PH203: Optics

Lecture #4

14.11.2018

Ray paths in a graded index medium:

$$n^2(x) = n_1^2 - \gamma^2 x^2$$

$$\left(\frac{dx}{dz}\right)^2 = \frac{n^2(x)}{\tilde{\beta}^2} - 1$$

$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{dn^2(x)}{dx}:$$

From Ray eq:

$$\left(\frac{dx}{dz}\right)^2 = \frac{n^2(x)}{\tilde{\beta}^2} - 1$$

$$\int \frac{dx}{\sqrt{n^2(x) - \tilde{\beta}^2}} = \pm\frac{1}{\tilde{\beta}}\int dz \quad \Rightarrow \quad \int \frac{dx}{\sqrt{n_1^2 - \gamma^2 x^2 - \tilde{\beta}^2}} = \pm\frac{1}{\tilde{\beta}}\int dz$$

$$\int \frac{dx}{\gamma\sqrt{\frac{n_1^2 - \tilde{\beta}^2}{\gamma^2} - x^2}} = \pm\frac{1}{\tilde{\beta}}\int dz \quad \Rightarrow \quad \int \frac{dx}{\sqrt{\frac{n_1^2 - \tilde{\beta}^2}{\gamma^2} - x^2}} = \pm\left(\frac{\gamma}{\tilde{\beta}}\right)\int dz \quad \Rightarrow \int \frac{dx}{\sqrt{x_0^2 - x^2}} = \pm\Gamma\int dz$$

$$\Gamma = \frac{\gamma}{\tilde{\beta}}$$

where

$$x_0 = \frac{\sqrt{n_1^2 - \tilde{\beta}^2}}{\gamma}$$

Let $x = x_0 \sin\theta \quad \Rightarrow dx = x_0\cos\theta\, d\theta \quad \Rightarrow$ Integral becomes $\int \frac{x_0\cos\theta\, d\theta}{x_0\cos\theta} = \pm\Gamma\int dz$

$$\Rightarrow \quad \theta = sin^{-1}\left(\frac{x}{x_0}\right) = \pm\Gamma(z - z_0) \quad \Rightarrow x = \pm x_0 \sin[\Gamma(z - z_0)]$$

Without any loss of generality, we can choose $z_0 = 0$

$$x_0 = \frac{\sqrt{n_1^2 - \tilde{\beta}^2}}{\gamma}$$

$\Rightarrow$ $x = x_0 \sin(\Gamma z)$ $\Rightarrow$ Ray paths are periodic with period

$$\Gamma = \frac{\gamma}{\tilde{\beta}} = \frac{\gamma}{n_1 \cos\theta}$$

Here x_0 embraces angle θ_1 through $\tilde{\beta}$

$$z_p = \frac{2\pi}{\Gamma} = \frac{2\pi n_1 \cos\theta_1}{\gamma}$$

In a parabolic index optical waveguide,

$$n^2(x) = n_1^2\left(1 - 2\Delta\left(\frac{x}{a}\right)^2\right), \quad |x| < a \qquad \text{core}$$

$$= n_1^2(1 - 2\Delta) = n_2^2, \qquad |x| \geq a \qquad \text{cladding}$$

$$\Rightarrow \quad \Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

Earlier we had

$$n^2(x) = n_1^2 - \gamma^2 x^2$$

Thus here

$$\gamma^2 = \frac{2\Delta\, n_1^2}{a} \qquad \Rightarrow \quad z_p = \frac{2\pi n_1 \cos\theta_1}{\gamma} = \frac{2\pi\, a \cancel{n_1} \cos\theta_1}{\cancel{n_1}\sqrt{2\Delta}} = \frac{2\pi\, a \cos\theta_1}{\sqrt{2\Delta}}$$

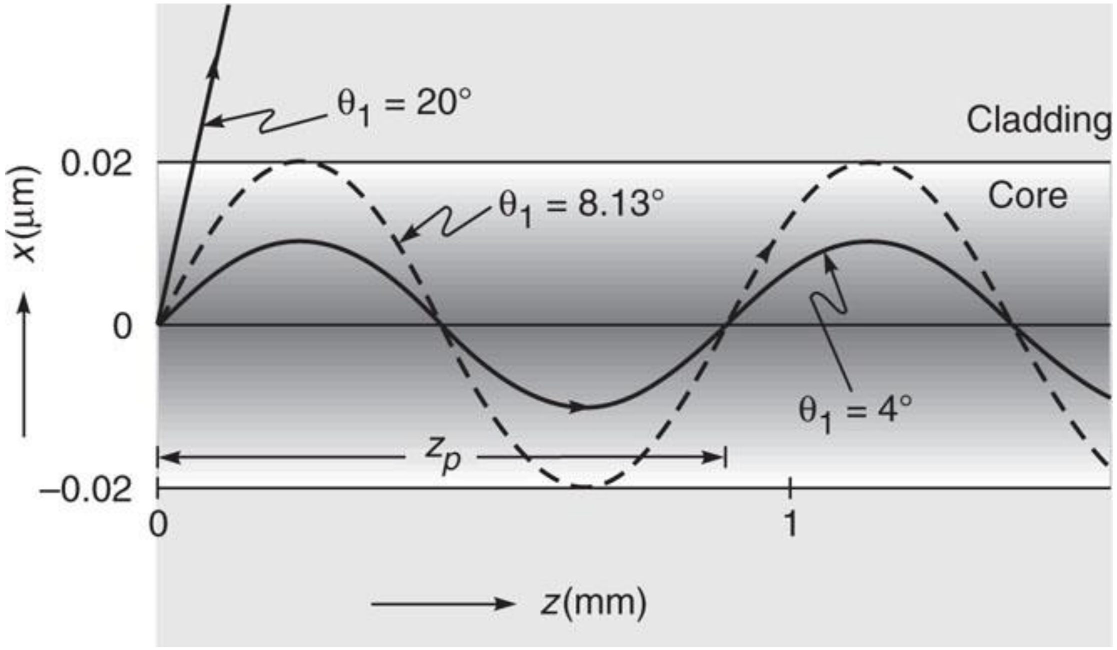
We had

$n(x)\cos\theta(x) = \tilde{\beta} \Rightarrow$ with increase in $x, n(x)$ reduces and $\theta(x)$ will decrease for corresponding increase in $\cos\theta$

$\Rightarrow$ when $\cos\theta(x)_{max} = 1 \Rightarrow \theta = 0^\circ \Rightarrow \tilde{\beta} = n_2,$ ray will become parallel to the axis as its slope is 0

Ray paths in a graded index medium having typical parameters: $n_1 = 1.5;\ \Delta = 1\% = 0.01;\ a = 25\ \mu\text{m}$

$\Rightarrow\ n_2 = 1.485;$



Since $$x_0 = \frac{\sqrt{n_1^2 - \tilde{\beta}^2}}{\gamma}$$

For real x_0 we must have $\tilde{\beta} < n_1$

Since smallest r.i. here is n_2, $\tilde{\beta} < n_2$

Hence, for guided rays, following condition must be satisfied:

$$n_2 < \tilde{\beta} \le n_1$$

For $\theta_1 = 4^\circ \Rightarrow \tilde{\beta} \approx 1.496; z_p = 0.8864\ \text{mm}$

$\theta_1 = 8.13^\circ \Rightarrow \tilde{\beta} \approx 1.485 = n_2; z_p = 0.8796\ \text{mm}$

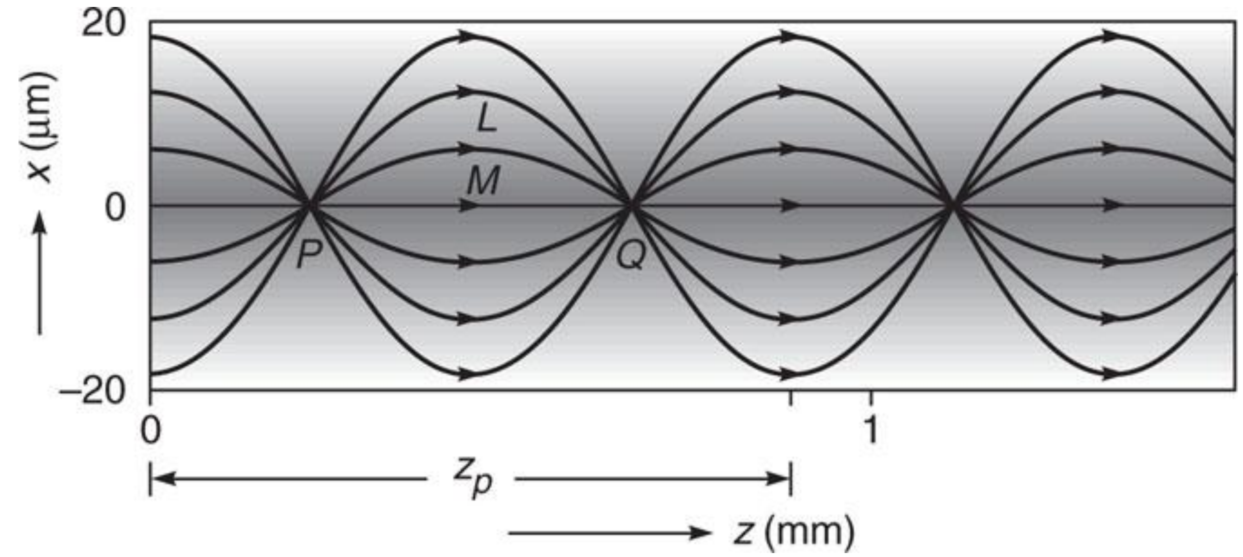
$\theta_1 = 20^\circ \Rightarrow \tilde{\beta} \approx 1.410 < n_2$

$$z_p = \frac{2\pi}{\Gamma} = \frac{2\pi\, a \cos\theta_1}{\sqrt{2\Delta}}$$

Under paraxial approximation, $\cos\theta_1 \approx 1$

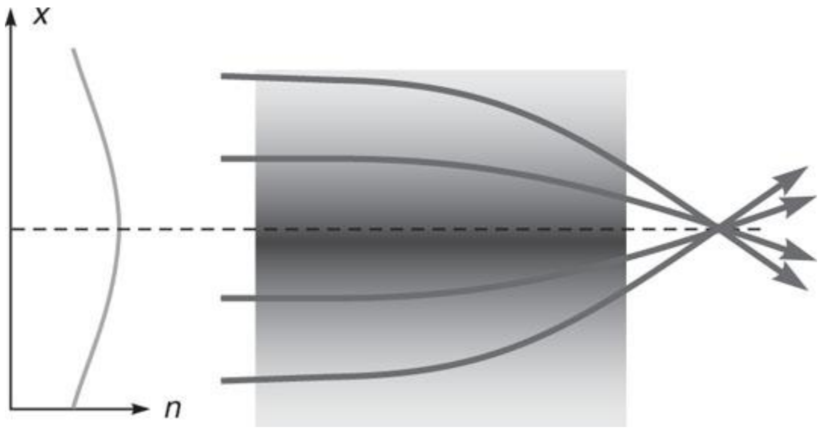
$$\Rightarrow\ z_p = \text{const}$$

For paraxial rays, all rays focuses at the same point before defocusing and focusing again



Medium acts as a lens called GRIN lenses

⇒ Example of a Self-focusing graded index medium



Reflection from the ionosphere (transmission of radio broadcasts)

R.I. in the ionosphere is given by

$$n^2(x) = 1- \frac{N_e(x)\, q^2}{m\varepsilon_0\omega^2} \equiv n_1^2 - gx; \quad x > 0$$

$$= n_1^2; \qquad x = 0$$

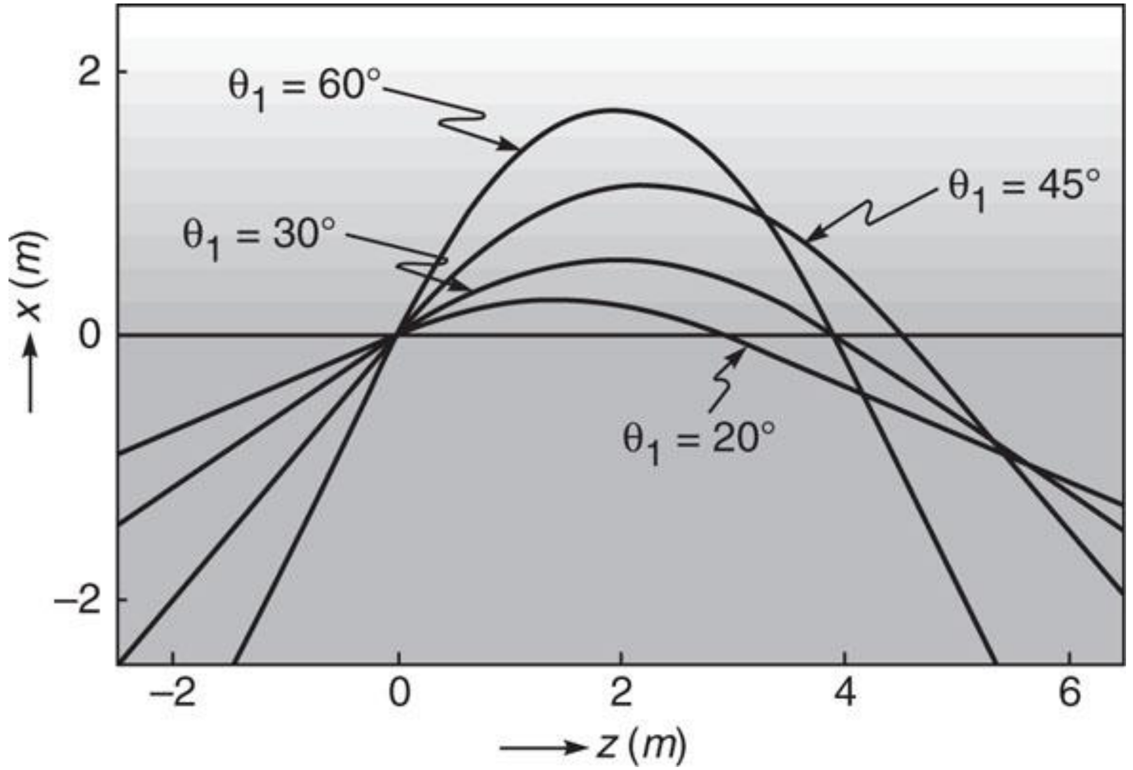
Solution of the ray eq:

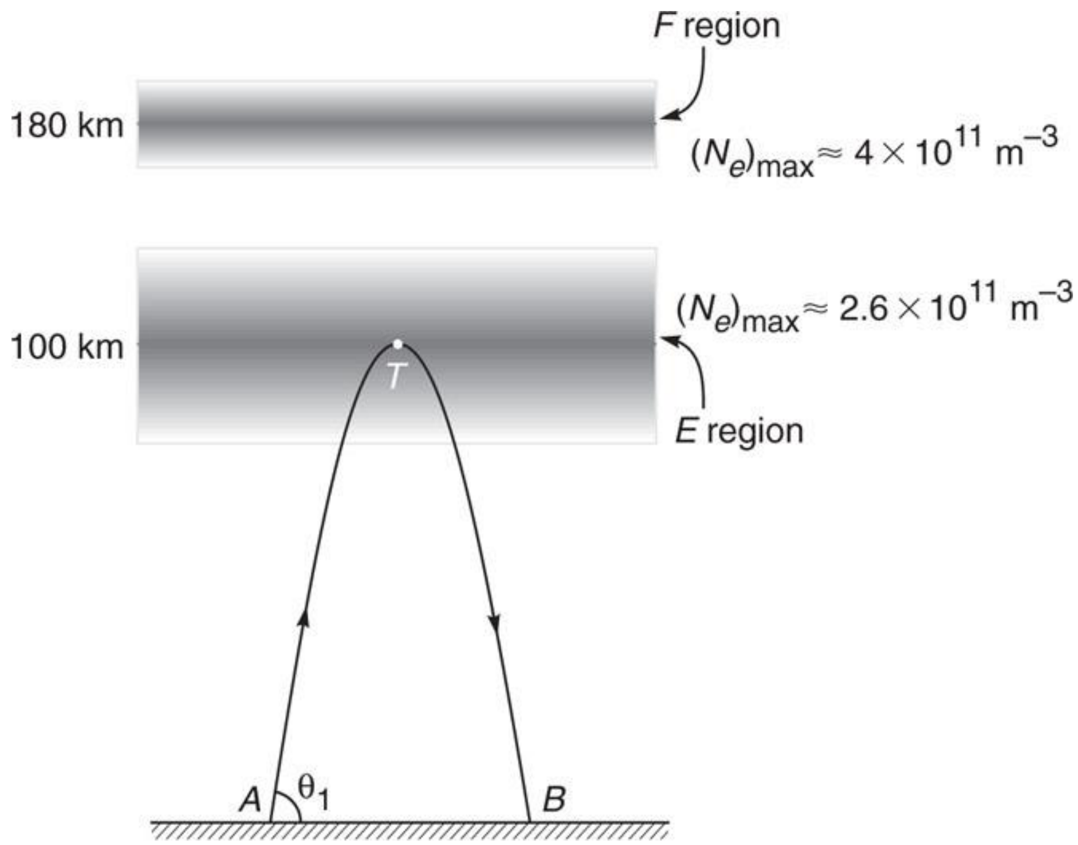
$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{dn^2(x)}{dx} = -\frac{g}{2\tilde{\beta}^2}$$

$$x(z) = (\tan\theta_1)z; \qquad z < 0$$

$$= -\frac{gz}{4\tilde{\beta}^2}(z - z_0); \qquad 0 < z < z_0$$

$$= -\frac{gz_0}{4\tilde{\beta}^2}(z - z_0); \qquad z > z_0$$


θ1 = 60°
θ1 = 45°
θ1 = 30°
θ1 = 20°
2
0
−2
x (m)
−2
0
2
4
6
z (m)


F region
180 km
(Ne)max ≈ 4 × 10^11 m^−3
(Ne)max ≈ 2.6 × 10^11 m^−3
100 km
T
E region
A
θ1
B

Interference of white light in a soap bubble

Subsequent slides to be continued and discussed on 15.11.18

Interference

https://www.youtube.com/watch?v=CAe3lkYNKt8

Interference

Consider superposition of two non-identical (amp & phase different) sinusoidal waves of same frequency:

$$x_1(t) = a_1 \cos(\omega t + \theta_1) = a_1 \cos\omega t \cos\theta_1 - a_1 \sin\omega t \sin\theta_1$$

$$x_2(t) = a_2 \cos(\omega t + \theta_2) = a_2 \cos\omega t \cos\theta_2 - a_2 \sin\omega t \sin\theta_2$$

Resulting displacement due to superposition of the two

$$x(t) = x_1(t) + x_2(t)$$

$$= \cos\omega t \underbrace{\left[a_1 \cos\theta_1 + a_2 \cos\theta_2\right]}_{= a\cos\theta} - \sin\omega t \left(\underbrace{\left[a_1 \sin\theta_1 + a_2 \sin\theta_2\right]}_{= a\sin\theta}\right)$$

$$\Rightarrow \; x(t) = a\cos(\omega t + \theta)$$

where $a \left(\cos^2\theta + \sin^2\theta\right)^{1/2} = \left[a_1^2 + a_2^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2)\right]^{1/2}$

We also get $\tan\theta = \dfrac{a_1 \sin\theta_1 + a_2 \sin\theta_2}{a_1 \cos\theta_1 + a_2 \cos\theta_2}$

From $$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos\left(\theta_1 - \theta_2\right)\right]^{1/2}$$

For phase difference $\theta_1 - \theta_2 = 0, 2\pi, 4\pi, \cdots \quad \Rightarrow \quad \theta_1 - \theta_2 = 2m\pi;\ \ m = 0,1,2, \dots.$

If we assume *a* to be always positive, from the above $a = [a_1^2 + a_2^2 + 2a_1a_2]^{1/2}$

$$\Rightarrow \ a = a_1 + a_2$$

$\Rightarrow$ If the two waves are in phase, two amplitudes add up to form the resultant amplitude:

called <u>constructive interference</u>

For $\theta_1 - \theta_2 = \pi, 3\pi, \cdots \quad \Rightarrow \quad \theta_1 - \theta_2 = (2m+1)\pi;\ \ m = 0,1,2, \dots.$

$$a = a_1 - a_2$$

called <u>destructive interference</u>

Whenever/wherever constructive interference takes place, we have an intensity maximum

Similarly in case of destructive interference, we have an intensity minimum

Whenever two waves superimpose they produce an intensity distribution having max and min

Intensity distribution is called interference pattern

Due to the very process of emission of light waves, interference is difficult to produce with two independent waves

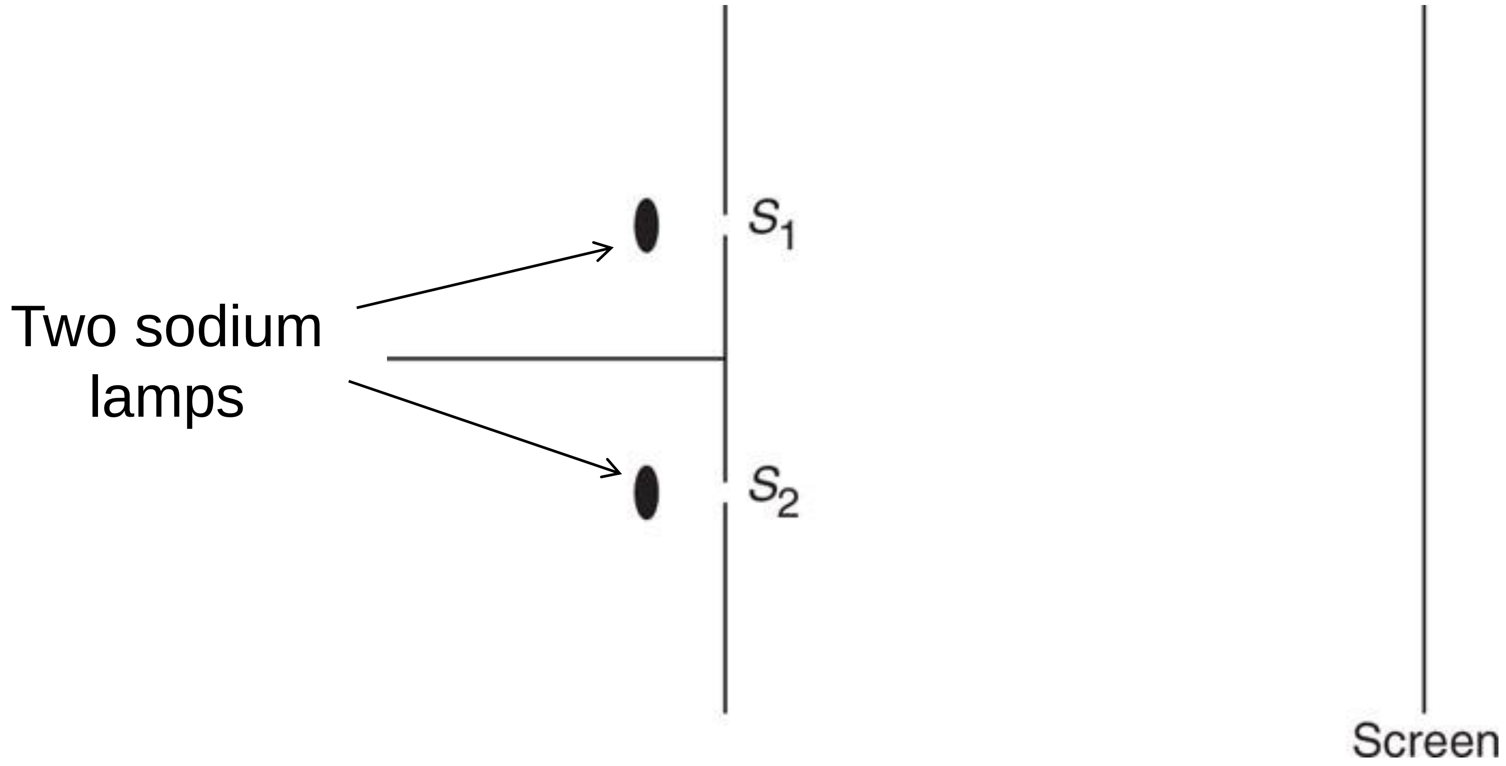
$\Rightarrow$ One derives interfering waves from a single source so as to maintain const phase relationship between the two

Two broad categories exist involving two beams:

1. division of wave front
2. division of amplitude

A third category is possible that involves

3. multiple beam interferometry



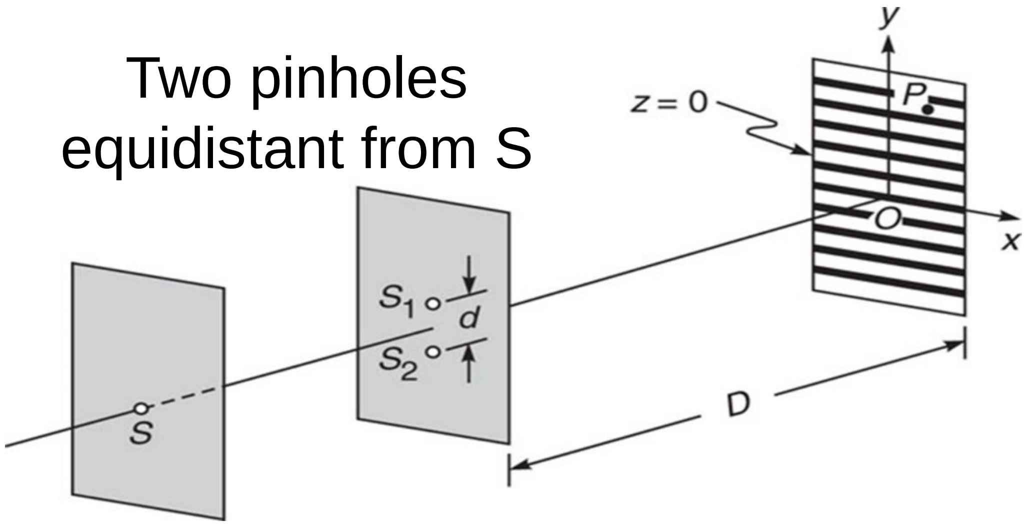
Light from an atom ≡ light pulse of ~ 10^{-10} sec

Human eyes cannot detect intensity changes that lasts for < 1/10 sec

⇒ A uniform intensity will be observed on the screen

Young's double hole experiment in 1801:

Two pinholes equidistant from S



Pinhole