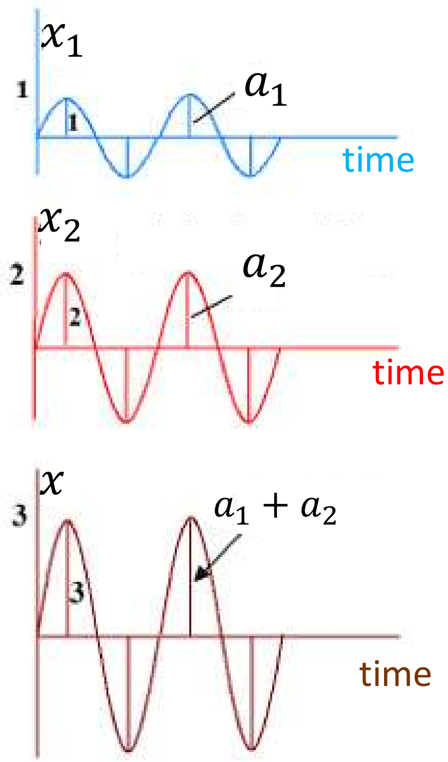


Fig.1 Constructive Interference

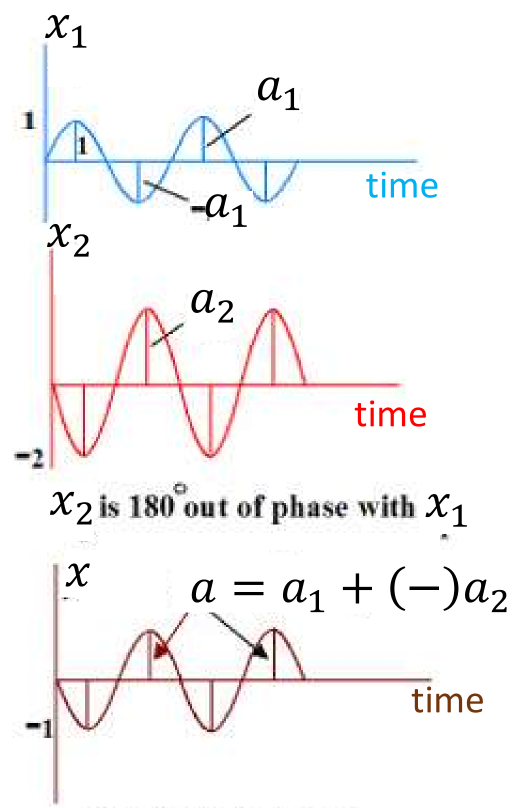


Fig.2 Destructive Interference

Whenever two waves superimpose they produce an intensity distribution having max and min

Intensity distribution is called <u>interference pattern</u>

Due to the very process of emission of light waves, interference is difficult to produce with two independent waves

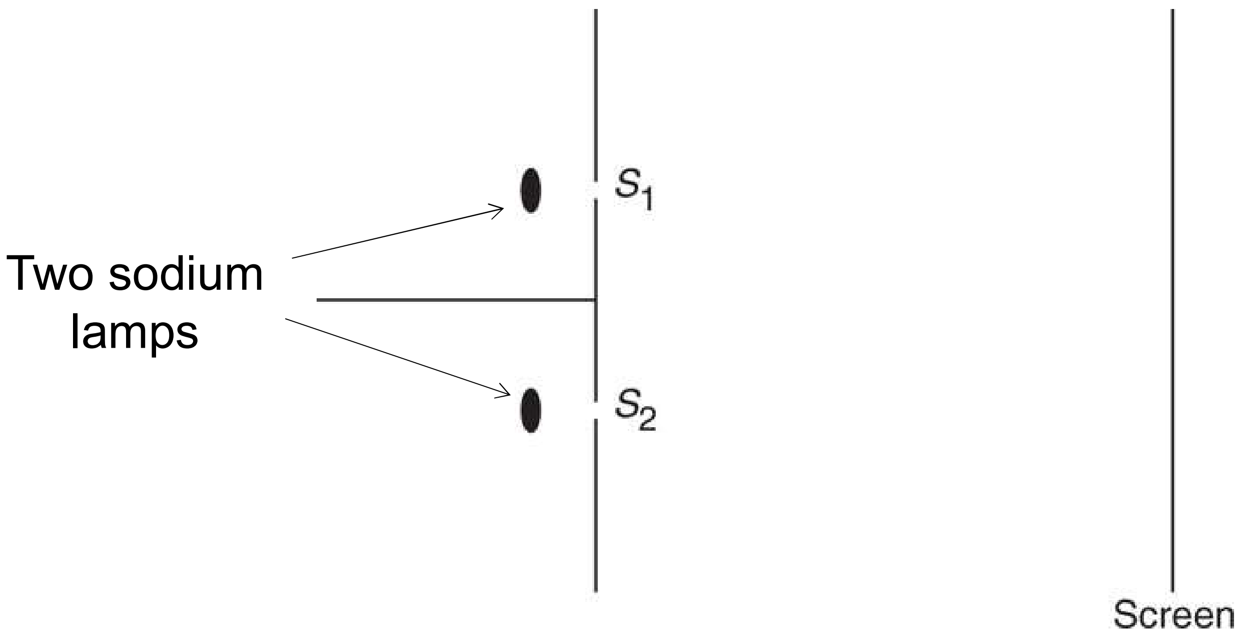
$\Rightarrow$ One derives interfering waves from a single source so as to maintain const phase relationship between the two

Two broad categories exist involving two beams:

1. division of wave front
2. division of amplitude

A third category is possible that involves

3. multiple beam interferometry

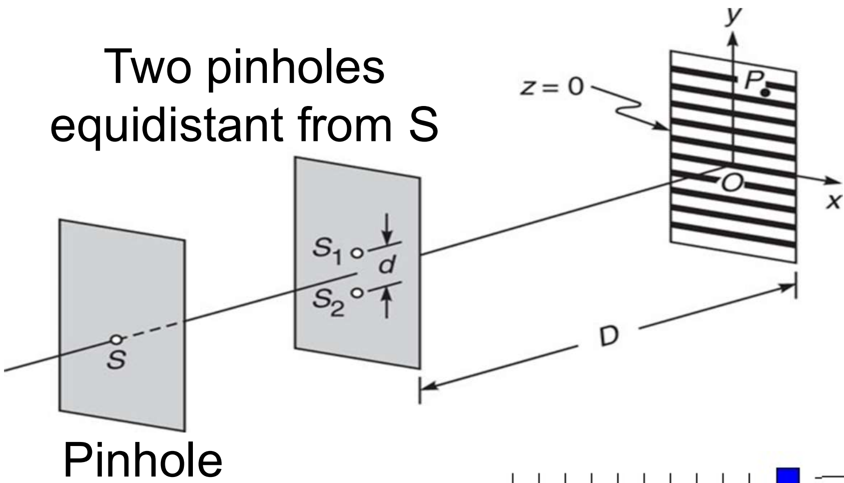


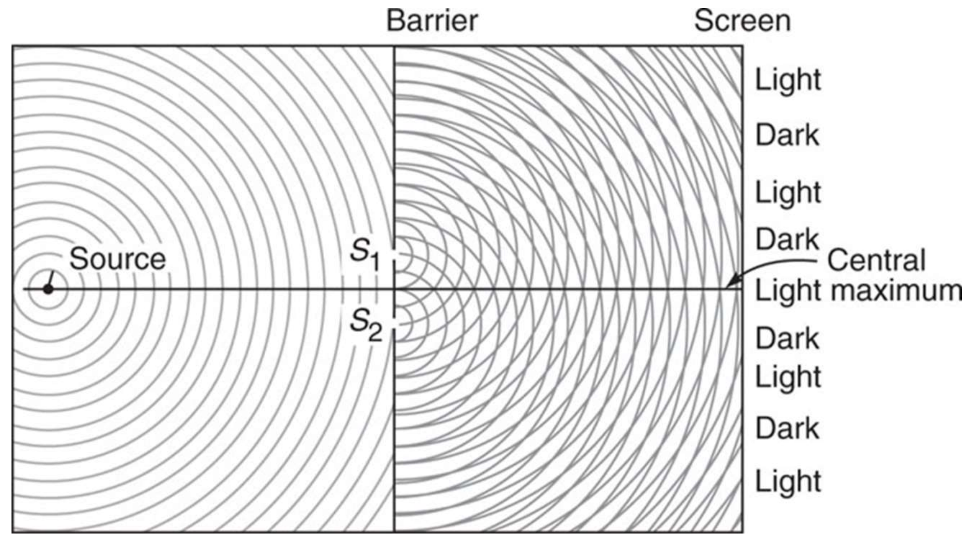
Light from an atom $\equiv$ light pulse of $\sim 10^{-10}$ sec

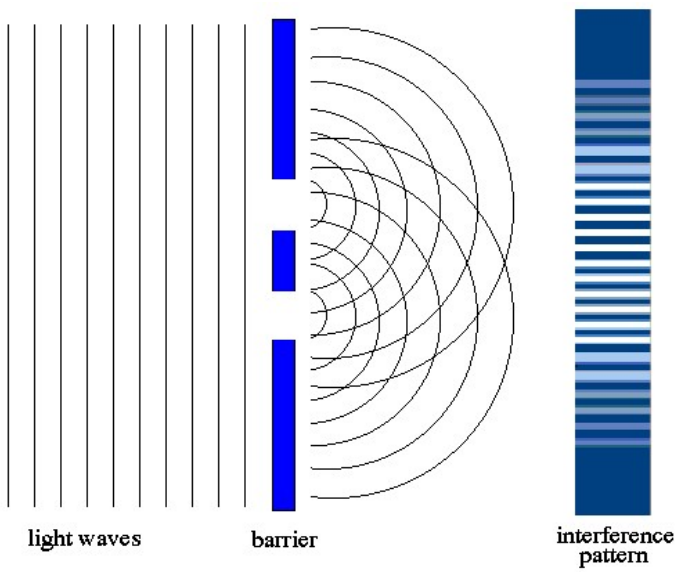
Human eyes cannot detect intensity changes that lasts for < 1/10 sec

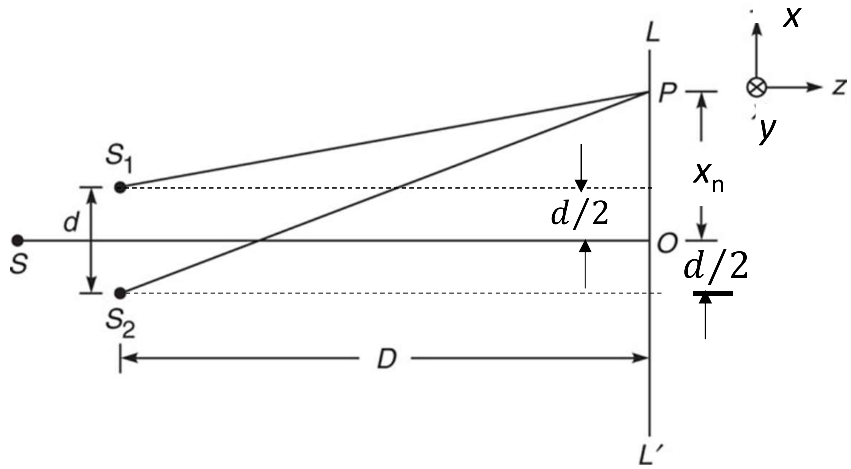
$\Rightarrow$ A uniform intensity will be observed on the screen

Young's double hole experiment in 1801:









For P to have a max, we must have light reaching it from $S_{1,2}$ in phase $\Rightarrow$ $k_o\,.(r.i.).\Delta = 2n\pi; n = 0,1,2, \ldots.$

$$\Rightarrow \quad \frac{\cancel{2\pi}}{\lambda}.1.\Delta = \cancel{2}n\cancel{\pi}$$

$\Rightarrow$ path difference between the two rays from $S_{1,2}$

$\Rightarrow \Delta = S_2P - S_1P = n\lambda$; $n = 0, 1, 2, \ldots.$

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x_n - \frac{d}{2}\right)^2\right] = 2x_n d$$

$$\Rightarrow \underbrace{(S_2P + S_1P)}_{\cong 2D} \times (S_2P - S_1P) = 2x_n d$$

$\cong 2D$ valid for $D \gg d$ and x_n

Example, for typical experimental values $d = 0.02$ cm, $D = 50$ cm, $OP(x_n) = 0.5$ cm

$$(S_2P + S_1P) = \sqrt{50^2 + (0.51)^2} + \sqrt{50^2 + (0.49)^2} \cong 100.005 \text{ cm} \cong 2D$$

$$\Rightarrow \quad (S_2P - S_1P) = \frac{\cancel{2}x_n d}{\cancel{2}D} \quad \Rightarrow \quad \underbrace{(S_2P - S_1P)}_{=\Delta} = \frac{x_n d}{D} \quad \Rightarrow \quad x_n = \frac{\Delta D}{d} = \frac{n\lambda D}{d}$$

$$x_n = \frac{n\lambda D}{d}$$

Thus distance (β) between two consecutive bright or dark fringes:

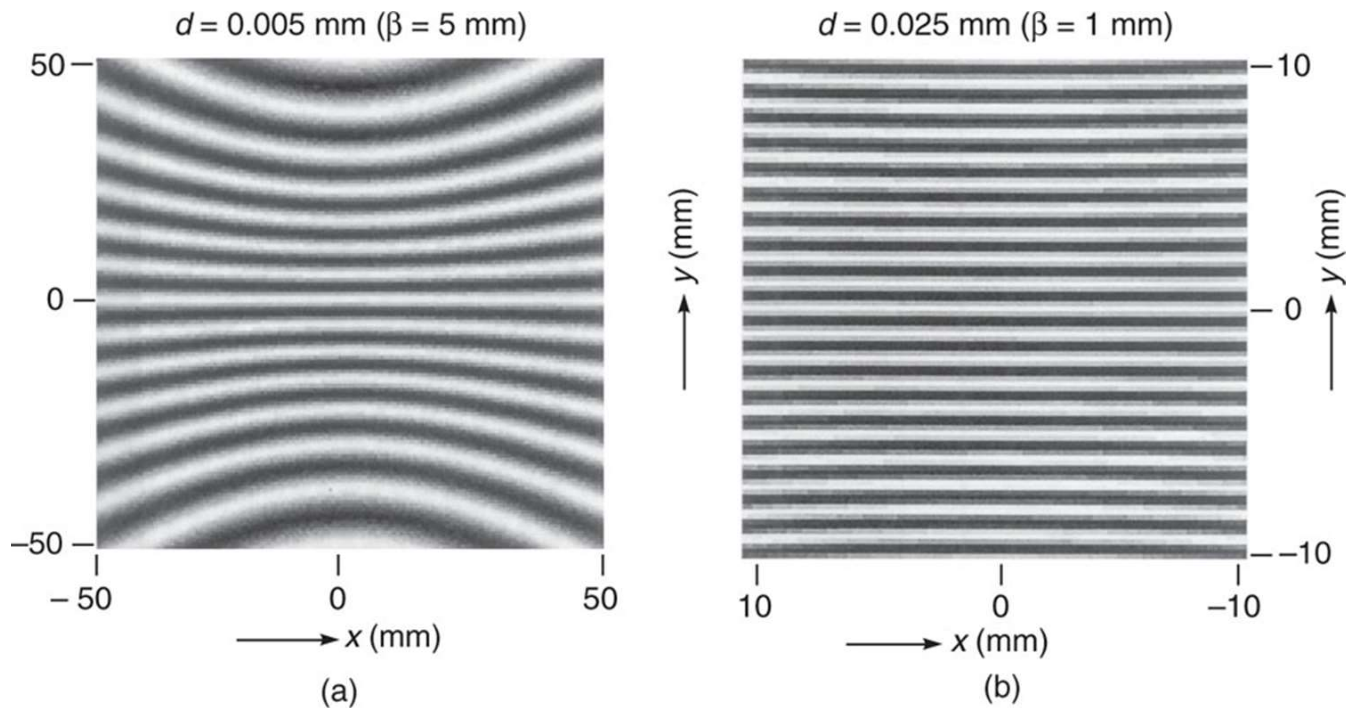
$\Rightarrow$ Fringe width: $$x_{n+1} - x_n = (\cancel{n} + 1 - \cancel{n})\frac{\lambda D}{d} = \frac{\lambda D}{d}$$

$\Rightarrow$ Bright and dark fringes will be equally spaced

The interference fringes due to two point sources will be hyperbolic in shape as shown on the figures

Computer generated fringe patterns:

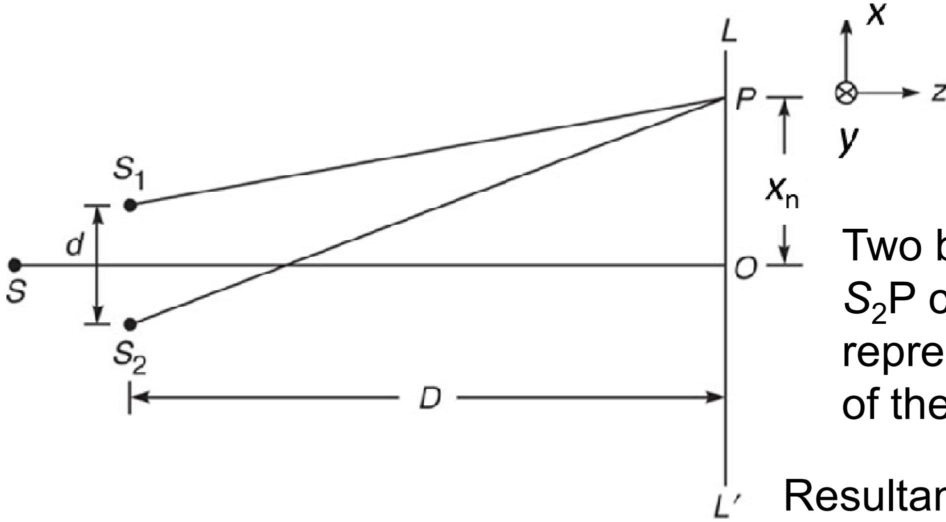
For λ = 500 nm, and D = 5 cm



Normally only a part of these fringes are seen on a screen and these appear as straight lines

- Fringe visibility: $\dfrac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$

Intensity distribution



For S_1P and S_2P >> $S_1\,S_2$ the two beams wld travel almost along the same direction

Two beams S_1P & S_2P can be represented in terms of their **E** fields as

$$\vec{E}_1 = \hat{x}E_{01}\cos\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right)$$

$$\vec{E}_2 = \hat{x}E_{02}\cos\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right)$$

Resultant field at P by superposition principle

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \hat{x}\left[E_{01}\cos\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right) + E_{02}\cos\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right)\right]$$

$\Rightarrow$ Intensity, $I = K\left|\vec{E}\right|^2$

From trigonometry, $\cos(A+B) = \cos A\cos B - \sin A\sin B$ and $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\Rightarrow \quad 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$I = K\left[E_{01}^2\cos^2\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right) + E_{02}^2\cos^2\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right) + 2E_{01}E_{02}\cos\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right)\cos\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right)\right]$$

3rd term:

$$E_{01}E_{02} \times 2\cos\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right)\cos\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right)$$

$$= E_{01}\ E_{02}\left\{\underbrace{\cos\left[\frac{2\pi}{\lambda_0}(S_1P + S_2P) - 2\omega t\right]}_{\cos\left[2\omega t - \frac{2\pi}{\lambda_0}(S_1P + S_2P)\right]} + \cos\left[\frac{2\pi}{\lambda_0}(S_2P - S_1P) - \cancel{\omega t} + \cancel{\omega t}\right]\right\}$$

$\Rightarrow$

$$I = K\left[E_{01}^2\cos^2\left(\tfrac{2\pi}{\lambda_0}S_1P - \omega t\right) + E_{02}^2\cos^2\left(\tfrac{2\pi}{\lambda_0}S_2P - \omega t\right) + E_{01}E_{02}\left\{\cos\left[2\omega t - \tfrac{2\pi}{\lambda_0}(S_1P + S_2P)\right] + \cos\left(\tfrac{2\pi}{\lambda_0}[S_2P - S_1P]\right)\right\}\right]$$

When a photodetector detects such a time varying intensity, it will respond only to the time average because optical frequency: $\omega_{\text{optical}} \approx 2\pi \times 10^{15}$

By definition, $\langle f(t)\rangle = \dfrac{1}{\tau}\displaystyle\int_{-\tau/2}^{\tau/2} f(t)dt \ \Rightarrow \ \langle\cos^2(\omega t - \theta)\rangle = \frac{1}{2}$; and $\langle\cos(2\omega t - \varphi)\rangle = 0$

$$\therefore\ \ I = \frac{1}{2}K(E_{01}^2 + E_{02}^2) + \sqrt{K}E_{01} \times \sqrt{K}E_{02}\cos\delta;\ \ \delta = \left(\frac{2\pi}{\lambda_0}\right)[S_2P - S_1P]$$

$$\therefore \quad I = \frac{1}{2}K(E_{01}^2 + E_{02}^2) + \sqrt{K}E_{01} \times \sqrt{K}E_{02}\cos\delta; \quad \delta = (2\pi/\lambda_0)[S_2P - S_1P]$$

$$\Rightarrow \quad I = I_1 + I_2 + 2 \times \sqrt{\frac{K}{2}}E_{01} \times \sqrt{\frac{K}{2}}E_{02}\cos\delta \;\Rightarrow\; I = I_1 + I_2 + 2 \times \sqrt{I_1 I_2}\cos\delta$$

where

$$I_1 = \frac{1}{2}KE_{01}^2; \quad I_2 = \frac{1}{2}KE_{02}^2$$

Intensity due to S_1 in the absence of S_2

Intensity due to S_2 in the absence of S_1

δ: represents phase diff between the light reaching *P* from $S_{1,2}$

$\because$ max of $\cos\delta$ are ± 1

$$\therefore \; I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2; \; I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta;\ \delta = \frac{2\pi}{\lambda}\left(S_2P - S_1P\right)$$

$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

Optical phase:

$$\delta = \frac{2\pi}{\lambda_0} \times n \times (\text{geometrical length})$$

Intensity maxima occur for $\delta = 2n\pi; n = 0,1,2, \dots .$

$$\Rightarrow \quad \delta = \frac{2\pi}{\lambda_0}(S_2P - S_1P) = 2n\pi$$

$$\Rightarrow \quad S_2P - S_1P = n\,.\,\cancel{2\pi} \times \frac{\lambda_0}{\cancel{2\pi}} = n\lambda_0; n = 0,1,2, \dots .$$

Intensity minima occur for

$$\delta = (2n+1)\pi;\ n = 0,1,2, \dots .$$

$$\Rightarrow \quad S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda_0;\ n = 0,1,2,$$

If

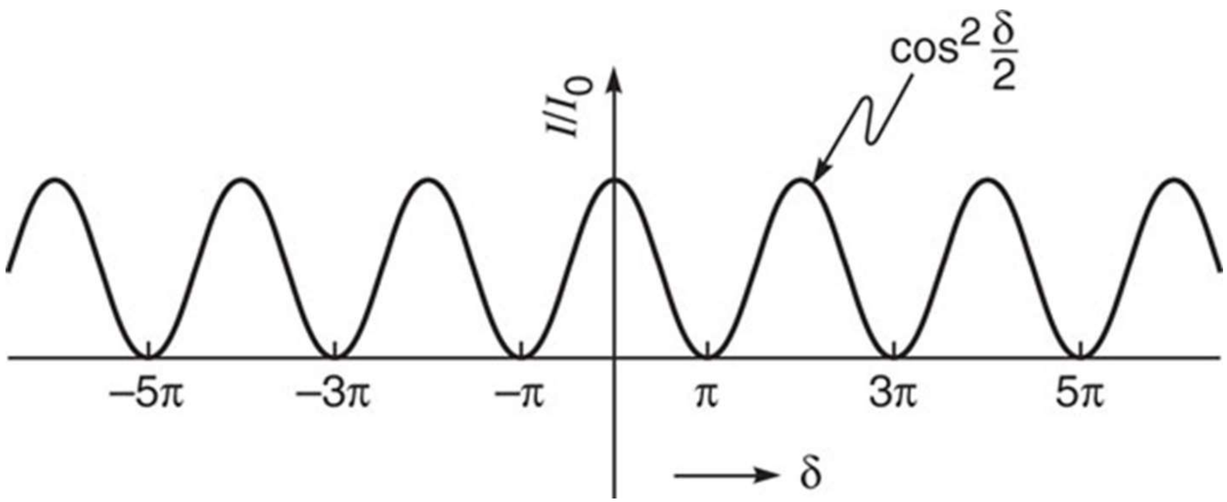
$I_1 = I_2 \quad \Rightarrow \quad I_{\min} = \left(\sqrt{I_1} - \sqrt{I_1}\right)^2 = 0$ In general, $I_1 \neq I_2 \ \Rightarrow$ Intensity is usually never 0 !

If S_1P and S_2P are large relative to separation *d* between the two sources, then

$$I_1 = I_2 = I_0$$

$$\Rightarrow I = 2I_0 + 2I_0 \cos\delta$$

$$\Rightarrow I = 2I_0\left(1+\cos\delta\right) = 2I_0 \times 2\cos^2\left(\frac{\delta}{2}\right) \quad \Rightarrow \frac{I}{I_0} = 4\cos^2\frac{\delta}{2}$$



$\cos^2$ fringe or pattern

Example

For a path difference Δ of $\lambda/5$, $\dfrac{I}{I_{\max}}$?

$$\because \frac{I}{I_0} = 4\cos^2\frac{\delta}{2} \quad \text{and} \quad \delta = \frac{2\pi}{\lambda_0}\left(S_2P - S_1P\right) \qquad \Rightarrow \delta = \frac{2\pi}{\lambda_0}\Delta = \frac{2\pi}{\cancel{\lambda_0}} \times \frac{\cancel{\lambda_0}}{5}$$

$$\Rightarrow I_{\max} = 4I_0 \qquad\qquad = \frac{2\pi}{5}$$

$$\Rightarrow \frac{I}{I_{\max}} = \frac{\cancel{4I_0} \times \cos^2\left(2\pi/10\right)}{\cancel{4I_0}} = \cos^2\left(0.628\right) \approx \left(0.809\right)^2 \approx 0.65$$