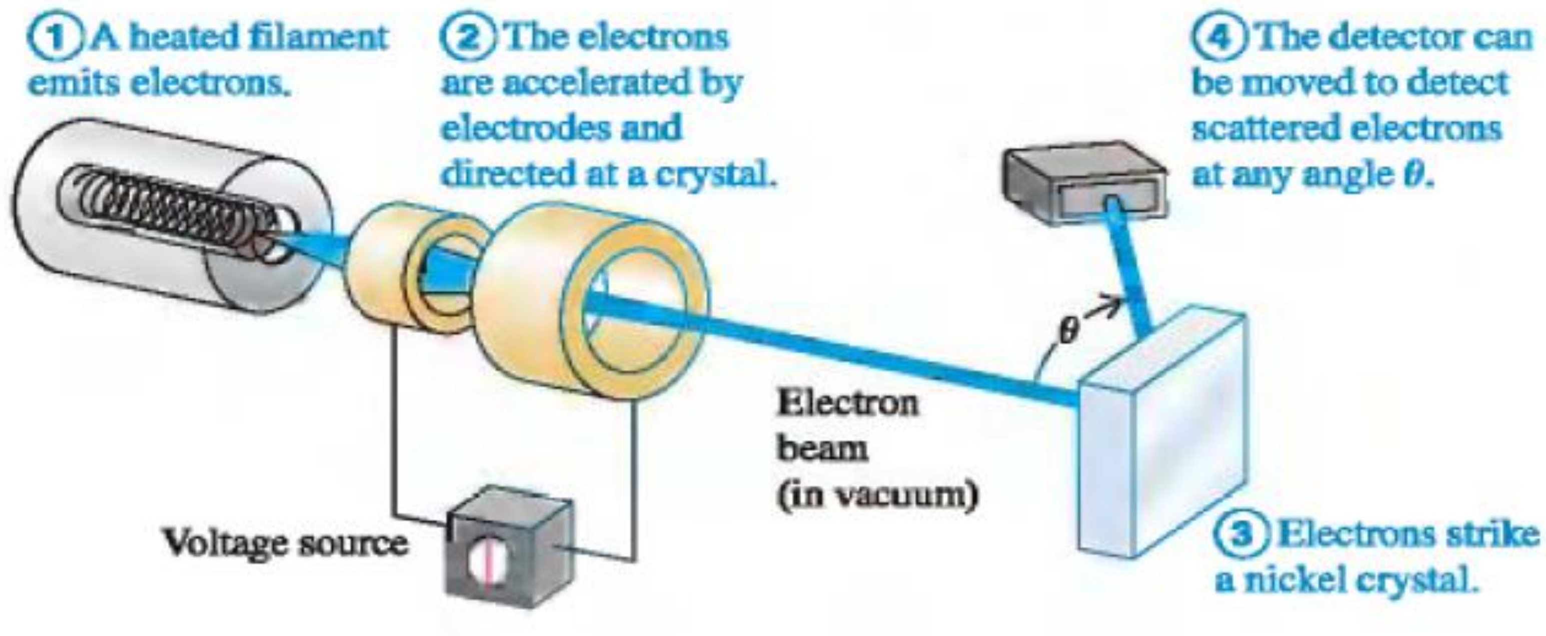
Modern Physics

Lecture 10

Davisson-Germer Experiment

- In 1927, *Clinton Davisson* and *Lester Germer* performed this experiment while working at the Bell Telephone Laboratories
- De Broglie hypothesis was proved through this experiment however by accident

Experiential Set up

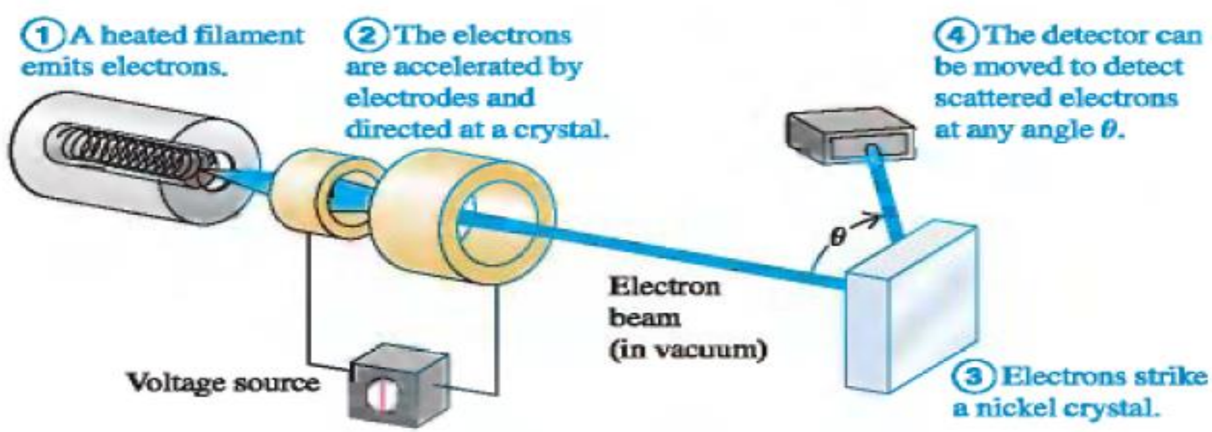


An apparatus similar to that used by Davisson and Germer to discover electron diffraction hence proving de Broglie hypothesis

They were studying the surface of a piece of nickel and observing the number of electrons bouncing off in various angles.

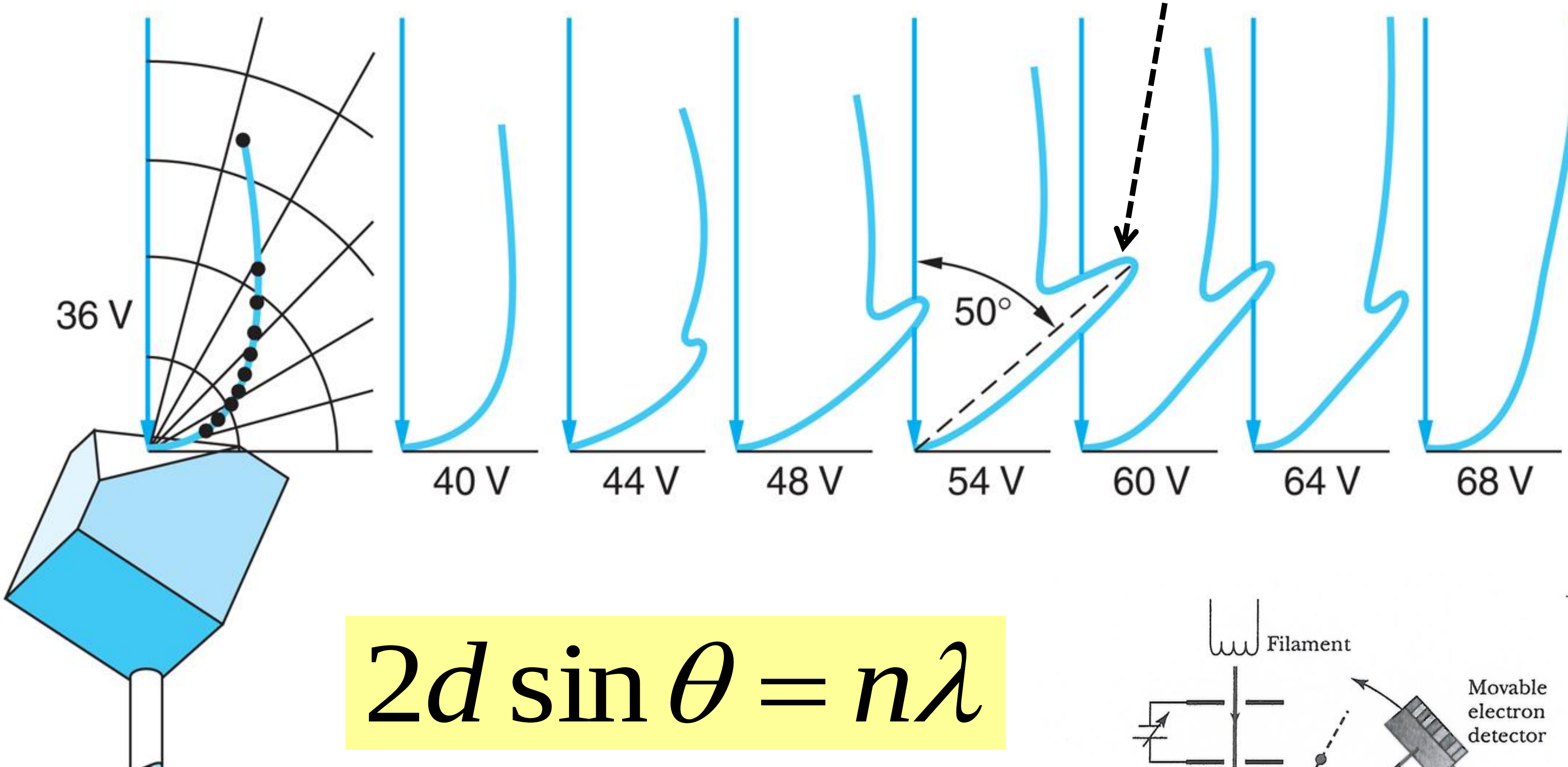
Variable parameters in the experiment

- Energy of incident beam
- Angle of incidence of electron beam
- Position of the detector



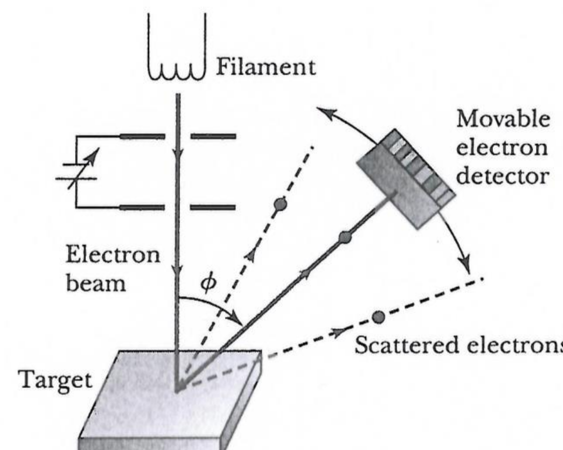
- During the experiment an accident that permitted air to enter the vacuum chamber, and an oxide film formed on the metal surface.
- To remove this film, Davisson and Germer baked the specimen in a high temperature oven, almost hot enough to melt it
- Unknown to them, this had the effect of creating large single-crystal regions with planes that were continuous over the width of the electron beam
- Distinct maxima due to Bragg scattering was observed in scattered beam

Results



$$2d\sin\theta = n\lambda$$

$$\lambda = \frac{1.226}{\sqrt{V}}$$

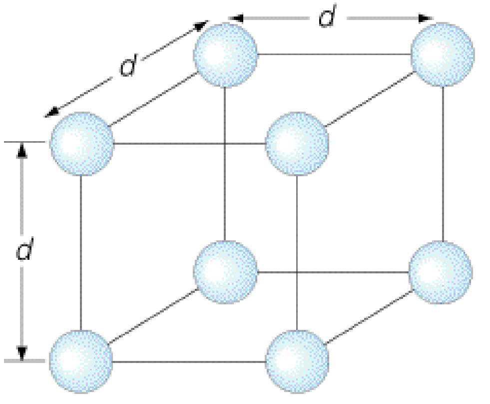


- The angular positions of the maxima depended on the accelerating voltage V used to produce the electron beam
- This was not the effect that they had been looking for, but they immediately recognized that the electron beam was being diffracted
- They had discovered a very direct experimental confirmation of the wave hypothesis

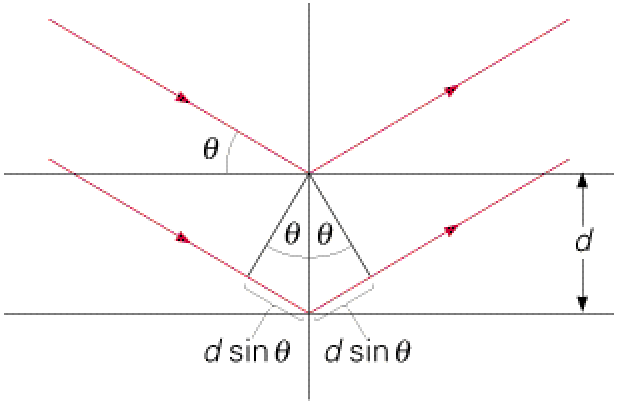
$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{1.226}{\sqrt{V}}$$

$$2d\sin\theta = n\lambda$$

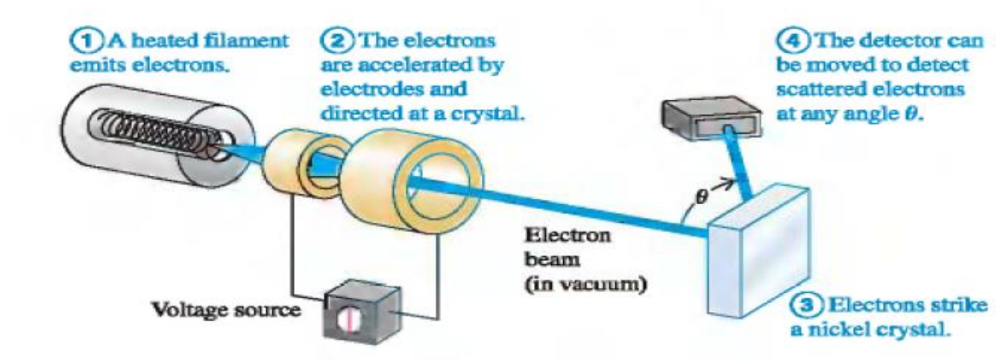


Atomic cubic lattice



Observation by Davisson and Germer

A beam of 54 eV electrons was directed perpendicularly at the nickel target and a sharp maximum in the electron distribution occurred at an angle of 50^0 with the original beam. The spacing of the planes measured is 0.091 nm. Validate de Broglie hypothesis.



If 50^0 is angle between incident and diffracted beam then angle of incidence and scattering will be 65^0

$$\theta = 65^0$$

For nickel target interlayer spacing is

$$d = 0.091\,nm$$

Using Bragg diffraction law

$$\lambda = 2d\sin\theta$$

$$\lambda = 2 * 0.091 * \sin(65)$$

$$\lambda = 0.165\text{nm}$$

From de Broglie hypothesis

$$\lambda = \frac{h}{p}$$

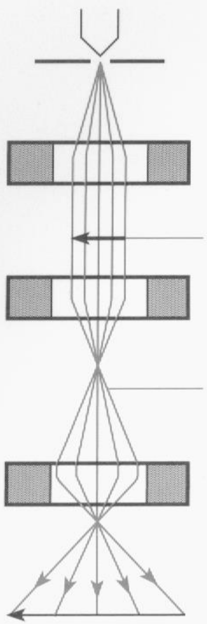
$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.64 \times 10^{-34}}{\sqrt{2 * 9.1 \times 10^{-31} * 54 * 1.6 \times 10^{-31}}}$$

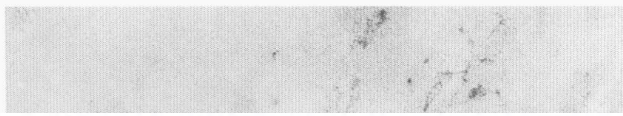
$$\lambda = 0.166\, nm$$

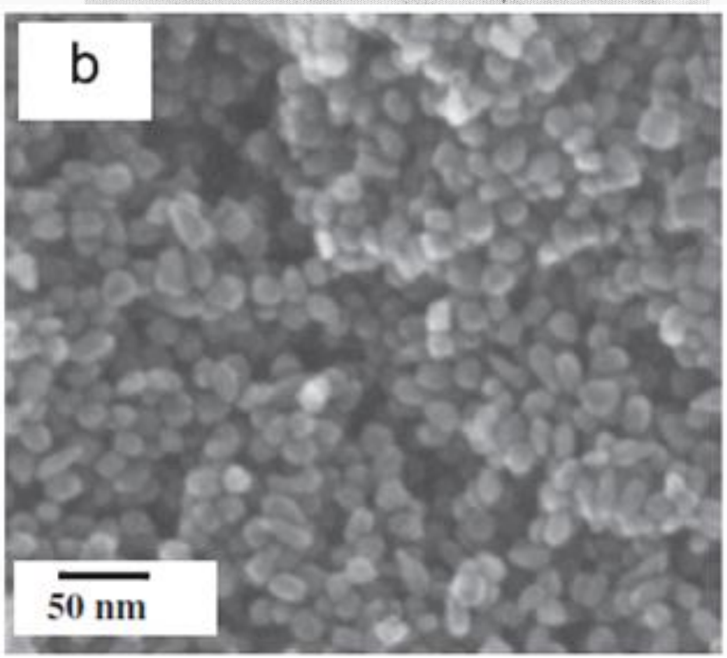
Hence de Broglie hypothesis is correct
Electrons can go through diffraction

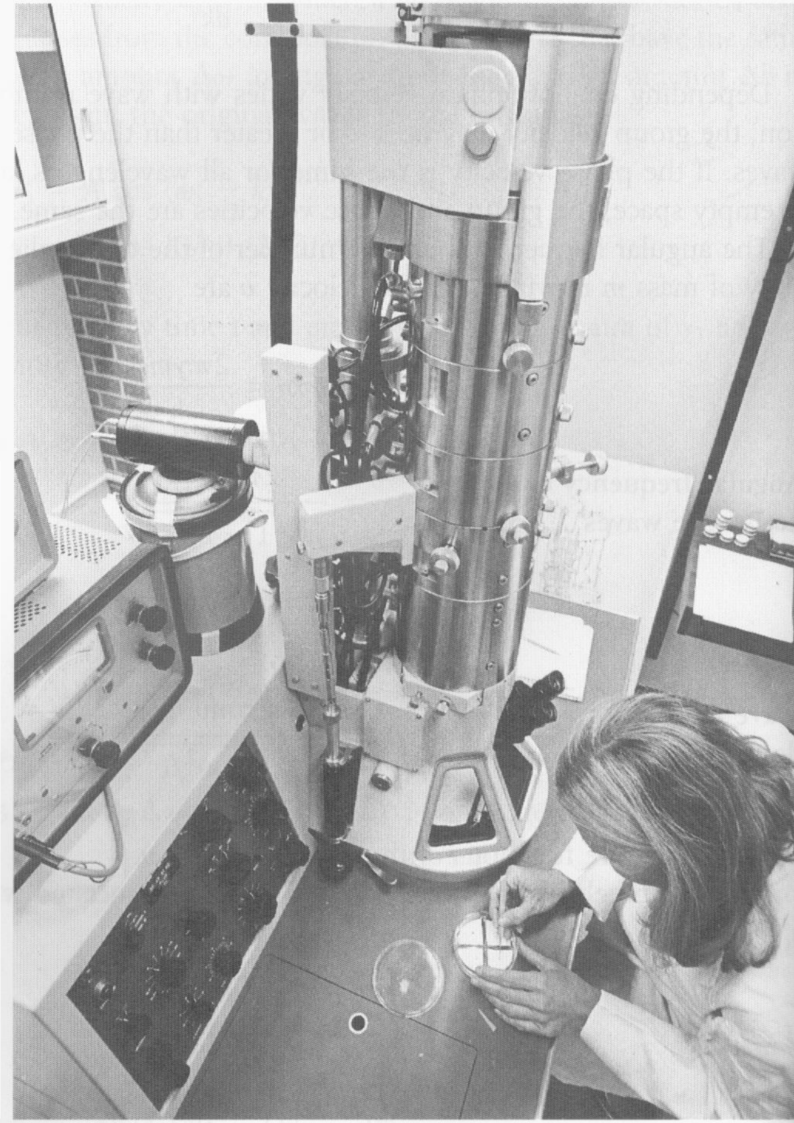
Electron microscope



ɔptical microscope, the elec-
ı microscope can produce
rp images at higher magnifi-
ons. The electron beam in an
tron microscope is focused
nagnetic fields.







An electron microscope.

SEM of an ant head

Why do you need electron microscope when you have a optical microscope

To resolve smaller feature sizes

Microscope resolution

$$d_0 \propto \lambda$$ (Approximately)

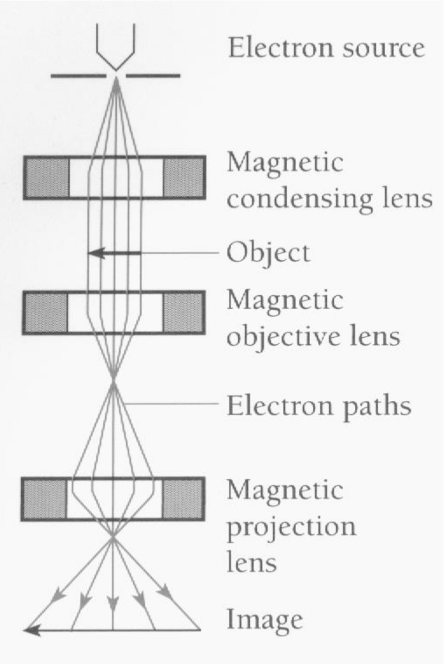
For optical microscope, $\lambda \approx 500\, nm, \quad d_0 \approx 500\, nm$

For electron microscope, $$\lambda = \frac{1.226}{\sqrt{V}}\, nm$$

$$If\ V = 1\, MeV = 1 \times 10^6$$

$$\lambda = 0.001226\, nm$$

$$\lambda = 1.226\ pm$$

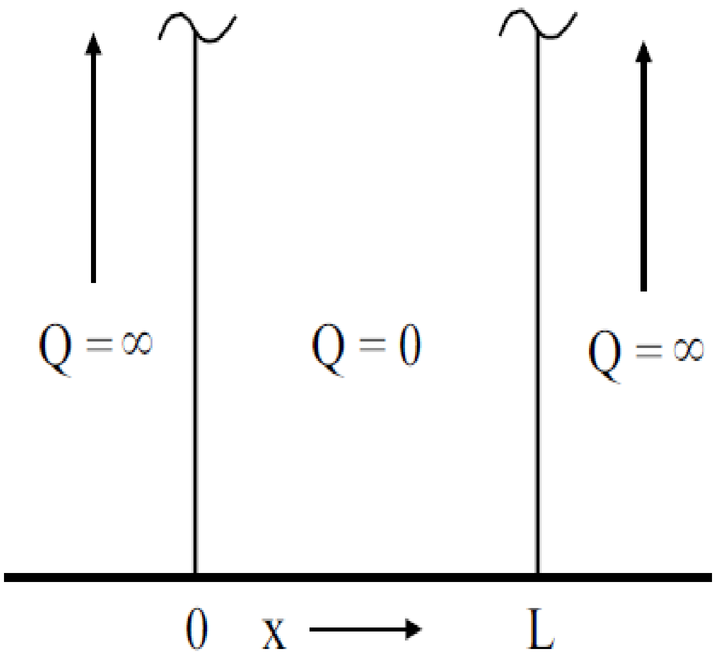


Hence resolution is improved

(Direct consequence of de Broglie relation)

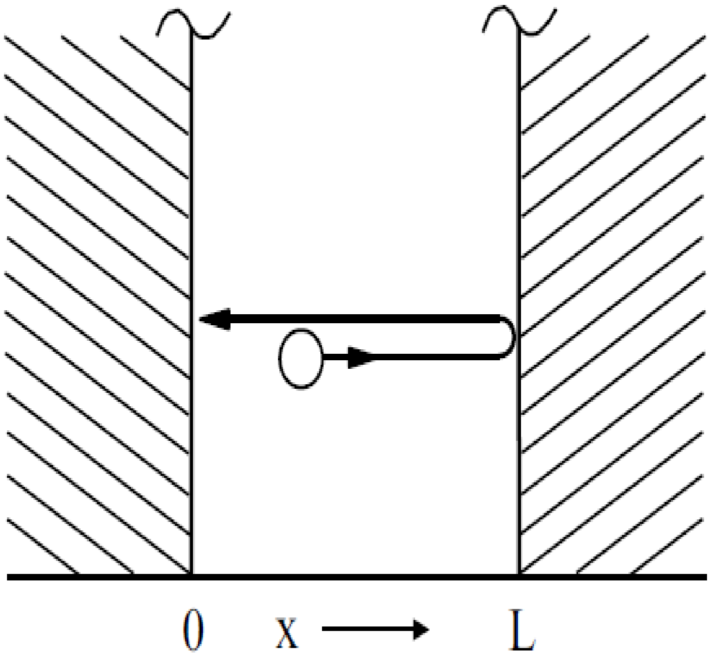
Particle in a Box

Definition of a perfect one D box



- Walls are infinitely high
- Infinitely massive
- Completely impenetrable
- No air resistance inside
- Potential energy is zero inside
- Potential energy is infinite outside
- Length of the box is *L*

The 'Classical' Case



A ball in a perfect 1D racquetball court

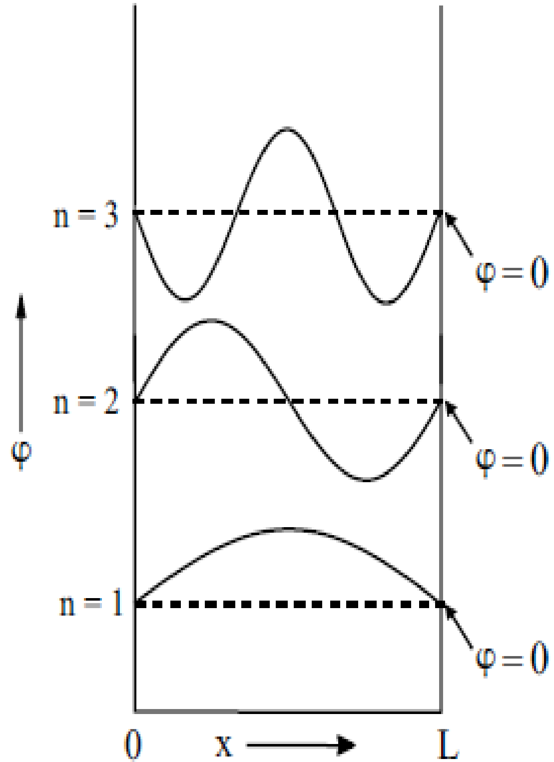
- Ball bounces back & forth between the wall
- This ball can take any energy level

The ‘Quantum’ Case

What happens if

- Length of court changes to nm level
- mass changes to electronic mass

Wave functions must be zero at the walls



$$\lambda = \frac{2L}{3}$$

$$\lambda = L$$

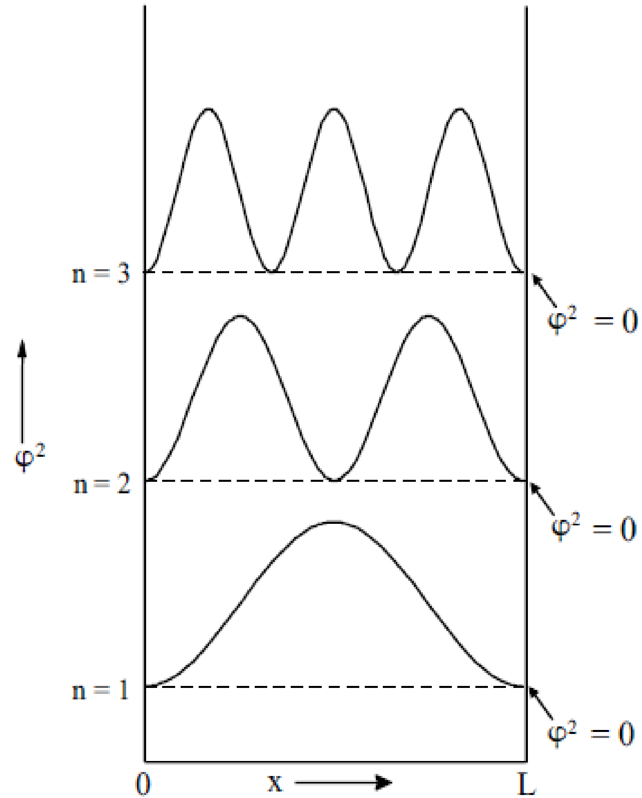
$$\lambda = 2L$$

$$\lambda_n = \frac{2L}{n}$$

- Three examples of wave functions, inside the box.
- Vertical axis is the amplitude of the wave function.
- Dashed line shows zero level for e- wave function which must be zero outside the box.
- Wave functions are continuous inside the box.

What are the n = 4 and n = 5 cases

Nodes are the points where the wave function crosses zero



- Squares of the first three wave functions, for the particle in a box.
- Vertical axis is the amplitude squared.
- Dashed line shows zero level.
- Square of the wave functions are always positive because they represent probabilities.
- However functions shown before can be positive or negative.

Energies are quantized

$$Kinetic\ Energy, E = \frac{1}{2}mv^2$$

$$Momentum,\ p = mv$$

$$E = \frac{p^2}{2m}$$

$$U\sin g\ de\ Broglie,\ p = \frac{h}{\lambda}$$

$$E = \frac{h^2}{2m\lambda^2}$$

For particle in a box,

$$\lambda_n = \frac{2L}{n}$$

$$E_n = \frac{h^2}{2m}\left(\frac{n}{2L}\right)^2$$

$$E_n = \frac{n^2 h^2}{8mL^2}; n = 1,2,3,.........$$

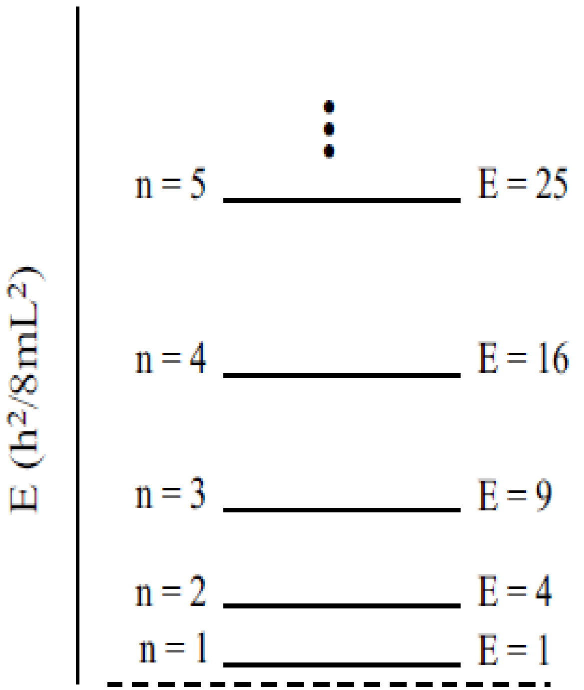
n is quantum number

E is quantized energy level

A Discreet set of energy levels

Therefore discrete set of energy levels for given mass m and given box length L. As n takes values 1, 2, 3 ….

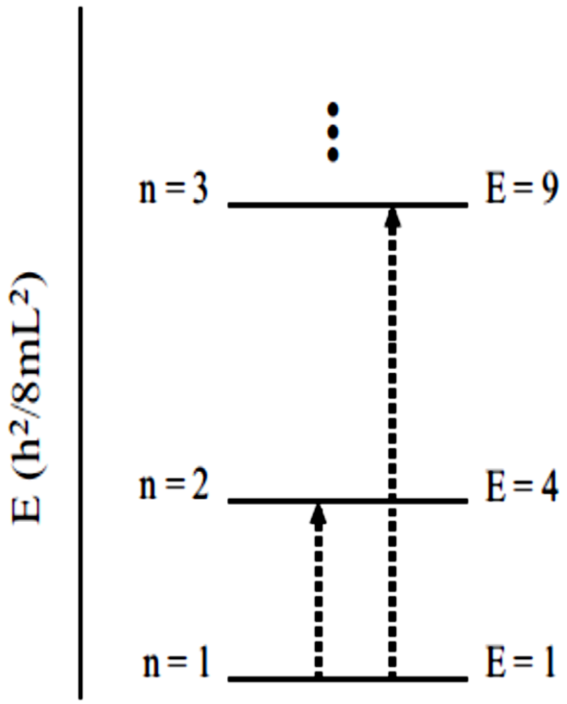
Energy levels are $$\frac{h^2}{8mL^2}, \frac{4h^2}{8mL^2}, \frac{9h^2}{8mL^2} ......$$



Particle in a box energy levels

- E increases with square of n (quantum number)
- E is plotted units of $h^2 8mL^2$
- E = 0 level is not permissible

Photon absorption



- Absorption of photon can take electron to higher energy levels
- For photon absorption energy must match ΔE

Particle in a box energy levels

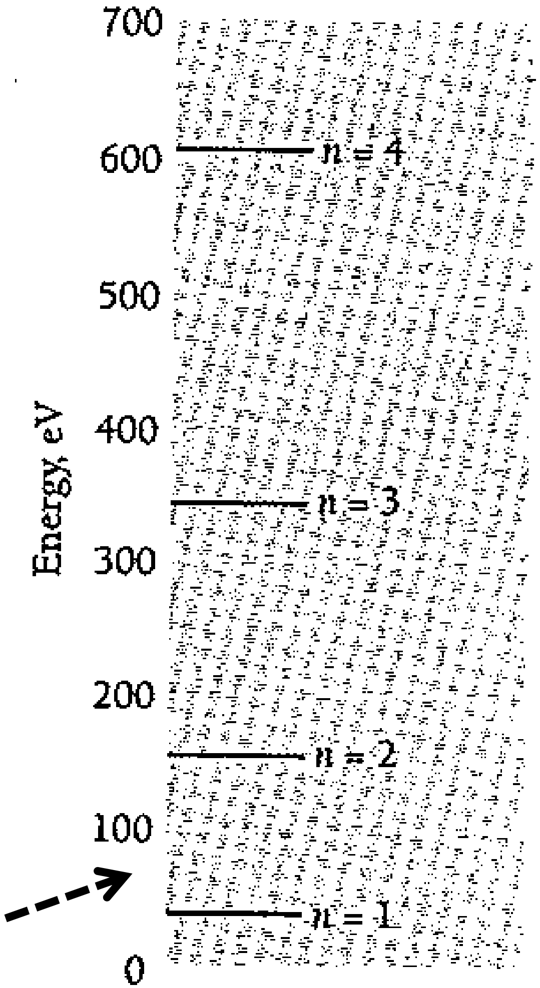
Example:

An electron is in a box 0.1 nm across, which is the order of magnitude of atomic dimensions. Find its permitted energies.

$$E_n = \frac{h^2}{2m}\left(\frac{n}{2L}\right)^2$$

$$E_n = \frac{n^2\left(6.64\times10^{-34}\right)^2}{8\left(9.1\times10^{-31}\right)\left(1.0\times10^{-10}\right)^2}$$

$$E_n = 38\, n^2 eV$$



For n = 1 minimum energy will be $E_1 = 38\, eV$

Energy levels of an electron confined in a box 0.1 nm length

What happens if a 10 g marble is kept inside a 10 cm box. Find the energy levels.

$$E_n = \frac{n^2\left(6.64\times10^{-34}\right)^2}{8\left(1.0\times10^{-2}\right)\left(1.0\times10^{-1}\right)^2}$$

$$= 5.5\times10^{-64}\, n^2\, J$$

This means $E_1 = 5.5\times10^{-64}\, J$ too small to detect

$$E_n - E_{n-1} = 5.5\times10^{-64}\left(n^2-(n-1)^2\right)J$$

$$= 5.5\times10^{-64}\left(2n-1\right)J$$

Therefore ΔE is too small to detect for classical objects

Particle in a Box

Summary

- This is a hypothetical case
- One of the few cases , **QM** problems can be solved analytically
- Useful tool for complicated **QM** problems
- Useful for box dimension of the order of nm
- Lowest energy cannot be zero