Modern Physics

Lecture 9

Nature of a wave

- A wave is described by frequency ν, wavelength λ, phase velocity u and intensity I
- A wave is spread out and occupies a relatively large region of space

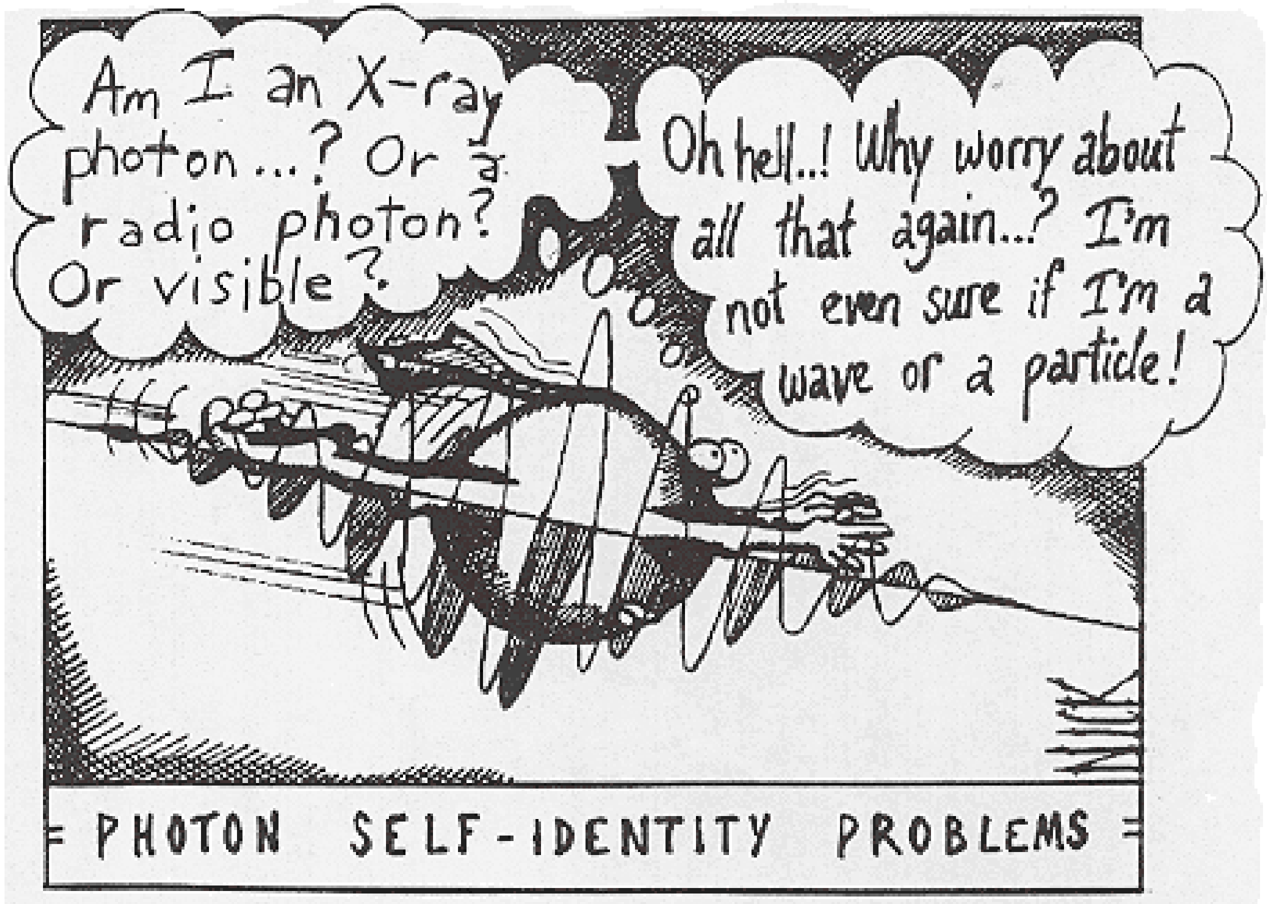
Nature of a particle

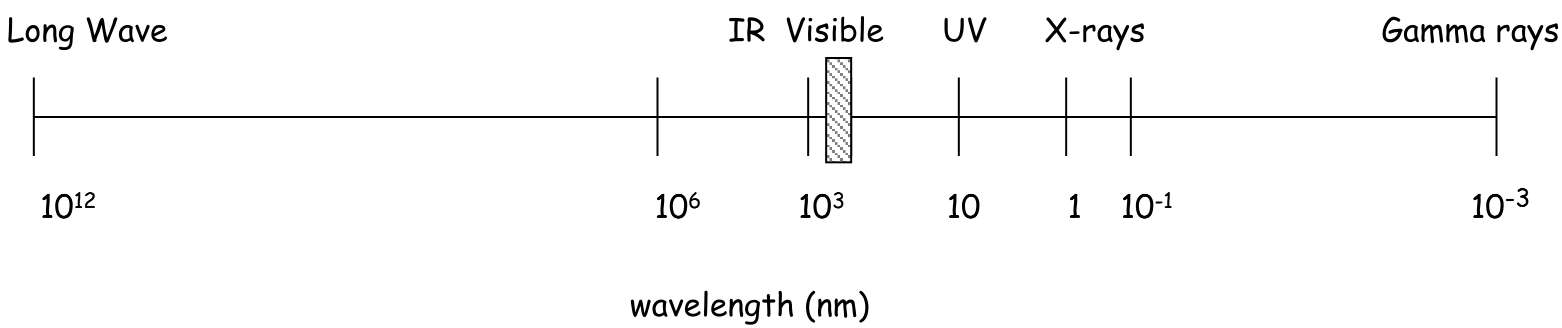
- A particle is specified by mass m, velocity v, momentum p, and energy E
- A particle occupies a definite position in space.

Light has dual nature

- Interference and Diffraction experiments showed the wave nature of light
- Blackbody radiation, Photoelectric effect and Compton Effect can be explained only by considering light as a stream of particles

So is light a wave or a particle?





Particles can show wave nature

LOUIS DE BROGLIE

“ If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature likes symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances”

wavelength

$$\lambda = \frac{h}{p}$$

momentum

Relates a particle-like property (p)
to a wave-like property (λ)

Kinetic Energy of particle

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\hbar^2 k^2}{2m}$$

If the de Broglie hypothesis is correct, then a stream of classical particles should show evidence of wave-like characteristics……………………………………..

What could be such particles

Electron proton etc

What happens if an electron is accelerated by certain voltage V

$$de\ Broglie\ Wavelength\ \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$Electron\ kinetic\ energy\ E$$

$$Accelerated\ by\ a\ potential\ difference\ V$$

$$Therefore\ \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

$$Substituting\ for\ h, m, and\ e\ we\ get$$

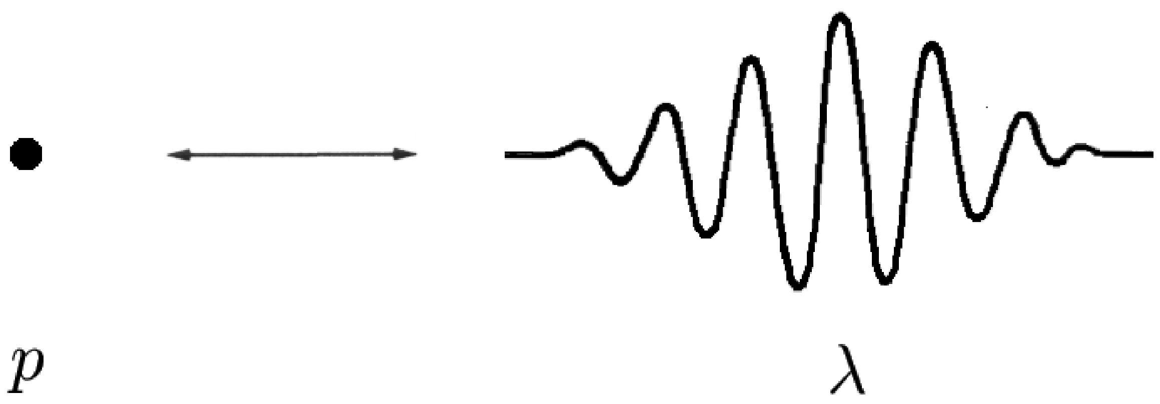
$$\lambda = \frac{6.625\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.602\times10^{-19}\times V}} \qquad \lambda = \frac{1.226}{\sqrt{V}}$$

$$If\ V = 100\ volts,\ \lambda = \frac{1.226}{\sqrt{100}} = 0.1226\ nm$$

de Broglie Wavelength

The Wave associated with the matter particle is called Matter Wave

The Wavelength associated is called de Broglie Wavelength



Example: de Broglie wavelength of an electron

Mass = 9.11 x 10^{-31} kg

Speed = 10^6 m / sec

$$\lambda = \frac{6.63 \times 10^{-34}\,\text{Joules}\cdot\text{sec}}{(9.11 \times 10^{-31}\,\text{kg})(10^{6}\,\text{m/sec})} = 7.28 \times 10^{-10}\,\text{m}$$

In which region of E spectrum does this wavelength belongs to ?

X-rays

Example: de Broglie wavelength of a ball

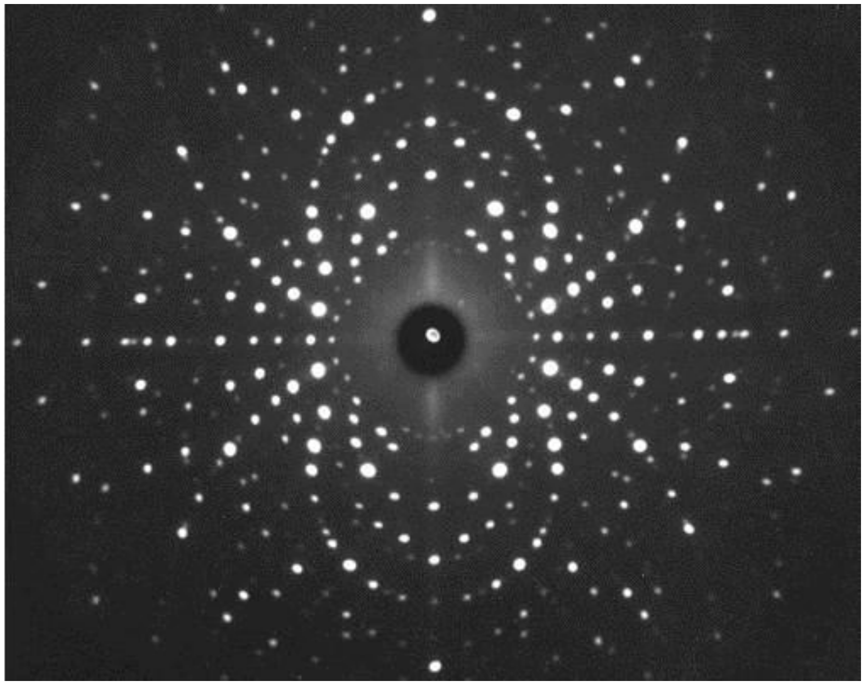
Mass = 1 kg

Speed = 1 m / sec

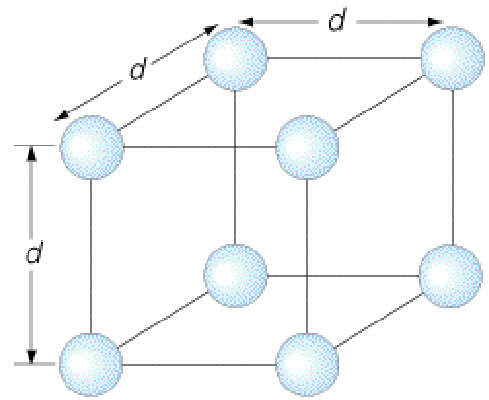
$$\lambda = \frac{6.63 \times 10^{-34}\,\text{Joules} \cdot \text{sec}}{(1\,\text{kg})(1\,\text{m/sec})} = 6.63 \times 10^{-34}\,\text{m}$$

Examples of Scattering

Bragg Scattering

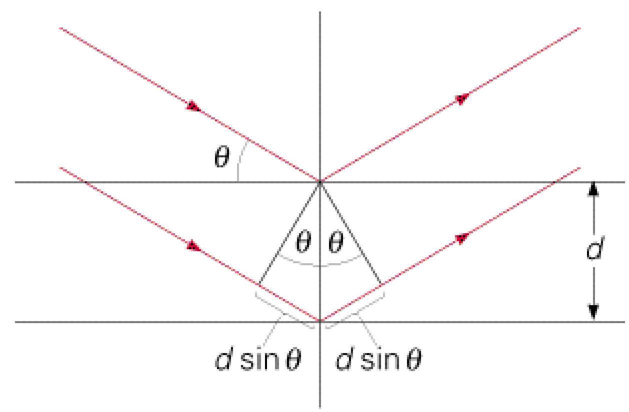


Bragg scattering of the atoms in a crystal using beam of electrons

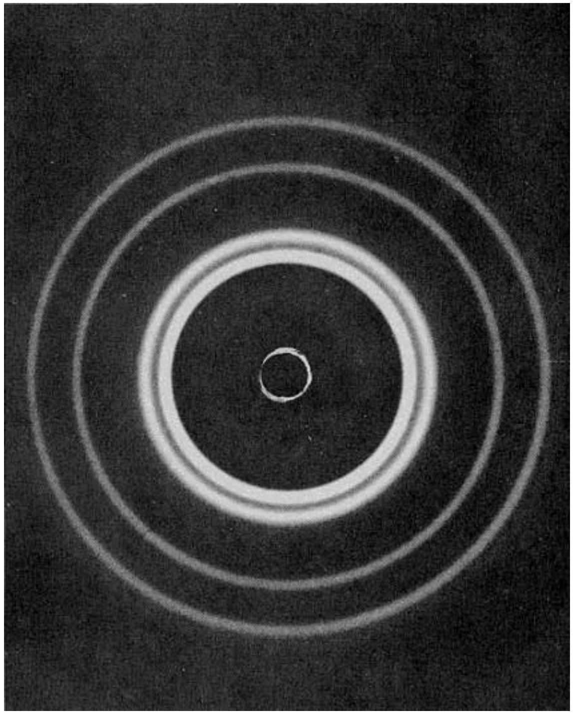


Atomic cubic lattice

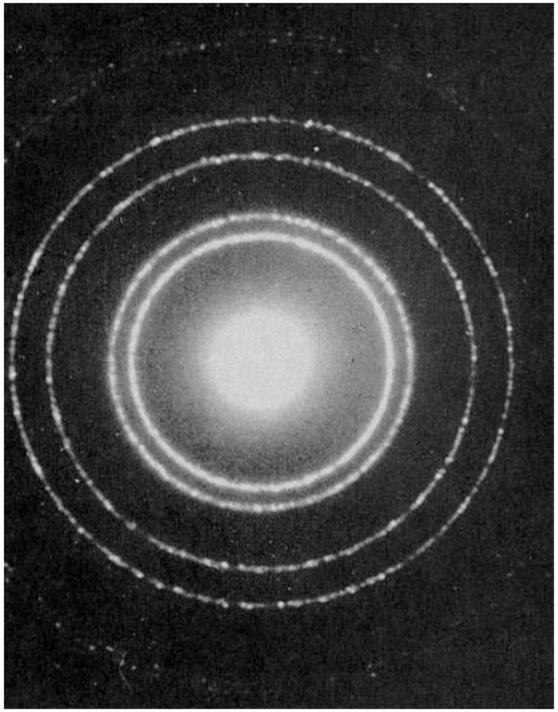
$$2d\sin\theta = n\lambda$$



The Diffraction



X-rays



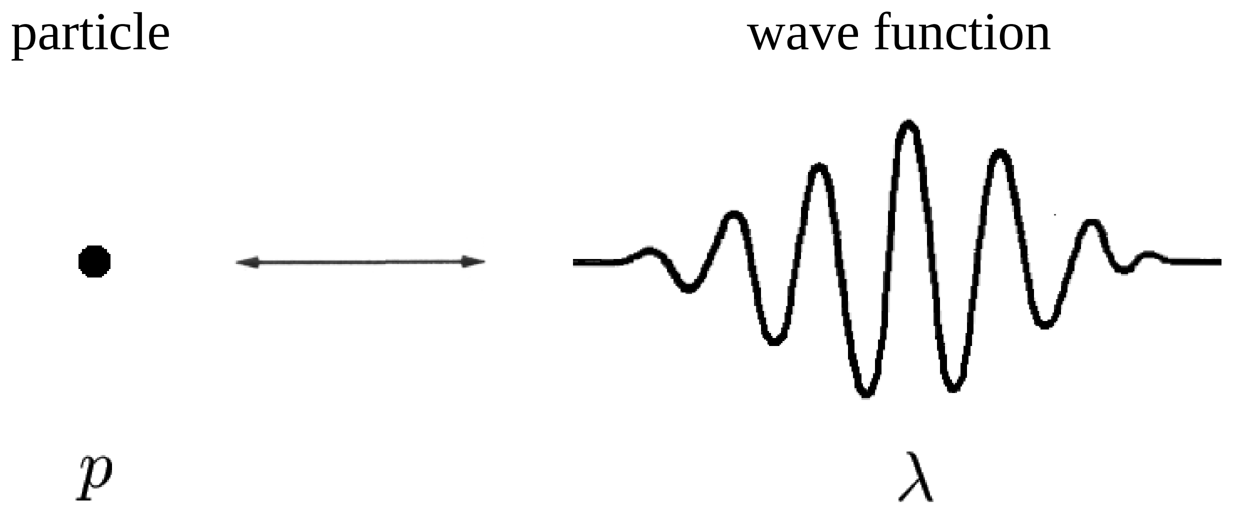
electrons

The diffraction patterns are similar because electrons have similar wavelengths to X-rays

Applications:

- **Bragg scattering**
- **Electron microscopes**
- **Electron- and proton-beam lithography**

Wave-Particle Duality



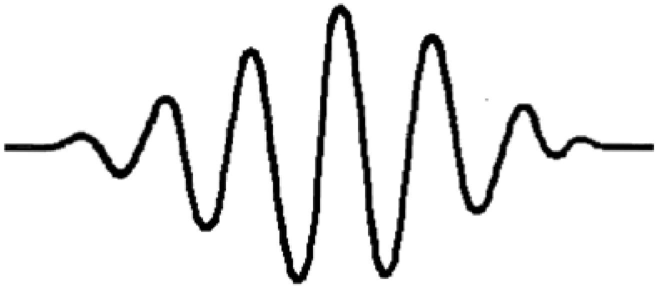
- **A particle can be defined by wave function**
- **This is called matter wave**

Wave Function

- Completely describes all the properties of a given particle

- Called ψ = ψ (*x*, *t*); is a complex function of position *x* and time *t*

$$\psi(x,t)=\psi_0 e^{i(kx-\omega t)}$$



Wave Function

$\psi(x,t)$ is not a measurable quantity

$|\psi(x,t)|^2$ is a measurable quantity, defines probability of finding a particle at time t and position x

Large value of $|\psi(x,t)|^2$ **means chances are high**

Small value of $|\psi(x,t)|^2$ **means chances are low**

Group and Phase velocity of a wave packet

A pure traveling wave is a function of ω and k as follows:

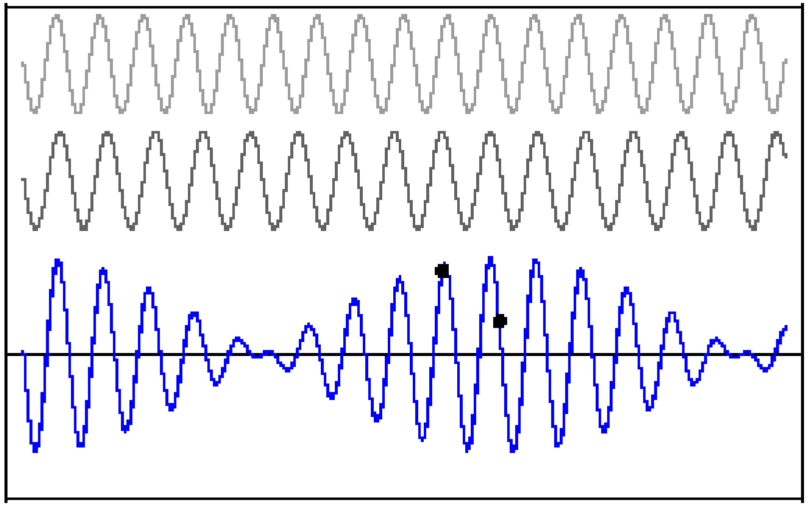
$$\psi(t,x) = \psi_0 \cos(\omega t - kx),$$

where ψ_0 is the maximum amplitude.

A wave packet is formed from the superposition of several such waves, with different ψ, ω, and k:

$$\psi(t,x) = \sum_n \psi_n \cos(\omega_n t - k_n x) \ .$$

Here is the result of superposing two sine waves whose amplitudes, velocities and propagation directions are the same, but their frequencies differ slightly. We can write:



$$A(t) = A_1(t) + A_2(t)$$

Where

$$A_1(t) = A\cos(\omega t - kx)$$

$$A_2(t) = A\cos((\omega + \Delta\omega)t - (k + \Delta k)x)$$

Resultant function will be,

$$A(t) = 2A\cos\frac{1}{2}[(2\omega + \Delta\omega)t - (2k + \Delta k)x]\cos\frac{1}{2}(\Delta\omega t - \Delta kx)$$

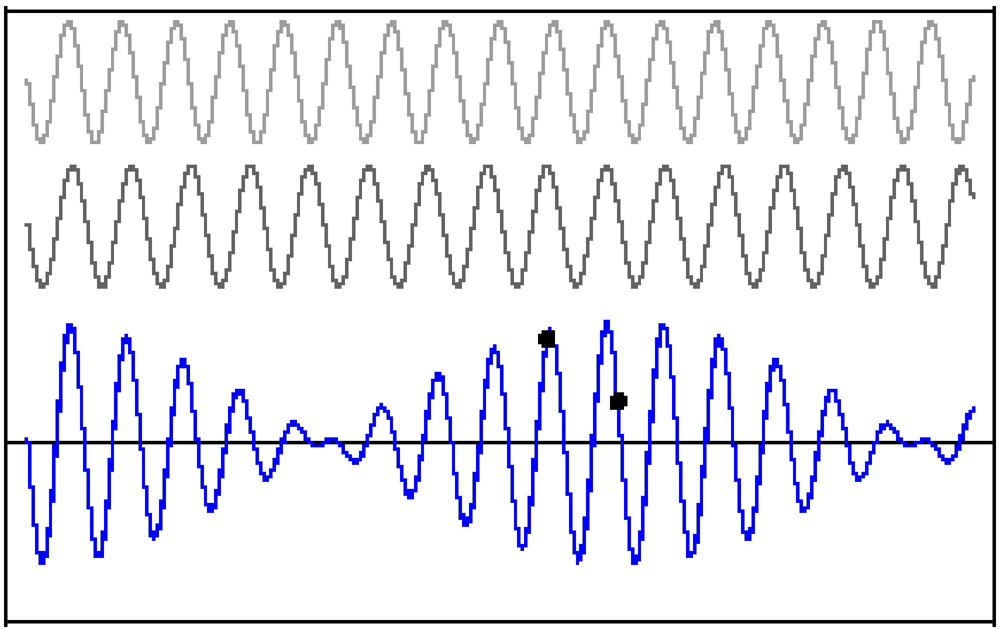
Considering $\Delta\omega << \omega$ **and** $\Delta k << k$ **we obtain**

$$2\omega + \Delta\omega \approx 2\omega$$

$$2k + \Delta k \approx 2k$$

Therefore resultant displacement

$$A(t) = 2A\cos(\omega t - kx)\cos(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x)$$



$$A(t) = 2A\cos(\omega t - kx)\cos(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x)$$

- **While the frequency of the first term is that of the phase,**
- **The frequency of the second term is that of the “envelope”, i.e. the group velocity.**

Phase velocity

$$v_p = \frac{\omega}{k}$$

Group velocity

$$v_g = \frac{\Delta\omega}{\Delta k}$$

The speed at which a given phase propagates does not coincide with the speed of the envelope.

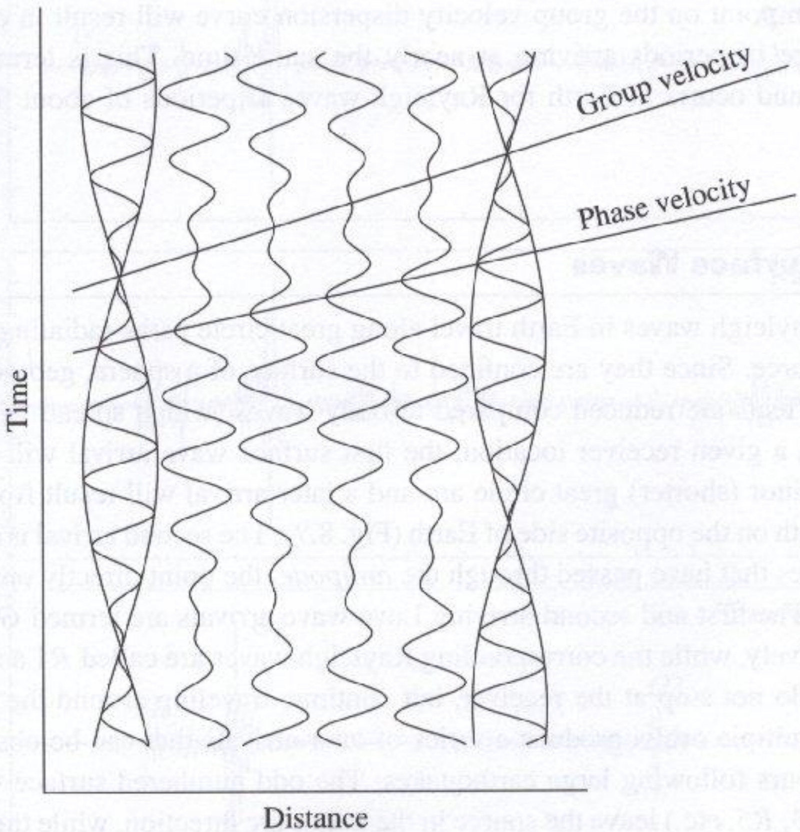


Fig. 8.5. The sum of two waves of slightly different frequencies results in a modulated wave. The group velocity is the velocity of the wave packets; the phase velocity is the velocity of the individual peaks.

The group velocity is the velocity with which the envelope of the wave packet, propagates through space.

The phase velocity is the velocity at which the phase of any one frequency component of the wave will propagate. You could pick one particular phase of the wave (for example the crest) and it would appear to travel at the phase velocity.