MA 203: Tutorial Sheet 2: Probability
Assignment submission deadline: 22/08/2018

∗ Problems to be submitted as Assignment

*1. Let A and B be two finite sets, with $|A| = m$ and $|B| = n$. How many distinct functions can be defined from set A to set B ?

2. Let A and B be two finite sets, with $|A| = m$ and $|B| = n$. How many distinct one-to-one functions can be defined from set A to set B ?

3. An urn contains 30 red balls and 70 green balls. What is the probability of getting exactly k red balls in a sample of size 20 if the sampling is done with replacement ? Assume $0 \leq k \leq 20$.

*4. An urn consists of 30 red balls and 70 green balls. What is the probability of getting exactly k red balls in a sample of size 20 if the sampling is done without replacement ?

5. How many distinct solutions does the following equation have

$$x_1 + x_2 + x_3 + x_4 = 100$$

such that $x_1 \in \{1, 2, 3, \cdots, 100\}$, $x_2 \in \{2, 3, \cdots, 100\}$, $x_3, x_4 \in \{0, 1, 2, \cdots, 100\}$.

6. Assume that there are k people in a room and we know that: (i) $k = 5$ with probability 1/4; (ii) $k = 10$ with probability 1/4; (iii) $k = 15$ with probability 1/2.

   (a) What is the probability that at least two of them have been born in the same month? Assume that all months are equally likely.

   (b) Given that we already know there are at least two people that celebrate their birthday in the same month, what is the probability that $k = 10$?

*7. Ten passengers get on an airport shuttle at the airport. The shuttle has a route that includes 5 hotels, and each passenger gets off the shuttle at his/her hotel. The driver records how many passengers leave the shuttle at each hotel. How many different possibilities exist?

8. There are 20 black cell phones and 30 white cell phones in a store. An employee takes 10 phones at random. Find the probability that

   (a) there will be exactly 4 black cell phones among the chosen phones;

   (b) there will be less than 3 black cell phones among the chosen phones.

9. The 52 cards in a shuffled deck are dealt equally among four players, call them A, B, C, and D. If A and B have exactly 7 spades, what is the probability that C has exactly 4 spades?

10. A multiple choice test has 10 questions with 3 choices each. How many ways are there to answer the test? What is the probability that two papers have the same answers?

11. Five balls are placed at random in five buckets. What is the probability that each bucket has a ball?

*12. A lot of 50 items has 40 good items and 10 bad items.

    (a) Suppose we test five samples from the lot, with replacement. Let X be the number of defective items in the sample. Find $P[X = k]$.

    (b) Suppose we test five samples from the lot, without replacement. Let Y be the number of defective items in the sample. Find $P[Y = k]$.

*13. A computer manufacturer uses chips from three sources. Chips from sources A, B, and C are defective with probabilities 0.005, 0.001, and 0.010, respectively. If a randomly selected chip is found to be defective, find the probability that the manufacturer was A; that the manufacturer was C. Assume that the proportions of chips from A, B, and C are 0.5, 0.1, and 0.4, respectively.

14. An urn contains two black balls and three white balls. Two balls are selected at random from the urn without replacement and the sequence of colors is noted.

    (a) Find the probability that both balls are black.

    (b) Find the probability of the event that the second ball is white.

15. A random experiment is repeated a large number of times and the occurrence of events A and B is noted. How would you test whether events A and B are independent?