Assignment 3 Solution

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1

Given that

$$f(x, y) = \frac{x^2y^2}{x^2y^2 + (x-y)^2}$$

whenever $x^2y^2 + (x-y)^2 \neq 0$. Now, applying the first limit, we get

$$\lim_{y \to 0} f = \frac{x^2y^2}{x^2y^2 + (x-y)^2} = \frac{0}{0 + (x)^2} = 0$$

Now, applying the outer limit we get

$$\lim_{x \to 0} \lim_{y \to 0} f = 0$$

Similarly, we get

$$\lim_{x \to 0} f = \frac{x^2y^2}{x^2y^2 + (x-y)^2} = \frac{0}{0 + (y)^2} = 0$$

And applying the outer limit we get

$$\lim_{y \to 0} \lim_{x \to 0} f = 0$$

Hence,

$$\lim_{y \to 0} \lim_{x \to 0} f = \lim_{x \to 0} \lim_{y \to 0} f$$

Now, $\lim_{(x,y) \to (0,0)} f(x, y)$ along the $y = x$ line, we get

$$\lim_{(x,y) \to (0,0)} f = \frac{x^4}{x^4} = 1$$

Similarly, $\lim_{(x,y) \to (0,0)} f(x, y)$ along the $y = 2x$ line, we get

$$\lim_{(x,y) \to (0,0)} f = \frac{4x^4}{4x^4 + x^2} = \frac{4x^2}{4x^2 + 1} = 0$$

Hence, limit does not exits at $(x, y) \to (0, 0)$.

2

See appended section.

3

Given that

$$f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}$$

We need to find the limit along $y = mx$ as $(x,y) \to (0,0)$

Replacing the value of y with mx in the given function, we get

$$f = \frac{x^2 - m^2x^2}{x^2 + m^2x^2}$$

Solving the limit, we get

$$\lim_{x\to 0} \frac{x^2 - m^2x^2}{x^2 + m^2x^2} = \frac{1 - m^2}{1 + m^2}$$

In order to define a $f(x,y)$ so as to make it continuous at $(0,0)$,

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

should be equal along every path $y = mx + c$ taken. Since this is not true in the given function, we cannot define $f(x,y)$ so as to make it continuous at $(0,0)$.

4

Given a scalar field

$$f(\mathbf{x}) = ||\mathbf{x}||^4$$

Now, assume that

$$g(t) = f(\mathbf{x} + t\mathbf{y})$$

Since

$$f(\mathbf{x}) = (\mathbf{x} \cdot \mathbf{x}) \times (\mathbf{x} \cdot \mathbf{x})$$

We get

$$g(t) = (\mathbf{x} + t\mathbf{y}) \cdot (\mathbf{x} + t\mathbf{y}) \times (\mathbf{x} + t\mathbf{y}) \cdot (\mathbf{x} + t\mathbf{y})$$

$$g(t) = (\mathbf{x} \cdot \mathbf{x} + 2t\mathbf{x} \cdot \mathbf{y} + t^2\mathbf{y} \cdot \mathbf{y}) \times (\mathbf{x} \cdot \mathbf{x} + 2t\mathbf{x} \cdot \mathbf{y} + t^2\mathbf{y} \cdot \mathbf{y})$$

$$g\prime(0) = f\prime(\mathbf{x}; \mathbf{y}) = 4||\mathbf{x}||^2(\mathbf{x} \cdot \mathbf{y})$$

5

5.1

Given that

$$f(x,y) = \frac{x}{\sqrt{x^2+y^2}}$$

Then the first order partial derivative can be calculated as

$$\frac{\partial f}{\partial x} = \frac{\partial \frac{x}{\sqrt{x^2+y^2}}}{\partial x}$$

Using division rule, we get

$$\frac{\partial f}{\partial x} = \frac{\frac{\partial}{\partial\, x}\left(x\right)\sqrt{x^2+y^2} - \frac{\partial}{\partial\, x}\left(\sqrt{x^2+y^2}\right)x}{\left(\sqrt{x^2+y^2}\right)^2}$$

$$\frac{\partial f}{\partial x} = \frac{1 \cdot \sqrt{x^2+y^2} - \frac{x}{\sqrt{x^2+y^2}}x}{\left(\sqrt{x^2+y^2}\right)^2} = \frac{y^2}{\left(x^2+y^2\right)\sqrt{x^2+y^2}}$$

The partial derivative with respect to y is

$$\frac{\partial f}{\partial y} = \frac{\partial \frac{x}{\sqrt{x^2+y^2}}}{\partial y}$$

$$\frac{\partial f}{\partial y} = x\frac{\partial}{\partial\, y}\left(\left(x^2+y^2\right)^{-\frac{1}{2}}\right)$$

And using chain rule, replacing $u = x^2 + y^2$, we get

$$\frac{\partial f}{\partial y} = x\frac{\partial}{\partial\, u}\left(u^{-\frac{1}{2}}\right)\frac{\partial}{\partial\, y}\left(x^2+y^2\right)$$

$$\frac{\partial f}{\partial y} = x\left(-\frac{1}{2u^{\frac{3}{2}}}\right) \cdot 2y$$

Replacing the back the value of u, we get

$$\frac{\partial f}{\partial y} = -\frac{xy}{\left(x^2+y^2\right)^{\frac{3}{2}}}$$

5.2

Given

$$f(x) = \vec{a}.\vec{x}$$

$\vec{a}$ being fixed, f is defined on R^n and $\vec{a} = a_1 i + a_2 j + ...$
Then, the partial derivative in the x direction is

$$\frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f((x,y,z...) + h(1,0,0,0...)) - f(x,y,z...)}{h}$$

$$= \lim_{h\to 0} \frac{\vec{a}\cdot(x+h,y,z..) - \vec{a}\cdot(x,y,z...)}{h}$$

$$= a_1$$

Then, the partial derivative in the y direction is

$$\frac{\partial f}{\partial y} = \lim_{h\to 0} \frac{f((x,y,z...) + h(0,1,0,0...)) - f(x,y,z...)}{h}$$

$$= \lim_{h\to 0} \frac{\vec{a}\cdot(x,y+h,z..) - \vec{a}\cdot(x,y,z...)}{h}$$

$$= a_2$$

And so on.

6

Given the function

$$f(x,y) = \frac{1}{y}\cos x^2$$

We have

$$D_2 f = \frac{\partial \frac{1}{y}\cos x^2}{\partial y} = \frac{-1}{y^2}\cos x^2$$

$$D_1 f = \frac{\partial \frac{1}{y}\cos x^2}{\partial x} = \frac{-2x}{y}\sin x^2$$

Then, the mixed partial derivatives $D_1(D_2 f)$ and $D_2(D_1 f)$ are given by

$$D_1(D_2 f) = \frac{2x}{y^2}\sin x^2$$

$$D_2(D_1 f) = \frac{2x}{y^2}\sin x^2$$

Since $D_1(D_2 f) = D_2(D_1 f)$ for all values of (x, y)
Hence, Proved

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Given the scalar field

$$f(x,y,z) = x^2 + 2y^2 + 3z^2$$

and the unit vector $\frac{i-j+2k}{\sqrt{6}}$

The directional derivative at $(1,1,0)$ in the direction of $\vec{v} = i - j + 2k$ equals to

$$DD = f((1,1,0);(\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{2}{\sqrt{6}})) = \lim_{h\to 0}\frac{f((1,1,0)+h(\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{2}{\sqrt{6}})) - f(1,1,0)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{(h+\sqrt{6})^2}{6} + \frac{(-h+\sqrt{6})^2}{6} + \frac{12h^2}{6} - 3}{h}$$

Applying L' Hopitals rule, we get

$$\lim_{h\to 0}\frac{15h^2 - 2\sqrt{6}h}{6h} = \frac{-2}{\sqrt{6}}$$

8

Given the scalar field

$$f(x,y,z) = axy^2 + byz + cz^2x^3$$

has a maximum value of 64 in a direction parallel to the z -axis.

The directional derivative at $(1,2,1)$ in the direction parallel to the z axis being $\vec{v} = k$ equals to

$$\Big(\frac{\partial f}{\partial x}i + \frac{\partial f}{\partial y}j + \frac{\partial f}{\partial z}k\Big)\cdot(k)$$

Replacing values, we get

$$\Big((ay^2 + 3cz^2x2)i + (2axy + bz)j + (by + 2czx^3)k\Big)\cdot\big(k\big)$$

where $(x,y,z) \to (0,0,1)$, we get

$$\Big((4a+3c)i + (4a-b)j + (2b-2c)k\Big)\cdot\big(k\big) = (\frac{2b-2c}{\sqrt{1}}) = 64$$

Hence, we get

$$b - c = 32$$

If maximum occurs along a direction, then the minimum occurs along a direction perpendicular to it.

$$4a + 3c = 0$$

$$4a - b = 0$$

Solving these equations, we get

$$a = 6, b = 24, c = -8$$

9

Given

$$\mathbf{r}(x, y, z) = xi + yj + zk$$

and

$$r(x, y, z) = ||\mathbf{r}(x, y, z)|| = \sqrt{x^2 + y^2 + z^2}$$

Also

$$r^n = \sqrt[n/2]{x^2 + y^2 + z^2}$$

Computing for some value n, a positive integer, the gradient of r^n

$$\nabla(r^n) = \Big(\frac{\partial r^n}{\partial x}i + \frac{\partial r^n}{\partial y}j + \frac{\partial r^n}{\partial z}k\Big)$$

$$\nabla(r^n) = \Big(\frac{2nx \sqrt[(n-2)/2]{x^2 + y^2 + z^2}}{2}i + \frac{2ny \sqrt[(n-2)/2]{x^2 + y^2 + z^2}}{2}j + \frac{2nz \sqrt[(n-2)/2]{x^2 + y^2 + z^2}}{2}k\Big)$$

$$\nabla(r^n) = \Big(nx \sqrt[(n-2)/2]{x^2 + y^2 + z^2}i + ny \sqrt[(n-2)/2]{x^2 + y^2 + z^2}j + nz \sqrt[(n-2)/2]{x^2 + y^2 + z^2}k\Big)$$

$$\nabla(r^n) = \Big(n \sqrt[(n-2)/2]{x^2 + y^2 + z^2}\Big) \times \Big(xi + yj + zk\Big)$$

$$\nabla(r^n) = \Big(nr^{n-2}\Big) \times \Big(xi + yj + zk\Big)$$

$$\nabla(r^n) = nr^{n-2}\mathbf{r}$$

10

Given a function $u = f(x, y)$, $x = X(t)$, $y = Y(t)$ define u as a function of t, say $u = F(t)$

10.1

Given

$$f(x, y) = x^2 + y^2, X(t) = t, Y(t) = t^2$$

Replacing the given values of (x, y) as $(X(t), Y(t))$, we get

$$u(t) = t^2 + t^4$$

$F\prime(t)$ can be calculated as

$$F\prime(t) = \frac{du(t)}{dt}$$

$$F\prime(t) = \frac{d(t^2 + t^4)}{dt}$$

$$F\prime(t) = 2t + 4t^3$$

$F\prime\prime(t)$ can be calculated as

$$F\prime\prime(t) = \frac{dF\prime(t)}{dt}$$

$$F\prime\prime(t) = \frac{d(2t + 4t^3)}{dt}$$

$$F\prime\prime(t) = 2 + 12t^2$$

10.2

Given

$$f(x, y) = e^{xy}\cos(xy^2), X(t) = \cos(t), Y(t) = \sin(t)$$

Replacing the given values of (x, y) as $(X(t), Y(t))$, we get

$$u(t) = e^{\cos(t)\sin(t)}\cos(\cos(t)(\sin(t))^2)$$

$F\prime(t)$ can be calculated as

$$F\prime(t) = \frac{du(t)}{dt}$$

$$F\prime(t) = \frac{d(e^{\cos(t)\sin(t)}\cos(\cos(t)(\sin(t))^2))}{dt}$$

Have to add.

11

See appended section

12

See appended section

13

See appended section

14

See appended section

15

See appended section

16

See appended section

17

See appended section

18

18.1

Given

$$f(x, y, z) = (y^2 z^2)i + 2yzjx^2k$$

along the path described by $\alpha(t) = ti + t^2j + t^3k$

The line integral of the vector field is

$$\int f(\alpha(t)) \cdot d\alpha = \int ((t^4 t^6)i + 2t^5 jt^2 k) \cdot (i + 2tj + 3t^2 k)dt$$

$$= \int (t^4 - t^6) + 4t^6 - 3t^4 dt$$

$$= \left(\frac{-2t^5}{5} + \frac{3t^7}{7}\right)$$

18.2

See appended section

19

19.1

Given

$$\int_C (x^2 - 2xy)dx + (y^2 - 2xy)dy$$

where C is a path from $(2,4)$ to $(1,1)$ along the parabola C $y = x^2$.

The parametric equation of the curve is $x = t$ and $y = t^2$, then the line integral is

$$= \int_{-2}^{1} (t^2 - 2t^3)dt + 2(t^5 - 2t^4)dt$$

$$= \int_{-2}^{1} (t^2 - 2t^3 + 2t^5 - 4t^4)dt$$

$$= \left(\frac{t^3}{3} - \frac{t^4}{2} + \frac{t^6}{3} - \frac{4t^5}{5}\right)_{-2}^{1}$$

$$= \frac{396}{10}$$

19.2

Given

$$\int_C \frac{(x+y)dx - (x-y)dy}{x^2 + y^2}$$

where C is is the circle $x^2 + y^2 = a^2$ traversed once in a counter-clockwise direction.

The parametric equation of the curve is $x = a\cos t$ and $y = a\sin t$, then the line integral is

$$= \int_0^{2\pi} \frac{-a\sin t(\cos t + \sin t)dt - a\cos t(\cos t - \sin t)dt}{a^2(\cos t^2 + \sin t^2)}$$

$$= \int_0^{2\pi} \frac{-a(\sin t^2 + \cos t^2)dt}{a^2}$$

$$= \int_0^{2\pi} \frac{-dt}{a}$$

$$= \int_0^{2\pi} \frac{-dt}{a}$$

$$= \left(\frac{-t}{a}\right)_0^{2\pi}$$

$$= \frac{-2\pi}{a}$$

20

20.1

Given the vector field

$$f(x,y) = (2xe^y + y)i + (x^2e^y + x2y)j$$

We have

$$f_1(x,y) = 2xe^y + y$$

and

$$f_2(x,y) = x^2e^y + x2y$$

Then, the partial derivatives D_2f_1 and D_1f_2 are given by

$$D_2f_1 = 2xe^y + 1$$

and

$$D_1f_2 = 2xe^y + 1$$

Since $D_2f_1 = D_1f_2$ for all values of (x,y), this vector field is a gradient on any open subset of R^2.

We know that

$$\frac{\partial\phi}{\partial x} = 2xe^y + 1$$

and

$$\frac{\partial\phi}{\partial y} = 2xe^y + 1$$

Using indefinite integrals and integrating the first of these equations with respect to x (holding y constant) we find

$$\phi(x,y) = \int (2xe^y + 1)dx + A(y) = x^2e^y + x + A(y)$$

and

$$\phi(x,y) = \int (2xe^y + 1)dy + B(x) = 2xe^y + y + B(x)$$

20.2

Given the vector field

$$f(x,y,z) = 2xy^3i + x^2z^3j + 3x^2yz^2k$$

We have

$$f_1(x,y) = 2xy^3$$
$$f_2(x,z) = x^2z^3$$
$$f_3(x,y,z) = 3x^2yz^2$$

Then, the partial derivatives D_3f_1, D_2f_2 and D_1f_3 are given by

$$D_3f_1 = 0$$

$$D_2f_2 = 0$$

$$D_1f_3 = 6xyz^2$$

Since $D_3f_1 = D_2f_2 = D_1f_3$ only when either x, y or z is equal to 0

Hence, this vector is not a gradient of a scalar field ϕ

21 Appendix

Due to time constrains, I haven't been able to type all the solutions for the assignments in LaTeX. The answers that refer to this section are in the other document.

Maths Assignment-3

ROCHAN AVLUR VENKAT (17XJ1A0543)

2) Given $f(x,y) = \begin{cases} x\sin\left(\frac{1}{y}\right) & y \neq 0 \\ 0 & y = 0 \end{cases}$

by squeezing rule

$$-1 < \sin\left(\frac{1}{y}\right) < 1 \qquad y \neq 0$$

$$-x < x\sin\left(\frac{1}{y}\right) < x$$

$$\underbrace{\lim_{(x,y)\to(0,0)} -x}_{0} < \lim_{(x,y)\to(0,0)} x\sin\left(\frac{1}{y}\right) < \underbrace{\lim_{(x,y)\to(0,0)} x}_{0}$$

$$\therefore \lim_{(x,y)\to(0,0)} x\sin\left(\frac{1}{y}\right) = 0.$$

$$\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right] \neq \lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right]$$

This limit does not exists as the value constantly oscillates between -1 and 1.

But $\lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right] = 0.$

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10.2) $x(t) = \cos t \quad y(t) = \sin t$

$$f(x(t), y(t)) = e^{\cos t \sin t} \cos(\cos t \cdot \sin^2 t)$$

$$f'(t) = \nabla f(\cos t, \sin t) \cdot (-\sin t, \cos t)$$

$$\nabla f = \left[e^{xy} y \cos(xy^2) + e^{xy}(-\sin(xy^2) \cdot y^2)\right]\hat{x} + \left[e^{xy} \cdot x \cos(xy^2) + e^{xy}(-\sin(xy^2) 2xy\right]\hat{y}$$

Simplifying, we get

$$f'(t) = e^{\cos t \sin t} \cos 2t \cos(\cos t \cdot \sin^2 t) + \sin(\cos t \sin^2 t)(\sin^3 t - \sin 2t \cos t)$$

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11) Given $S_1 = (x-c)^2 + y^2 + z^2 = 3 \qquad \nabla S_1 = 2(x-1), 2y, 2z$

$S_2 = x^2 + (y-1)^2 + z^2 = 1 \qquad \nabla S_2 = 2x, 2(y-1), 2z$

$\nabla f \cdot (\bar{x} - \bar{a})$ is equation of a plane.

Since is is given that the planes are perpendicular, product of DR's are zero.

$$\nabla S_1 \cdot \nabla S_2 = 0$$

$$\Rightarrow x^2 + y^2 + z^2 = xc + y \quad \text{——— (i)}$$

Take $x^2 + (y-1)^2 + z^2 = 1$

$$\Rightarrow x^2 + y^2 + z^2 = 2y \quad \text{——— (ii)}$$

using (i) and (ii) ; $2y = xc + y$, $cx = y$.

Consider S_1

$$x^2 - 2xc + c^2 + y^2 + z^2 = 3$$

$$2y \cdot 2xc + c^2 = 3$$

$$c^2 = 3 \;, \; c = \pm\sqrt{3}$$

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12) (i) $f(x,y) = x^3 + y^3 - 3xy$

$\nabla f = 0$

$\Rightarrow$ $\cancel{3x^2 + 3y^2}$ $(3x^2 - 3y,\ 3y^2 - 3x) = 0$

points are $(0,0)$ and $(1,1)$

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 6x & -3 \\ -3 & 6y \end{bmatrix}$$

for $(0,0)$ $\Delta H < 0$

$\therefore$ saddle point.

for $(1,1)$ $\Delta H > 0$

$\therefore$ minima exist.

(ii) $z = \sin x \cosh y$

$\nabla f = 0$

$[\cosh y \cos x,\ \sin x \sinh y]$

$x = \pi/2 + n\pi$ $\quad$ $n\pi$

critical points

$x = n\pi + \pi/2 \quad y = 0$

$n = 0, 1, 2, 3$

$$H = \begin{bmatrix} -\cosh y \sin x & \cos x \sinh y \\ \cos x \sinh y & \sin x \cosh y \end{bmatrix}$$ at $(n\pi + \pi/2, 0)$

$\Delta H < 0$,

$\therefore$ saddle points exist.

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13) $f(x,y) = xy - x^3y - xy^3$; $0 \le y \le 1$ and $0 \le x \le 1$.

<u>find</u>:- relative and absolute extrima values and saddle points.

$$\frac{\partial f}{\partial x} = y - 3x^2y - y^3 \quad , \quad \frac{\partial f}{\partial y} = x - 3y^2x - x^3$$

$$\frac{\partial f}{\partial x} = 0 \quad , \quad \frac{\partial f}{\partial y} = 0$$

$$y - 3x^2y - y^3 = 0$$
$$x - 3y^2x - x^3 = 0$$

Solving :, we get $(0,0)$, $(-1/2, -1/2)$, $(1/2, 1/2)$

$\times$ not possible due to hypothesis.

$$H = \begin{bmatrix} -6xy & 1-3x^2-3y^2 \\ 1-3x^2-3y^2 & -6xy \end{bmatrix}$$

at $(0,0)$; $\Delta H < 0$

$\therefore$ it has a saddle point.

at $(1/2, 1/2)$ $\Delta H > 0$

$\therefore$ it has maxima.

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14) $5x^2 + 6xy + 5y^2 = 8$

$\nabla g = (10x + 6y, 10y + 6x)$

Using Lagrange's multiplies method

$f(x,y) = \sqrt{x^2+y^2}$

$\nabla f(x,y) = \lambda \nabla g$

$\therefore (2x, 2y) = \lambda(10x+6y, 10y+6x)$

$2x = \lambda(10x+6y)$

$2y = \lambda(10y+6x)$

$\Rightarrow \frac{x}{y} = \frac{10x+6y}{10y+6x}$

$\Rightarrow 10xy + 6x^2 = 10xy + 6y^2$

$\Rightarrow x^2 - y^2 = 0$

$\Rightarrow (x+y)(x-y) = 0$

$x = y$ or $x = -y$

$10x^2 + 6x^2 = 8$ | $10x^2 - 6y^2 = 8$

$x = \pm 1/\sqrt{2}$ | $x = \pm\sqrt{2}$

$\therefore (x,y) = \left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$; $(\pm\sqrt{2}, \mp\sqrt{2})$

min | max

15) $f(x,y,z) = x - 2y + 2z \quad ; \quad x^2+y^2+z^2-1=0$

$$\nabla g = (2x, 2y, 2z)$$

$$\nabla f = (1, -2, 2)$$

$$\lambda(\nabla g) = (\nabla f)$$

$$(1, -2, -2) = \lambda(2x, 2y, 2z)$$

$$x = \frac{1}{2\lambda} \quad , y = \frac{1}{-\lambda} \quad , z = \frac{1}{\lambda}$$

$$x^2+y^2+z^2=1$$

$$\frac{1}{4\lambda^2} + \frac{1}{\lambda^2} + \frac{1}{\lambda^2} = 1 \quad \Rightarrow \quad \lambda = \pm 3/2$$

$$(x, y, z) = (\pm 1/3 , \mp 2/3 , \pm 2/3)$$

extreme values

$$f(1/3 , -2/3 , 2/3) = 3$$

$$f(-1/3 , 2/3 , -2/3) = -3$$

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16) $f(x,y,z) = x^a y^b z^c$; $x+y+z=1$

$\nabla f = (ax^{a-1}y^b z^c, \; bx^a y^{b-1} z^c, \; cx^a y^b z^{c-1})$

$\nabla g = (1,1,1)$

Using Lagrange's multipliers method

$\nabla f = \lambda \nabla g$

$(ax^{a-1}y^b z^c, \; bx^a y^{b-1} z^c, \; cx^a y^b z^{c-1}) = \lambda(1,1,1)$
① ② ③

$\therefore$ all three points are equal.

① = ② ② = ③

$ay = bx$ $bz = by$

$\Rightarrow \frac{a}{x} = \frac{b}{y} = \frac{c}{z} = \alpha$

$x = \frac{a}{\alpha}$, $y = \frac{b}{\alpha}$, $z = \frac{c}{\alpha}$

from constraints

$\frac{a+b+c}{\alpha} = 1$

extreme occurs at $x = \frac{a}{a+b+c}$, $y = \frac{b}{a+b+c}$, $z = \frac{c}{a+b+c}$

$\therefore$ max value is $\left(\frac{a}{a+b+c}\right)^a \left(\frac{b}{a+b+c}\right)^b \left(\frac{c}{a+b+c}\right)^c$

$= \frac{a^a b^b c^c}{(a+b+c)^{(a+b+c)}}$

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17) $C : x^2+4y^2=4$

length of the line from point (x_1, y_1) to line $x+y=4$

is $\frac{|x_1+y_1-4|}{\sqrt{2}}$

$\Rightarrow f(x,y) = \frac{|x_1+y_1-4|}{\sqrt{2}}$; $g(x) = x^2+4y^2-4$ ---- (i)

$\nabla f = (1,1)$ $\quad \nabla g = (2x, 8y)$

using Lagrange multiplier method

$\nabla f = \lambda \nabla g$

$(1,1) = \lambda(2x, 8y)$

$\Rightarrow x = 1/2\lambda \; , y = 1/8\lambda$

But from (i)

$\Rightarrow 1/4\lambda^2 + \frac{4\times 1}{64\lambda^2} = 4$

$\Rightarrow \lambda = \pm\frac{\sqrt{5}}{8}$

$\Rightarrow (x,y) = \left(\frac{4}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) ; \left(\frac{-4}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

max value for $(x,y) = \left(\frac{\sqrt{5}+4}{\sqrt{2}}\right)$

min value for $(x,y) = \left(\frac{4-\sqrt{5}}{\sqrt{2}}\right)$

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18) (iii) $f(x,y) = 2xy\hat{i} + (x^2+z)\hat{j} + y\hat{k}$ from $(1,0,2)$ to $(3,4,1)$

$(3,4,1)$ are DR's of line

The line $\alpha(t) = (1,0,2) + t(2,4,1)$

$x(t) = 1+2t$

$y(t) = 4t$

$z(t) = 2-t$

t goes from 0 to 1.

$$f(\alpha(t)) = 2(1+2t)4t\,\hat{i} + ((1+2t)^2 + (2-t))\hat{j} + 4t\hat{k}$$

$$= (8t+16t^2)\hat{i} + (3+4t^2+3t)\hat{j} + 4t\hat{k}$$

$$\bar{\alpha}'(t) = (2,4,-1)$$

$$\int_0^1 f(\alpha(t)) \cdot \bar{\alpha}'(t)\,dt = \int_0^1 (24t + 48t^2 + 12)\,dt$$

$$= \left[12t^2 + 16t^3 + 12t\right]_0^1$$

$$= \underline{\underline{40}}$$

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