PH202: Optics

Lecture #7

21.11.2018

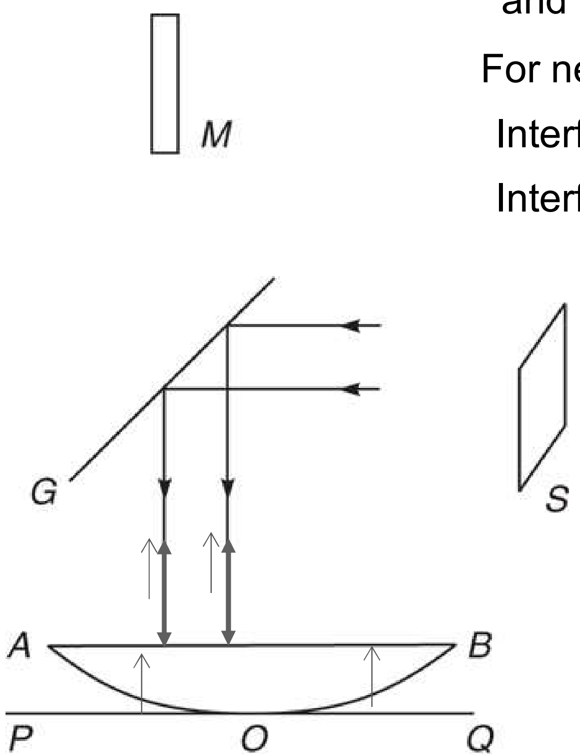
Newton's rings

A thin air film of r.i. (n = 1) of variable thickness (t) is entrapped between the lens and the glass plate: t is 0 at the point of contact O and increases away from O

For near-normal incidence, and for points close to O, opt. path difference $\approx 2nt$

Interference takes place between light reflected from *AOB* and *POQ*

Interfering light which is reflected from *POQ* accumulates an additional phase of π



$\therefore$ for maxima: $2t = \left(m + \frac{1}{2}\right)\lambda; m = 0,1,2, \ldots$

for minima: $2t = m\lambda; m = 1,2, \ldots$

Due to the spherical surface of the lens, t will be const over a circle with O as its center $\Rightarrow$ we will get concentric dark and bright fringes in the form of rings

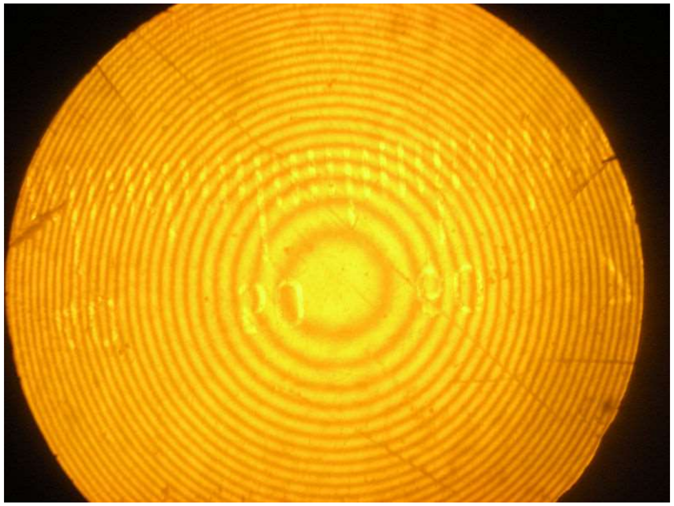
M : A travelling microscope

AOB : A plano-convex lens

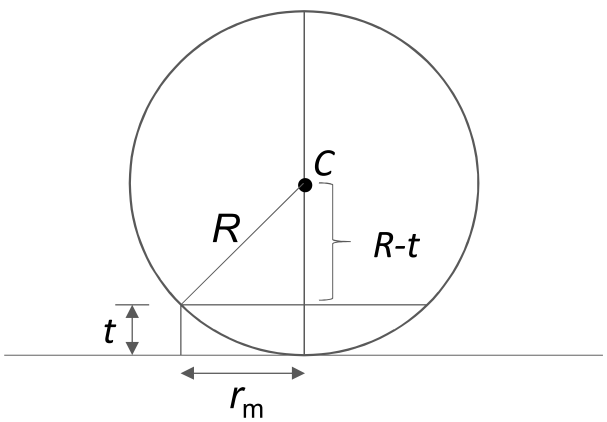
POQ : A plane glass plate

S : An extended light source

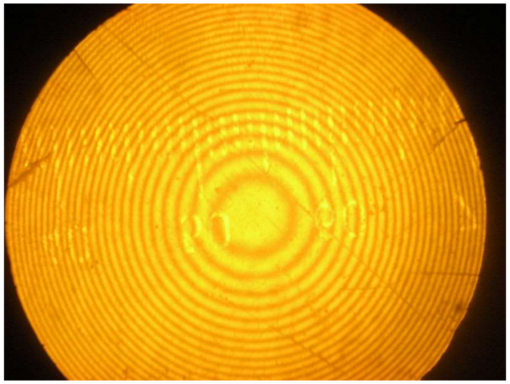
G : A glass plate beam splitter



Radius r_m of the m^{th} dark ring:



From the figure



$$(R-t)^2 + r_m^2 = R^2$$

$$\Rightarrow \quad \cancel{R^2} \cong r_m^2 + \cancel{R^2} - 2tR + t^2$$

$$\Rightarrow \quad r_m^2 \cong t(2R-t)$$

Typically, R ~100 cm; $t \sim 10^{-3}$ cm $\Rightarrow$ t can be neglected rel to $2R$

$$\Rightarrow \quad 2t = \frac{r_m^2}{R} = m\lambda \quad \Rightarrow r_m = \sqrt{mR\lambda}\,; m = 1,2,\ldots$$ (for m^{th} dark ring)

$\Rightarrow$ Radii of the dark rings vary as sq root of natural numbers

$\Rightarrow$ Rings will become close to each other as the radius increases

In expt, diameters of m^{th} and $(m + p)^{th}$ rings are measured (p ~ 10) and from the following relation source wavelength λ is measured:

$$\left(D_{m+p}\right)^2 - (D_m)^2 = 4(\cancel{m} + p - \cancel{m})R\lambda$$

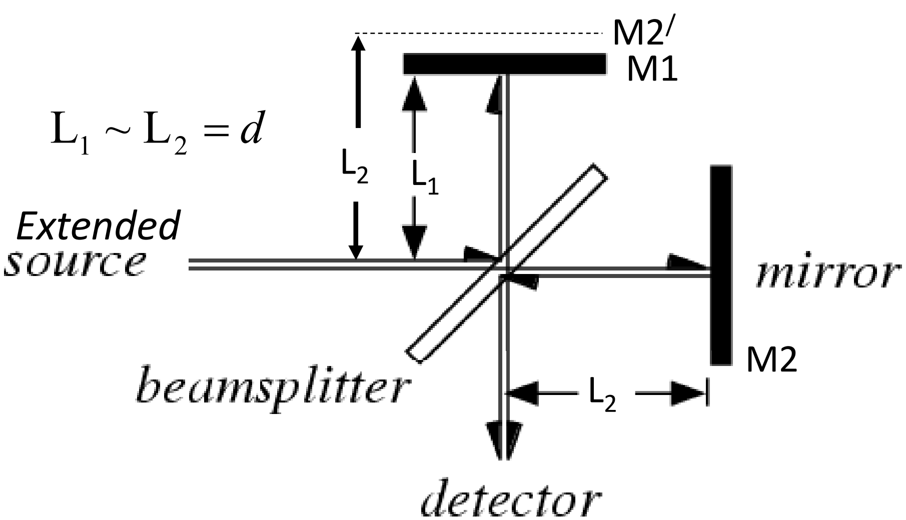
$$\Rightarrow \quad \lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

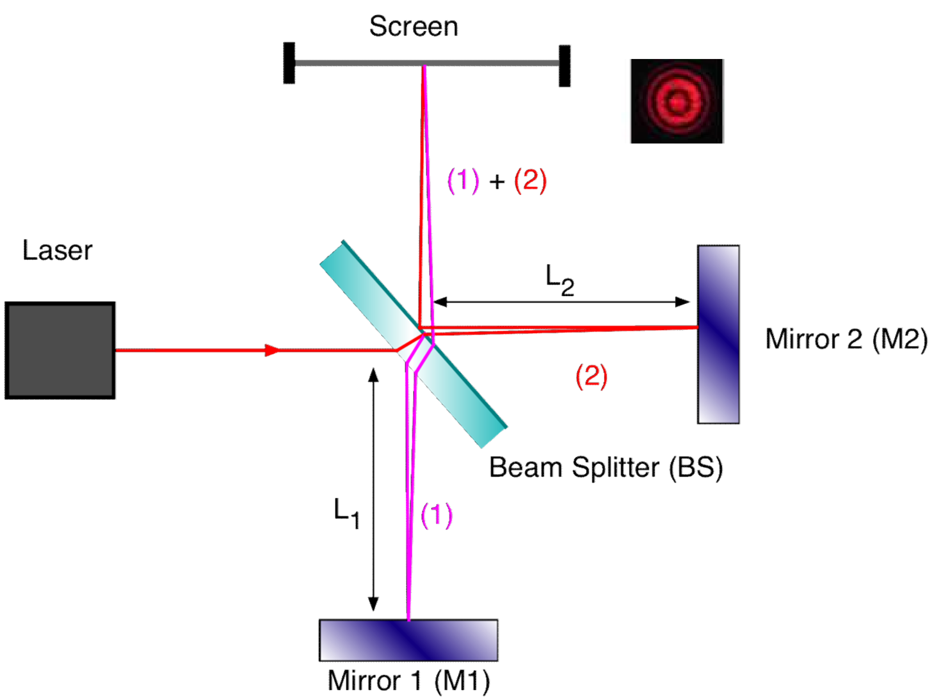
Michelson interferometer

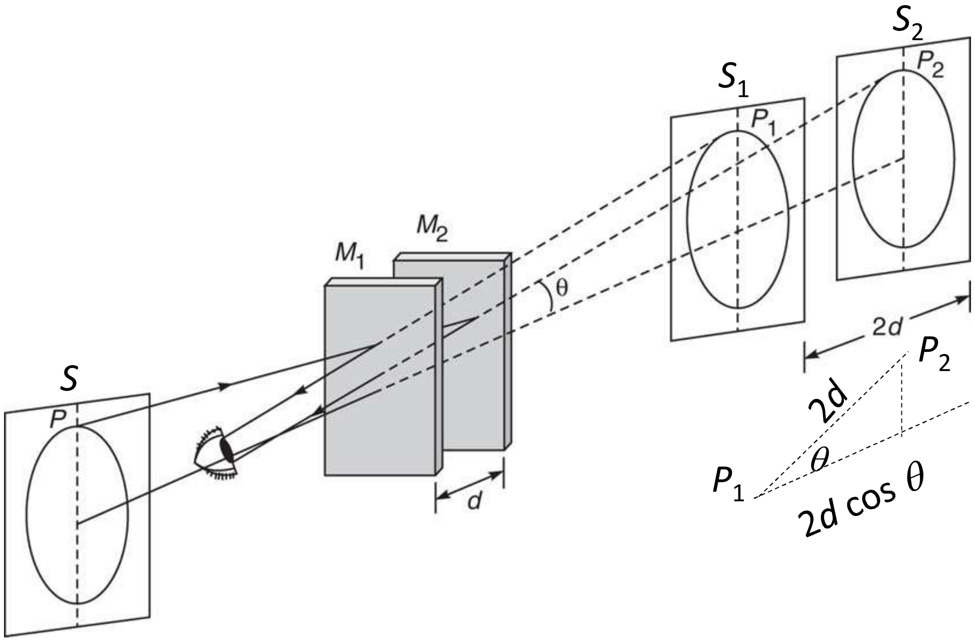
"Following a method suggested by Fizeau in 1868, Professor Michelson has produced what is perhaps the most ingenious and sensational instrument in the service of astronomy – the Interferometer"

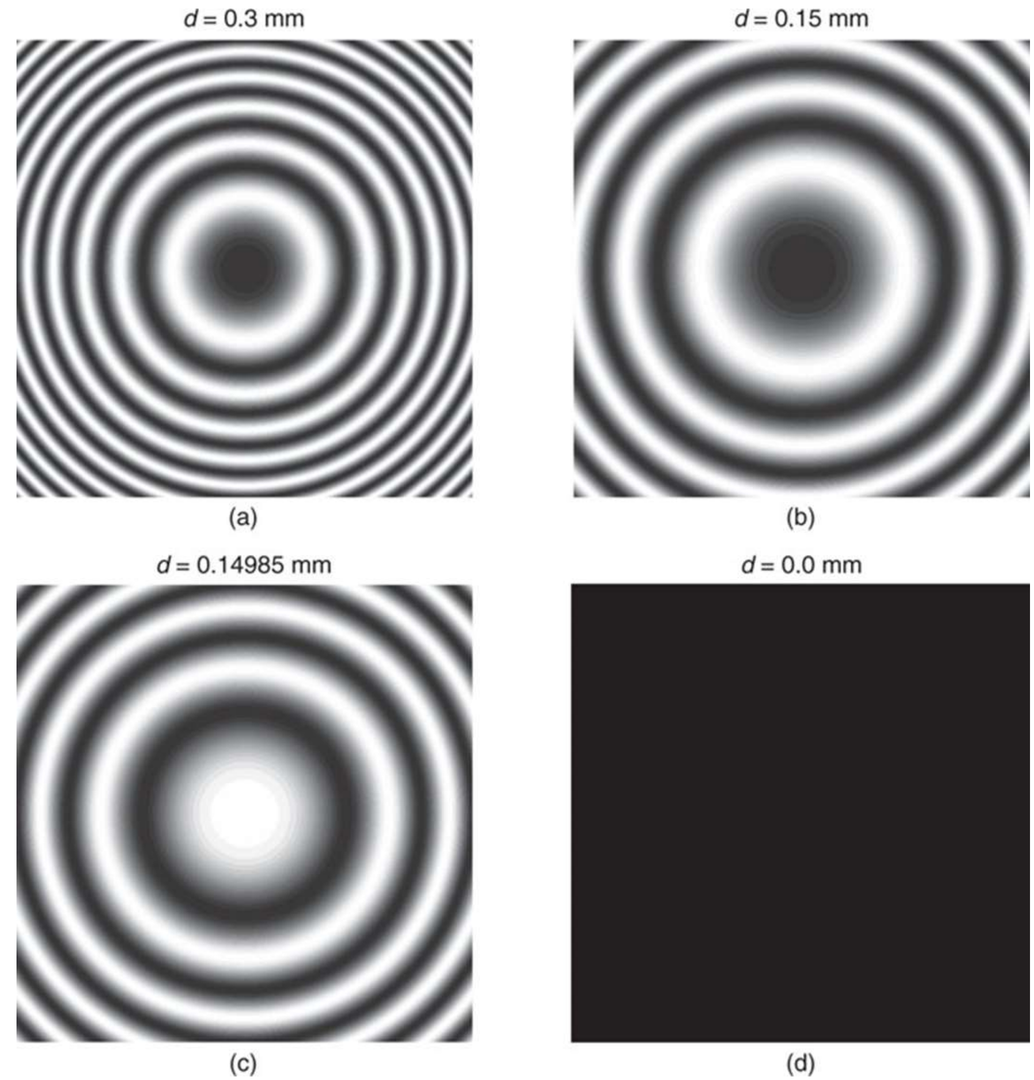
- Sir James Jeans in "The Universe Around us", Cambridge Univ Press (1930)

$$L_1 \sim L_2 = d$$





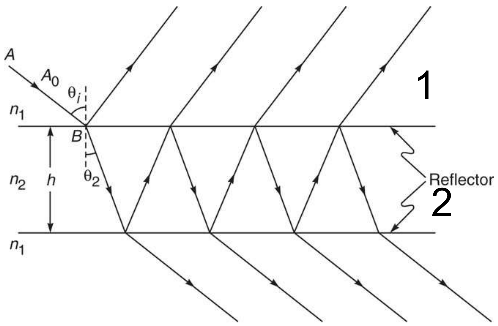




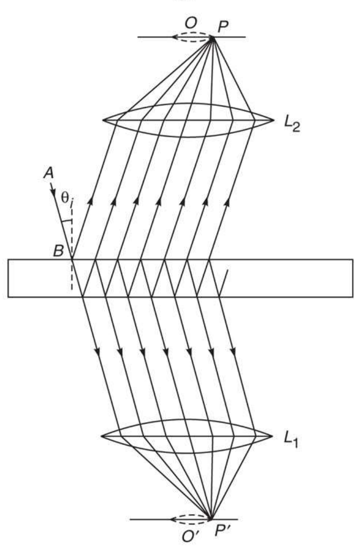
Computer generated interference patterns from a Michelson interferometer for different *d*'s

Multiple beam interferometry

A third variety of interferometry involves multiple beams derived from same source through division of amplitude by multiple reflections



(a)



(b)

Consider incidence of a plane wave of amplitude A_0 from medium 1 of r.i. n_1 at an angle θ_i on a glass plate of thickness h and of r.i. n_2

If r_1 and t_1 represent reflection and transmission coeffs of the plate as incident light from medium 1 enters medium 2, then amps of successive reflected waves will be

$$A_0 r_1, A_0 t_1 r_2 t_2 e^{i\delta}, A_0 t_1 r_2 r_2 r_2\ t_2 e^{2i\delta}, \ldots..$$

where

$$\delta = \frac{2\pi}{\lambda_0} \times \Delta = \frac{2\pi}{\lambda_0}(2h\, n_2 \cos\theta_2) = \frac{4\pi h\ n_2 \cos\theta_2}{\lambda_0}$$

$\Rightarrow$ Resultant amplitude of the reflected waves from the interface between the medium 1 and medium 2:

$$A_r = A_0\left(r_1 + t_1 t_2 r_2 e^{i\delta}\left[1 + r_2^2 e^{i\delta} + r_2^4 e^{2i\delta} + \ \ldots.\right]\right) = A_0\left(r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}\right)$$

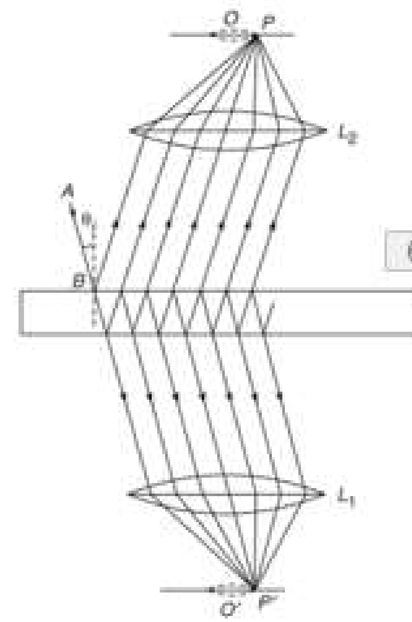
$\Rightarrow$ Net reflectivity $\mathcal{R}$:

$$\left|\frac{A_r}{A_0}\right|^2 = R\left|\frac{1-e^{i\delta}}{1-Re^{i\delta}}\right|^2; \quad R = r_1^2 = r_2^2$$

$\mathcal{R}$ can be shown to be :

$$\mathcal{R} = \frac{F\sin^2\frac{\delta}{2}}{1+F\sin^2\frac{\delta}{2}} \quad \text{where} \quad F = \frac{4R}{(1-R)^2}$$

F is called coefficient of finesse



Similarly, amplitude of successive transmitted waves (assuming zero phase for the 1st transmitted wave):

$$A_0t_1t_2, A_0t_1t_2r_2^2e^{i\delta}, \ldots\ldots$$

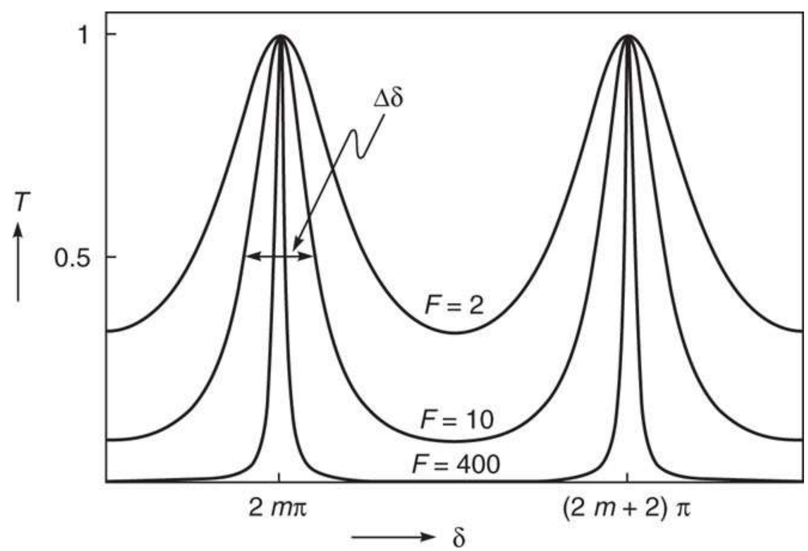
It can be shown that transmittivity T of the film/glass plate will be given by $T = \left|\frac{A_t}{A_0}\right|^2 = \frac{1}{1+F\ \sin^2\frac{\delta}{2}}$

Also for

$$\delta = 2m\pi, m = 1,2,3, \ldots$$

$$T = \frac{1}{1 + F \sin^2 m\pi} = 1$$

transmittivity T as a function of δ for different F is plotted in the figure



⇒ Transmission resonances become sharper with increase in F

These results are useful in designing resonators used to construct lasers

DIFFRACTION

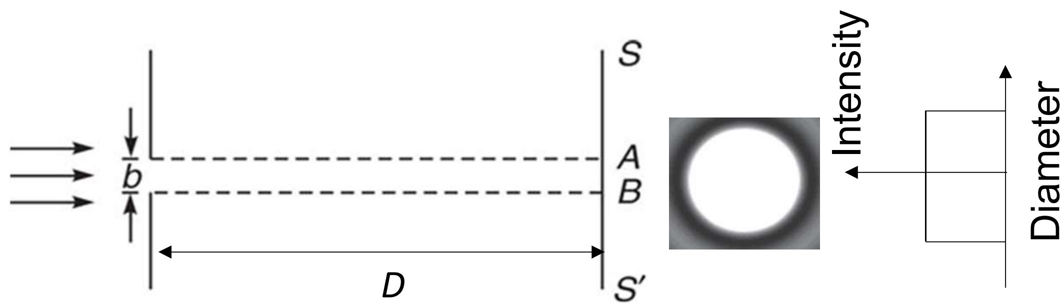
DIFFRACTION

"No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them. The best we can do is, roughly speaking, is to say that when there are only a few sources, say two, then the result is called interference, but if there is a large number of them, it seems that the word diffraction is more often used."

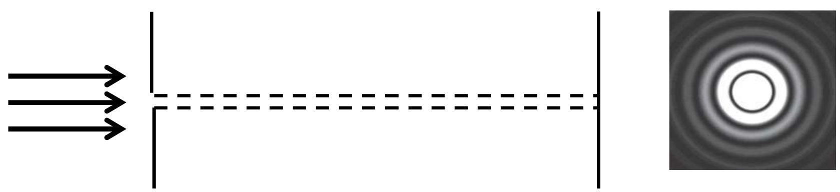
– Richard Feynman

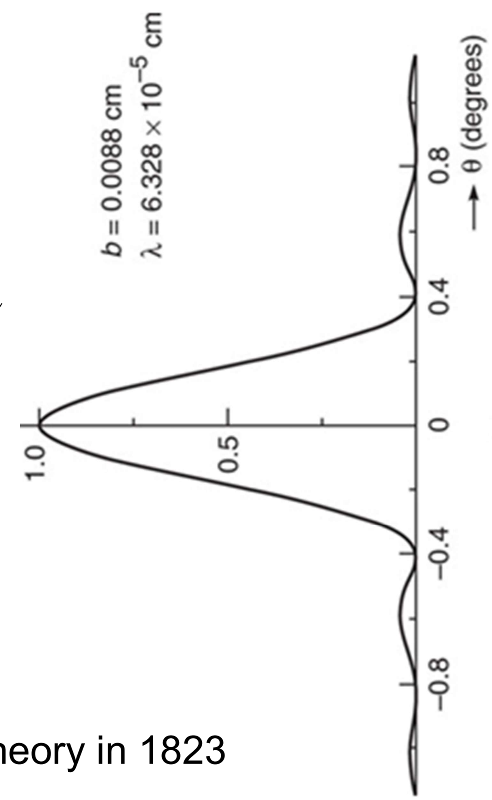
Recollect

A plane wave of wavelength λ incident on a circular aperture of diameter $2b$ (>> > λ) on a screen and a screen is placed at a distance D behind the aperture what will you observe?



If size b of the aperture is significantly reduced to few times λ i.e. comparable to λ





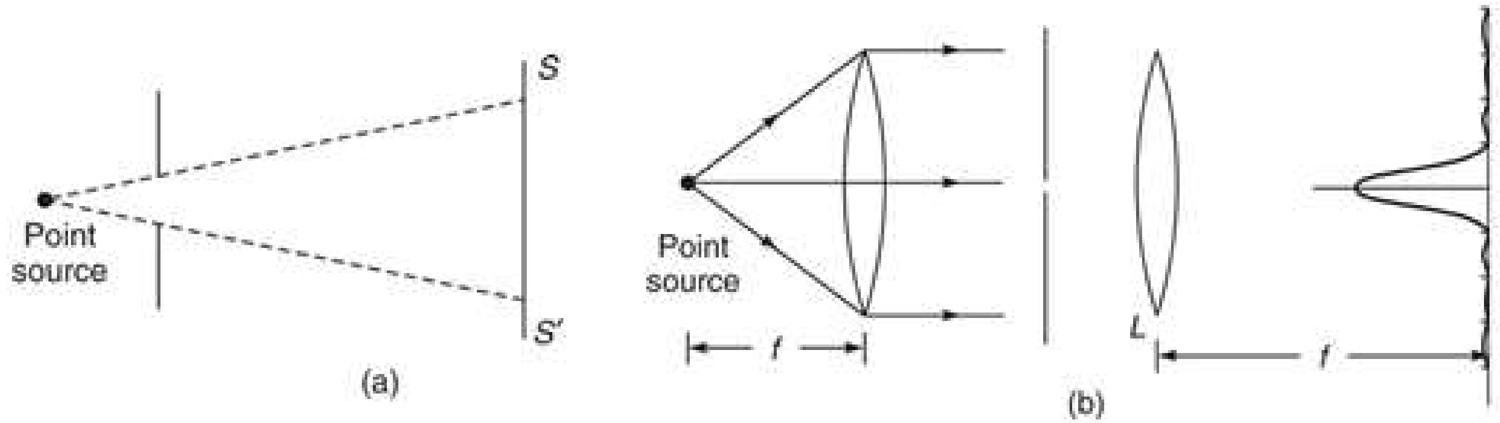
$\Rightarrow$ Diffraction due to a circular aperture

1835: George Airy calculated diffraction pattern due to a circular aperture

1819: Joseph Fraunhofer demonstrated diffraction by a grating and published his theory in 1823

Two classes of diffraction:

a) Fresnel diffraction: source and observation screen are at a finite distances from the aperture



b) Fraunhofer diffraction: source and observation screen are at infinite distances from the aperture

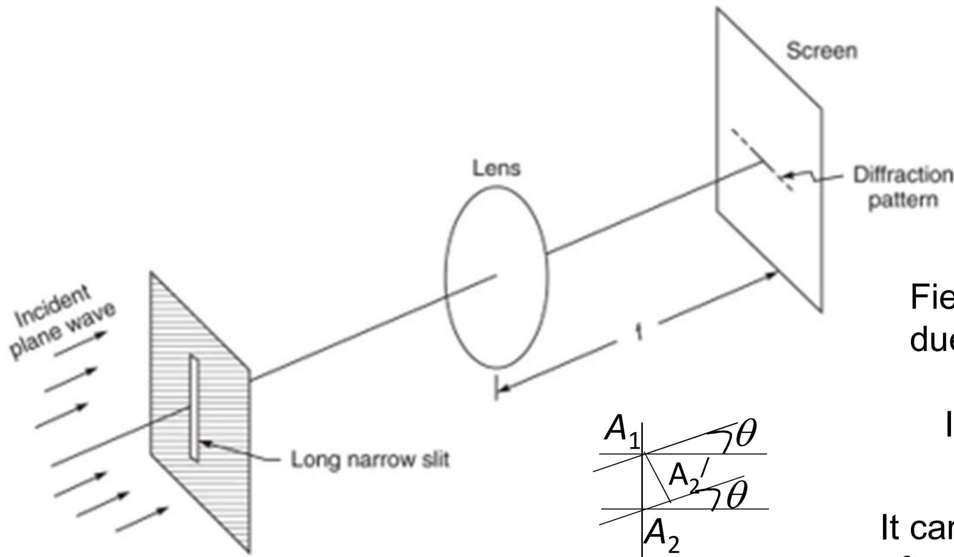
How to realize infinite distance within a laboratory?

Use a pair of convex lenses or a laser light, which emits almost parallel light

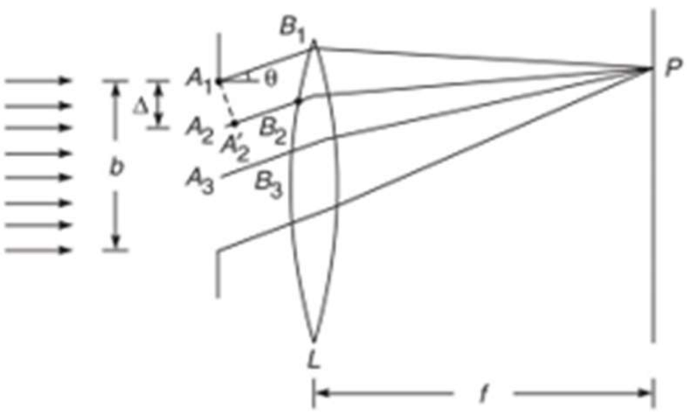
How do you check if a beam of light is parallel or not?

Fraunhofer diffraction is much easier to study/model as compared to Fresnel diffraction

Fraunhofer diffraction by a single slit:



(a)



(b)

Let the slit width be *b*

To obtain the intensity at *P* at the focal plane of Lens

Assume the slit consists of *N* number of equally spaced (Δ) point sources $A_{1,2,3,}$ ….

Fields due to sources $A_{1,2,3,}$ … will superimpose at *P* with different phases due to slight path length difference between consecutive point sources

If we assume *n* number of point sources ⇒ $b = (N-1)\Delta$

It can be shown through detailed algebra (as given at the end after the after the final discussions on single slit diffraction)

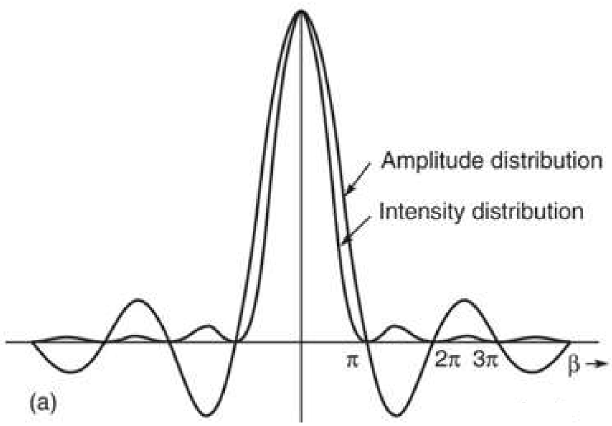
$$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) \Rightarrow$$

$$(N-1)\frac{\phi}{2} \approx N\frac{\phi}{2} = \frac{\pi}{\lambda_0} b\sin\theta = \beta$$

where

$A = aN;$ *a* represents amplitude of light emitted by each point source

Intensity: $|E|^2 = I = \left|A\frac{\sin\beta}{\beta}e^{i(\omega t-\beta)}\right|^2 = I_0\frac{sin^2\beta}{\beta^2}$



From

$$I = I_0 \frac{sin^2\beta}{\beta^2}$$ for single-slit diff pattern

I will be minimum for $\beta = m\pi; m \neq 0$

$$\beta = \frac{\cancel{\pi}}{\lambda_0} b \sin\theta = m\cancel{\pi}\,; m = 0, 1, 2, 3, \ldots .$$

$\Rightarrow$

$b \sin\theta = m\lambda_0; m = \pm 1, \pm 2, \pm 3, \ldots .$: represents condition for min

$\Rightarrow$ 1st minimum appears at $\theta = \sin^{-1}\left(\frac{\lambda_0}{b}\right)$

2nd minimum appears at $\theta = \sin^{-1}\left(\frac{2\lambda_0}{b}\right)$; ....

What will be the max value of *m* allowed ?

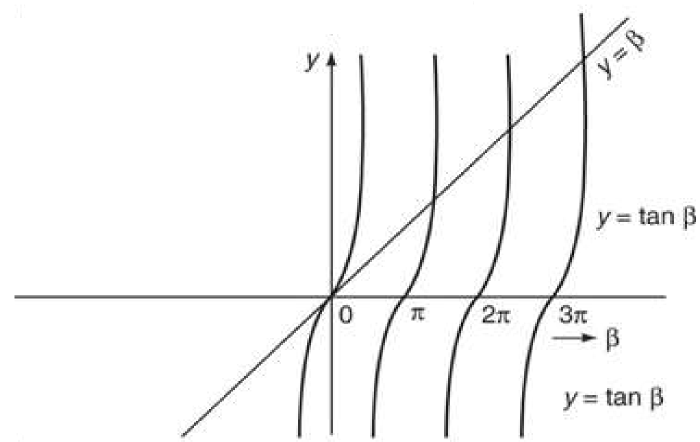
Integer $\leq \frac{b}{\lambda_0}$

For maxima, we differentiate $I = I_0 \dfrac{\sin^2\beta}{\beta^2}$

$$\frac{dI}{d\beta} = I_0\left[\frac{2\sin\beta\cos\ \beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3}\right] = 0 \quad \Rightarrow \quad 2\cos\beta\left[\beta\sin\beta - \frac{\sin^2\beta}{\cos\beta}\right] = 0$$

$\Rightarrow$ Either $\cos\beta = 0$ or $\sin\beta[\beta - \tan\beta] = 0$

But $\sin\beta = 0$ corresponds to minima: $\beta = m\pi; m \neq 0$



Thus maxima will be given by roots of the transcendental eq:

$$\beta - \tan\beta = 0 \ \ i.e.\ \tan\beta = \beta$$

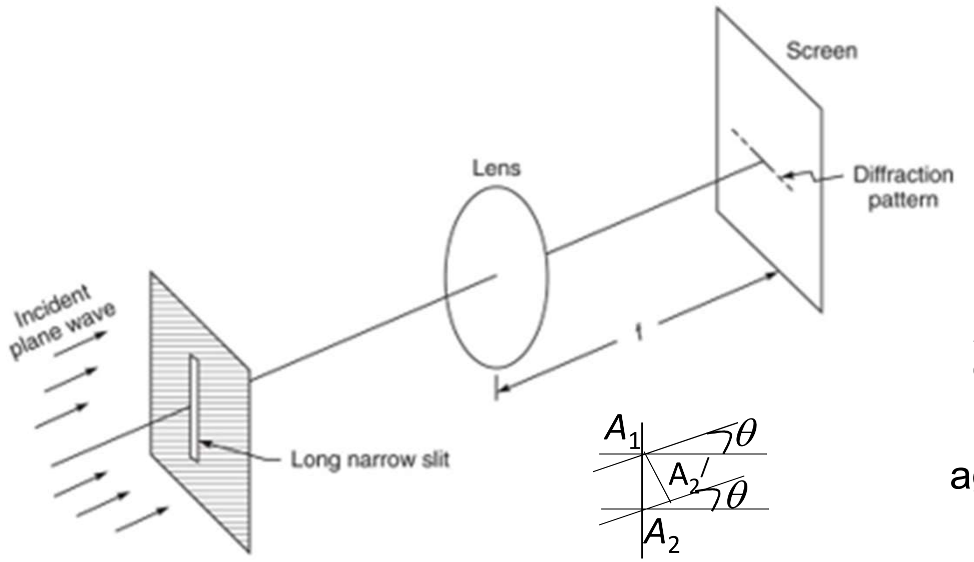
Intersections of the plots of LHs and RHS yields roots as

$$\beta = 1.43\pi, 2.46\pi, \ldots\ldots$$

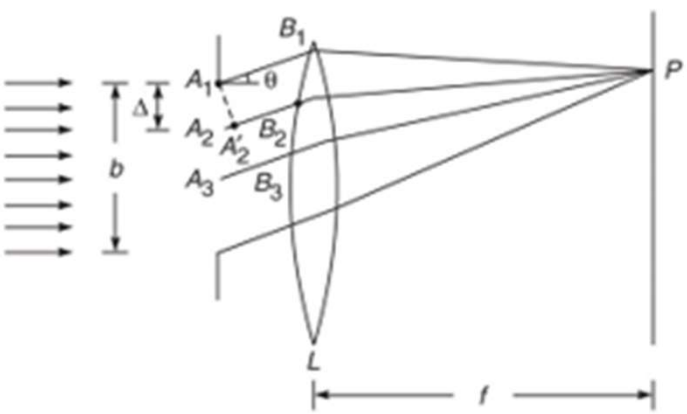
$$\Rightarrow \quad I = I_0\frac{\sin^2 1.43\pi}{(1.43\pi)^2} = 0.0496 \text{ for the 1st maxima}$$

Next three slides present details of calculation for the amp and intensity of the diffraction pattern by a single slit

Fraunhofer diffraction by a single slit:



(a)

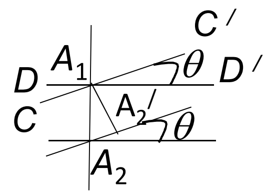


(b)

Let the slit width be b

To obtain the intensity at P at the focal plane of Lens

Assume the slit consists of N number of equally spaced (Δ) point sources $A_{1,2,3,}$ ….



$$\angle C'A_1D' = \theta = \angle CA_1D$$

$$\angle CA_1A_2 = \frac{\pi}{2} - \theta = \angle A_1A_2A_2'$$

A_2' is foot of the normal from A_1 on $A_2'B_2$

again

$$\angle A_1A_2A_2' + \angle A_2A_1A_2' = \cancel{\frac{\pi}{2}} = \cancel{\pi/2} - \theta + \angle A_2A_1A_2'$$

$$\Rightarrow \angle A_2'\,A_1A_2 = \theta$$

$$\Rightarrow A_2A_2'/\Delta = \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$\Rightarrow$ Corresponding phase diff: $\phi = \dfrac{2\pi}{\lambda_0}\Delta\sin\theta$

Fields due to sources $A_{1,2,3,}$ … will superimpose at P with different phases due to slight path length difference between consecutive point sources

If we assume n number of point sources $\Rightarrow$

$$b = (N-1)\Delta$$

Likewise, subsequent rays will also be differing in phase from the previous one – all of which will superimpose at *P*

Thus the resultant electric field of the light at *P* will be $E = a\cos\omega t + a\cos(\omega t - \phi) + a\cos(\omega t - 2\phi) + \ ...$

$$E = a[\cos\omega t + \cos(\omega t - \phi) + \cos(\omega t - 2\phi) + \ .... + \cos(\omega t - (N-1)\phi)]$$

From complex analysis one can represent

$E_1 = a\cos\omega t$ as real part of $E_1 = ae^{i\omega t}$

Likewise

$E_2 = a\cos(\omega t - \phi_1)$ is the real part of $ae^{i(\omega t - \phi_1)}$

$\vdots$

Thus one can express total *E* at *P* as

$$E = E_1 + E_2 + E_3 + \ ..... = a\,e^{i\omega t}\left[1 + e^{-i\phi} + e^{-2i\phi} + \cdots + e^{-i(N-1)\phi}\right]$$

$$= a\,e^{i\omega t}\frac{1 - e^{-iN\phi}}{1 - e^{-i\phi}} = a\,e^{i\omega t}\frac{e^{-iN\phi/2}}{e^{-i\phi/2}} \times \frac{e^{+iN\phi/2} - e^{-iN\phi/2}/2i}{e^{+i\phi/2} - e^{-i\phi/2}/2i}$$

$$= a\frac{\sin\dfrac{N\phi}{2}}{\sin\dfrac{\phi}{2}} \times \frac{e^{i\left[\omega t - \frac{N\phi}{2}\right]}}{e^{-i\frac{\phi}{2}}} = a\frac{\sin\dfrac{N\phi}{2}}{\sin\dfrac{\phi}{2}} \times e^{i\left[\omega t - (N-1)\frac{\phi}{2}\right]}$$

Thus

$$E = a\frac{\sin\frac{N\phi}{2}}{\sin\frac{\phi}{2}} \times e^{i\left[\omega t-(N-1)\frac{\phi}{2}\right]} = E_\theta \cos\left[\omega t - (N-1)\frac{\phi}{2}\right]; \quad E_\theta = a\frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\frac{\phi}{2}} \ldots\ldots\ldots \quad \text{(X)}$$

In the limit, $N \to \infty$ and $\Delta \to 0 \Rightarrow N\Delta \to b$

$$\frac{N\phi}{2} = \frac{N}{\cancel{2}} \times \frac{\cancel{2}\pi}{\lambda_0}\Delta \sin\theta = \frac{\pi}{\lambda_0} N\Delta \sin\theta = \frac{\pi}{\lambda_0} b \sin\theta$$

Moreover

$$\phi = \frac{2\pi}{\lambda_0}\Delta \sin\theta = \frac{2\pi}{\lambda_0} \times \frac{b}{N} \times \sin\theta \to 0 \text{ for large } N \Rightarrow \sin\frac{\phi}{2} \approx \frac{\phi}{2}$$

Hence

$$E_\theta \approx a\frac{\sin\frac{N\phi}{2}}{\frac{\phi}{2}} = a\frac{\sin\left(\frac{\pi}{\lambda_0} b \sin\theta\right)}{\frac{\pi}{N\lambda_0} b \sin\theta} = aN\frac{\sin\left(\frac{\pi}{\lambda_0} b \sin\theta\right)}{\frac{\pi}{\lambda_0} b \sin\theta} = A\frac{\sin\beta}{\beta}; \ \beta = \frac{\pi}{\lambda_0} b \sin\theta$$

$\Rightarrow$ From (X) $E = A\dfrac{\sin\beta}{\beta}\cos(\omega t - \beta) \Rightarrow$ Intensity: $|E|^2 = I = \left|A\dfrac{\sin\beta}{\beta}e^{i(\omega t-\beta)}\right|^2 = I_0\dfrac{sin^2\beta}{\beta^2}$

$$(N-1)\frac{\phi}{2} \approx N\frac{\phi}{2} = \frac{\pi}{\lambda_0} b \sin\theta = \beta$$