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MA 204: Tutorial Sheet 2

Prob 1) Suppose that r is a double zero of the function f. Thus, $f(r) = f'(r) = 0 \neq f''(r)$. Show that if f'' is continuous, then in Newton's method we shall have $e_{n+1} \approx \frac{1}{2}e_n$ (linear convergence).

Prob 2) Prove that if r is a zero of multiplicity m of the function f, then quadratic convergence in Newton's iteration will be restored by making the following modification:

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}.$$

Prob 3) Consider the equation

$$G(x, y) = 3x^7 + 2y^5 - x^3 + y^3 - 3 = 0,$$

where y is defined implicitly as a function of x. Using Newton's method, produce a table of x versus y. For doing this, start at $x = 0$ and proceed in steps of 0.1 to $x = 1$.

Prob 4) Perform two iterations of Newton's method on these systems:

a) Starting with (0,1)

$$\begin{aligned} 4x_1^2 - x_2^2 &= 0 \\ 4x_1x_2^2 - x_1 &= 0 \end{aligned}$$

a) Starting with (0,0,1)

$$\begin{aligned} xy - z^2 &= 1 \\ xyz - x^2 + y^2 &= 2 \\ e^x - e^y + z &= 3 \end{aligned}$$

Prob 5) Show that the formula for the Secant method can be written in the following mathematical form

$$x_{n+1} = \frac{x_{n-1}f(x_n) - x_nf(x_{n-1})}{f(x_n) - f(x_{n-1})}, (n \geq 1).$$

Explain why this formula is inferior to the original one.

Prob 6) Discuss the rate of convergence for the Secant and Regula-Falsi method.

Prob 7) Solve the following problems using Secant and Regula-Falsi method (at least 3 iterations)

(i) $x^3 - \sinh x + 4x^2 + 6x + 9 = 0.$

(ii) $x^5 + x^3 + 3 = 0$ with $x_0 = -1$ and $x_1 = 1$.

Prob 8) Consider an iteration function of the form

$$F(x) = x + f(x)g(x),$$

where $f(r) = 0$ and $f'(r) \neq 0$. Find the precise conditions on the function g so that the method of functional iteration $x_{n+1} = F(x_n)$ will converge cubically to r if started near r.

Prob 9) Let F be continuously differentiable in an open interval, and suppose that F has a fixed point s in this open interval. Prove that if $| F'(s) | < 1$, then the sequence defined by $x_{n+1} = F(x_n)$ will converge to s if started sufficiently close to s.

Lab Exercises

Ex 1) Write a code for the method proposed in Problem 2 and use it to get an approximate root of the equation $x^3 - 3x^2 + 4 = 0$ near $x = 2$.

Ex 2) Write a code to produce a table of x versus y using Newton's method, where y is defined implicitly as a function of x. Consider the equation

$$x^3 - 2y^2 + y - 2x + 1 = 0.$$

Start at $x = 0$, proceeding in steps of 0.1 to $x = 10$.

Ex 3) Write a code to solve system of nonlinear equations by using Newton's Method and apply it to the problem 4.

Ex 4) Write codes for solving Problem 7 by using Secant and Regula-Falsi methods.