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MA 204: Tutorial Sheet 4

Prob 1) Prove the following facts:

- a) If U is upper triangular and invertible, then U^{-1} is also upper triangular.
- b) The inverse of a unit lower triangular matrix is unit lower triangular.

Prob 2) Prove that if A is invertible and has an LU -decomposition, then all principal minors of A are nonsingular.

Prob 3) Prove or disprove: If a singular matrix has a Doolittle factorization, then that factorization is not unique.

Prob 4) Find the Doolittle, Crout and Cholesky factorizations of the matrix

$$A = \begin{bmatrix} 60 & 30 & 20 \\ 30 & 20 & 15 \\ 20 & 15 & 12 \end{bmatrix}.$$

Prob 5) Solve the following linear system thrice. First, use basic Gauss Elimination and give the factorization $A = LU$. Second, use Gauss Elimination with partial pivoting and finally use Gauss elimination with complete pivoting.

$$\begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1/3 \end{bmatrix}.$$

Prob 6) Assume that $0 < \epsilon < 2^{-22}$. If the Gaussian algorithm without pivoting is used to solve the system

$$\begin{bmatrix} \epsilon & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

on the Marc-32, what will be the solution vector (x_1, x_2) ?

Prob 7) A **Tridiagonal** matrix is characterized by the condition $a_{ij} = 0$ if $|i - j| \geq 2$. Write an algorithm to solve a system $Ax = b$, where the coefficient matrix $A = [a_{ij}]_{n \times n}$ has tridiagonal structure. Further, apply the same on the following system

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 4 \end{bmatrix}.$$

Lab Exercises

- Ex 1) Write a code for getting Doolittle's, Crout's and Cholesky factorization of a nonsingular matrix A for which all leading principal minors are nonsingular. Test the result with the problem 4.
- Ex 2) Write three separate codes for solving system of linear equations. First, by using Gauss Elimination without pivoting. Second, by using Gauss Elimination with Partial pivoting and finally by using Gauss Elimination with Complete pivoting.
- Ex 3) Write a programme to solve a system of linear equation $Ax = b$, where A has a Tridiagonal structure and the matrix A is strictly diagonally dominant, i.e.,

$$|a_{ii}| > \sum_{j=1, j \neq i}^n |a_{ij}| \quad (1 \leq i \leq n).$$

Apply the code to solve the problem 7.