Lecture 7

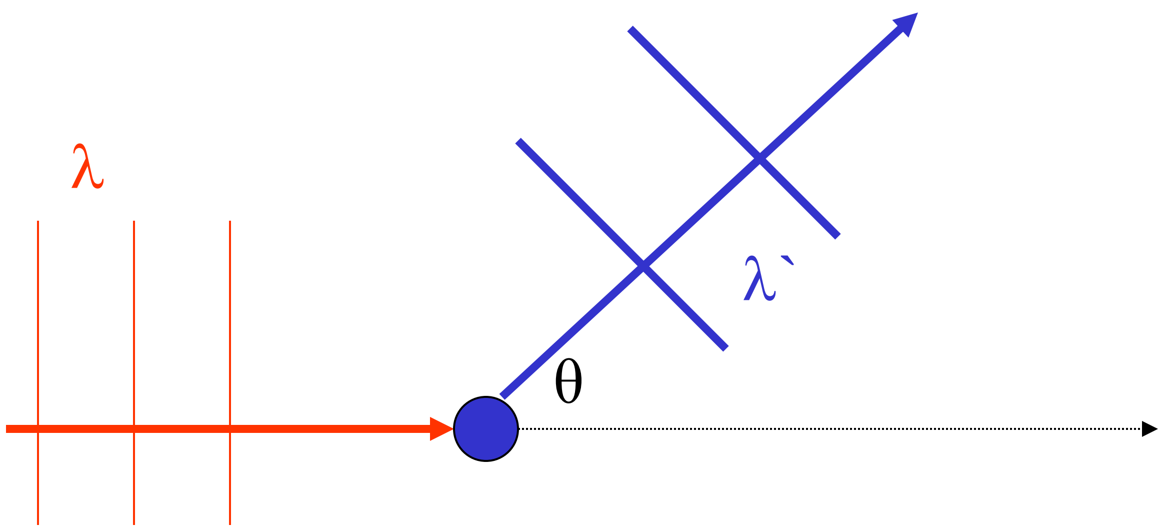
Compton Effect

Compton Effect

- Scattering of radiation from an electron
- Compton effect provides a direct confirmation of particle nature of radiation.

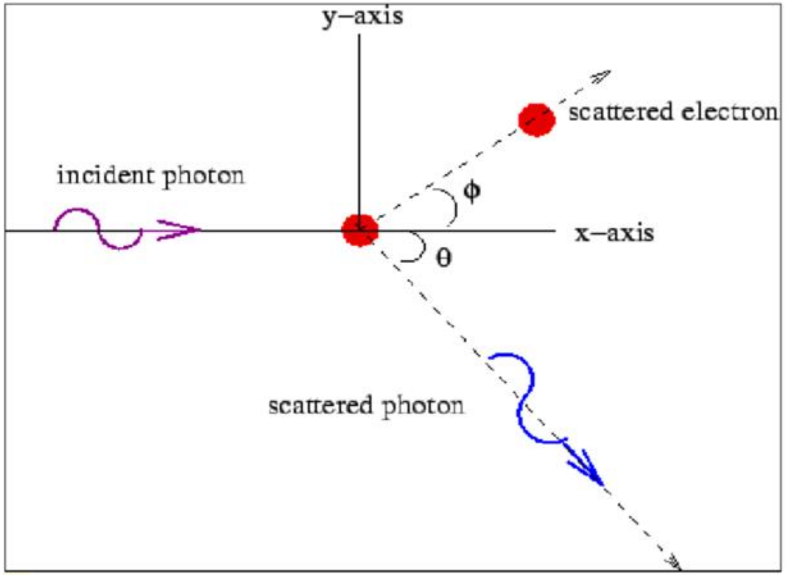
Compton Effect

- 1923 Compton performed an experiment which supported this idea
- directed a beam of x-rays of wavelength λ onto a carbon target precisely graphite target
- x-rays are scattered in different directions



$\lambda = 71.1$ pm (10^{-12} m)

λ` has 2 peaks



$E_i = h\nu_0$ and $p_i = \dfrac{h\nu_0}{c}$ Energy and momentum of incident photon

x-axis direction of incident electron

p_e momentum of scattered electron

p_f momentum of scattered photon

Relativistic energy expression

$$E^2 = \mathrm{m}^2 c^4 + p^2 c^2$$

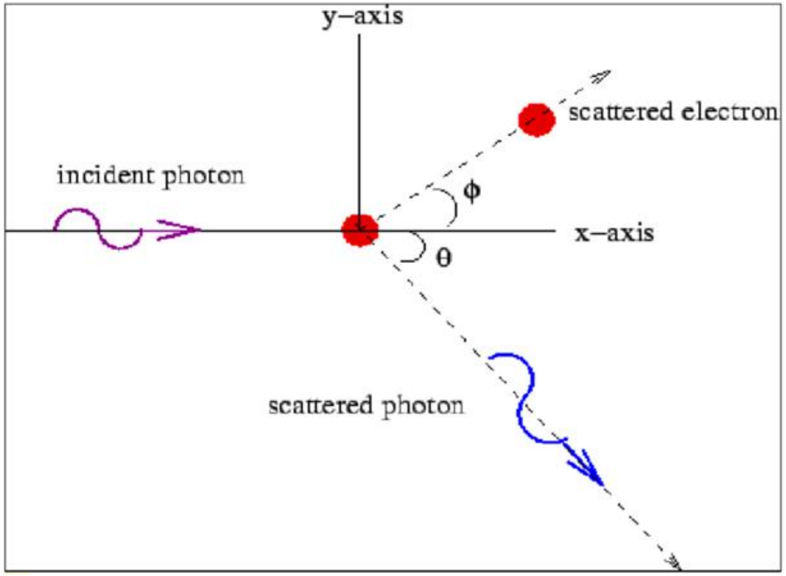
Considering zero mass (m = 0) for photon

$$E = pc$$

Therefore,

$$p = \frac{h\nu}{c}$$ for photon

From conservation of momentum



x-direction:

$$p_i = p_f Cos\theta + p_e Cos\varphi \qquad (1)$$

y-direction:

$$0 = -p_f Sin\theta + p_e Sin\varphi \qquad (2)$$

Equation 1 can be written as,

$$p_i - p_f Cos\theta = p_e Cos\varphi$$

Similarly Equation 2 can be written as,

$$p_f Sin\theta = p_e Sin\varphi$$

Squaring and adding above two relations we get

$$p_e^2 (Cos^2\varphi + Sin^2\varphi) = (p_i - p_f Cos\theta)^2 + p_f^2 Sin^2\theta$$

$$p_e^2 = p_i^2 - 2 p_i p_f Cos\theta + p_f^2 Cos^2\theta + p_f^2 Sin^2\theta$$

$$p_e^2 = p_i^2 - 2 p_i p_f Cos\theta + p_f^2 \qquad (3)$$

Total initial energy of electron at rest

$$E = \mathrm{m}_0 c^2 \qquad (4)$$

Relativistic final energy of electron

$$E^2 = \mathrm{m}_0^2 c^4 + p_e^2 c^2 \qquad (5)$$

Thus from conservation of energy

X-ray photon energy + Electron rest energy

= scattered x ray energy + electron total energy

$$h\nu_0 + \mathrm{m}_0 c^2 = h\nu + \sqrt{\mathrm{m}_0^2 c^4 + p_e^2 c^2}$$

Rearranging and squaring both sides

$$(h\nu_0 - h\nu + \mathrm{m}_0 c^2)^2 = \mathrm{m}_0^2 c^4 + p_e^2 c^2 \qquad (6)$$

Expanding the squared term

$$(h\nu_0 - h\nu)^2 + \mathrm{m}_0^2 c^4 + 2\mathrm{m}_0 c^2 (h\nu_0 - h\nu) = \mathrm{m}_0^2 c^4 + p_e^2 c^2$$

Cancelling the term containing $\mathrm{m}_0 c^2$

$$(h\nu_0 - h\nu)^2 + 2\mathrm{m}_0 c^2 (h\nu_0 - h\nu) = p_e^2 c^2 \qquad (7)$$

On substituting p_e from equation 3 derived earlier in equation 7

$$p_e^2 = p_i^2 - 2 p_i p_f Cos\theta + p_f^2$$

$$p_i^2 c^2 + p_f^2 c^2 - 2 p_i p_f c^2 Cos\theta = (h\nu_0 - h\nu)^2 + 2\mathrm{m}_0 c^2 (h\nu_0 - h\nu)$$

(8)

Considering momentum expressions derived earlier

$$p_i = \frac{h\nu_0}{c} \quad \text{and} \quad p_f = \frac{h\nu}{c}$$

$$\frac{h^2\nu_0^2}{\cancel{c^2}}\cancel{c^2} + \frac{h^2\nu^2}{\cancel{c^2}}\cancel{c^2} - 2\frac{h\nu_0}{\cancel{c}}\frac{h\nu}{\cancel{c}}\cancel{c^2}Cos\theta$$
$$= (h\nu_0 - h\nu)^2 + 2\mathrm{m}_0c^2(h\nu_0 - h\nu)$$

$$h^2\nu_0^2 + h^2\nu^2 - 2h^2\nu_0\nu Cos\theta$$
$$= (h\nu_0 - h\nu)^2 + 2\mathrm{m}_0c^2(h\nu_0 - h\nu)$$

$$\cancel{h^2\nu_0^2} + \cancel{h^2\nu^2} - 2h^2\nu_0\nu Cos\theta$$
$$= \cancel{h^2\nu_0^2} + \cancel{h^2\nu^2} - 2h^2\nu\nu_0 + 2\mathrm{m}_0c^2(h\nu_0 - h\nu)$$

$$\cancel{2}h^{\cancel{2}}\nu\nu_0 - \cancel{2}h^{\cancel{2}}\nu_0\nu Cos\theta = \cancel{2}\mathrm{m}_0c^2(\cancel{h}\nu_0 - \cancel{h}\nu)$$

$$h\nu\nu_0(1 - Cos\theta) = \mathrm{m}_0c^2(\nu_0 - \nu) \qquad (9)$$

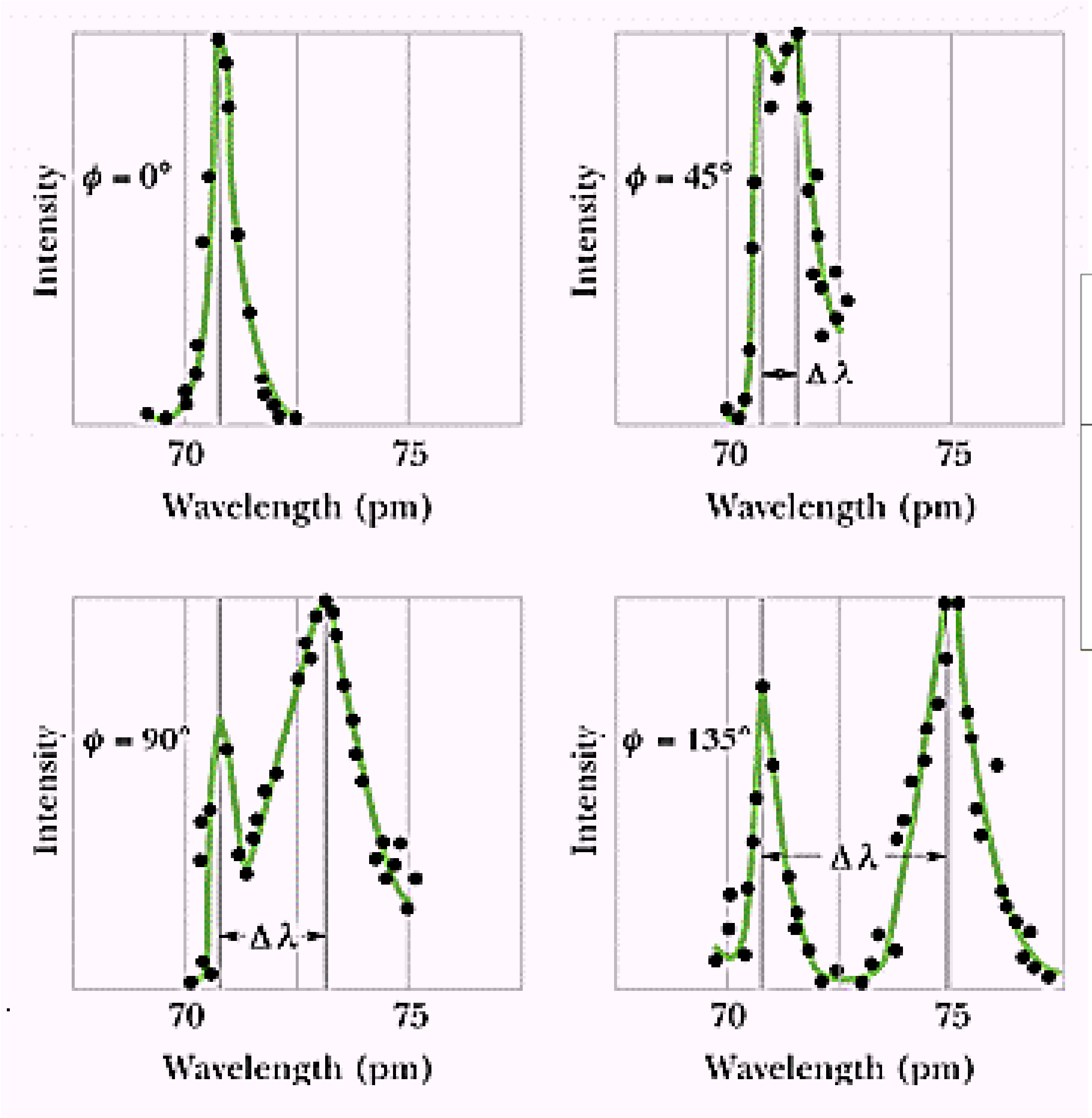
Using $\lambda_s = \dfrac{c}{\nu}$ in relation 9

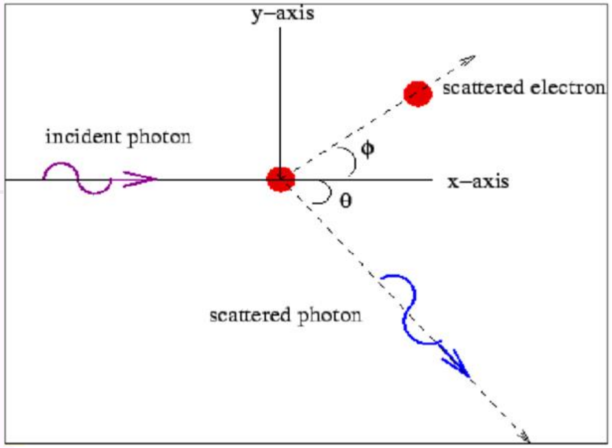
$$\lambda_s - \lambda_0 = \frac{h}{\mathrm{m}_0c}(1 - \cos\theta)$$

Compton scattering formula

Compton wavelength

Detector in different scattering angle





Compton Scattering

- Wavelength λ` of scattered x-rays has two peaks
- these occur at λ and $\lambda + \Delta\lambda$
- $\Delta\lambda > 0$ is the Compton shift

- Quantum picture:
- a *single* photon interacts with electrons in the target
- light behaves like a 'particle" of energy $E=hf=hc/\lambda$ and momentum $p=h/\lambda$ => a collision

Problem

- An x-ray beam of wavelength 0.01 nm strikes a target containing free electrons. Consider the xrays scattered back at 180^0
- Determine (a) change in wavelength of the xrays (b) change in photon energy between incident and scattered beams

Solution

- X-ray beam has λ=.01 nm = 10 pm
- $\phi=180^0$
- $\lambda` - \lambda = (h/m_e c)(1 - \cos\phi)$
- $= \lambda_c (1 - \cos\phi)$ λ_c is Compton wavelength of the electron
- (a) $\Delta\lambda=(h/cm_e)(1-\cos(180))= 2h/cm_e$
  $=2(6.63\text{x}10^{-34})/[(3\text{x}10^8)(9.11\text{x}10^{-31})]$
  $=2(2.43 \text{ pm})=$ 4.86 pm
- (b) $\Delta E=\{ hc/ \lambda` - hc/\lambda\}$
  $=(6.63\text{x}10^{-34})(3\text{x}10^8)\{1/14.86 - 1/10\}/(10^{-12})$
  $=-.65\text{x}10^{-14}$ J $= -.41\text{x}10^5$ eV = -41 keV

PROBLEM

Is Compton effect easier to observe with I.R., visible, UV or X-rays ? Why?

$$\lambda_s - \lambda_0 = \frac{h}{\mathrm{m}_0 c}(1 - \cos\theta)$$

$$\frac{h}{\mathrm{m}_0 c} = \frac{6.64\times10^{-34}}{9.1\times10^{-31}\times10^{8}} = 0.73\times10^{-11}$$

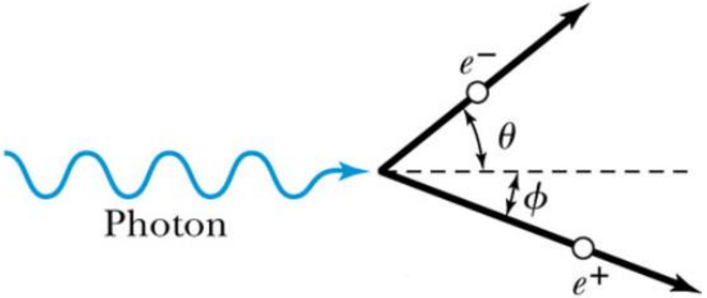
$$\frac{h}{\mathrm{m}_0 c} = 7.3\times10^{-12} m = 7.3 pm$$

Wavelength of X-ray of the order of $(10-100)\,pm$

Pair Production

Conversion of X-ray or γ-ray photon into particle and anti particle pair

- Electron positron pair
- Proton anti proton pair
- Muon anti muon pair



- Energy is conserved
- Charge is conserved
- Momentum is conserved

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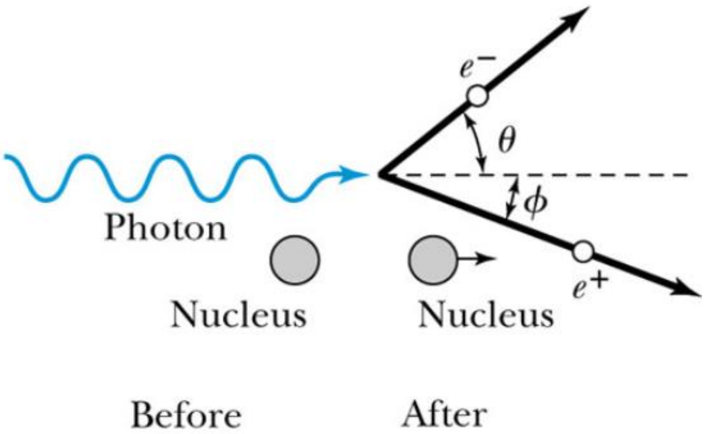

- Energy is conserved
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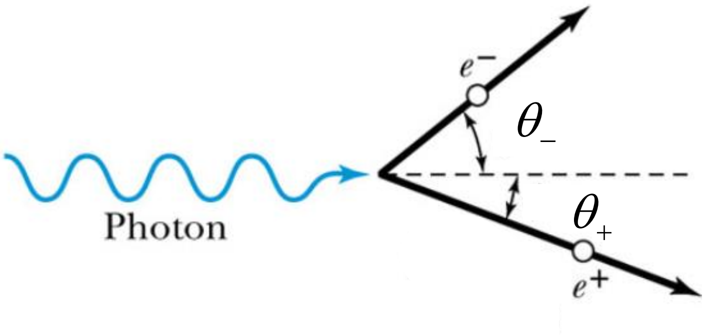
Pair Production

- Energy and momentum conservations give

$$\text{Energy} \quad hf = E_- + E_+$$

$$\text{Momentum(x)} \quad \frac{hf}{c} = p_- \cos\theta_- + p_+ \cos\theta_+$$

$$\text{Momentum(y)} \quad 0 = p_- \sin\theta_- + p_+ \sin\theta_+$$



- Energy conservation can be re-written

$$hf = \sqrt{p_-^2c^2 + m^2c^4} + \sqrt{p_+^2c^2 + m^2c^4}$$

- But momentum conservation (x) gives

$$hf_{\max} = p_-c + p_+c$$

- Thus energy and momentum are not simultaneously conserved

A nucleus is require to conserve momentum

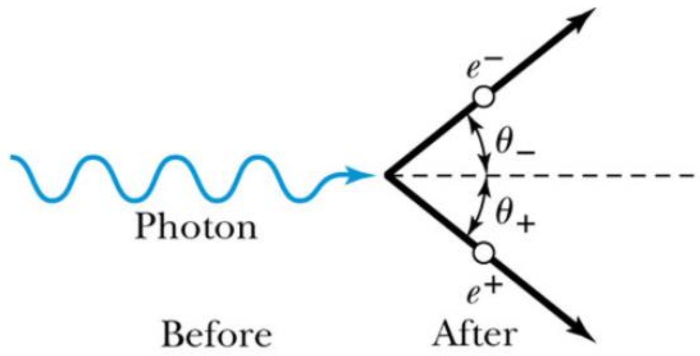
A nucleus is require to conserve momentum

$$hf_{\max} = p_{-}c + p_{+}c + M_{nucleus}$$

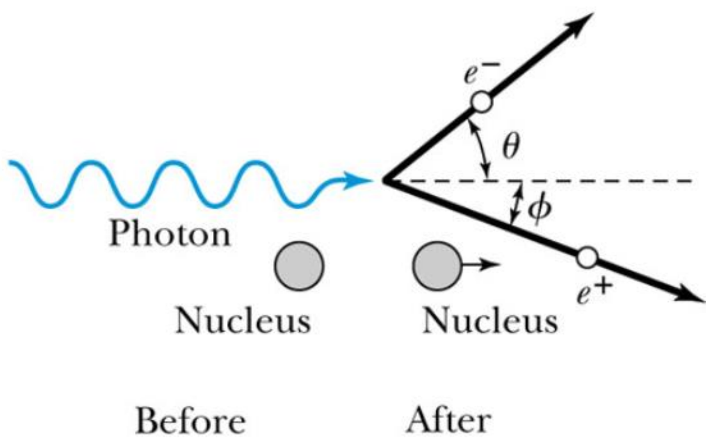
Energy and momentum are simultaneously conserved
Hence pair production is possible

- In order to create a pair, the photon must have energy equivalence > $2m_e$ = 1.022 MeV
- In order to conserve energy and momentum, pair production must take place in the vicinity of a nucleus

Pair Production



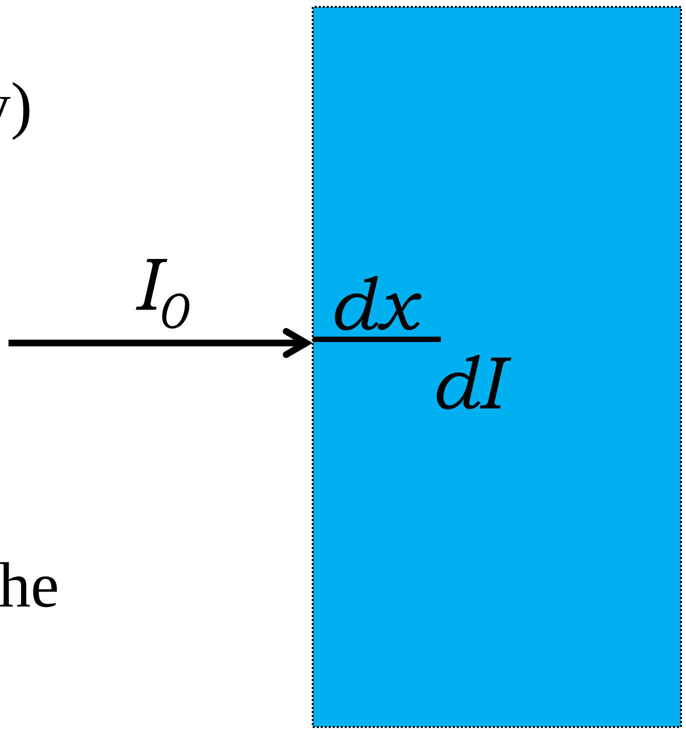
(a) Free space (**cannot occur**)



(b) Beside nucleus

Photon Absorption

How a photon (typically X-ray or γ-ray) losses energy in a medium



I be the intensity of the beam

Fraction of energy lost is proportional to the distance travelled inside the medium

$$-\frac{dI}{I} \propto dx$$

$$-\frac{dI}{I} = \mu dx$$

μ is called the attenuation coeficient

Solving the above relation

$$I = I_0 e^{-\mu x}$$

Radiation intensity

Taking log in both sides and simplifying we get,

$$x = \frac{\ln(I_0 / I)}{\mu}$$

Absorber thickness

- Initial intensity
- Intensity at some distance
- Hence thickness is known