Modern Physics

Lecture 13

Application of uncertainty principle

Planck's constant is so small that we generally do not encounter the uncertainty principle in Newtonian mechanics…

…but its consequences are manifested in materials we constantly use in everyday life!

Simple Harmonic Oscillator

Total energy of the oscillator,

$$E_T = \frac{1}{2} m\omega^2 x^2 + \frac{p^2}{2m}$$

From uncertainty principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$\Delta x \rightarrow$ Uncertainty in position

$\Delta p \rightarrow$ Uncertainty in momentum

Total energy becomes,

$$E_T = \frac{1}{2} m\omega^2 (\Delta x)^2 + \frac{(\Delta p)^2}{2m}$$

$$\frac{1}{2} m\omega^2 (\Delta x)^2 + \frac{(\Delta p)^2}{2m} \geq \frac{1}{2} m\omega^2 (\Delta x)^2 + \frac{\hbar^2}{8m(\Delta x)^2}$$

This mean Minimum energy can not be zero
Even if uncertainty in position goes to zero

What will be the minimum energy of the oscillator

$$\frac{dE_T}{d(\Delta x)} = 0$$

$$\frac{dE_T}{d(\Delta x)} = \frac{1}{2} m\omega^2 2\Delta x - \frac{\hbar^2}{8m(\Delta x)^3} = 0$$

Solving this,

$$(\Delta x)^2 = \frac{\hbar}{2m\omega}$$

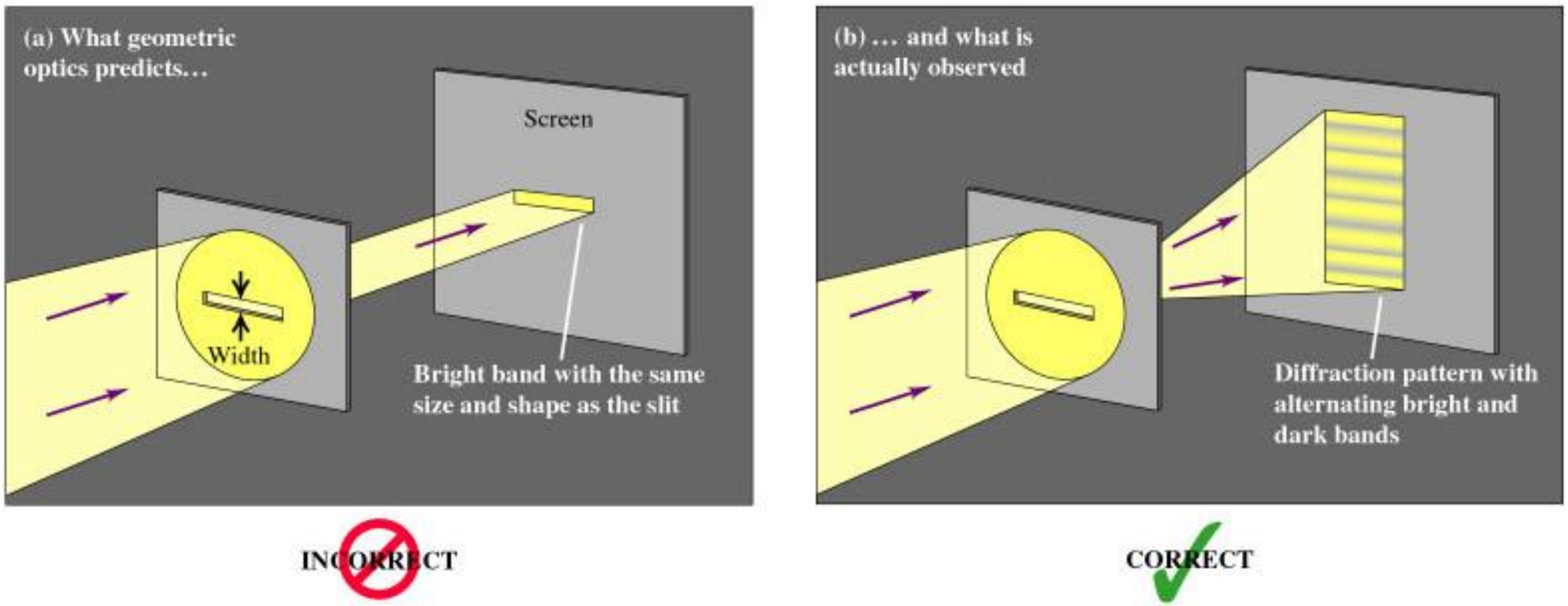
Substituting this value in total energy expression,

$$E_T \geq \frac{1}{2} m\omega^2 (\Delta x)^2 + \frac{\hbar^2}{8m(\Delta x)^2} \qquad E_T \geq \frac{\hbar\omega}{2}$$

This is the minimum energy of harmonic oscillator which can not be zero

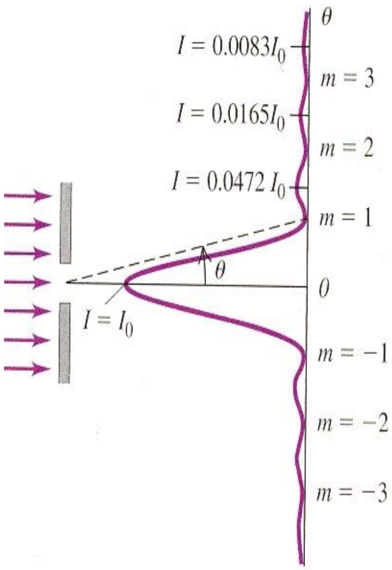
Diffraction from Heisenburg Uncertainty principle

Single Slit Diffraction



“geometrical” picture breaks down when slit width becomes comparable with wavelength

• We can define the width of the central maximum to be the distance between the m = +1 minimum and the m=-1 minimum:



Intensity distribution

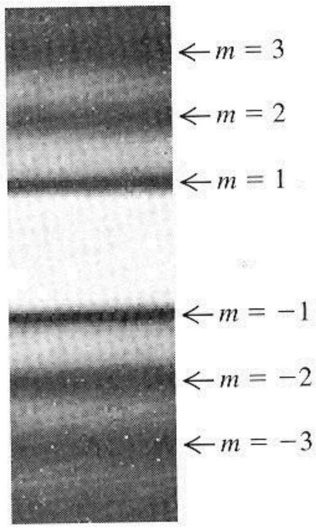
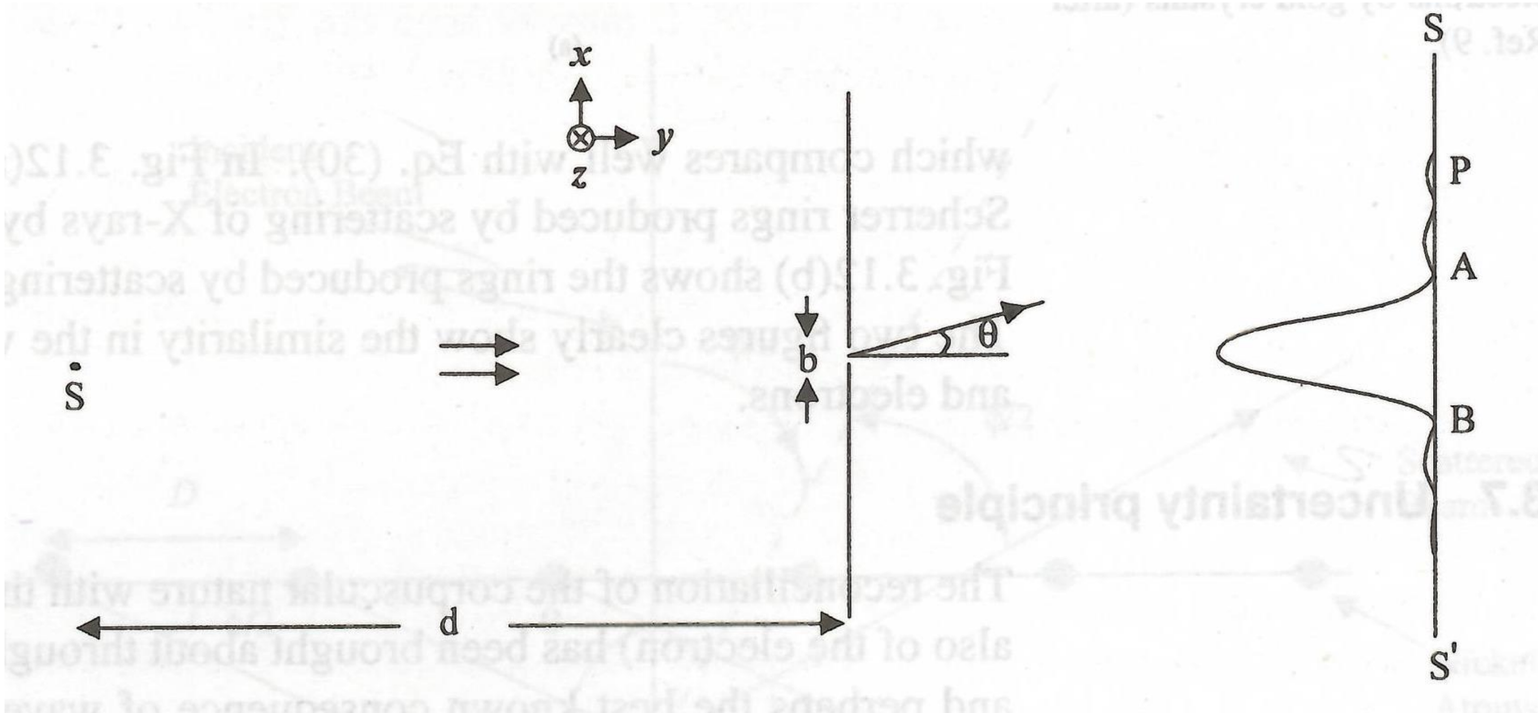


image of diffraction pattern



$$p_x = p\frac{b}{d}$$

$b \longrightarrow$ Slit width

$d \longrightarrow$ Distance of source

$$p_x = \frac{h\nu}{c}\frac{b}{d}$$

After passing through the slit uncertainty incorporated is Δx

This means $\Delta x \approx b$

Therefore from uncertainty,

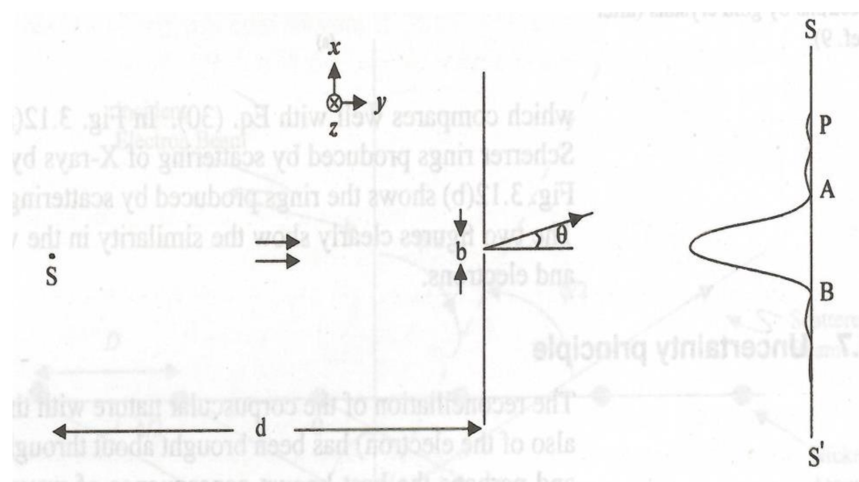
$$\Delta p_x \approx \frac{\hbar}{b}$$

This means just passing through the slit electron gains a momentum Δp_x

But $p_x = p\sin\theta$

$$p\sin\theta \approx \frac{\hbar}{b}$$

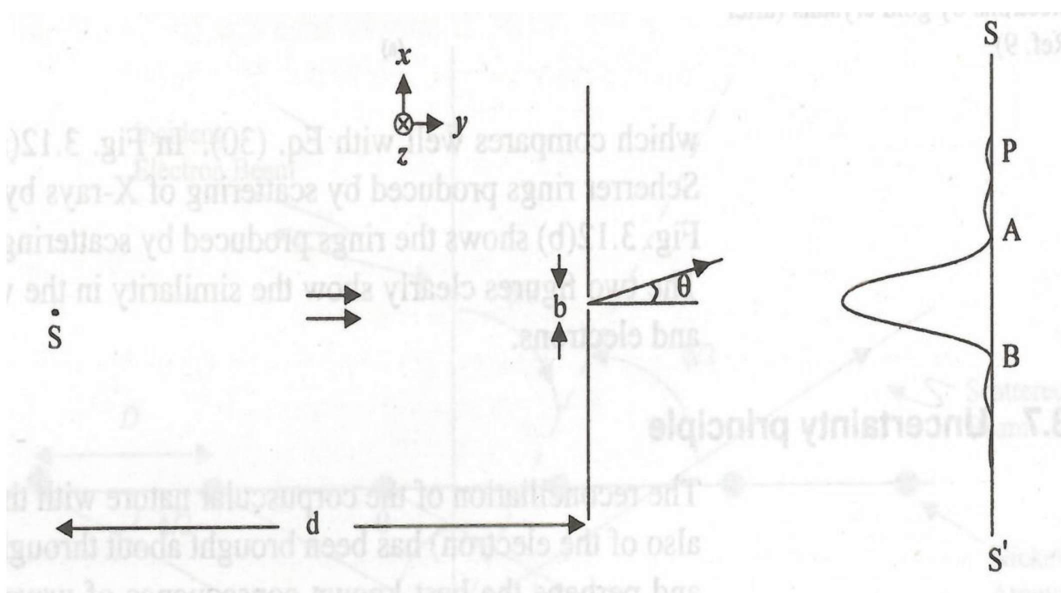
$$\sin\theta \approx \frac{\hbar}{pb}$$



$$\sin\theta \approx \frac{\hbar}{pb}$$

- Probability of photon travelling at θ is inversely proportional to slit width (b)
- Smaller the b, greater θ means possibility of reaching photon deep inside geometrical shadow

Diffraction phenomena



Using de Broglie $(p = \frac{\hbar}{\lambda})$ in $\sin\theta \approx \frac{\hbar}{pb}$

We get,

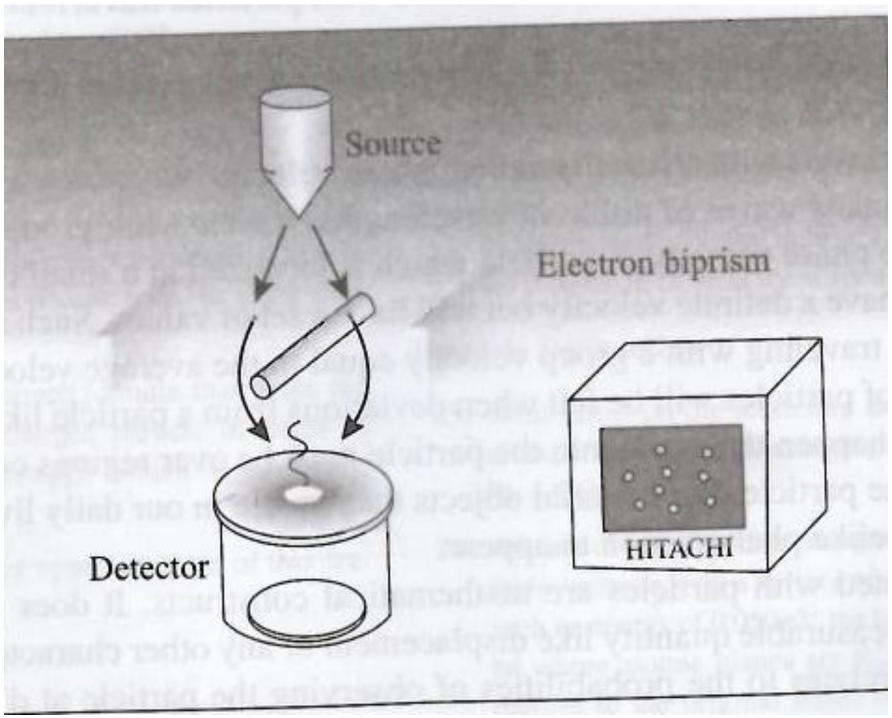
$$\sin\theta \approx \frac{\lambda}{b}$$

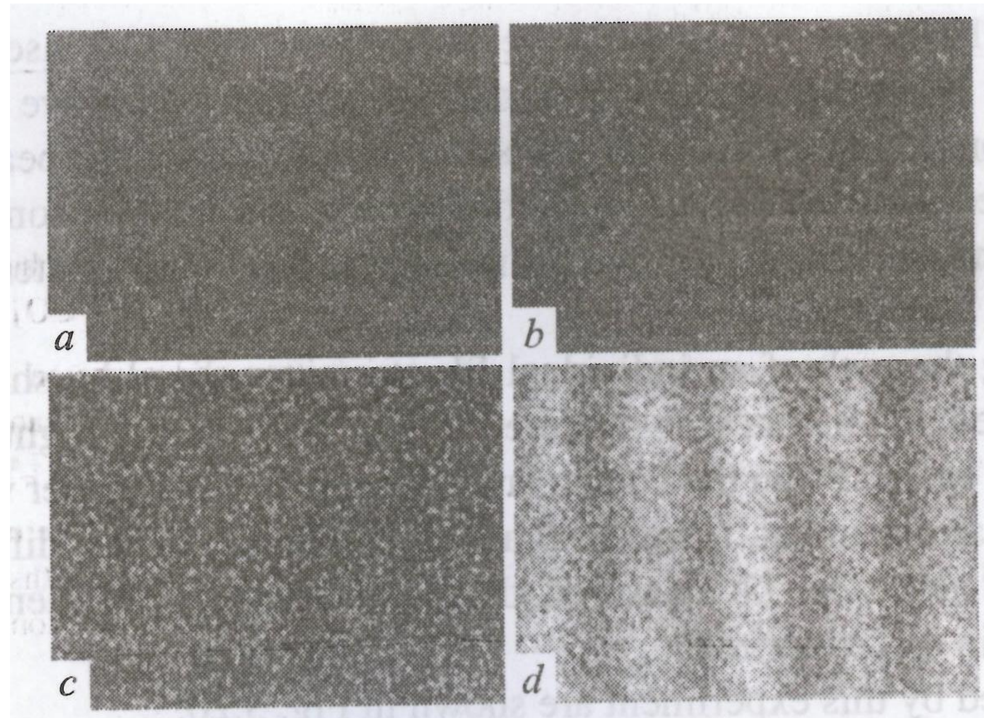
$$b\sin\theta \approx \lambda$$

Diffraction equation

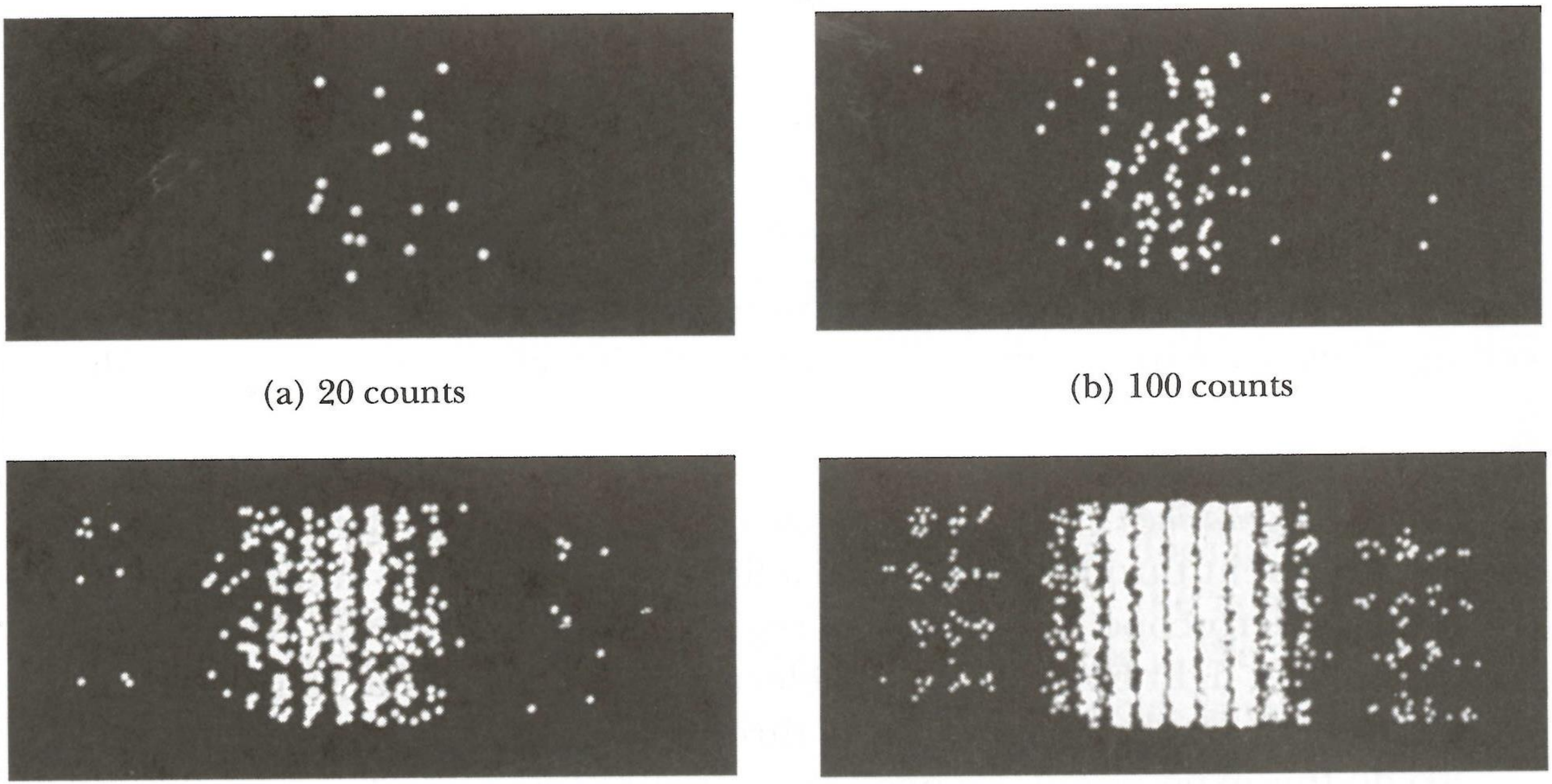
Tonumura Experiment

Dual nature of electrons





- Quantum mechanics is probabilistic and requires large number of observations



(a) 20 counts

(b) 100 counts

(c) 500 counts

(d) ~4000 counts

Simulation and photos courtesy of Julian V. Noble.

A computer simulation