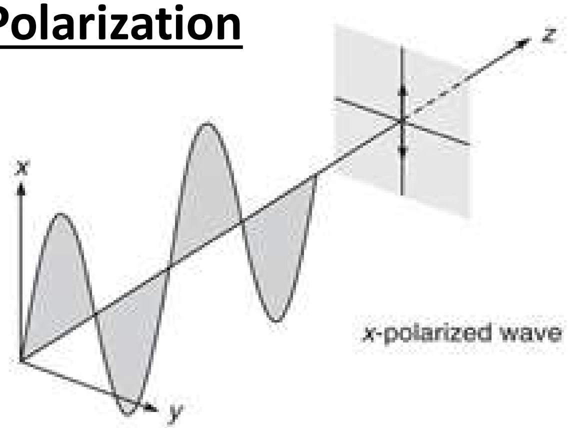
PH203: Optics

Lecture #9

23.11.2018

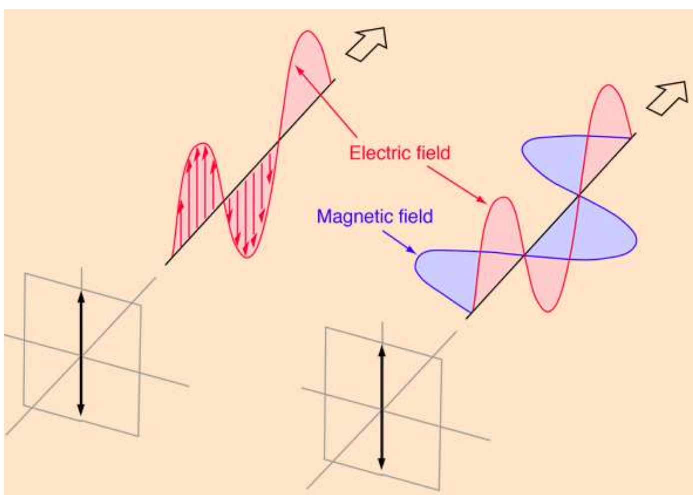
POLARIZATION

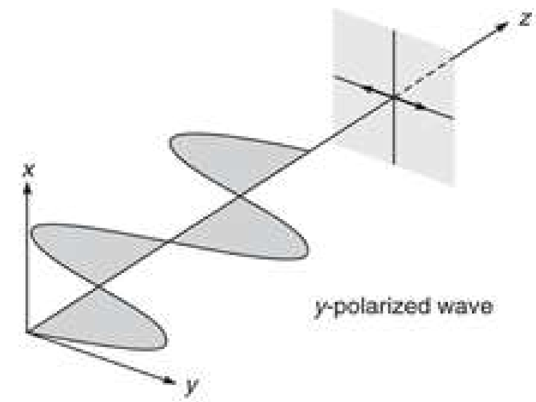
Polarization



The wave is traveling along *z* and is confined in the *xz* plane

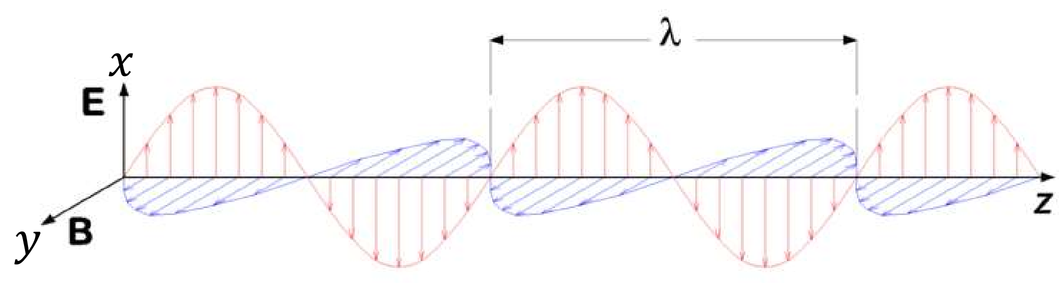
$$\vec{E} = \hat{x}E_0\cos(kz - \omega t + \phi_0);\, k = \frac{2\pi}{\lambda_0}n$$





The wave is traveling along *z* and is confined in the *yz* plane

$$\vec{E} = \hat{y}E_0\cos(kz - \omega t + \phi_0);\, k = \frac{2\pi}{\lambda_0}n$$



Consider an unpolarized wave to be incident on a wire grid polarizer

- unpolarized wave $\Rightarrow$ **E** is vibrating randomly in all directions transverse to the *z* direction

- Component of vibrating **E** along *y* gets absorbed due to Joule heating

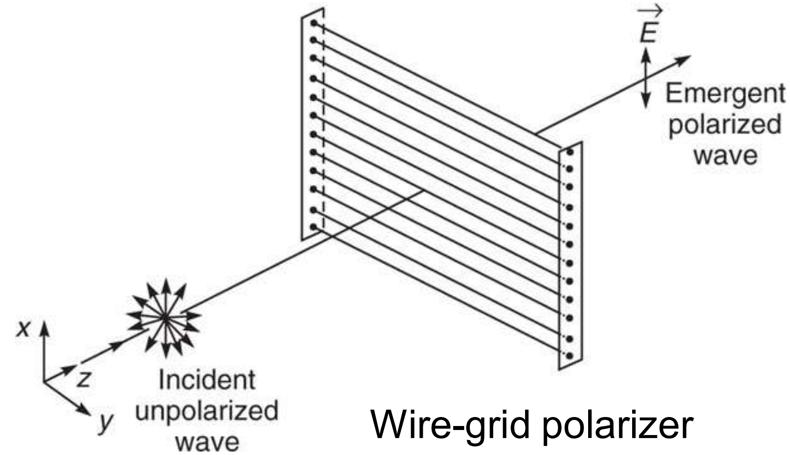
- Assuming the separation between the wires, which are very thin is $\sim \lambda$, *x* component of **E** passes through!
  $\Rightarrow$ the em wave becomes linearly polarized with **E** along $\hat{x}$

- At optical frequencies, since the $\lambda \sim$ is very short sub-micrometer, it is very difficult to fabricate such a wire grid polarizer

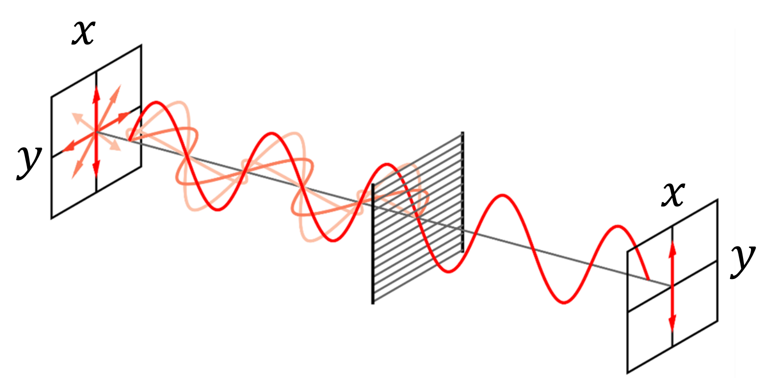
- Such a polarizer can instead be produced through use of long chain polymer molecules containing atoms of e.g. iodine, which can provide high conductivity along the length of the chain

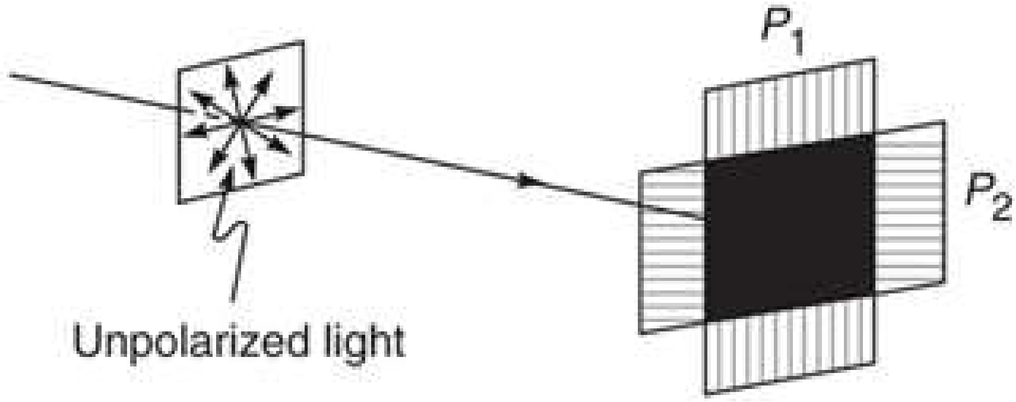
- $\Rightarrow$ Component parallel to the chain of vibrating **E** along *y* gets absorbed due to Joule heating

- Called a **polaroid** sheet



Wire-grid polarizer

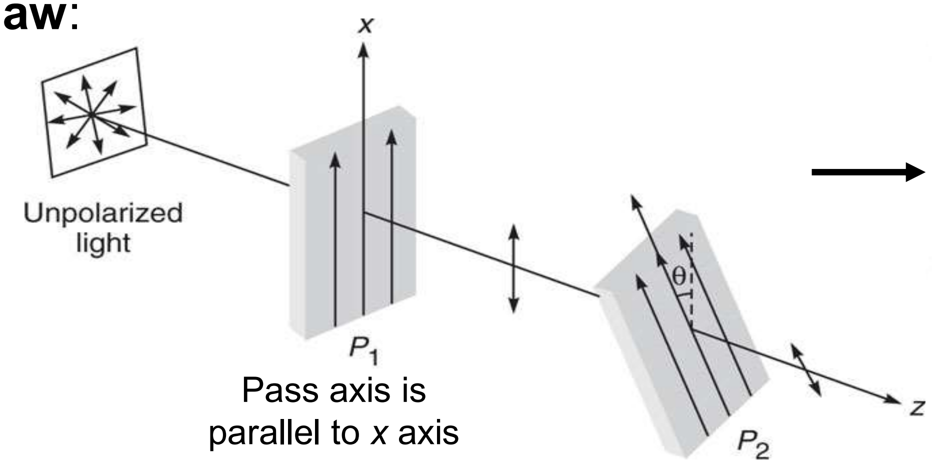


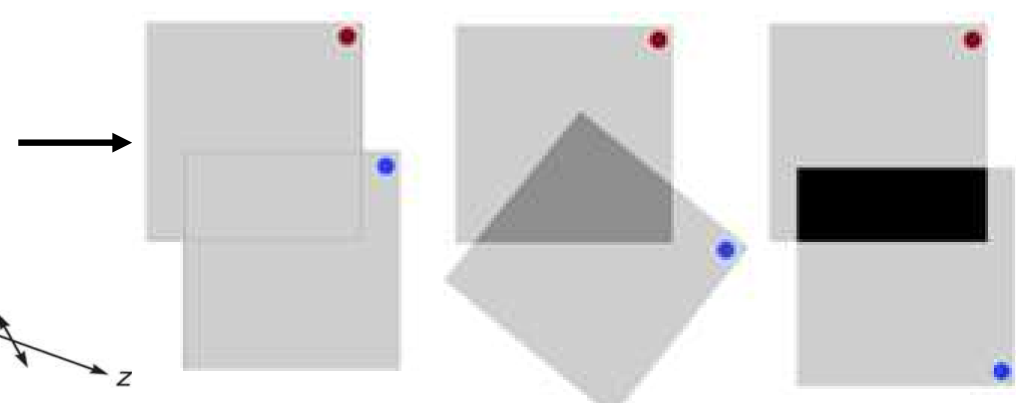


If two polaroids with their pass axis normal to each other are placed against each other, no light will be transmitted and the overlapping portion will appear dark.

In general, the transmitted intensity will depend on orientation of P_2 with respect to P_1

Malus's law:





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If incident electric field is E_0 then the amplitude of the emergent light from polaroid P_2 will be $E_0 \cos\theta \Rightarrow$ corresponding intensity: $I = I_0 \cos^2\theta$; I_0 represents intensity at $\theta = 0$

Consider time variation of the superposition of two waves :

$$\left.\begin{aligned}\vec{E}_1 &= \hat{x} a \cos(kz - \omega t)\\ \vec{E}_2 &= \hat{y} b \cos(kz - \omega t + \theta)\end{aligned}\right\} \quad (1)$$

at an arbitrary plane *z* = 0 perpendicular to the *z* axis i.e. *xy*-plane

Superposition of the two will yield resultant field as $\vec{E} = \vec{E}_1 + \vec{E}_2$

Let us consider x and y components of $\vec{E}$ as $E_{x,y}$ at $z = 0$

$$E_x = a\ \cos(-\omega t) = a\ \cos \omega t$$

$$E_y = b\ \cos(-\omega t + \theta) = b\ \cos(\omega t - \theta)$$

For $\theta = p\pi$; p =0, $\pm$1, $\pm$2,

$$E_y = (-1)^p b \cos \omega t$$

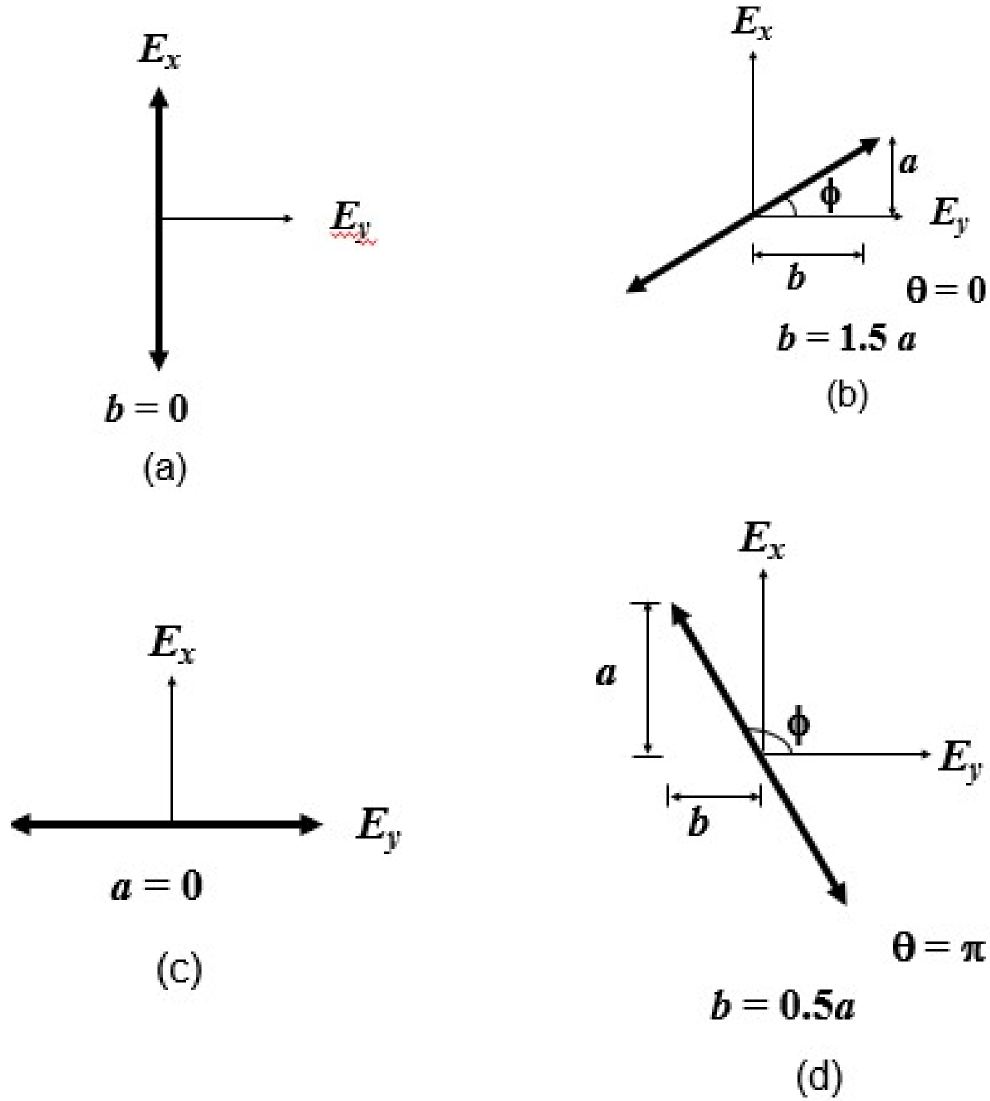
$$\Rightarrow \frac{E_x}{E_y} = \pm \frac{a}{b} (\text{independent of } t) \quad + : p = \text{even (in phase)}\ \ \& - : p = \text{odd (out of phase)}$$

$$\Rightarrow E_x = \pm \underbrace{\frac{a}{b}}_{\text{slope}} E_y \equiv \pm m E_y$$

$\Rightarrow$ Eq of a st line in the E_x- E_y plane making an angle ϕ with the E_y axis

$$\Rightarrow \quad \phi = \tan^{-1}\left(\pm \frac{a}{b}\right)$$

$\Rightarrow$ Superposition of two linearly polarized waves in phase (or π out of phase) with their **E** at right angles to each other is again a linearly polarized wave with its **E** oscillating in a new direction different from either of them



$$E_x = a \cos \omega t$$

$$E_y = b \cos(\omega t - \theta)$$

For $\theta = p\pi$; p =0, ±1, ±2,

$$E_x = \pm \frac{a}{b} E_y$$

$$\phi = \tan^{-1}\left(\pm \frac{a}{b}\right)$$

If $\theta \neq p\,\pi$ in general resultant **E** will not be a st line

$$E_x = a\ \cos\omega t$$
$$E_y = b\ \cos(\omega t - \theta)$$

For $\theta = \pi/2$, and $a = b$:

$$E_x = a\cos\omega t$$
$$E_y = a\sin\omega t \qquad \Rightarrow \qquad E_x^2 + E_y^2 = a^2$$: eq of a circle

At $t = 0,\ \pi/2\omega,\ \pi/\omega,\ 3\pi/2\omega$

i) $E_x = a\,;\ E_y = 0$

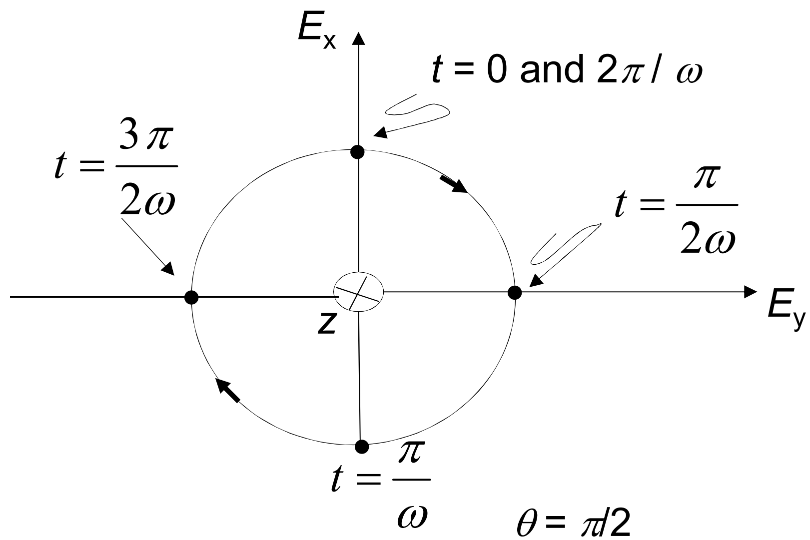
ii) $E_x = 0\,;\ E_y = a$

iii) $E_x = -a\,;\ E_y = 0$

iv) $E_x = 0\,;\ E_y = -a$

Again at $t = 2\pi/\omega,\ 4\pi/\omega,\ 6\pi/\omega$

v) $E_x = a\,;\ E_y = 0$



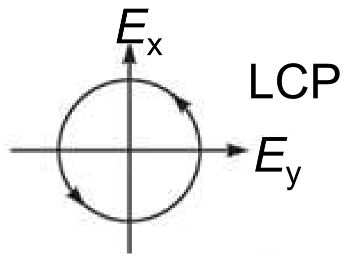
RCP

$$E_x = a\cos\omega t$$
$$E_y = b\cos\left(-\omega t + \theta\right)$$

$$E_x = a\ \cos\omega t$$
$$E_y = b\ \cos(\omega t - \theta)$$

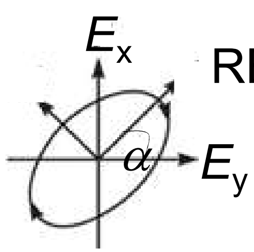
For θ = 3π / 2 and b = a :

$$E_{\mathrm{x}} = a\cos\omega t$$
$$E_{\mathrm{y}} = -a\sin\omega t$$



For θ = π/3 and b = a :

$$E_{\mathrm{x}} = a\cos\omega t$$
$$E_{\mathrm{y}} = a\cos\left(\omega t - \frac{\pi}{3}\right)$$



θ = π / 3

At t = 0, π /3ω, π /2ω, π /ω

i) $E_x = a\,;\, E_y = \dfrac{a}{2}$

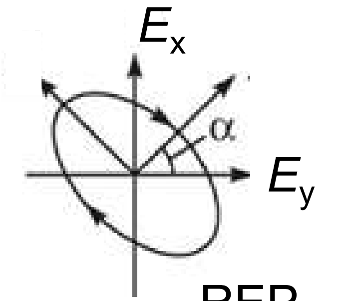
ii) $E_x = \dfrac{a}{2}\,;\, E_y = a$

iii) $E_x = 0\,;\, E_y = \dfrac{\sqrt{3}}{2}a$

iv) $E_x = -a\,;\, E_y = -\dfrac{1}{2}a$

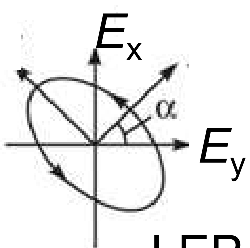
Tip of the **E** rotates cw on the circumference of an ellipse $\Rightarrow$ REP

For θ = $2\pi/3$ and $b = a$:



REP

For θ = $4\pi/3$ and $b = a$:



LEP

Find SOP of

$$E_x = E_0 \sin\left(kz - \omega t + \frac{\pi}{4}\right); \quad E_y = \frac{E_0}{\sqrt{2}} \sin(kz - \omega t)$$

At $t = 0$, $E_x = \frac{E_0}{\sqrt{2}}$; $E_y = 0$

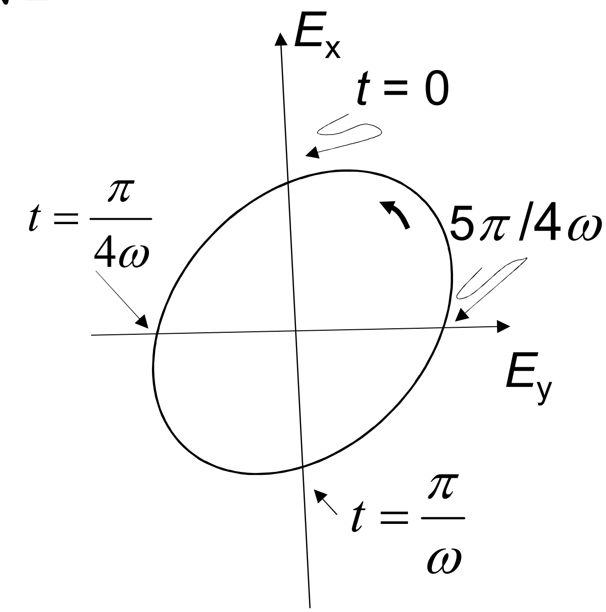
At $t = \pi/4\omega$, $E_x = 0$; $E_y = -\frac{E_0}{2}$

At $t = \pi/2\omega$, $E_x = -\frac{E_0}{\sqrt{2}}$; $E_y = -\frac{E_0}{\sqrt{2}}$

At $t = 3\pi/4\omega$, $E_x = -E_0$; $E_y = -\frac{E_0}{2}$

At $t = \pi/\omega$, $E_x = -\frac{E_0}{\sqrt{2}}$; $E_y = 0$

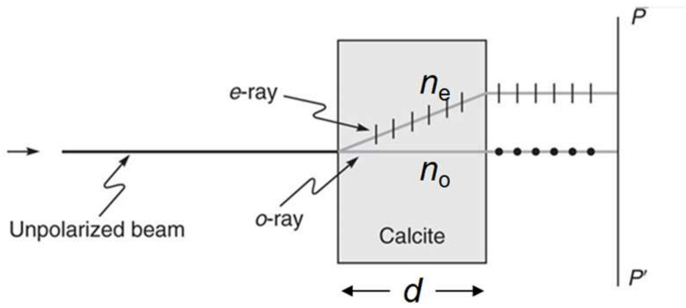
At $t = 5\pi/4\omega$, $E_x = 0$; $E_y = \frac{E_0}{2}$



LEP

Polarization by double refraction:

Anisotropic medium e.g. calcite, quartz are characterized by two characteristic refractive indices depending on direction of propagation of the em wave



- Ordinary ray/wave follows Snell's law
- Extra-ordinary ray/wave does not follow Snell's law

- $n_e < n_o$: negative crystal
- $n_e > n_o$: positive crystal

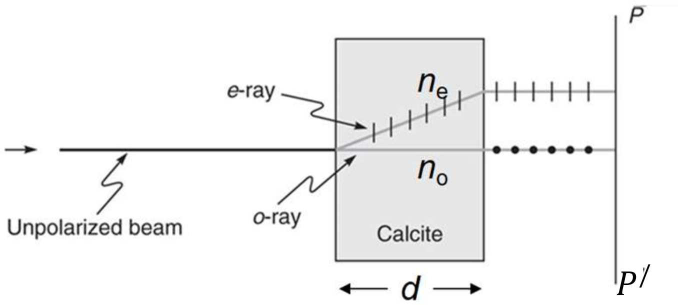
For a thickness *d* of an anisotropic crystal, phase diff between *o*-ray and *e*-ray is: $k_0\left(n_e - n_o\right)d$

- A quarter wave plate (QWP) based on an anisotropic medium, introduces a phase difference of $\pi/2$ between the *o*-ray and *e*-ray

$$\Rightarrow \frac{2\cancel{\pi}}{\lambda_0}\left(n_e - n_o\right) d_{\mathrm{QWP}} = \frac{\cancel{\pi}}{2} \qquad \Rightarrow \left(n_e - n_o\right) d_{\mathrm{QWP}} = \frac{\lambda_0}{4}$$

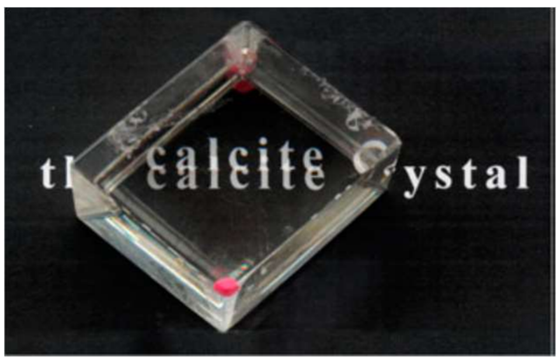
- A half wave plate (HWP) based on an anisotropic medium, introduces a phase difference of π between the *o*-ray and *e*-ray such that

$$\frac{2\cancel{\pi}}{\lambda_0}\left(n_e - n_o\right) d_{\mathrm{HWP}} = \cancel{\pi} \quad \Rightarrow \left(n_e - n_o\right) d_{\mathrm{HWP}} = \frac{\lambda_0}{2}$$



If a polaroid $PP^/$ is kept at the position shown with its pass axis perpendicular to the plane of the paper, e-ray will be blocked and o-ray will pass through

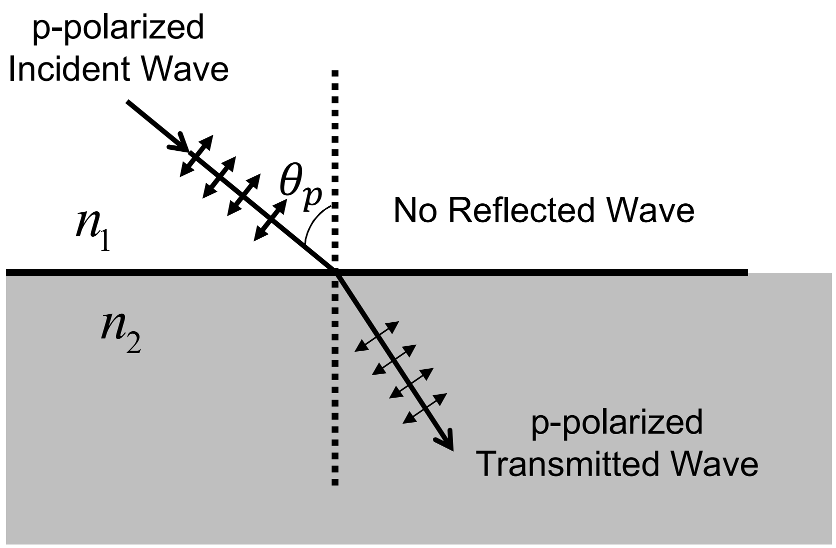
If a polaroid $PP^/$ is kept at the position shown with its pass axis parallel to the plane of the paper, o-ray will be blocked and e-ray will pass through

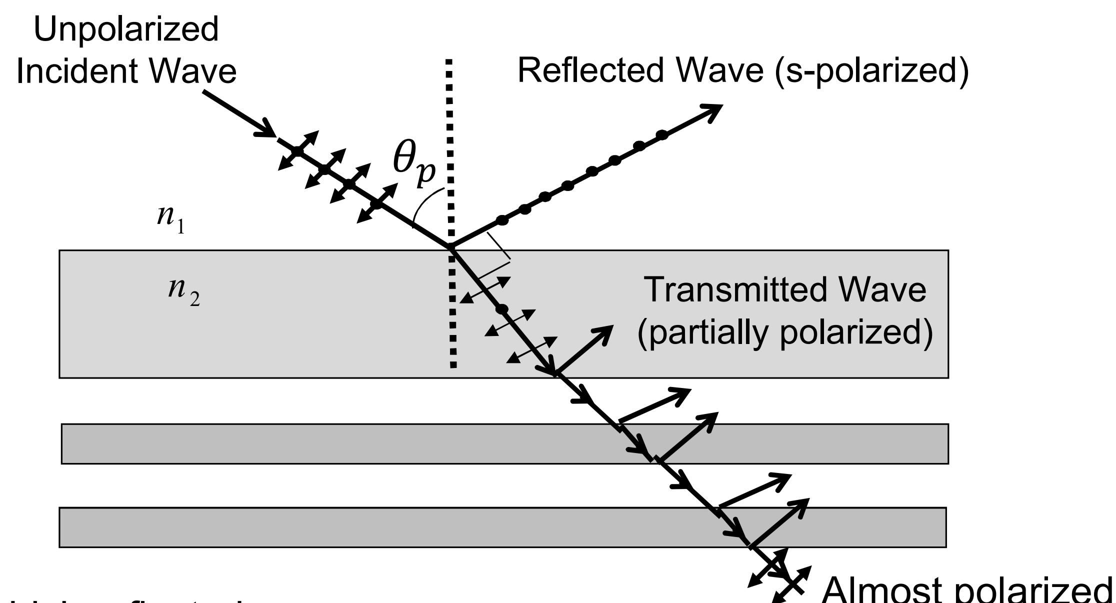


Polarization by reflection:

For an angle of incidence, $\theta_p = \tan^{-1}\dfrac{n_2}{n_1}$

Reflection coefficient is 0



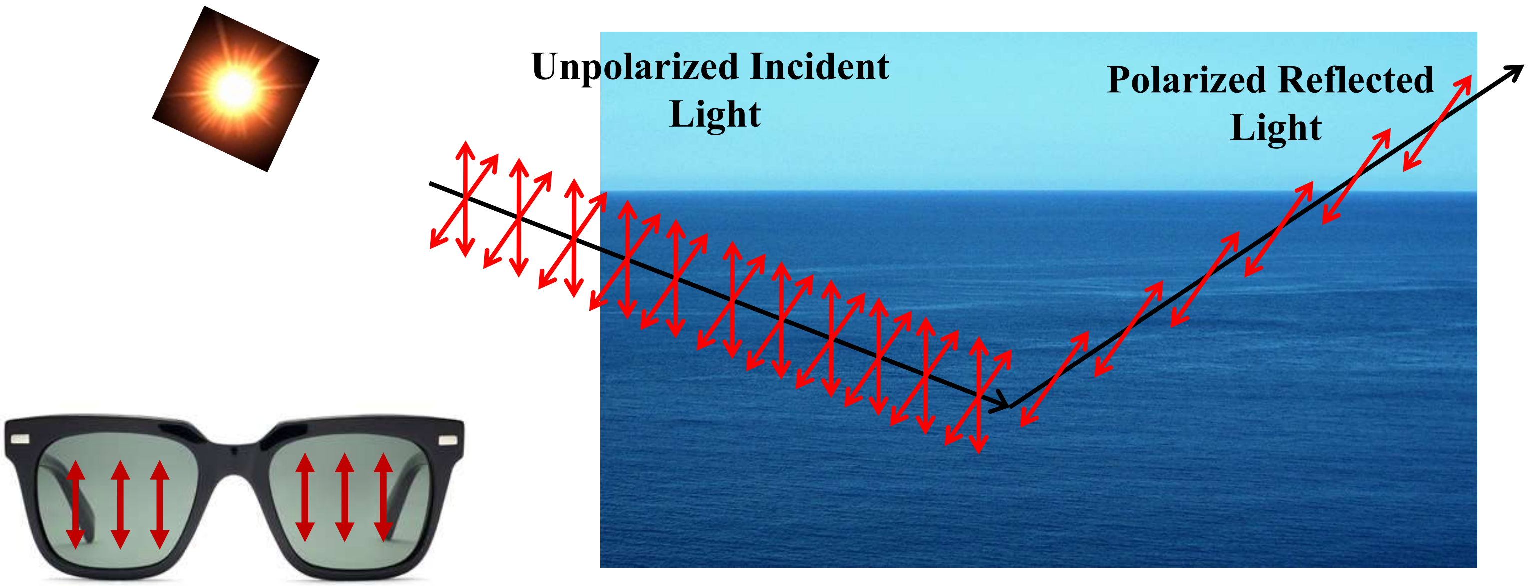


$\tan\theta_p = \frac{n_2}{n_1}$: Brewster law

θ_p is called polarization angle or Brewster angle for which reflected and transmitted waves are perpendicular to each other

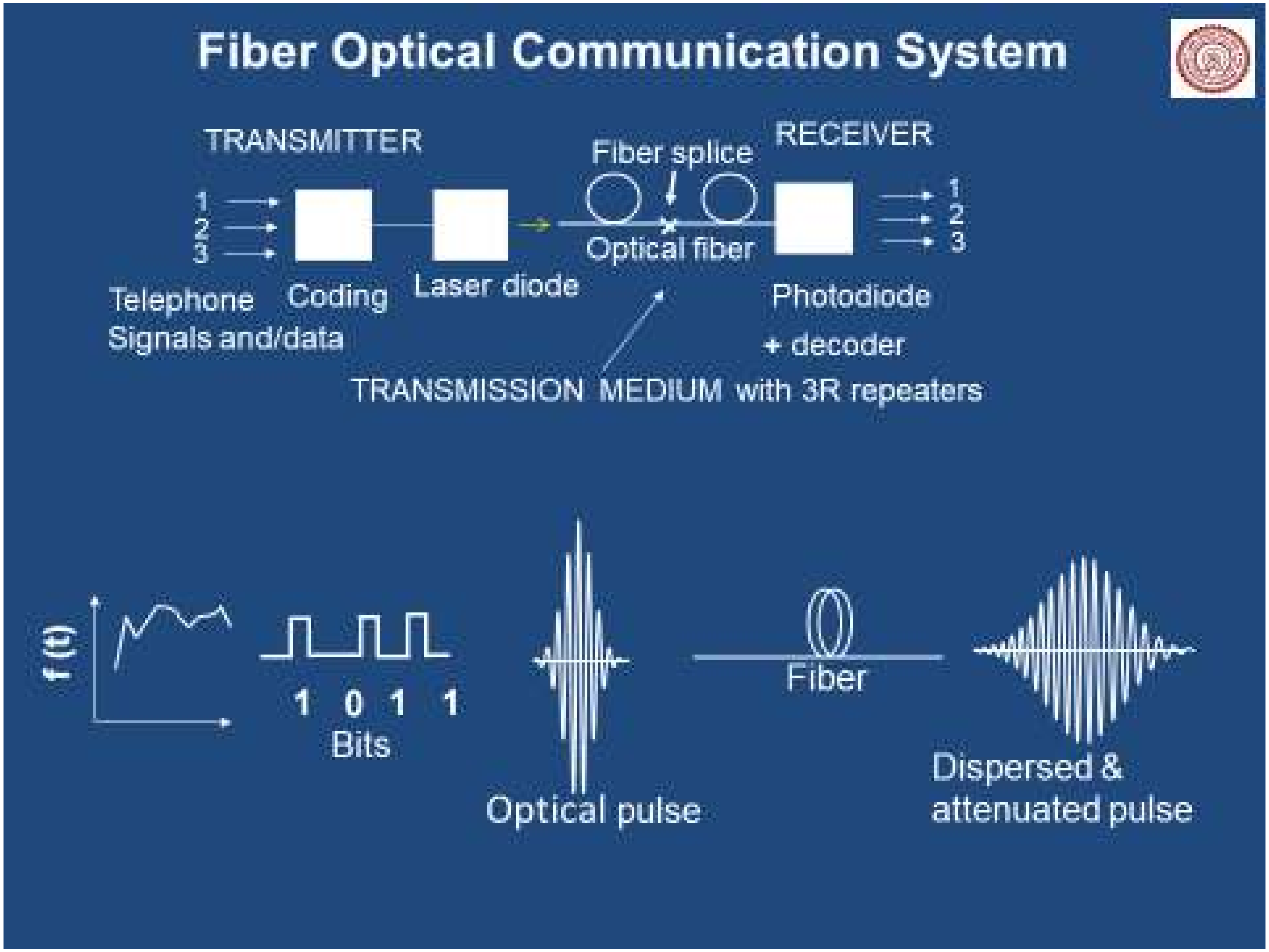
For $n_1 = 1, n_2 = 1.33, \Rightarrow \theta_p \approx 53°$

For $n_2 = 1.5, \theta_p \approx 56°$



(a) (b)

A (commercially available) polarized sunglass blocks the horizontal component and allows only the vertical component to pass through. (b) If the sunlight is incident on the water surface at an angle close to the Brewster angle, then the reflected light will be almost polarized and if we now wear polarized sunglasses, the glare, i.e., the light reflected from the water surface will not be seen. Polarized sunglasses are often used by fishermen to remove the glare on the surface and see the fish inside water.


Fiber Optical Communication System
TRANSMITTER
Fiber splice
RECEIVER
1
2
3
Optical fiber
1
2
3
Telephone
Signals and/data
Coding
Laser diode
Photodiode
+ decoder
TRANSMISSION MEDIUM with 3R repeaters
f (t)
1 0 1 1
Bits
Optical pulse
Fiber
Dispersed &
attenuated pulse

Light-guiding conduit

Jean-Daniel Colladon
(1802-1893)

Demonstrated optical confinement of light in a jet stream of water in 1841 during a course in Mechanics

"...once entering the stream [the light rays] encounter its surface under an angle small enough to experience a total internal reflection; the same effect repeats at each new point of incidence, such that the light circulates in the transparent jet like in a canal, and follows all the turns." (Comptes Rendus, 15, 800-802 Oct. 24, 1842)

"…one of the most beautiful, and most curious experiments that one can perform in a course on optics." (Comptes Rendus, 15, 800-802 Oct. 24, 1842)

….and 167 years later

The 2009 Nobel Prize in Physics

Charles K. Kao

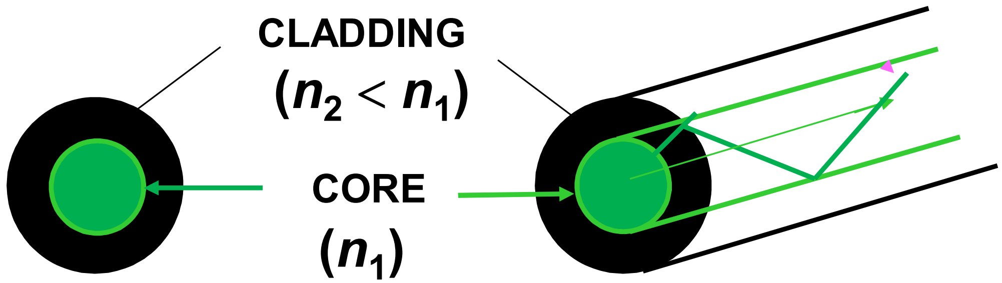
"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"

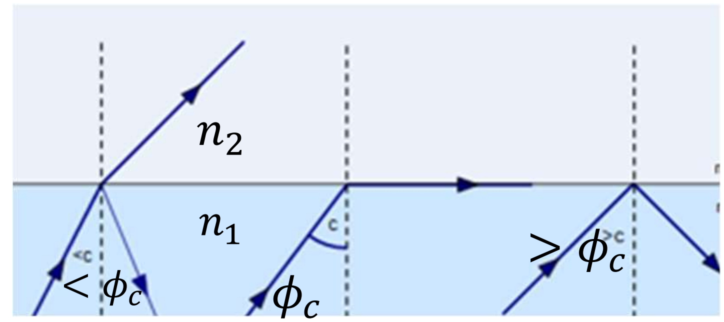
- **Nobel Foundation Press release:**

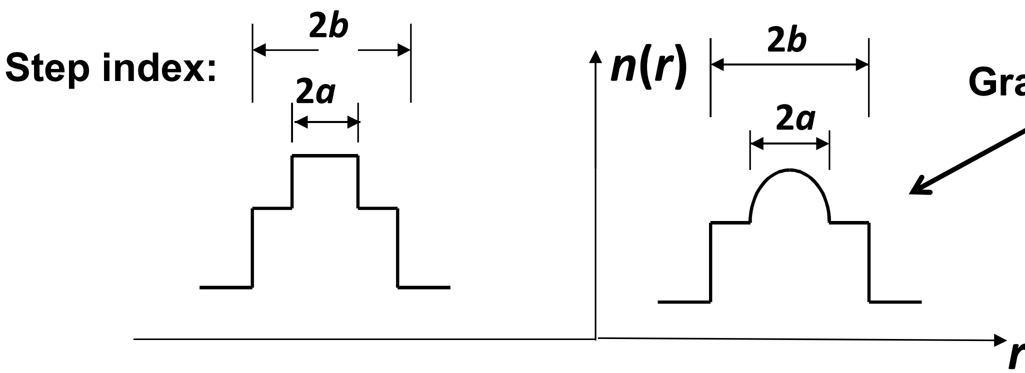
"If we were to unravel all of the glass fibers that wind around the globe, we would get a single thread over one billion kilometers long, which is enough to encircle the globe more than 25,000 times…

….and is increasing by thousands of kilometers every hour"

Optical Fiber Geometry







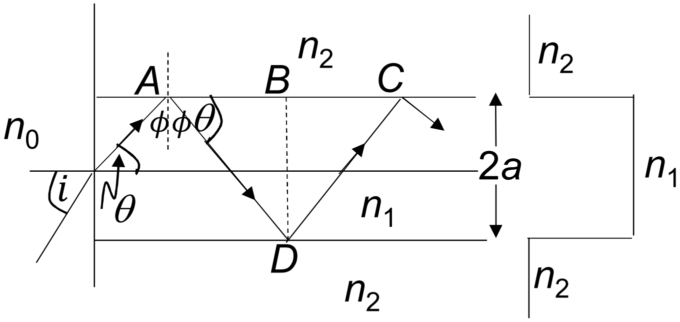
Graded index:

$$n^2(r) = n_1^2\left[1 - 2\Delta\left(\frac{r}{a}\right)^2\right]; \quad r < a$$

$$n_2^2 = n_1^2[1 - 2\Delta]; \quad r \geq a$$

- **Multimode fiber: 2*a* = 50 μm (Telecom)**
  = 62.5 μm (for LANs)
- **Single-mode fiber: 2*a* = 8 ~ 10 μm,**
  [@ λ ~ 1310 nm to 1550 nm]

2*b* = 125 ± 1 μm



From the figure, $$\frac{\sin i}{\sin\theta} = \frac{n_i}{n_0} \quad \Rightarrow n_0 \sin i = n_1 \sin\theta$$

Naturally, for TIR,

$$\sin\phi\left[= \sin\left({}^{\pi}/_{2} - \theta\right)\right] > \frac{n_2}{n_1}$$

$$\Rightarrow \quad \cos\theta > \frac{n_2}{n_1}$$

$$\Rightarrow \quad \sqrt{1-\sin^2\theta} > \frac{n_2}{n_1} \quad \Rightarrow \; 1 - \sin^2\theta > \frac{n_2^2}{n_1^2}$$

$$\Rightarrow \quad \sin\theta < \left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2} \quad \Rightarrow \; n_0 \frac{\sin i}{n_1} < \left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2} \quad \Rightarrow \; \sin i < \frac{n_1}{n_0}\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^{1/2}$$

or

$$\sin i < \left(\cancel{n_1^2}\,\frac{n_1^2 - n_2^2}{n_0^2 \times \cancel{n_1^2}}\right)^{\frac{1}{2}} \quad \Rightarrow \; n_0 \sin i < (n_1^2 - n_2^2)^{1/2}$$

Since $n_0 = 1$ $\Rightarrow$ For a ray to be guided through the fiber $\sin i_{max} = (n_1^2 - n_2^2)^{\frac{1}{2}} = \mathrm{NA}$

Typical value of NA of a multimode fiber: ~ 0.2 and of single-mode fiber : ~ 0.1

Loss/attenuation:

Basic raw material used in making an optical fiber is SiO_2

Core of an optical fiber is typically doped with GeO_2 by adding a small percentage of GeO_2 to SiO_2 to increase r.i. of SiO_2 up to ~ 1% over that of the cladding, which is SiO_2

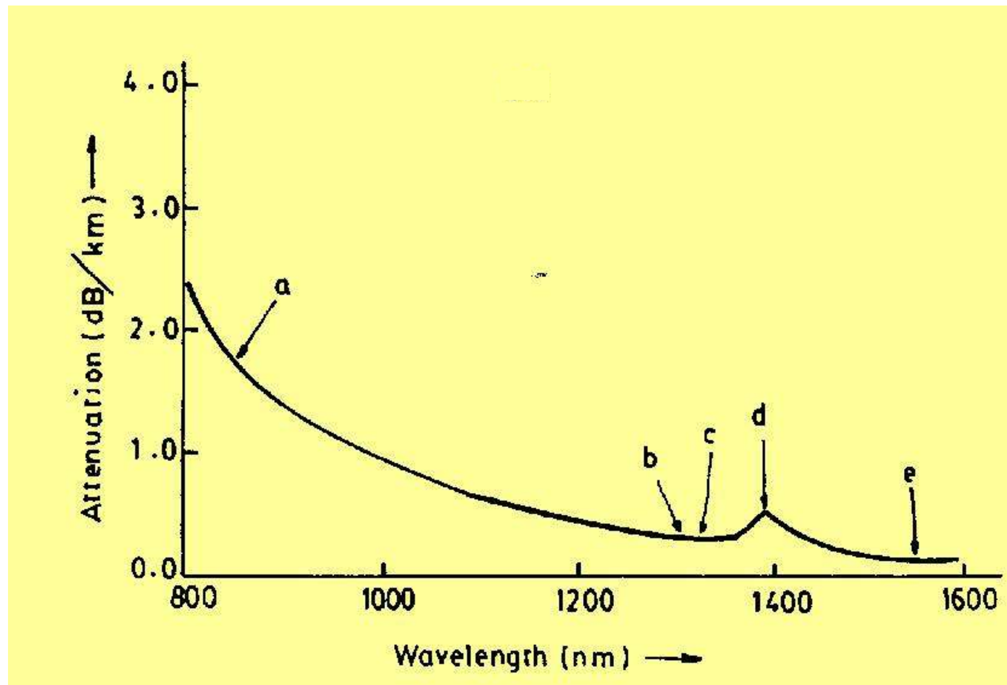
Loss is defined through

$$\mathrm{dB/km} = \left(10\log_{10}\frac{P_1}{P_2}\right)/L(\text{in km})$$

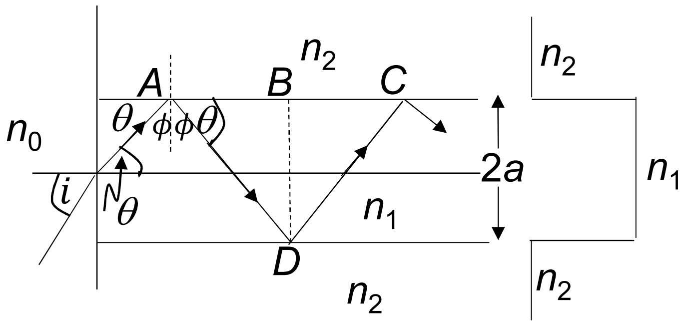
$$\Rightarrow 0.2 = 10\log_{10}\frac{1}{P_{out}} \;\Rightarrow \frac{1}{P_{out}} = 10^{0.02}$$

$$\Rightarrow \quad P_{out} = \frac{1}{10^{0.02}} = 0.956$$

$\Rightarrow$ Approx. 96% of the input power will be available after propagation through 1 km of the fiber.



Dispersion in a step index fiber:



Time taken to cover the distance *AC*:

$$t_{AC} = \frac{AD + DC}{c/n_1} = \frac{\frac{AB}{\cos\theta} + \frac{BC}{\cos\theta}}{c/n_1} = \frac{\frac{AC}{\cos\theta}}{c/n_1} = \frac{n_1 AC}{c\cos\theta}$$

$\Rightarrow$ Time taken to cover length *L* of the fiber: $t_L = \dfrac{n_1 L}{c\cos\theta}$

$\Rightarrow t$ depends on ray launch angle

If all rays within the NA of the fiber is launched simultaneously, time difference between extreme rays to cover length *L*:

$$t_{max} = \frac{n_1 L}{c\cos(\pi/2 - \phi_c)} = \frac{n_1 L}{c\sin\phi_c} = \frac{n_1 L}{c\sin\sin^{-1}\frac{n_2}{n_1}} = \frac{n_1^2 L}{c n_2}$$

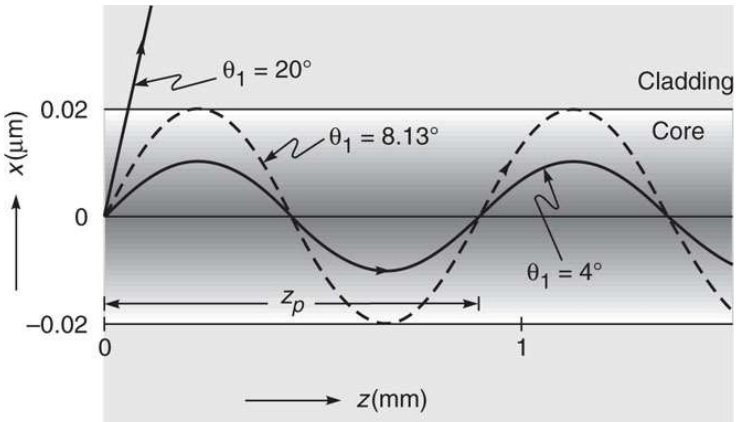
$$t_{min} = \frac{n_1 L}{c}$$

$$\Rightarrow t_{max} - t_{min} = \frac{n_1 L}{c}\left[\frac{n_1}{n_2} - 1\right] = \frac{n_1 L}{c}\left(\frac{n_1 - n_2}{n_2}\right) = \frac{n_1 L}{c}\Delta$$

This time difference leads t broadening of a temporal pulse as it propagates through a fiber

This phenomenon limits pulse transmission rate

Dispersion in a graded index fiber:



Rays travelling at an angle relative to the axial ray travels at a region whose r.i. is lower than the axal region and hence will travel faster rel to the axial ray

$\Rightarrow$ Leads to effective compensation of the time difference and hence less broadening of a temporal pulse