5- Physical Pendulum

The experiment is carried out in multiple stages by slightly modifying the experimental set up. Each set up has definitive objective as given as below

Objectives of the experiment (Setup – 1)

- To measure the oscillation period of a simple pendulum.
- Determining reduced pendulum length

Apparatus:

- Physical pendulum, Rotary Motion Sensor S, Sensor-CASSY, weights.

Principle:

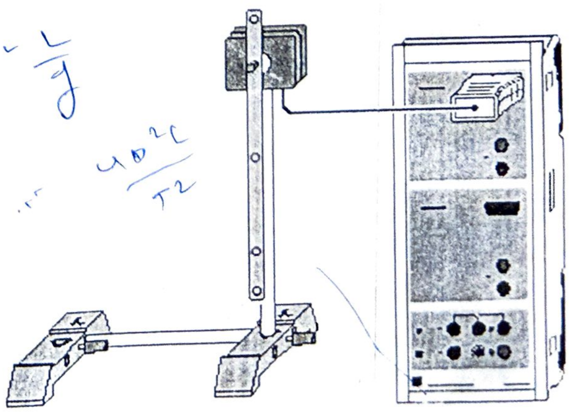
When a pendulum is displaced sideways from its resting equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position. When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth. The time for one complete cycle, a left swing and a right swing, is called the period. The period depends on the length of the pendulum, and also to a slight degree on the amplitude, the width of the pendulum's swing.

The period of a pendulum or any oscillatory motion is the time required for one complete cycle, that is, the time to go back and forth once. If the amplitude of motion of the swinging pendulum is small, then the pendulum behaves approximately as a simple harmonic oscillator, and the period T of the pendulum is given approximately by

$$T=2\pi\sqrt{\frac{L}{g}}$$

$T^2 = 4\pi^2 \frac{L}{g}$

$g = \frac{4\pi^2 L}{T^2}$



Where;
T = period of oscillation
L = pendulum length
g = acceleration due to gravity

Procedure:

Step 1: Initial setup

- Connect the sensor CASSY with computer.
- Attach pendulum to the axle of motion sensor and to the sensor CASSY.
- Open CASSY Lab software and load settings for respective experiments.

Step 2: Oscillations of a rod pendulum

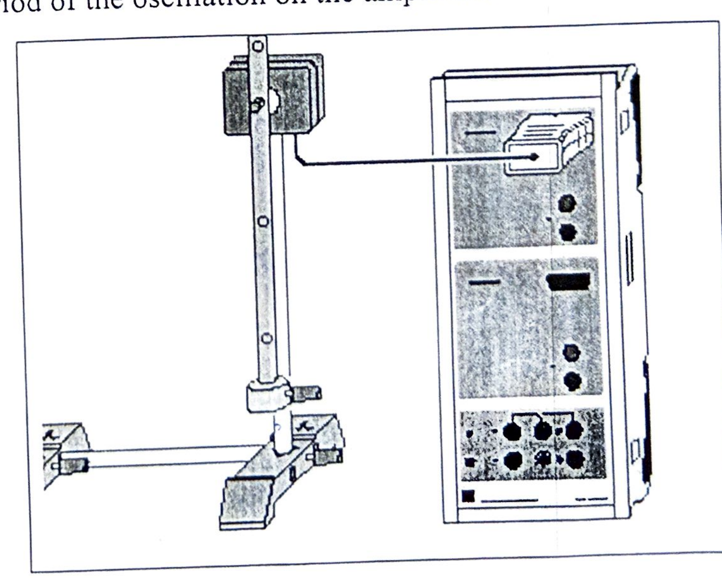
- Load settings for P1513.
- Define the zero point in the equilibrium position of the pendulum (→ 0 ← in Settings αA1)
- Deflect the pendulum by approx. 5° only and release
- Start the measurement with ⏲. The measurement will stop automatically after 10 s.
- Draw vertical lines at 10th oscillation and note down the value for 10 oscillations.
- Period of oscillation can be determined from the above value by dividing it by 10.
- Save the experiment by clicking "SAVE' button.
- Repeat the measurement without the mass or with a further mass added.

Observation and Evaluation 1:

Find out the reduced length from the time period and acceleration due to gravity.

<u>Objectives of the experiment (Setup – 2)</u>

- Analyze the dependency of the period of the oscillation on the amplitude



Experimental set-up – 2
Experiment Procedure:

Step 1: Initial setup (as drawn above)

- Connect the sensor CASSY with computer.
- Attach pendulum to the axle of motion sensor and to the sensor CASSY.
- Attach a mass at the bottom of the pendulum.
- Open CASSY Lab software and load settings for respective experiments.

Step 2: Oscillations of a rod pendulum

- Load settings for P1514.
- Define the zero point in the equilibrium position of the pendulum (correct→ 0→correct offset→ in Settings A_{A1})
- Deflect the pendulum by approx. 30° only and release.
- Start the measurement with once the oscillation settled to a constant value.
- Stop the measurement with as soon as amplitude fall below 5°.
- Right click on the graph, go to "Fit Function" → "Free Fit" → "Continue with Range Marking"
- Mark the required curve section from the graph.
- The result will be displayed at the bottom left corner, A
- Save the experiment by clicking "SAVE' button.

Observation and Evaluation 1:

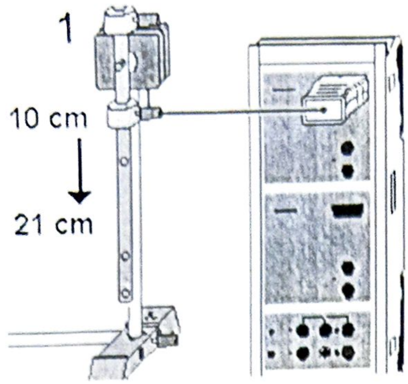
During the measurement, the amplitude decreases slowly. This causes a minor reduction in oscillation period. The theoretical connection between amplitude and period is
$T = T_0 \cdot (1 + (1/2)\, 2 \cdot (\sin(\alpha/2)\, 2 + (3/4)\, 2 \cdot (\sin(\alpha/2)\, 4 + (5/6)\, 2 \cdot (\sin(\alpha/2)\, 6 + \ldots))))$

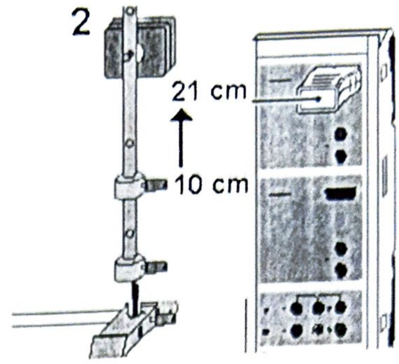
Perform free fit using CASSY Lab software and comment on the relation

<u>Objectives of the experiment (Setup – 3)</u>

- Determine the acceleration due to gravity by means of a reversible pendulum

Experimental set up-3





Experiment Procedure:

Step 1: Initial setup – Fig. 1.

- Connect the sensor CASSY with computer.
- Attach pendulum to the axle of motion sensor and to the sensor CASSY.
- Attach 1st mass at the top and 2nd mass at x= 10 cm from top of the pendulum.
- Open CASSY Lab software and load settings for respective experiments.

Step 2:

- Load settings for P1515.
- Define the zero point in the equilibrium position of the pendulum (correct→ 0→correct offset→ in Settings A_{A1})
- Deflect the pendulum by approx. 10° only and release.
- Record the measurement with ⏱ once the oscillation falls below 5°.
- Push the 2nd pendulum mass 1 cm down from x=10 cm and measure again. Repeat until x = 21 cm is reached.
- Arrange the setup as shown in Setup - 3.
- Repeat the measurement by pushing the mass upward up to 21 cm.
- Right click on the graph → "set marker" → "Horizontal Line" → draw a line connecting two intersections.
- Note down the value displayed at bottom left, T = .898s.
- Save the experiment by clicking "SAVE' button.

Observation and Evaluation 1: In the graphic display, two intersections of the oscillation period curves can be seen. In both intersections the period of oscillation and therefore the reduced pendulum length are equal. It corresponds to the displacement of the two axes of rotation, which is L_r= 0.20 m. Compute the acceleration due to gravity from the known T, L_r values.

4- Young's Modulus

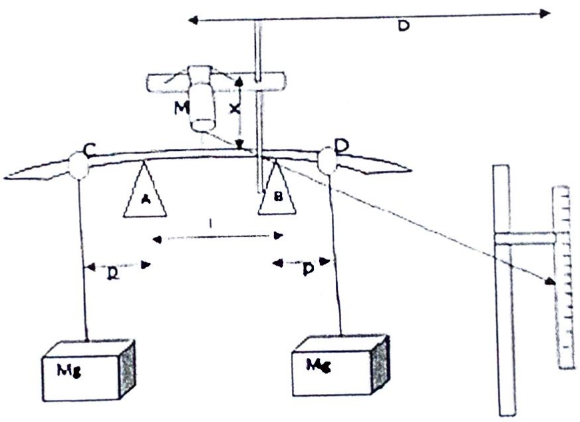
Objectives of the experiment:

Determine the Young's Modulus of the material of the bar subjected to uniform bending by measuring the elevation at the center using optic lever.

Apparatus:

Laser, Laser mount, Knife edges, slotted weights, Material bar

Principle:



$\frac{12}{6} = 2$

Young's Modulus setup

Consider a bar of thickness d and breadth b is supposed symmetrically between two knife edges at a distance l apart and loaded with equal weights Mg and Mg at the ends at equal distance p from each knife edges. The elevation at the midpoint is given by,

$$z = \frac{Mgpl^2}{8Y\left(bd^3/12\right)}$$

100 g / 4mm

The Young's Modulus of the material of the bar is

$\frac{g}{2} \times \frac{100}{h} \times$

100g 0.1 / 4mm 0.004 / 100

$$Y = \frac{3Mgpl^2}{2bd^3}$$

If the optical lever, scale and laser arrangement are used for measuring the elevation, the angle tilted by the bar is

$= 214.9875 \times 10^7$

$\frac{9.8 \times 0.09 \times (0.26)^2}{5 \times 10^{-2} \times (3 \times 10^{-9})}$

2.14×10^7 $\frac{68{,}796}{320}$ $\frac{3}{2} \times \frac{100}{h} \times$

$98 \times 9 \times 26$

$\frac{22{,}932 \times 10^{-5}}{40 \times 10^{-17}}$

$\frac{3}{6}$

$$q = z/x$$

Where x is the perpendicular distance to the legs of the optical lever, If y is to shift on the scale arranged at a distance D from the laser module of the optical lever then

$$q = y/2D$$

$$Z = xy/2D$$

Thus,

$$Y = \frac{3MgpDl^2}{xybd^3}$$

Procedure:

With the weight hanger of mass W_0 alone to the bar, note the reading on scale corresponding to the laser spot.

Add the mass M in step up to 7m and the scale readings are noted.

The experiment is repeated by unloading the masses in steps and the mean value of the scale reading for each mass is noted.

From this, the mean shift y for a mass M and the pDl^2/y are calculated. The breadth (b) of the bar is measured using Vernier calipers. Its thickness (d) is measured using screw gauge and hence the Young's modulus of the given material of the bar is calculated using equation (2).

Observations and Calculations:

Trial No	Distance between		Mass suspended m(kg)	Scale reading (cm)			Shift for M(=4m)	Mean shift	pDl^2/y
	Scale and optical lever D(m)	Knife edges l(m)		Loading	Unloading	Mean	Y(cm)	Y(cm) y_1-y_2	(m^2)
1			W0 W0+M W0+2M W0+3M W0+4M W0+5M W0+6M W0+7M	X0 X1 X2 X3 X4 X5 X6 X7		X4-X0 X5-X1 X6-X2 X7-X3			

Results:

Thickness of the material of the bar "d" = ... mm, Breadth of the material of the bar "b" = ...mm, Young's Modulus of the material Y=...............N/m^2

6- Doppler Effect

Objectives of the experiment:

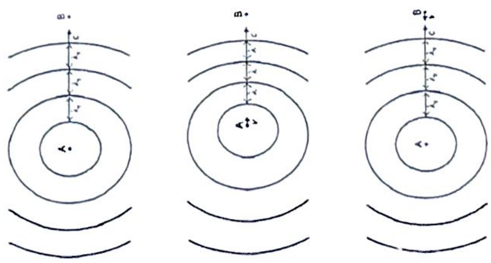
- Determine the velocity of the moving trolley (both in the forward and backward direction)
- Measuring the change of frequency perceived by an observer at rest as a function of the velocity v of the source of ultrasonic waves.
- Validating the proportionality between the change of frequency Δf and the velocity v of the source of ultrasonic waves.
- Determining the velocity of sound c in air.

Apparatus:

- Ultrasonic transducers, Generator, Amplifier, Trolley, Counter, Stopclock, Metal rail

Principle:

This experiment is aimed towards understanding the Doppler Effect phenomena. Doppler Effect can be described as the effect produced by a moving source of waves in which there is an apparent upward shift in frequency for observers towards whom the source is approaching and an apparent downward shift in frequency for observers from whom the source is receding.



The experimental set up consists of two equal transducers, one serves as transmitter (sound source) and the other as receiver (observer) depending on their connection. One transducer is attached to a trolley with electric drive, the other is fixed to a stand rod. The frequency of the observed signal is measured with a high-resolution digital counter. In order to determine the velocity, v, of the moving transducer, the time, Δt, in which the trolley covers a given distance, Δs, is measured with a stop clock.

$$\mathrm{v} = \frac{\Delta s}{\Delta t} \qquad (1)$$

Here v is the velocity of the trolley; Δs = distance travelled by the trolley in a time, Δt

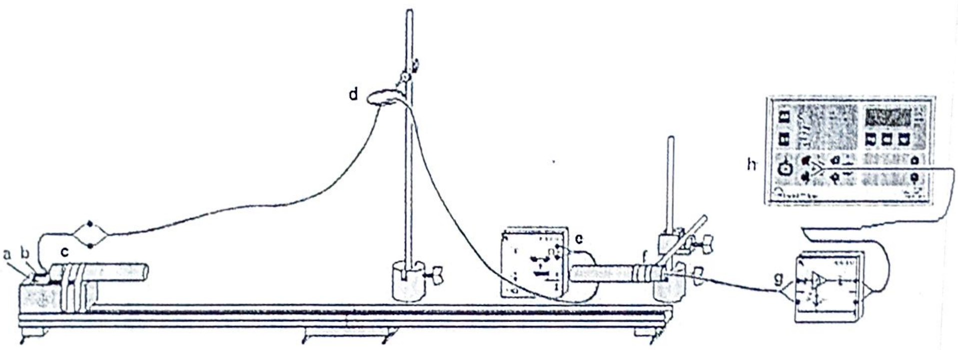
The wave fronts starting from a sound source with a frequency, f_0, and a fixed observer in the axis receives a wave at a different frequency, f, which depends on the velocity of the emitter

$$\Delta f = f - f_0 = f_0\left(\frac{v}{c}\right) \tag{2}$$

Where, Δf = change in frequency, f_0 = rest frequency, f = moving frequency, c = velocity of sound wave

Procedure:

Block diagram of a typical experimental set up with the relevant connections are shown below:



In the above diagram, c = source/emitter transducer; f = receiver transducer; a = velocity controller; b = three way switch; e = ultra sound generator; g = signal amplifier; h = frequency counter.

This experiment be performed by following the steps given below:

Step 1: Adjusting resonance frequency with oscilloscope

- Keep both transducers at same height and 1 m away.
- Set the generator frequency to 40 kHz.
- Connect a cable from output of generator to channel one of oscilloscope.
- Connect a cable from amplifier output to channel two of oscilloscope.
- Observe the frequency reading in both channels and make it approximately equal by adjusting the gain regulator on the amplifier.
- Once the frequency in channel one and two are almost equal remove all connections to oscilloscope and start performing the experiment.

Step 2: Measuring the change of frequency with the source of ultrasonic waves moving

- Set the velocity of the trolley using the knob. Let it be higher velocity first.

- Start the trolley with start/stop switch.
- Measure the time "Δt" taken to cover distance "Δs" (1m) using a stopwatch.
- Measure the rest frequency f_0 with counter.
- Press the trolley switch and move towards right. Note down the frequency f while moving.
- Stop the trolley and note down the rest frequency f_0 again.
- Press the trolley switch and move towards left. Note down the frequency f while moving towards left.

38.453

Step 3:

- Set the velocity of the trolley to a smaller value.
- First measure the velocity and repeat step 2 for three more velocities.

1. Observations and Calculations:

Time, Δt s	direction	Rest frequency, f_0 Hz	Frequency, f Hz
	Right		
	Left		
	Right		
	Left		
	Right		
	Left		
	Right		
	Left		

(a) Determining the velocity and change of frequency and verify the functional relationship between the two

Velocity v, $\Delta s/\Delta t$ m/s	$\Delta f = f - f_0$ Hz

Tabulate the change of frequency, $\Delta f = f - f_0$ as a function of the velocity 'v' of the trolley and plot against v *vs* Δf. Validate the relation between them and check whether it is consistent with the known theory.

(b) With the help of above plot, compute the velocity of sound in air. How does it compare with the theoretical values of the sound velocity?

$\Delta f = f_0 \, v/c$

4.2 3.6 4.0 3.4

$\frac{\Delta f}{f_0} x = y$

Fitting a Straight Line to a Set of Data

In some of the experiments, a straight line is to be fitted to the experimental data. This can be done using the **Least Square Fitting Method**. The method can be used to fit any polynomial, but you would require it only for fitting a straight line. In the following the method for this purpose is described.

Let $(x_i, y_i\ ;\ \ i = 1,2,\ldots N)$ be the given set of data from a measurement. You are required to fit a straight line to this data. The best straight line, according to the least square method, can be fitted as follows.

Let the equation of the straight line to be fitted be written as

$$y = mx + c$$

where the slope m and the intercept c are to be determined from the given data. This is done using the least square method which gives the following working formulae:

$$m = \frac{AC - DN}{A^2 - NB} \quad \text{and} \quad c = \frac{C - mA}{N}$$

where
$$A = \sum_{i=1}^{N} x_i \qquad B = \sum_{i=1}^{N} x_i^2 \qquad C = \sum_{i=1}^{N} y_i \qquad D = \sum_{i=1}^{N} x_i y_i$$

Example: Fit a straight line to the following data:

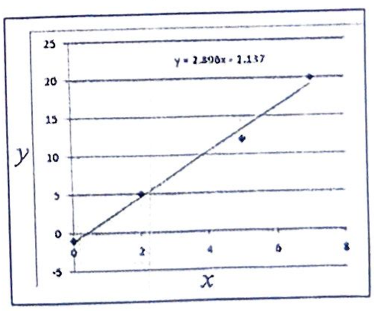
x	0	2	5	7
y	-1	5	12	20

x	x^2	y	xy
0	0	-1	0
2	4	5	10
5	25	12	60
7	49	20	140
Sum= 14	78	36	210
A	B	C	D

$$m = \frac{AC - DN}{A^2 - NB} = \frac{14X36 - 210X4}{14X14 - 4X78} = \frac{-336}{-116} = \frac{84}{29} = 2.8966$$

$$c = \frac{C - mA}{N} = \frac{36 - 14X336/116}{4} = \frac{36X116 - 14X336}{4X116} = \frac{-33}{29} = -1.1379$$

Thus the equation of the straight line is: $y = (84x - 33)/29$



A comparison of the given data and the fitted line:

x	0	2	5	7
y	-1	5	12	20
y_{fit}	-1.138	4.655	13.35	19.18

The straight line actually does not pass through any of the given points yet this the best fit line (please see the figure). In this case, the sum of the square of errors is minimum, i.e. $\sum (y - y_{fit})^2$, is minimum. Thus, it is not required that the straight line passes through any of the points for it to be the best fit line.

0.0044. 1.0044.

$$\frac{-0.0913}{-0.1519}$$

8- Energy and momentum in elastic and inelastic collisions

Objectives:

1. To calculate the velocities before and after collisions in case of both elastic and inelastic collisions.
2. To verify law of conservation of momenta and energies.

Experimental Setup:

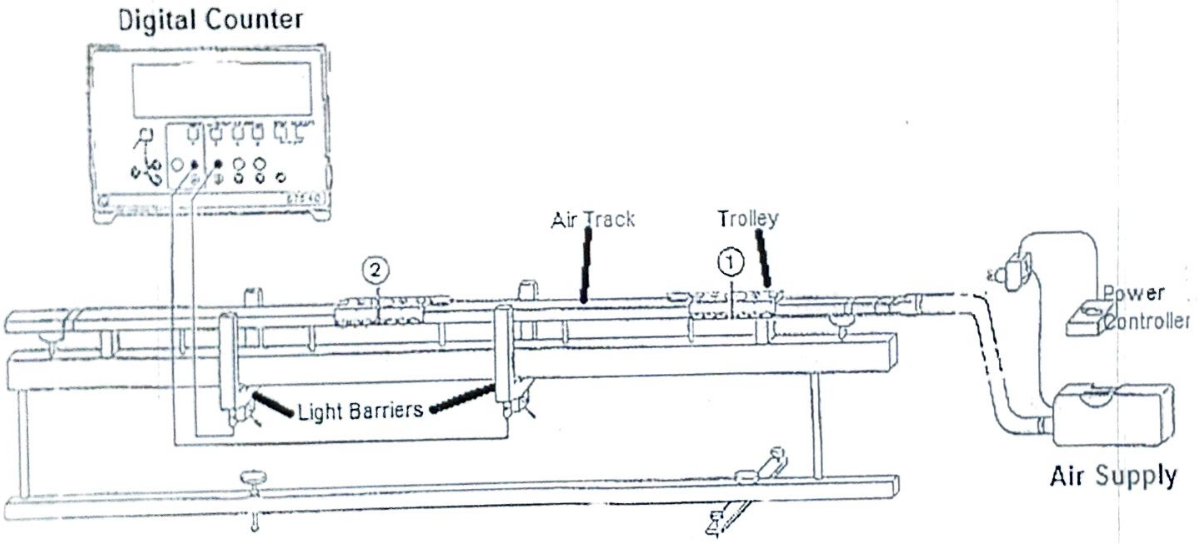
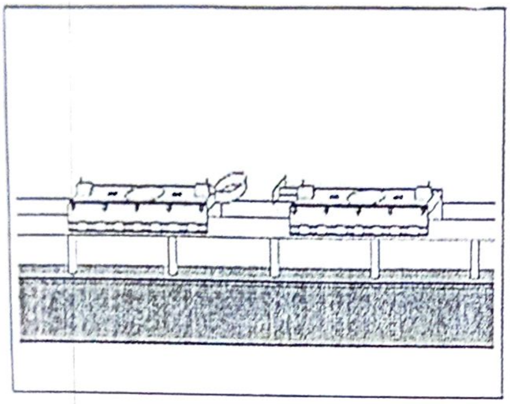


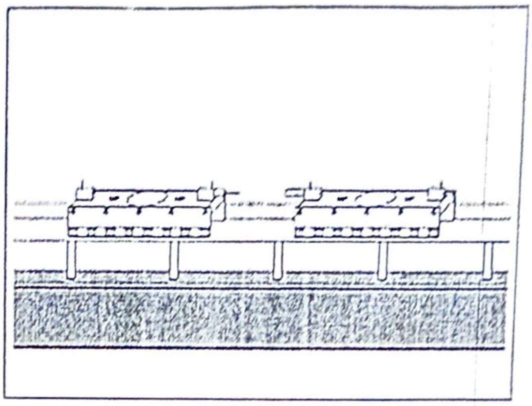
Fig.1.

- Arrange the setup as shown in Fig.1.
- Connect both light barriers to the digital counter.
- Connect the air supply to the power controller.
- Attach interrupter flags to both sliders.
- Weigh the sliders and write down the masses.
- Keep the sliders on the track and increase the air supply slowly until the slider moves on the track without friction.

In case of collisions, two sliders are mounted on the air track. The first slider is equipped with an impact spring and second one with an impact plate in case of elastic collisions as shown below. In case of inelastic collisions, the first slider is equipped with a needle and the second one with a tube filled with plasticine as shown below.



a). Elastic collision



b). Inelastic collision

Two light barriers are mounted at a distance to calculate the velocities before and after collision.

Theory:

Consider a single slider of mass M on a nearly frictionless air track. A small mass m is attached to the slider by a string passing over a pulley. The gravitational force on the small mass is equal in magnitude to its weight mg.
We can say that the gravitational force causes the entire system of mass $M+m$ to accelerate. Newton's second law can then be written as

$$mg = (M+m)a \qquad (1)$$

Thus, the acceleration is given by

$$a = \frac{mg}{(M+m)} \qquad (2)$$

An elastic collision is one in which the internal forces are conservative, so that both mechanical and internal energy separately stay constant throughout collision. In this case, equation for conservation of momentum is written as

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \tag{3}$$

Since kinetic energy is also conserved,

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \tag{4}$$

Solving above equations together yields the velocities before and after collision.

$$\begin{aligned} v_1' &= \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \\ v_2' &= \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2 \end{aligned} \tag{5}$$

In the case of inelastic collisions, the objects can collide and stick together. In inelastic collision, the kinetic energy is not conserved. Instead, some of the initial energy goes into other forms, such as heating the objects. In this case, the conservation of momentum is written as,

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v' \tag{6}$$

where v' is the final velocity of the two masses.

In this experiment, we record the values of initial and final velocities and explore the conservation laws. These values are compared with the theoretical values given above.

Experimental Procedure:

1. To start with, one of the slider is placed on the air bench. The mass, m and the width of the slider, w are recorded.

2. The slider is released from the holding magnet gently with minimum amount of force to keep the velocity constant. As the slider passes through the forked barrier, the light is obstructed, which is sent as a signal to the computer. The velocity of the slider to pass through the barrier is noted from the setup. Since the width is known, the velocity of the slider is calculated from $v=w/t$. The velocity of the slider recorded at various distances from the holding magnet should remain constant. This can be a check for us to see if the air bench is without friction.

3. A mass, m (say 10 gm) is attached to the slider by a string passing over a pulley at the end of the bench. This accelerates the slider with a uniform acceleration given by eqn (1). Velocities of the slider are recorded at increasing distances or time intervals from the holding magnet. This gives the acceleration of the slider at different time intervals, which is verified with the eqn (1).

4. Two sliders of masses $m1$ and $m2$ are placed on the air bench. Place the light barriers E and F between the two sliders as shown below. As the sliders cross the barriers, the velocities v_1 (v_E) and v_2 (v_F) are recorded. The first one to cross the light barrier is assigned v_E and consequently the second one v_F. After collision, v_1' and v_2' are recorded in a similar way as the sliders move away from each other in case of elastic collision.

5. In case of inelastic collisions, the sliders stick together and move in one direction recorded by a single light barrier. The experiment is repeated a number of times and the velocities are verified with the theoretical values.

6. The values of momentum and kinetic energy are obtained to verify conservation laws as given in the theory.

Observations: Table 1 (single slider):

S. No	m_1	v_1	Acceleration, a	Acceleration, a(theoretical)
1.				
2.				
3.				
4.				

Table 2 (collisions):

S. No	v_1	v_2	v_1'	v_2'	Theoretical values	
					v_1'(th)	v_2'(th)
1.						
2.						
3.						
4.						

Calculations:

0.2245
0.1905
~~0.1725~~
0.172
0.1805

Elastic collisions:

1. Calculate the percent difference between the theoretical and experimental values of v_1' and v_2' using equation 5.
2. Calculate the initial and final total linear momentum (P_i and P_f) and the kinetic energies (KE_i and KE_f).
3. Plot P_f as a function of P_i for each case. Determine the slope of this line and record it on your data sheet.
4. Plot KE_f as a function of KE_i for each case. Determine the slope of this line and record it on your data sheet.

Inelastic Collisions:

1. Calculate the percent difference between the theoretical and experimental values of v' using the following equation for theoretical values.

$$v' = \frac{m_1}{m_1 + m_2} v_1 + \frac{m_2}{m_1 + m_2} v_2$$

2. Calculate the initial and final total linear momentum (P_i and P_f) and the kinetic energies (KE_i and KE_f).
3. Plot P_f as a function of P_i for each case. Determine the slope of this line and record it on your data sheet.

Questions/Exercises:

1. What happens when the above experiments are performed with different masses on the sliders?
2. Suppose the air bench is tilted, what would be the effect on the conservation of momentum between the two sliders?
3. How does friction affect the conservation of momentum?
4. Show that, if: (1) the second slider has no initial velocity, (2) they collide inelastically, and (3) the sliders were of equal mass, then the ratio of final kinetic energy to the initial kinetic energy is ½.

1.006
0.928
0.078

1- Maxwell's Wheel

Objectives of the experiment:

- Determination of moment of inertia of the Maxwell's wheel.
- Verify the transformation of potential energy into translational and rotational energy.
- Validate the concept of conservation of energy

Apparatus:

- Maxwell Wheel setup; Forked Light Barrier with cable; Counter; Magnetic switch; Scale with pointers; Weighing balance; Vernier Caliper

Principle:

Maxwell's wheel experiment is used to demonstrate the conservation of mechanical energy. At any given height, the wheel possesses certain potential energy (*PE*). When it is released, the PE gets transformed into kinetic energy due to translational motion (KE_{trans}) and rotational motion (KE_{rot}).

$$PE = KE_{trans} + KE_{rot} \tag{1}$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \tag{2}$$

Where, I = Moment of inertia of the wheel, m = mass of the wheel, r = radius of the spindle v = velocity of the wheel, g = acceleration due to gravity, h = height position, ω = angular velocity.

In the standstill position, both v and ω are zero and hence the wheel possess only potential energy.

The linear and angular velocities can be related by the radius of the spindle;

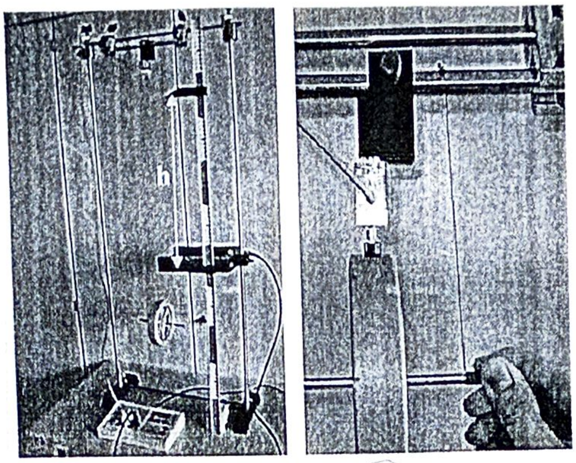
$$v = r.\omega \tag{3}$$

From equation 3 and 4, the moment of inertia, *I*, can be written as,

$$I = mr^2\left(\frac{2gh}{v^2} - 1\right) \tag{4}$$

Procedure:

The experiment is carried out in two steps:



Step 1: Measure the time "t" required for the height "h" between start to the light barrier

- Connect the key switch to port **E** of the counter and the light barrier to port **F** of the counter.
- Set the counter MODE as $t_{E \rightarrow F}$ and press start button of the counter
- Bring the wheel little up so that it presses the key switch
- Release the wheel and observe the counter (counter starts time counting)
- As soon as the wheel spindle pass the light barrier sensor, counting stops.
- Note down the time ***t*** as displayed in the counter

Note: Vary the wheel height (between the switch and the light barrier) from 15 cm to 55 cm in steps of 5 cm.

Step 2: Measure the velocity of the wheel/spindle "v" at the light barrier

- Remove key switch from port E and connect the light barrier to port E of the counter.
- Bring the wheel to the top position touching key switch
- Set the counter MODE as t_E and press start button of the counter
- Release the wheel
- Observe the counter displaying counting of time taken for the wheel spindle to pass through it "Δt
- Note down the value of "Δt" and calculate velocity $v = d/\Delta t$ (measure the diameter 'd' of the spindle)
- Calculate the inertia of the wheel using equation (4)

Observations and Calculations:

Height, h (cm)	Time, t (s)	Time, Δt (ms)	Velocity, v (m/s)	Inertia, I (Kg/m^2)

a) Examination of Dynamics:

- Plot a graph with time, *t*, on X-Axis and the distance, *h*, and velocity, **v**, on Y-Axis. What kind of relation you expect between ***t vs h*** and ***t vs* v**. What kind of functional relationship do you expect from these plot? Is it consistent with the theory you know before?
- Calculate the mean and standard deviation of moment of Inertia

b) Transformation **of energy:**

- Using equation (2), (3) and (4) and from the result for the inertia, *I*, calculate the potential energy **PE** and the kinetic energy $\mathbf{KE_{trans}}$ and $\mathbf{KE_{rot}}$. Tabulate the results.
- Plot all the calculated energies vs time, *t* and height, *h*, comment on your results. Which part KE is dominant? And why?

Height, h (cm)	Time, t (s)	PE J	KE_{trans} J	KE_{rot} J

$N = kg\,m/s^2$

Fitting a Straight Line to a Set of Data

In some of the experiments, a straight line is to be fitted to the experimental data. This can be done using the Least Square Fitting Method. The method can be used to fit any polynomial, but you would require it only for fitting a straight line. In the following the method for this purpose is described.

Let $(x_i, y_i;\ i = 1,2,\ldots N)$ be the given set of data from a measurement. You are required to fit a straight line to this data. The best straight line, according to the least square method, can be fitted as follows.

Let the equation of the straight line to be fitted be written as

$$y = mx + c$$

where the slope m and the intercept c are to be determined from the given data. This is done using the least square method which gives the following working formulae:

$$m = \frac{AC - DN}{A^2 - NB} \quad \text{and} \quad c = \frac{C - mA}{N}$$

where $\quad A = \sum_{i=1}^{N} x_i \qquad B = \sum_{i=1}^{N} x_i^2 \qquad C = \sum_{i=1}^{N} y_i \qquad D = \sum_{i=1}^{N} x_i y_i$

Example: Fit a straight line to the following data:

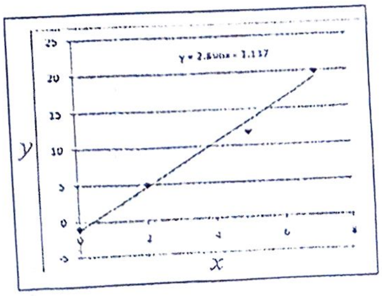
x	0	2	5	7
y	-1	5	12	20

x	x^2	y	xy
0	0	-1	0
2	4	5	10
5	25	12	60
7	49	20	140
Sum= 14	78	36	210
A	B	C	D

$$m = \frac{AC - DN}{A^2 - NB} = \frac{14X36 - 210X4}{14X14 - 4X78} = \frac{-336}{-116} = \frac{84}{29} = 2.8966$$

$$c = \frac{C - mA}{N} = \frac{36 - 14X336/116}{4} = \frac{36X116 - 14X336}{4X116} = \frac{-33}{29} = -1.1379$$

Thus the equation of the straight line is: $\quad y = (84x - 33)/29$



A comparison of the given data and the fitted line:

x	0	2	5	7
y	-1	5	12	20
y_{fit}	-1.138	4.655	13.35	19.18

The straight line actually does not pass through any of the given points yet this the best fit line (please see the figure). In this case, the sum of the square of errors is minimum, i.e. $\sum (y - y_{fit})^2$, is minimum. Thus, it is not required that the straight line passes through any of the points for it to be the best fit line.

3- Thermal Conductivity by Lee's method

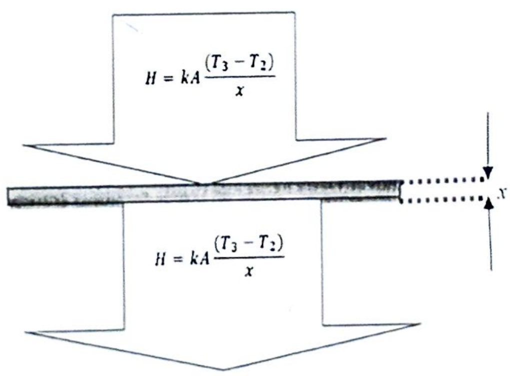
Objectives of the experiment:

Determine the thermal conductivity of a bad conductor (glass, wood, nylon) in form of a disc using Lee's method.

Apparatus:

Lee's apparatus and the experimental specimen in the form of a disc.
Weighing balance
Heater

Principle:



Thermal conductivity

According to Fourier's law of heat transfer, conductive heat flow occurs in direction of the decreasing temperature because higher temperature is associated with higher molecular energy. This expresses conductive heat transfer as

$$H = kA\frac{(T_2 - T_3)}{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where H is the steady state rate of heat transfer, k is the thermal conductivity of the sample, A is the cross sectional area and $(T_2 - T_3)$ is the temperature difference across the sample thickness 'x', assuming that the heat loss from the sides of the sample is negligible. To keep the loss from the sides at a minimal level, the sample is made in form of a thin disk with large cross sectional area compared to the area exposed at the edge. Keeping 'A' large and 'x' small results in a large rate of energy transfer across the sample. Keeping 'x' small also means that the apparatus reaches a steady state (when temperature T_2 and T_3 are constant) more quickly.

Amount of heat flowing through the specimen per second, H is given by Eq. (1). When the apparatus is in steady state (temperatures T_2 and T_3 constant), the rate of heat conduction into the brass disc is equal to the rate of heat loss from the bottom of it. The rate of heat loss can be determined by measuring how fast the disc cools at the previous (steady state) temperature T_3. If the mass and specific heat of the lower disc are m and s, respectively and the rate of cooling at T_3 is dT/dt then the amount of heat radiated per second is,

$$H = ms\frac{dT}{dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Equating (1) and (2) and simplifying, k can be determined as,

$$k = \frac{ms\frac{dT}{dt}x}{A(T_2 - T_3)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Procedure:

Place the insulator in between T_2 and T_3.

Set the maximum temperature.

Switch on the heater and stat the heating up to a steady temperature, say, 50 degree Celsius.

Maintain this temperature, say T_2 = 50 degree.

Now, T_3 will be less than T_2, say, 40 degree.

Record the readings T_2 and T_3.

Remove the insulator and make contact with both discs.

Now the temperature T_3 will start increasing.

Allow T_3 to increase 5 degree from the steady temperature.

Stop heating. Note the readings Temperature and Time.

Take the readings in equal intervals of temperature fall.

Plot the graph Temperature vs. Time.

Find the slope of the curve from the graph

Observations and Calculations:

70
68.5
55
29

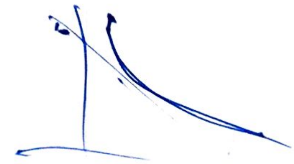
S.I. no	Temperature (°C)	Time (sec)

59:30
60

Mass of the metal disc $m = 490$ g

Specific heat capacity of the metal disc (brass) $s = 375$ J/ Kg

Radius of the metal disc $r = 38$ mm



Contact Area of the metal disc $A = \pi r^2$

Height of the metal disc $h = 23.1$ mm

Thickness of the sample $x = \ldots\ldots$ m

Temperature of the Heating disc $T_2 = \ldots\ldots\ldots$°C

Temperature of the Lee's disc $T_3 = \ldots\ldots\ldots$°C

Temperature difference: $(T_2 - T_3)$

Rate of cooling of the Lee's Disc dT/dt from graph = ...

Thermal conductivity of the sample is given by, $k = \frac{ms\frac{dT}{dt}x}{A(T_2-T_3)}$

Results:

Thermal conductivity of the sample = ...W m^{-1} K^{-1}

Fitting a Straight Line to a Set of Data

In some of the experiments, a straight line is to be fitted to the experimental data. This can be done using the Least Square Fitting Method. The method can be used to fit any polynomial, but you would require it only for fitting a straight line. In the following the method for this purpose is described.

Let $(x_i, y_i;\ i = 1,2,\ldots N)$ be the given set of data from a measurement. You are required to fit a straight line to this data. The best straight line, according to the least square method, can be fitted as follows.

Let the equation of the straight line to be fitted be written as

$$y = mx + c$$

where the slope m and the intercept c are to be determined from the given data. This is done using the least square method which gives the following working formulae:

$$m = \frac{AC - DN}{A^2 - NB} \quad \text{and} \quad c = \frac{C - mA}{N}$$

where $\quad A = \sum_{i=1}^{N} x_i \qquad B = \sum_{i=1}^{N} x_i^2 \qquad C = \sum_{i=1}^{N} y_i \qquad D = \sum_{i=1}^{N} x_i y_i$

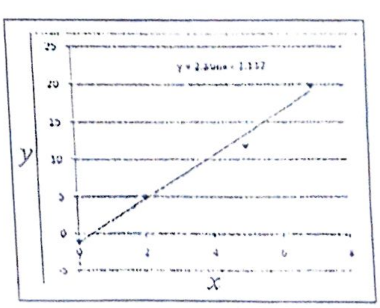
Example: Fit a straight line to the following data:

x	0	2	5	7
y	-1	5	12	20

x	x^2	y	xy
0	0	-1	0
2	4	5	10
5	25	12	60
7	49	20	140
Sum= 14	78	36	210
A	B	C	D

$$m = \frac{AC - DN}{A^2 - NB} = \frac{14X36 - 210X4}{14X14 - 4X78} = \frac{-336}{-116} = \frac{84}{29} = 2.8966$$

$$c = \frac{C - mA}{N} = \frac{36 - 14X336/116}{4} = \frac{36X116 - 14X336}{4X116} = \frac{-33}{29} = -1.1379$$



Thus the equation of the straight line is: $\quad y = (84x - 33)/29$

A comparison of the given data and the fitted line:

x	0	2	5	7
y	-1	5	12	20
y_{fit}	-1.138	4.655	13.35	19.18

The straight line actually does not pass through any of the given points yet this the best fit line (please see the figure). In this case, the sum of the square of errors is minimum, i.e. $\sum (y - y_{fit})^2$, is minimum. Thus, it is not required that the straight line passes through any of the points for it to be the best fit line.

$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x}{\partial x^2} + V\psi(x$

2- Velocity of sound using Kundt's tube

Objectives of the experiment:

Determine the velocity of sound in air at room temperature and calculate γ for air

Apparatus:

Brass tube containing loud speaker and microphone audio signal generator, oscilloscope and frequency counter

Principle:

This experiment is aimed towards understanding the Doppler Effect phenomena. For a standing wave pattern set in air column inside the hollow tube, successive maxima are separated by $\lambda/2$, where λ is the wavelength. If the position of successive maxima are given by 'y', then a plot of 'y' vs. the number of maxima n (1, 2, 3, etc) is straight line with slope $\lambda/2$. For a frequency 'ν' of the sound wave the velocity is given by

$$V=\nu\lambda \quad \text{.................(1)}$$

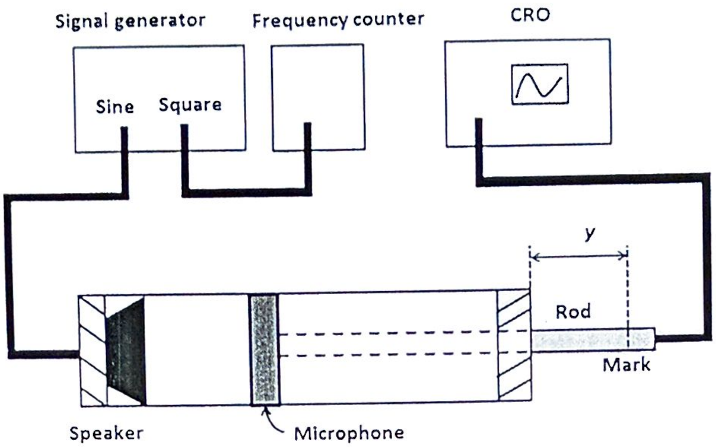
It can be shown that

$$V=\sqrt{\frac{\gamma RT}{M}} \quad \text{.................(2)}$$

Where $\gamma = C_p/C_v$, R=8.32 J/moleK and M is the molecular weight expressed in kg. Assuming air to be 78% N_2 and 22% O_2, γ can be calculated at room temperature (~30-35°C).

Procedure:

The method used in this experiment is a modern version of Kundt's tube. It consists of a glass tube with fixed loudspeaker at one end and a microphone (mic) attached to a movable rod at other end. Sound from the loudspeaker produces a standing wave pattern in the tube and is detected by the mic which is moved in the tube using a rod. The signal from the mic is viewed using an oscilloscope (CRO). An audio signal generator is used to drive the loud speaker. Successive maxima, which are separated by $\lambda/2$, are located by moving the rod and observing the signal on the CRO. For each maximum, a measurement (y) is made of the distance between a point on the movable rod and the end of the tube.



Experimental setup

Steps

Connect the sine wave output of the signal generator to the speaker, mic to the CRO and square wave output of the signal generator to frequency counter.

Check that the signal on the CRO is sinusoidal, stable and undistorted.

Move the rod along the tube and check that signal goes through maxima and minima.

Take ν=2.5 kHz. For each maximum, use the mark on the rod to measure the distance (y) between this mark and the end of the tube. Take as many reading as you can. Enter the data in the form of a table with column for y and n.

Repeat step (d) for any other one convenient frequency between 2 and 4 kHz.

Observations and Calculations:

Using the least squares method to fit a straight line for y vs. n data, calculate the slop and the error in the slop. Make an appropriate table so that all the quantities appearing in these equations can be tabulated. Also write down the column sums (Σx_i, $\Sigma x_i y_i$ $etc.$) for all these quantities. From the slope calculate the wavelength λ and hence the velocity V for each frequency and also the error in the velocity in each case. You may assume that there is negligible error in measurement of frequency. Are the measured velocities consistent with their respective errors?

Using the formula $V=\sqrt{\frac{\gamma RT}{M}}$, calculate γ for each frequency. Do your γ value compare with what you would expect for air?

Plot y vs. n data on a graph sheet. Plot the least squares fit line on the same graph paper and check how the experiments data points are placed with respect to the least squares fit line.

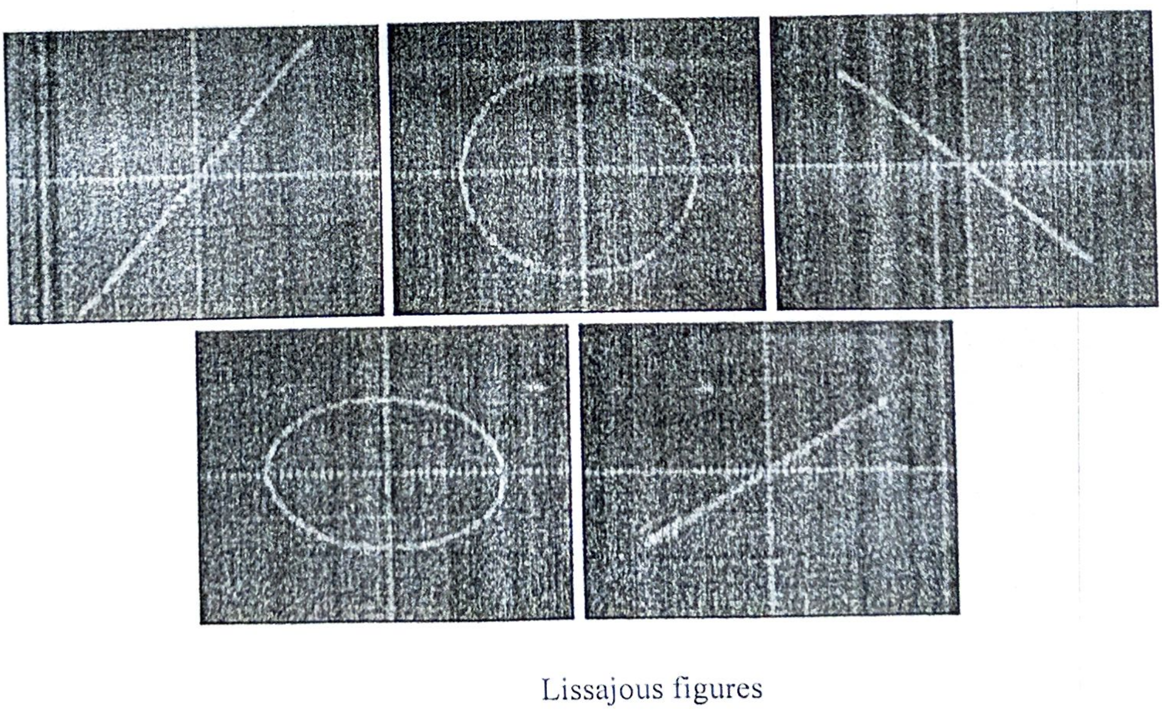
Precautions:

Make sure that the connections are proper before switching on the power. Microphone cable goes to the CRO and is terminated in a BNC connector.

For a set of readings, audio signal level should not be adjusted.

The signal at the CRO should be an undistorted sine wave.

Note: Velocity of sound can also be measured by using Lissajous figures.



Lissajous figures

Fitting a Straight Line to a Set of Data

In some of the experiments, a straight line is to be fitted to the experimental data. This can be done using the Least Square Fitting Method. The method can be used to fit any polynomial, but you would require it only for fitting a straight line. In the following the method for this purpose is described.

Let $(x_i, y_i;\ \ i = 1,2,\ldots N)$ be the given set of data from a measurement. You are required to fit a straight line to this data. The best straight line, according to the least square method, can be fitted as follows.

Let the equation of the straight line to be fitted be written as

$$y = mx + c$$

where the slope m and the intercept c are to be determined from the given data. This is done using the least square method which gives the following working formulae:

$$m = \frac{AC - DN}{A^2 - NB} \quad \text{and} \quad c = \frac{C - mA}{N}$$

where $\quad A = \sum_{i=1}^{N} x_i \qquad B = \sum_{i=1}^{N} x_i^2 \qquad C = \sum_{i=1}^{N} y_i \qquad D = \sum_{i=1}^{N} x_i y_i$

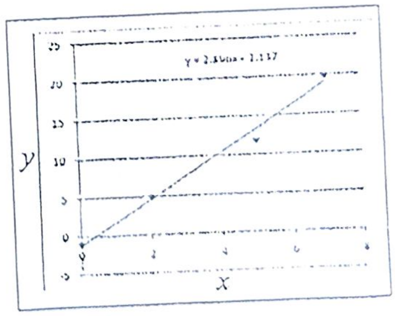
Example: Fit a straight line to the following data:

x	0	2	5	7
y	-1	5	12	20

x	x^2	y	xy
0	0	-1	0
2	4	5	10
5	25	12	60
7	49	20	140
Sum= 14	78	36	210
A	B	C	D

$$m = \frac{AC - DN}{A^2 - NB} = \frac{14X36 - 210X4}{14X14 - 4X78} = \frac{-336}{-116} = \frac{84}{29} = 2.8966$$

$$c = \frac{C - mA}{N} = \frac{36 - 14X336/116}{4} = \frac{36X116 - 14X336}{4X116} = \frac{-33}{29} = -1.1379$$



Thus the equation of the straight line is: $\qquad y = (84x - 33)/29$

A comparison of the given data and the fitted line:

x	0	2	5	7
y	-1	5	12	20
y_{fit}	-1.138	4.655	13.35	19.18

The straight line actually does not pass through any of the given points yet this the best fit line (please see the figure). In this case, the sum of the square of errors is minimum, i.e. $\sum(y - y_{fit})^2$, is minimum. Thus, it is not required that the straight line passes through any of the points for it to be the best fit line.

Experiment: Helmholtz coil

Course: PH201 Lab

3^{rd} Semester

Helmholtz coil: a bit of history.

"*Historically Helmholtz coil is a device famous for producing a region of nearly uniform magnetic field which is named after the German physicist Hermann Von Helmholtz. Besides creating uniform magnetic fields, Helmholtz coils are also used in scientific apparatus to cancel external magnetic fields, such as the Earth's magnetic field.* ***Hermann Ludwig Ferdinand von Helmholtz*** *(August 31, 1821 – September 8, 1894) was a German physician and physicist who made significant contributions to several widely varied areas of modern science. He is known for his theories on the conservation of energy, work in electrodynamics, chemical thermodynamics, and on the mechanical foundation of thermodynamics. The largest German association of research institutions, the Helmholtz Association was named after him.*"

OBJECTIVES:

To study the magnetic field distributions inside Helmholtz Coil.

1. To measure the magnetic flux density along the x-axis of the circular coils when the distance between them $a = R$ (R = radius of the coils) and when it is greater ($a = 2R$ or $a = 1.5R$) and lesser ($a = R/2$ or $a = 3R/4$) than this.

2. To measure the spatial distribution of the magnetic flux density when the distance between coils is $a = R$. Using the rotational symmetry of the set-up derive the following

   a. Measurement of radial component B_r
   b. Measurement of the axial component B_x

3. To measure the radial components B_{r1} and B_{r2} of the two individual coils in the plane midway between them and to demonstrate the overlapping of the two fields at $B_r = 0$.

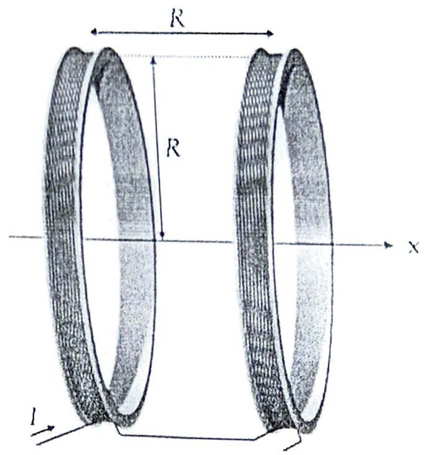


Fig. 1: Schematic showing the Helmholtz coil

INTRODUCTION:

A magnetic field is created whenever charge is in motion, either moving in some space, conductor or spinning around itself. A moving charge in space or in a conductor constitutes what is called a "current" (denoted by the symbol *I*) and is measured in *coulombs/sec* or *amperes*. The strength of magnetic field is measured at a point in space (often called the *field point*). Helmholtz coil is a useful experiment for getting a fairly uniform magnetic field employing a pair of circular coils on a common axis with equal currents flowing in the same direction; see Figs. 1 and Fig. 2. In other words Holmholtz coil consists of two solenoid electromagnets on the same axis (Fig. 1). For a given coil radius, the optimum separation needed to give the most uniform central field can be calculated. Theoretically this separation is equal to the radius of the coils. The magnetic field lines for such geometry are illustrated elaborately in Fig. 2 below. In the case of the Helmholtz coils, the field points of interest are located in the mid-plane between the two coils and the strength of the magnetic field is dependent upon three quantities: the current I, the number of turns N in each coil, and the radius r of the coil (Fig. 2). The most astonishing factor about Helmholtz coil lies in its ability to yield very uniform magnetic field. However, the question that naturally arises, "how uniform the magnetic field is and how is it spatially distributed?" We shall find these answers in this experiment. The spatial distributions of the field strength between a pair of coils in the Helmholtz arrangement will be measured in this experiment. The spacing at which a uniform magnetic field can be produced will be investigated and the superposition of the two individual fields to form the combined field due to the pair of coils will also be demonstrated.

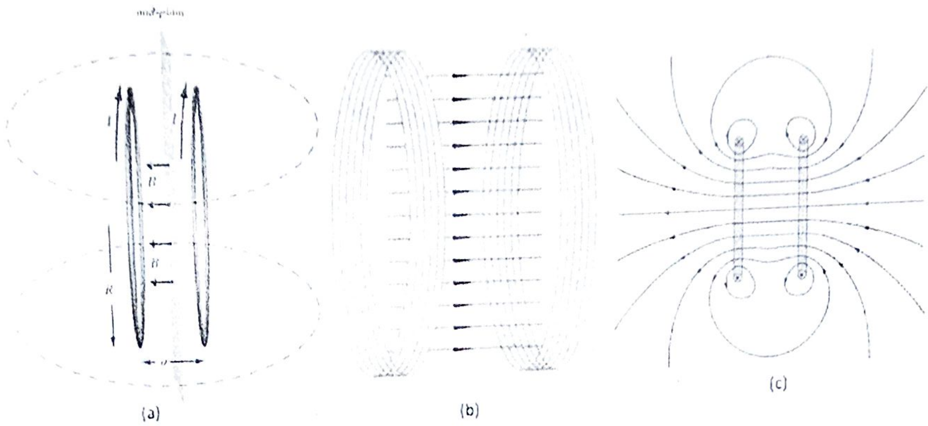


Fig. 2: Magnetic field line distributions arising due to two coils of equal diameters in Helmholtz coils.

THEORY:

The magnetic field of the coil arrangement shown is rotationally symmetrical about the axis of the coils, which is chosen as the x-axis of a system of cylindrical coordinates (x, r, Φ). The origin is at the centre of the system. The magnetic flux density does not depend on the azimuth angle Φ because of cylindrical symmetry, so only the components $B_x(x, r)$ and $B_r(x, r)$ are measured. We computed the field on the axis of a circular current loop. When a current I exists in a loop of radius R, the magnetic field strength B along an axis through the center of the loop and a distance x from the center of the loop is given by

$$B = \frac{\frac{\mu_0 I}{2R^2}}{(x^2 + R^2)^{3/2}} \tag{1}$$

In the derivation of Equation 1 the assumption made is that the origin is located at the loop center. If the origin does not coincide with the center of the loop, but is instead a distance x_o from the loop center, we must replace x in Equation 1 by (x - x_o). Moreover, if the loop actually consists of N turns, each carrying a current I_o, we can replace the current I by NI_o: with these substitutions, Equation 1 becomes

$$B = \frac{\frac{\mu_0 N I_0}{2R^2}}{((x - x_0)^2 + R^2)^{3/2}} \tag{2}$$

This equation can be written more compactly by defining B_o as the field at the center of the loop (x = x_o)

$$B_0 = \frac{\mu_0 N I_0}{2R} \tag{3}$$

So that,

$$B = B_0 \frac{R^3}{((x - x_0)^2 + R^2)^{3/2}} \tag{4}$$

HELMHOLTZ COILS: Helmholtz coils are constructed from two circular coils of wire, each perpendicular to the same axis, and each carrying the same amount of current in the same direction. As shown in Fig. 3, the coils are separated by a distance R, which is also the radius of each coil. We can use Eq. 4 to find an expression for the B field at any point P on the axis of the coils. If the magnetic field strength due to coil 1 is B_1 and that due to coil 2 is B_2, then by superposition

$$B = B_1 + B_2$$

In this configuration, it is convenient to specify x_o not at the center of a single coil, but rather at the midpoint between the two coils. Therefore, in the equation for B_1, x_o must be replaced by $x_o - R/2$, and for B_2, x_o must be replaced by $x_o + R/2$. Also note that B_o as defined above does not correspond to the field at this new position x_o.

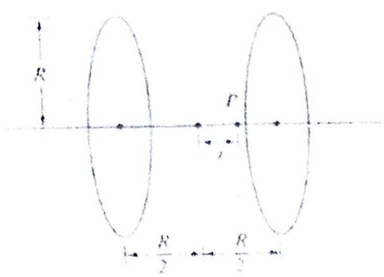


Fig. 3: Defining length x (axial distance) from the origin which in this case is mid point between the coils.

DESCRIPTION OF THE APPARATUS:

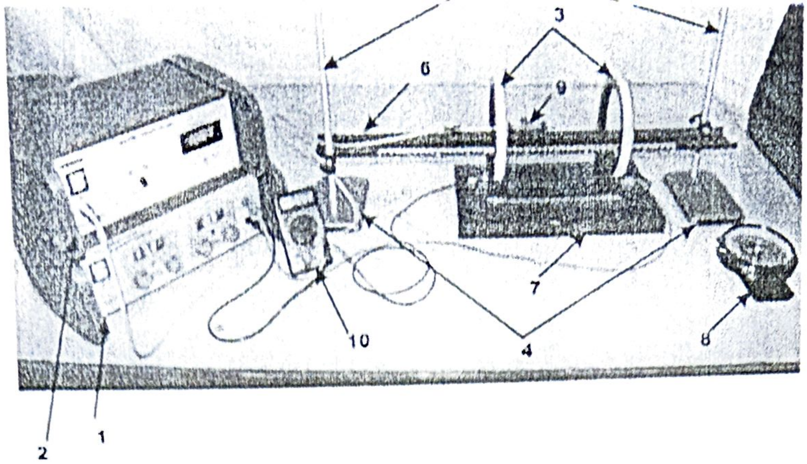


Fig. 4: Photograph of the set up indicating each component.

Sl. No.	Item name	Quantity
1	Power Supply DC 0-16V, 5 Amp	1
2	Digital Gauss Meter with Axial Probe	1
3	Coil N =390, Dia =150mm	2
4	Support Base	2
5	Support Rod	2
6	U Channel Big	1
7	U Channel Small	1
8	Deflection Compass with Base	1
9	Axial probe holder	1
10	Multimeter	1
11	Connecting Leads	4

EXPERIMENTAL PROCEDURE:

Connect the coils in series so that current flows in the same direction in both the coils as shown in Fig. 5. The current magnitude through the coils is to be fixed at 0.5 A. Power supply is employed to provide the constant current through the coils. Magnetic fields will be measured with the help of axial Hall probe as supplied [indicated as 9 in (Fig. 4)]. The magnetic field induced inside the Helmholtz coil arrangement is rotationally symmetric with respect to the axis of the coils, which is chosen as the x-axis of a system of cylindrical coordinates (x, r, ϕ). The origin is considered at the center of the system for the ease of interpreting the results. The magnetic field does not depend on the angle ϕ (because of cylindrical symmetry involved), so only the components B_x (x, r) and B_r (x, r) will be studied. Set up the experiment as shown in Fig. 5 below. Clamp the axial probe on to a support base provided and level horizontally with the axis of the coils.

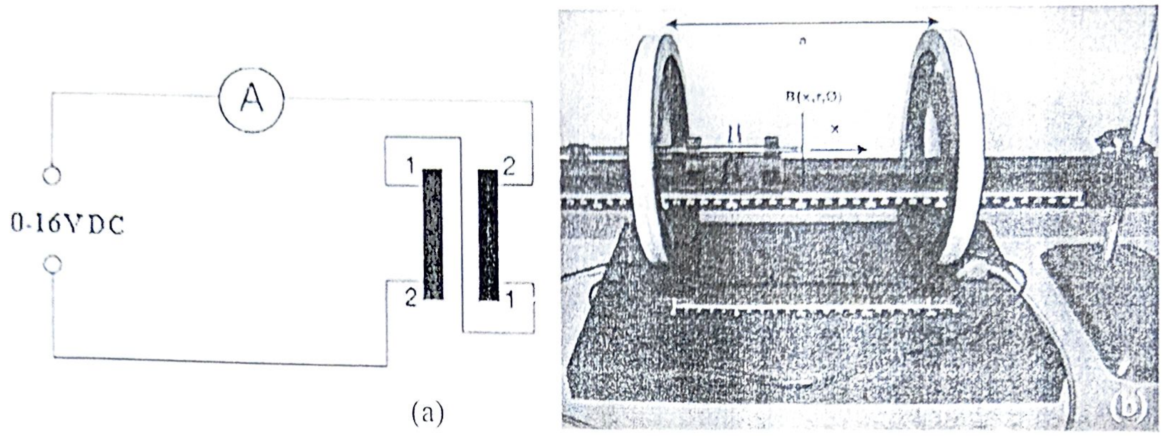


Fig. 5: Schematics showing (a) the electrical circuit for Helmholtz coil experiment and (b) Holmholtz coil arrangement for B field along x-axis

1. Because of symmetry the magnetic flux density has only the axial component B_x along the x-axis. Figure 5 shows the set up involving the coils, probe and rules. Measure the values of B (x, $r = 0$) when the distance between the coils are $a = R$ and, $a < R$ (for example, $a = R/2$ and $a = 3R/4$) and $a > R$ (for example, $a = 1.5R$ and $a = 2R$). Tabulate your measured results according to the table given below.

For series connection

Distance	x (cm)	B at $a = R/2$ (Gauss)	B at $a = 3R/4$ (Gauss)	B at $a = R$ (Gauss)	B at $a = 1.5R$ (Gauss)	B at $a = 2R$ (Gauss)

Plot curves for all the five cases.

The magnetic field along the axis of the two identical coils at a distance 'a' apart is derived as

$$B(x, r = 0) = \frac{\mu_0 NI}{2R}\left(\frac{1}{(1+Z_1^2)^{3/2}} + \frac{1}{(1+Z_2^2)^{3/2}}\right))$$

where

$$Z_1 = \frac{(x + a/2)}{R} \text{ and } Z_2 = \frac{(x - a/2)}{R}$$

7.5cm

When $x = 0$, magnetic flux density has a maximum value when $a < R$ and a minimum value when $a > R$. The curves are to be plotted from measurements data as taken in the above table; when $a = R$, the field is supposed to be uniform in the range $-R/2 < x < +R/2$

Magnetic flux density at the mid-point when $a = R$ is measured as

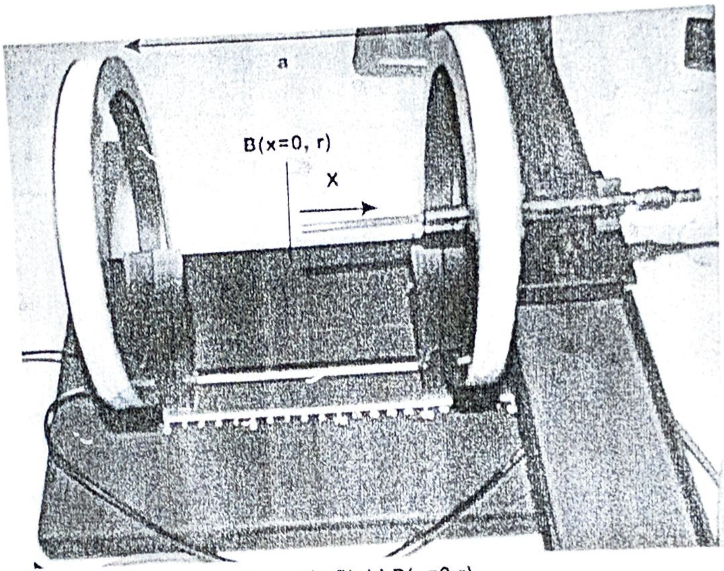
For parallel connection

Distance	x (cm)	B at $a = R/2$ (Gauss)	B at $a = 3R/4$ (Gauss)	B at $a = R$ (Gauss)	B at $a = 1.5R$ (Gauss)	B at $a = 2R$ (Gauss)

Plot the curves

2. Keep the coils at distance $a = R$.

a. Measure $B_x(x, r)$ as shown in Fig. 6 below. You can change the r-coordinate by moving the probe whereas the x-coordinate can be changed by moving the coils. Measure $B_x(0, r)$ by moving the probe as indicated in the figure below and then Plot $B_x(0, r)$. Check for the maximum value of magnetic flux density in this configuration.



Magnetic Field B(x=0,r)

Fig. 6: Configuration to probe magnetic flux in the radial direction (B_r).

At distance $a = R$,

Distance (r) cm	$Bx(0, r)$ (Gauss)

b. Turn the pair of coils through 90° (see figure below). Now probe is in a direction perpendicular to the first configuration hence find the magnetic field (B_r) along x-axis.

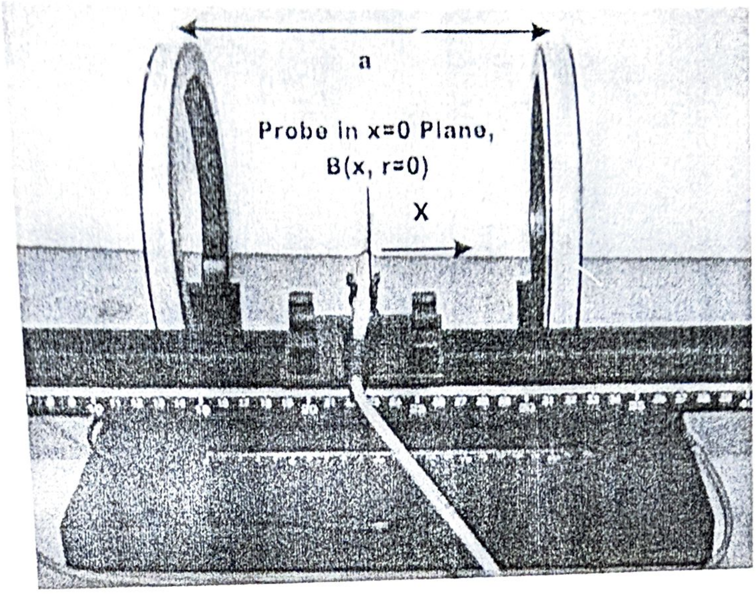


Fig. 7: Evaluation of B perpendicular to the previous measurement along the x-axis.

Distance (x) cm	Bx (r, 0) (Gauss)

Plot $B_x(r, 0)$ with respect to x.

Draw your own conclusions and remarks about the experiment and the results obtained by you.

9- Vibrating String Experiment

Objectives:

1. Generate standing waves on an elastic string as a function of the excitation frequency f.
2. Deduce the wave velocity v of the string.

Apparatus:

Rubber string, STE Motor, Function generator, Measuring tape

Principle:

In this experiment, a vertically mounted rubber string is caused to oscillate by means of an electric motor with an oscillation lever at one end of the string. The excitation frequency is continuously adjustable using a function generator.

When the string is fixed at both ends, reflections occur at the ends. Standing waves form at certain frequencies as stationary oscillation patterns. The distance between two oscillation nodes or two antinodes of a standing wave corresponds to half the wavelength. Only oscillation nodes can form at the fixed ends. For a standing wave with n oscillation antinodes on a string of length s, we can say

$$s = n\frac{\lambda_n}{2} \text{, where n= 1,2,3,...} \qquad (1)$$

We also know that propagation velocity v is related to the oscillation frequency, f_n and wave length as,

$$v = f_n \lambda_n \qquad (2)$$

As the phase velocity, v does not change, the excitation frequencies are given by

$$f_n = \frac{v}{2s}n \qquad (3)$$

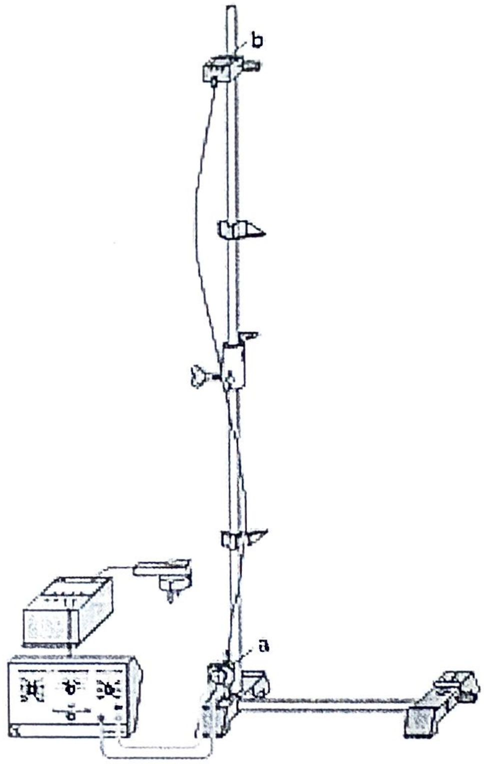
Experiment Procedure:

0.75

Arrange the setup as described below:

- Tie the rubber string of length 75 cm (un-stretched).
- Stretch it by about 5 -10 % by moving the clamp block shown in the figure at 'b'.

- Connect the function generator with STE Motor at 'a'.
- Set the output voltage of the function generator to 3 V.
- Set the frequency form as '~' sign wave
- Set the frequency range as ' x 10'



Step 1: Measurement with rubber string

- Set the frequency of the function generator so that a standing wave with two oscillation antinodes is formed, and optimize the oscillation by moving the clamping block.
- Measure the length 's' of the extended rubber string.
- Turn the frequency control knob to the minimum position (full left stop).
- Slowly increase the frequency 'f' and carefully seek those frequencies at which standing waves with n= 1, 2, 3, 4 and 5 antinodes are formed.
- Note down the corresponding frequencies.

Observations:

- Note down the values in a tabulated form as shown below.
- Plot the values of f vs n in a graph sheet

- Take the slope of the curve from which phase velocity, v is calculated.

# of anti-nodes **N**	**Frequency, f** **Hz**
1	
2	
3	
4	
5	

Important note:

Depending on the length and the tension of the rubber string, it can be difficult to generate exactly one antinode.
Vary the string tension as necessary.

The above experiment can also be performed with helical springs fixed at both ends to investigate longitudinal waves.

Fitting a Straight Line to a Set of Data

In some of the experiments, a straight line is to be fitted to the experimental data. This can be done using the Least Square Fitting Method. The method can be used to fit any polynomial, but you would require it only for fitting a straight line. In the following the method for this purpose is described.

Let $(x_i, y_i;\ i = 1,2,\ldots N)$ be the given set of data from a measurement. You are required to fit a straight line to this data. The best straight line, according to the least square method, can be fitted as follows.

Let the equation of the straight line to be fitted be written as

$$y = mx + c$$

where the slope m and the intercept c are to be determined from the given data. This is done using the least square method which gives the following working formulae:

$$m = \frac{AC - DN}{A^2 - NB} \quad \text{and} \quad c = \frac{C - mA}{N}$$

where $A = \sum_{i=1}^{N} x_i \qquad B = \sum_{i=1}^{N} x_i^2 \qquad C = \sum_{i=1}^{N} y_i \qquad D = \sum_{i=1}^{N} x_i y_i$

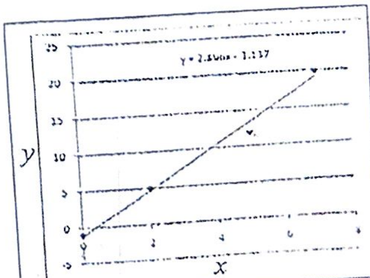
Example: Fit a straight line to the following data:

x	0	2	5	7
y	-1	5	12	20

x	x^2	y	xy
0	0	-1	0
2	4	5	10
5	25	12	60
7	49	20	140
Sum= 14	78	36	210
A	B	C	D

$$m = \frac{AC - DN}{A^2 - NB} = \frac{14X36 - 210X4}{14X14 - 4X78} = \frac{-336}{-116} = \frac{84}{29} = 2.8966$$

$$c = \frac{C - mA}{N} = \frac{36 - 14X336/116}{4} = \frac{36X116 - 14X336}{4X116} = \frac{-33}{29} = -1.1379$$



Thus the equation of the straight line is: $y = (84x - 33)/29$

A comparison of the given data and the fitted line:

x	0	2	5	7
y	-1	5	12	20
y_{fit}	-1.138	4.655	13.35	19.18

The straight line actually does not pass through any of the given points yet this the best fit line (please see the figure). In this case, the sum of the square of errors is minimum, i.e. $\sum (y - y_{fit})^2$, is minimum. Thus, it is not required that the straight line passes through any of the points for it to be the best fit line.