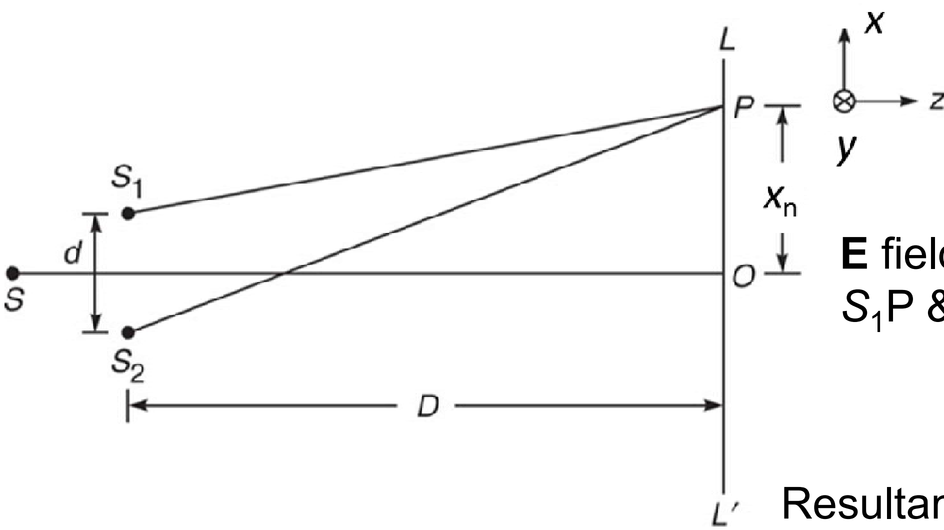
PH203: Optics

Lecture #6

16.11.2018

Intensity distribution



For S_1P and S_2P >> S_1 S_2 the two beams wld travel almost along the same direction

E fields of the beams S_1P & S_2P:

$$\vec{E}_1 = \hat{x}E_{01}\cos\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right)$$

$$\vec{E}_2 = \hat{x}E_{02}\cos\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right)$$

Resultant field at *P* by superposition principle

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \hat{x}\left[E_{01}\cos\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right) + E_{02}\cos\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right)\right]$$

$\Rightarrow$ Intensity, $I = K\left|\vec{E}\right|^2$

From trigonometry, $\cos(A+B) = \cos A\cos B - \sin A\sin B$ and $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\Rightarrow \quad 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$I = K\left[E_{01}^2\cos^2\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right) + E_{02}^2\cos^2\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right) + 2E_{01}E_{02}\cos\left(\frac{2\pi}{\lambda_0}S_1P - \omega t\right)\cos\left(\frac{2\pi}{\lambda_0}S_2P - \omega t\right)\right]$$

$\Rightarrow$

$$I = K\left[E_{01}^2\cos^2\left(\frac{2\pi}{\lambda_0}S_1P-\omega t\right)+E_{02}^2\cos^2\left(\frac{2\pi}{\lambda_0}S_2P-\omega t\right)+E_{01}E_{02}\left\{\cos\left[2\omega t-\frac{2\pi}{\lambda_0}(S_1P+S_2P)\right]+\cos\left(\underbrace{\frac{2\pi}{\lambda_0}[S_2P-S_1P]}_{\delta}\right)\right\}\right]$$

When a photodetector detects such a time varying intensity, it will respond only to the time average because optical frequency:

$$\omega_{\text{optical}} \approx 2\pi\times 10^{15}\ \text{Hz}$$

By definition,

$$\langle f(t)\rangle = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} f(t)dt \;\Rightarrow\; \langle\cos^2(\omega t-\theta)\rangle = \frac{1}{2};\ \text{and}\ \langle\cos(2\omega t-\varphi)\rangle = 0$$

$$\therefore\; I = \frac{1}{2}K(E_{01}^2+E_{02}^2)+\frac{1}{2}\sqrt{K}E_{01}\times\sqrt{K}E_{02}\times 2\cos\delta;$$

$$\delta = \left(\frac{2\pi}{\lambda_0}\right)[S_2P-S_1P] = 2m\pi; m = 0,1,2,\ldots\quad \text{: for maxima}\;\Rightarrow\quad S_2P-S_1P = m\lambda_0$$

and for minima: $\delta = (2m+1)\pi \;\Rightarrow\; S_2P-S_1P = \left(m+\frac{1}{2}\right)\lambda_0$

$\Rightarrow$

$$I = I_1+I_2+2\times\sqrt{\frac{K}{2}}E_{01}\times\sqrt{\frac{K}{2}}E_{02}\cos\delta \;\Rightarrow\; I = I_1+I_2+2\times\sqrt{I_1I_2}\cos\delta$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta; \ \delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$$

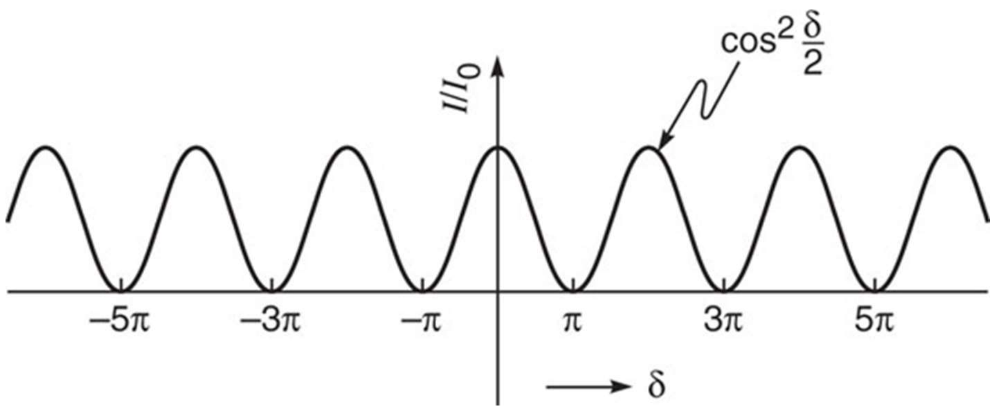
$\because$ max of $\cos\delta$ are ± 1 corresponding to $m = 0$ for max or min

$\therefore \ I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2; I_{\mathrm{mi}} \ = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

If

$I_1 = I_2 \ \Rightarrow \ I_{\min} = \left(\sqrt{I_1} - \sqrt{I_1}\right)^2 = 0$ In general, $I_1 \neq I_2 \ \Rightarrow$ Intensity is usually never 0 !

For $I_1 = I_2 = I_0, \quad I = 2I_0 + 2I_0\cos\delta \ \Rightarrow \ \frac{I}{I_0} = 2(1 + \cos\delta) \ = 2 \times 2cos^2\frac{\delta}{2} = 4cos^2\frac{\delta}{2}$



cos² fringe or pattern

Example

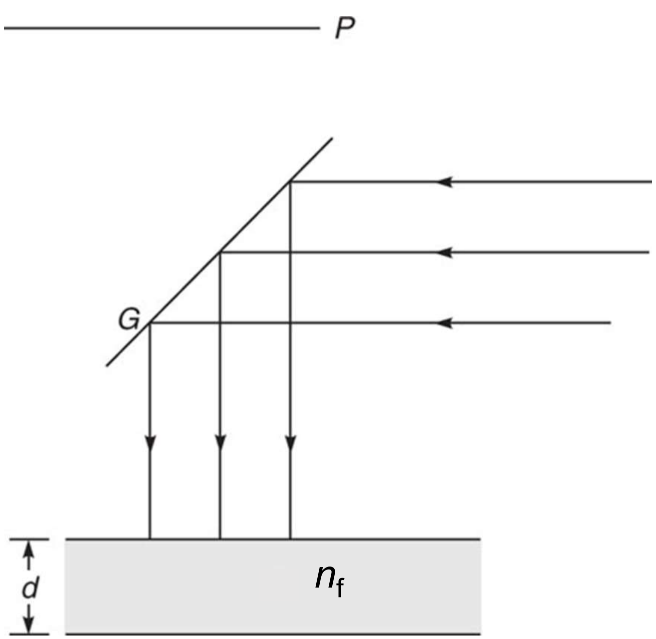
For a path difference Δ of $\lambda/5$, $\dfrac{I}{I_{\max}}$?

$$\because \frac{I}{I_0} = 4\cos^2\frac{\delta}{2} \qquad \text{and}\, \delta = \frac{2\pi}{\lambda_0}\left(S_2P - S_1P\right) = \frac{2\pi}{\lambda_0}\Delta \qquad \Rightarrow \delta = \frac{2\pi}{\lambda_0}\delta = \frac{2\pi}{\cancel{\lambda_0}} \times \frac{\cancel{\lambda_0}}{5}$$

$$\Rightarrow I_{\max} = 4I_0 \qquad\qquad = \frac{2\pi}{5}$$

$$\Rightarrow \frac{I}{I_{\max}} = \frac{\cancel{4I_0} \times \cos^2(2\pi/10)}{\cancel{4I_0}} = \cos^2(0.628) \approx (0.809)^2 \approx 0.65$$

Interference by division of amplitude



Wave reflected from the upper and lower surface of the thin film will interfere at the photographic plate *P*

Optical path difference from the one reflecting from the upper surface and the one from the lower surface: $n_f.2d$ because the film thickness *d* is traversed twice

Additionally the beam reflected from the upper surface undergoes an additional phase change of π as it is reflected from the interface of air and film of higher r.i. (can be proved from Lloyd's Mirrror expt.)

Since for constructive interference, phase difference should be $2m\pi$

$$\Rightarrow \frac{2\pi}{\lambda_0} \times 2n_f \times d - \pi = 2m\pi \Rightarrow \frac{\cancel{2\pi}}{\lambda_0} \times 2n_f \times d = \cancel{2\pi}\left(m + \frac{1}{2}\right); m = 0,1,2, \dots.$$

$\Rightarrow 2n_f \times d = \left(m + \frac{1}{2}\right)\lambda_0; m = 0,1,2, \dots.$: for constructive interference

$$2n_f \times d = \left(m + \tfrac{1}{2}\right)\lambda_0; m = 0,1,2, \ldots.$$: for constructive interference

For destructive interference:

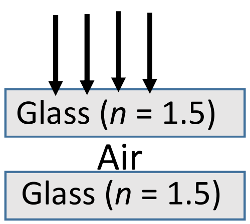
$$\frac{2\pi}{\lambda_0} \times 2n_f \times d - \pi = (2n + 1)\pi; n = 0,1,2, \ldots.$$

$$\Rightarrow \quad \cancel{2}\pi \times 2n_f \times d = \cancel{2}\pi \times (n + 1)\lambda_0; n = 0,1,2, \ldots.$$

$$\Rightarrow \quad 2n_f \times d = (n + 1)\lambda_0; n = 0,1,2, \ldots. = m\lambda_0; m = 1,2, \ldots.$$

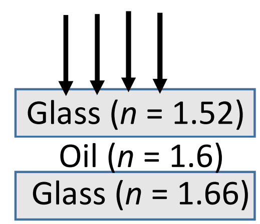
$$\Rightarrow 2n_f d = m\lambda_0; m = 1,2, \ldots.$$: for destructive interference

If an air film is sandwiched between two glass plates:



- Extra phase change of π is only at air-glass interface

$\Rightarrow$ No change in interference condition

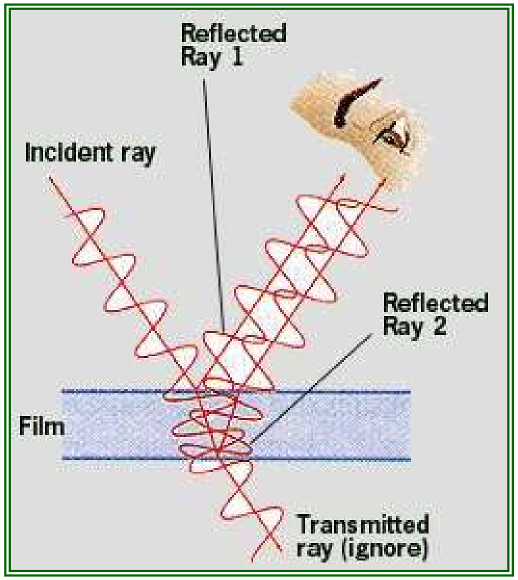


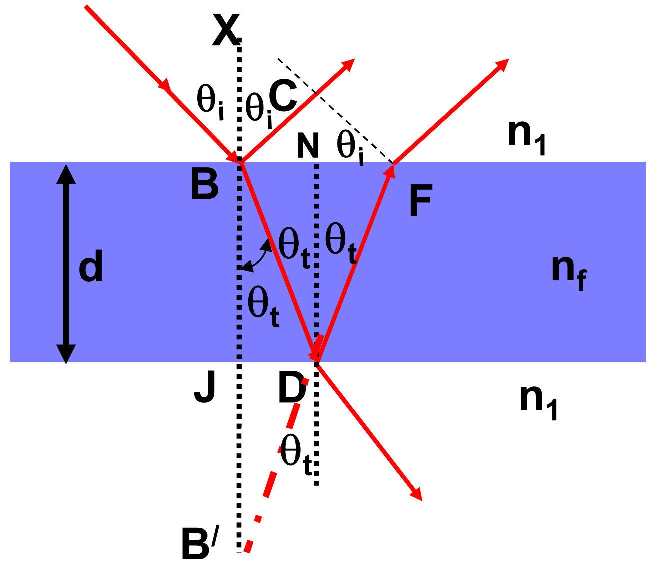
?

Interference condition will be reversed

Check

Oblique incidence (cosine law):





C: foot of the normal from *F*

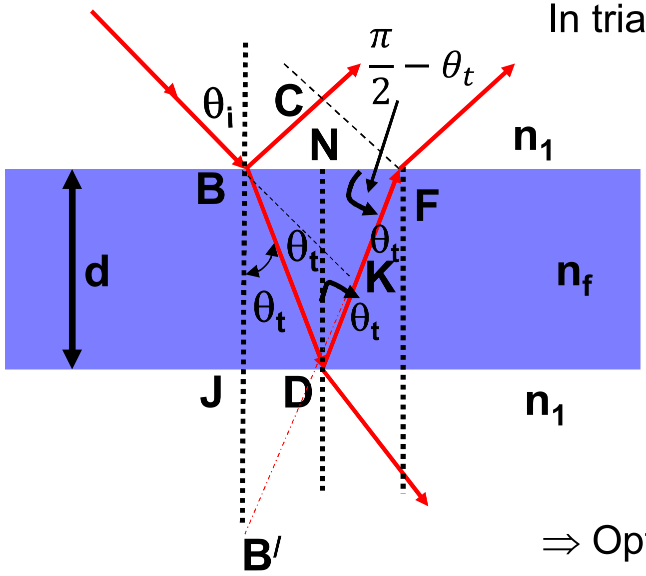
Wave reflected from the upper surface of the film and the one reflected from the lower surface interfere

$\Rightarrow$ Path difference between them: $\Delta = n_f(BD + DF) - n_1 BC$

$\angle$JBD = $\angle$BDN = $\angle$NDF = θ_t

$\angle$BDJ = $\frac{\pi}{2} - \theta_t$ and $\angle$B$^/$DJ = $\pi - \left[\left(\frac{\pi}{2} - \theta_t\right) + \theta_t + \theta_t\right] = \frac{\pi}{2} - \theta_t$ $\Rightarrow$ $BD = B^/D$ and $BJ = JB^/ = d$

$\Rightarrow$ $BD + DF = B^/D + DF = B^/F$ $\Rightarrow \Delta = n_f B^/F - n_1 BC$ but $\angle$CFB = $\frac{\pi}{2} - \left(\frac{\pi}{2} - \theta_i\right) = \theta_i$



In triangle BCF, $\frac{BC}{BF} = \sin\theta_i$ In triangle BKF, $\frac{KF}{BF} = \cos\left(\frac{\pi}{2} - \theta_t\right) = \sin\theta_t$

$$\therefore\ BC = BF\sin\theta_i = \frac{KF}{\sin\theta_t}\sin\theta_i = KF\frac{n_f}{n_1}$$

Thus optical path difference:

$$\Delta = n_f B^/F - n_1 BC = n_f B^/F - \cancel{n_1}\frac{n_f}{\cancel{n_1}}KF$$

$$= n_f B^/K = 2dn_f\cos\theta_t$$

$\Rightarrow$ Opt. phase diff δ: $\frac{2\pi}{\lambda_0}\Delta - \pi = \frac{2\pi}{\lambda_0} \times 2dn_f\cos\theta_t - \pi$

For constructive interference, $\delta = \frac{2\pi}{\lambda_0} \times 2dn_f\cos\theta_t - \pi = 2m\pi; m = 0,1,2,\ldots$

$$\Rightarrow \frac{\cancel{2\pi}}{\lambda_0} \times 2dn_f\cos\theta_t = \cancel{2\pi}\left(m + \frac{1}{2}\right)$$

$\Rightarrow \Delta = 2dn_f\cos\theta_t = \left(m + \frac{1}{2}\right)\lambda_0$: maxima

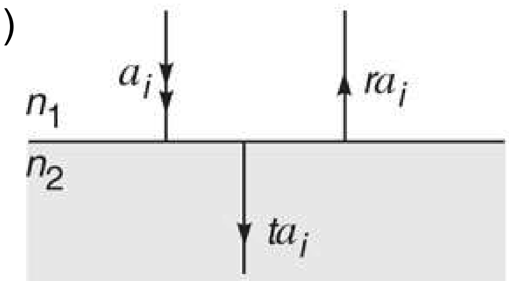
$= m\lambda_0$: minima

Called Cosine law

Non-reflecting films:

Consider two media having r.i.'s $n_{1,2}$ separated by an interface

a)



Light is incident normally a) from a medium of r.i. n_1 on a medium of r.i. n_2

From Stoke's relations, one can show:

Amplitudes of reflected and transmitted light are given by:

$$a_r = \frac{n_1 - n_2}{n_1 + n_2} a_i$$

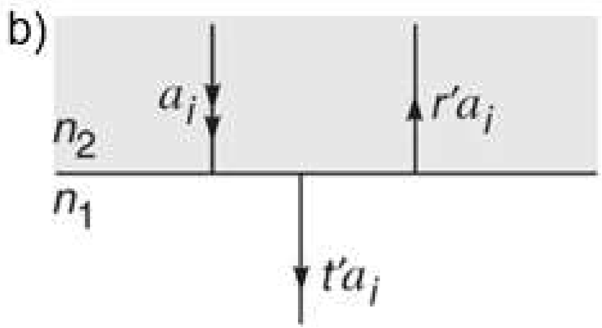
$$a_t = \frac{2n_1}{n_1 + n_2} a_i$$

$\Rightarrow$ If $n_2 > n_1, a_r$ is negative $\Rightarrow$ A phase change of π takes place

Amplitude reflection and transmission coefficients are given by:

$$\frac{a_r}{a_i} = r = \frac{n_1 - n_2}{n_1 + n_2}; \quad \frac{a_t}{a_i} = t = \frac{2n_1}{n_1 + n_2}$$

Corresponding quantities, when light is incident from a medium of r.i. n_2 on a medium of r.i. n_1:



$$r^{/} = \frac{n_2 - n_1}{n_1 + n_2} = -r;$$

$$t^{/} = \frac{2n_2}{n_1 + n_2}$$

$$\Rightarrow \quad 1 - tt^{/} = 1 - \frac{4n_1 n_2}{(n_1 + n_2)^2} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = r^2$$

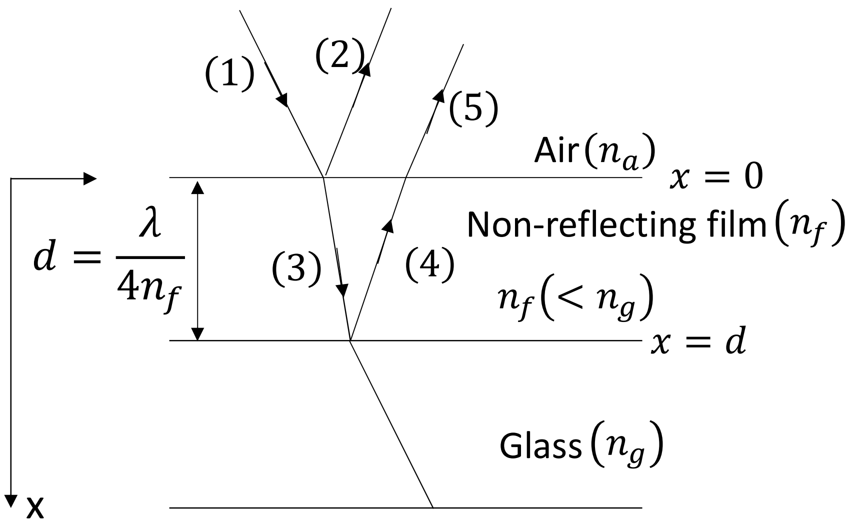
In many optical instruments there could be several interfaces, loss in intensity due to reflection at each of these could lead to substantial loss

Example, for air-crown glass ($n = 1.5$): $r = \left(\frac{1.5-1}{1.5+1}\right)^2 = \frac{0.25}{6.25} = 0.04 \Rightarrow 4\%$

of the incident light is reflected at each such reflection

For flint glass having $n = 1.67$, it would be about 6%

In order to reduce these losses, lens e.g. spectacles surfaces are coated with a thin non-reflecting film of r.i. (e.g. MgF_2 of n_f = 1.38) less than that of the glass



Abrupt phase change of π occurs at both interfaces: air-film and film-glass

$\Rightarrow$ Condition for destructive interference for near normal incidence i.e. $\cos\theta \approx 1$:

$$2n_f d \cong \left(m + \frac{1}{2}\right)\lambda$$

$\Rightarrow$ For m = 0

$$2n_f d \cong \frac{\lambda}{2} \Rightarrow d \cong \frac{\lambda}{4n_f}$$

Thus, for $\lambda \sim 5 \times 10^{-5}$ cm,

$$d \cong \frac{5\times10^{-5}}{4\times1.38} \approx 9 \times 10^{-6} \text{ cm} = 0.09\ \mu\text{m}$$

Visual benefits/advantage of lenses with anti-reflective (AR) coating $\Rightarrow$ sharper vision with less glare when driving at night in low-light conditions and greater comfort during prolonged computer use (compared with wearing eyeglass lenses without AR coating)

From Stoke's relations, it can be shown that required n_f is

$$n_f = \sqrt{n_a n_g}$$

with $n_a = 1, n_g = 1.5, n_f = 1.38,$

Reflectivity will be ~ 1.3% in contrast to ~ 4% without AR coating

Ideal value should have been $n_f = 1.2247!$

Newton's rings

A thin air film of r.i. (n = 1) of variable thickness (t) is entrapped between the lens and the glass plate: t is 0 at the point of contact O and increases away from O

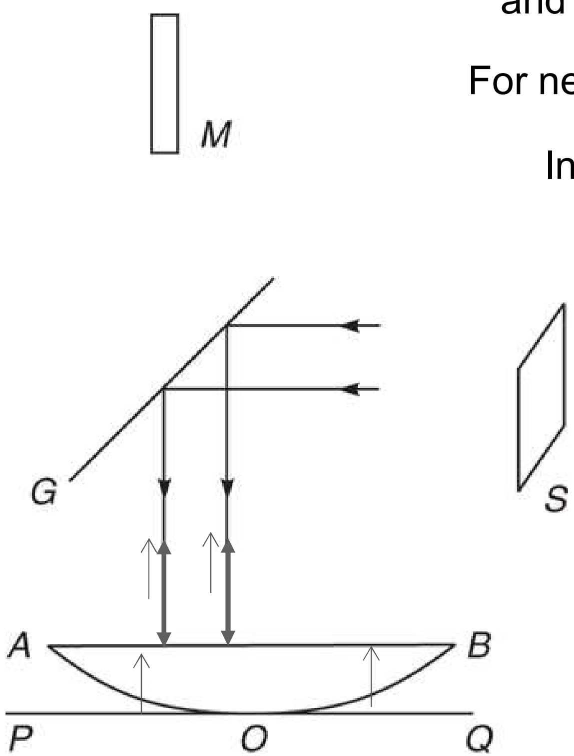
For near-normal incidence, and for points close to O, opt. path difference $\approx 2nt$

Interference takes place between light reflected from AOB and POQ

$\therefore$ for maxima: $2t = \left(m + \frac{1}{2}\right)\lambda; m = 0,1,2, \ldots$

for minima: $2t = m\lambda; m = 1,2, \ldots$

Due to the spherical surface f the lens, t will be const over a circle with O as its center $\Rightarrow$ we will get concentric dark and bright fringes in the form of rings



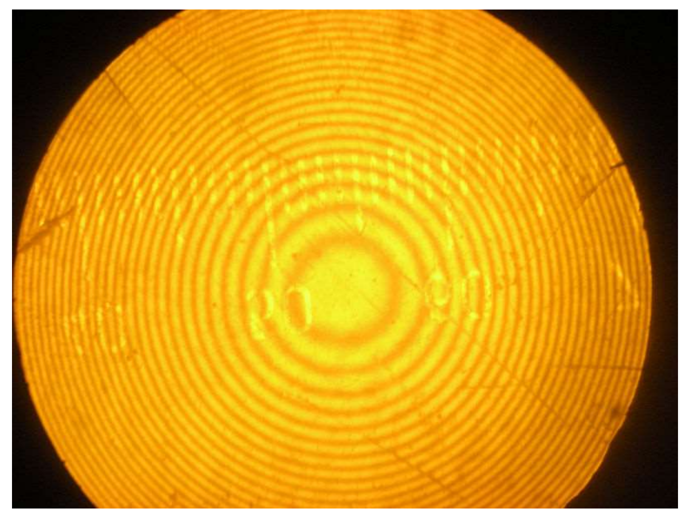
M : A travelling microscope

AOB : A plano-convex lens

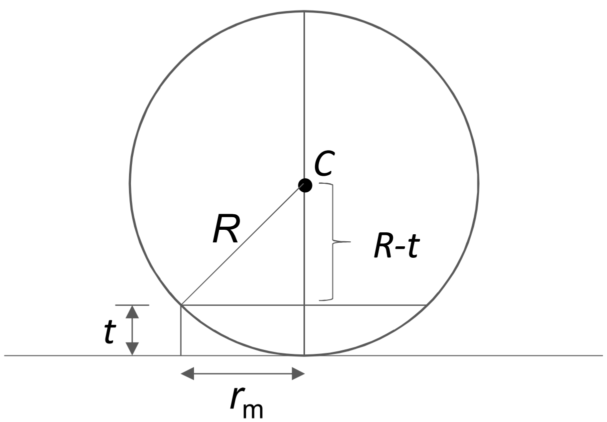
POQ : A plane glass plate

S : An extended light source

G : A glass plate beam splitter



Radius of the m^{th} dark ring:



From the figure

$$(R-t)^2 + r_m^2 = R^2$$

$$\Rightarrow \quad \cancel{R^2} \cong r_m^2 + \cancel{R^2} - 2tR + t^2$$

$$\Rightarrow \quad r_m^2 \cong t(2R - t)$$

Typically, R ~100 cm; $t \sim 10^{-3}$ cm $\Rightarrow$ t can be neglected rel to $2R$

$$\Rightarrow \quad 2t = \frac{r_m^2}{R} = m\lambda \quad \Rightarrow \; r_m = \sqrt{mR\lambda}\,; m = 1,2, \ldots$$ (for m^{th} dark ring)

$\Rightarrow$ Radii of the dark rings vary as sq root of natural numbers

In expt, diameters of m^{th} and $(m + p)^{th}$ rings are measured ($p \sim 10$) and from the following relation source wavelength λ is measured:

$$\left(D_{m+p}\right)^2 - (D_m)^2 = 4(\cancel{m} + p - \cancel{m})R\lambda$$

$$\Rightarrow \quad \lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$