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MA 203: Problem Sheet 8: Statistics

1. A voltage measurement consists of the sum of a constant unknown voltage and a Gaussian distributed noise voltage of zero mean and variance 10 μV^2. Thirty independent measurements are made and a sample mean of 100 μV is obtained. Find the corresponding 95% confidence interval.

2. The lifetime of 225 light bulbs is measured and the sample mean and sample variance are found to be 223 hr and 100 hr, respectively.

   (a) Find a 95% confidence interval for the mean lifetime.

   (b) Find a 95% confidence interval for the variance of the lifetime.

3. Let X be a Gaussian random variable with unknown mean and unknown variance. A set of 10 independent measurements of X yields

$$\sum_{j=1}^{10} X_j = 350, \quad \sum_{j=1}^{10} X_j^2 = 12,645.$$

   (a) Find a 90% confidence interval for the mean of X.

   (b) Find a 90% confidence interval for the variance of X.

4. A new Web page design is intended to increase the rate at which customers place orders. Prior to the new design, the number of orders in an hour was a Poisson random variable with mean 30. Eight one-hour measurements with the new design find an average of 32 orders completed per hour.

   (a) At a 5% significance level, do the data support the claim that the order placement rate has increased?

   (b) Repeat part (a) at a 1% significance level.

5. The output of a receiver is the sum of the input voltage and a Gaussian random variable with zero mean and variance 4 V^2. A scientist suspects that the receiver input is not properly calibrated and has a nonzero input voltage in the absence of a true input signal.

   (a) Find a 1% significance level test involving n independent measurements of the output to test the scientist's hunch.

   (b) What is the outcome of the test if 10 measurements yield a sample mean of -0.75 V ?

   (c) Find the probability of a Type II error if there is indeed an input voltage of 1 V.

6. The breaking strength of plastic bags is a Gaussian random variable. Bags from company 1 have a mean strength of 8 kg and a variance of 1 kg^2; bags from company 2 have a mean strength of 9 kg and a variance of 1 kg^2. We are interested in determining whether a batch of bags comes from company 1 (null hypothesis). Find a hypothesis test and determine the number of bags that needs to be tested so that is 1% and the probability of detection is 99%.

7. Light Internet users have session times that are exponentially distributed with mean 2 hours, and heavy Internet users have session times that are exponentially distributed with mean 4 hours.

   (a) Use the Neyman-Pearson method to find a hypothesis test to determine whether a given user is a light user. Design the test for $\alpha = 5\%$.

   (b) What is the probability of detecting heavy users?

8. When operating correctly (null hypothesis), wires from a production line have a mean diameter of 2 mm, but under a certain fault condition the wires have a mean diameter of 1.75 mm. The diameters are Gaussian distributed with variance 0.04 mm^2. A batch of 10 sample wires is selected and the sample mean is found to be 1.82 mm.

   (a) Design a test to determine whether the line is operating correctly. Assume a false alarm probability of 5%.

   (b) What is the probability of detecting the fault condition?

9. Coin 1 is fair and coin 2 has probability of heads 3/4. A test involves flipping a coin repeatedly until the first occurrence of heads.The number of tosses is observed.

   (a) Can you design a test to determine whether the fair coin is in use? Assume $\alpha = 5\%$. What is the probability of detecting the biased coin?

   (b) Repeat part (a) if the biased coin has probability 1/4.