Q1

$x(t)$ and $y(t)$ are WSS processes with zero mean and $R_{xx}(\tau)$ and $R_{yy}(\tau)$ as their correlation functions respectively.

Find $Z(t)$ a new process generated using $x(t)$ and $y(t)$ which is (i) WSS (ii) $R_{zz}(\tau) = R_{xx}(\tau)\, R_{yy}(\tau)$.

Q2

A discrete random process has $R(m) = a^{|m|}$ $m = 0, - \infty$

Find it's spectrum $S(w)$. Is it periodic ?

Q3

$x(t)$ and $y(t)$ are independent WSS random processes with zero means and same correlation function $R(\tau)$.

Let $z(t) = x(t)\cos\omega t + y(t)\sin\omega t$

(i) Is $z(t)$ WSS?

(ii) If $R(\tau) = \sigma^2 e^{-\alpha|\tau|}$, find $R_{zz}(\tau)$.

Q4 If $y(n) = x(n) - x(n-1)$,

find $R_{yy}(m)$ and $S_{YY}(z)$ and $S_{YY}(\omega)$

Q5

The joint pdf of n, real Gauss random variables with zero means $X_1, X_2, X_i, X_j, \dots X_n$ is given by:

$$p_{\underline{x}}(x) = \frac{1}{\sqrt{(2\pi)^n \cdot \Delta}} \cdot e^{-x^T R^{-1} x/2}$$

where
$$R = \begin{bmatrix} R_x(1,1) & R_x(1,2) & \dots & R_x(1,n) \\ \vdots & & & \\ R_x(n,1) & \dots & \dots & R_x(n,n) \end{bmatrix}$$

where $R_x(i,j) = E[x_i x_j]$ and $\Delta = |R|$

Note $R_x(i,i) = \sigma_i^2$ and $R_x(i,j) = R_x(j,i)$

i) Now let $x(t)$ be a random process — zero mean Gauss. If the process is sampled at $t = i$ with $i = 1, \dots n$ (and i are integers) and the process with is WSS write R matrix for such process.

ii) If the WSS process has $R_x(k) = e^{-|k|}$ express $p_{\underline{x}}(x)$ for $n = 2$

iii) Show that uncorrelated X_i s are independent if they are Gauss

Q6

show that if $S = \int_0^{10} x(t)\,dt$, then $E[S^2] = \int_{-10}^{10} (10 - |\tau|)\, R_x(\tau)\, d\tau$

Find mean of S if $E[X(t)] = 8$

Find variance of S if in addition given $R_x(\tau) = 64 + 10e^{-2|\tau|}$

Q7

- A student claims that $R_X(\tau)$ shown below is the correlation function of a real process

$$R_X(\tau) = 1 \quad -1 < \tau < 1$$
$$= 0 \quad \text{otherwise}$$

Do you agree ?

- If $S_Z(w)$ is the spectral density of $Z(t) = aX(t) + bY(t)$ where a, b are arbitrary real constants, $X(t)$ and $Y(t)$ are jointly W.S.S, what is the condition on $S_{XX}(w)$, $S_{YY}(w)$ and $S_{XY}(w)$

Q_8 If processes $X(t)$ and $Y(t)$ are WSS and

$E[|X(0) - Y(0)|^2] = 0$ Then $R_{XX}(\tau) = R_{XY}(\tau) = R_{YY}(\tau)$

Q9

A discrete process X_n is given by

$$X_n = 2\cos\left(\frac{\pi n}{8} + \phi\right)$$

where ϕ is a uniformly distributed r.v

$$P_\phi(\phi) = \frac{1}{2\pi} \quad -\pi < \phi < \pi$$

- Find mean of X_n
- Find $R_X(m,n)$
- Is X_n WSS

Q10

$X(t) = A\cos(2t + \phi)$

where A uniformly distributed $0 < \alpha < 1$

and ϕ is also uniformly distributed $0 < \phi < 2\pi$

- find $E[X(t)]$, $E[X(t+\tau)X(t)]$
- Is $X(t)$ WSS

Q11 If $x(t)$ is zero mean Gauss, and $y(t) = x^2(t)$

then $S_y(\omega) = 2\pi R_x^2(0) u_0(\omega) + 2\, S_x(\omega) \otimes S_x(\omega)$

Q12

If $S_x(\omega) = \dfrac{P}{\omega^2 + \omega_0^2}$, find $E[X^2(t)]$.

and $S_x(\omega) = \dfrac{1}{(4+\omega^2)^2}$

Q13

If $Z_n = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

and $x_1, \ldots x_n$ are samples of a WSS process $X(t)$ with $N(0,1)$ at integral $t = 1, \ldots n$, find mean, variance of Z_n

Can you find pdf of Z_n

Note: $N(0,1)$ stands for Normal (zero mean, unit variance)

If X(t) is a WSS process with $R_X\ (\tau)$ show that :

$$P\{|X(t+\tau) - X(t)| > \alpha\} \leq \frac{2R_X(0) - 2R_X(\tau)}{\alpha^2}$$