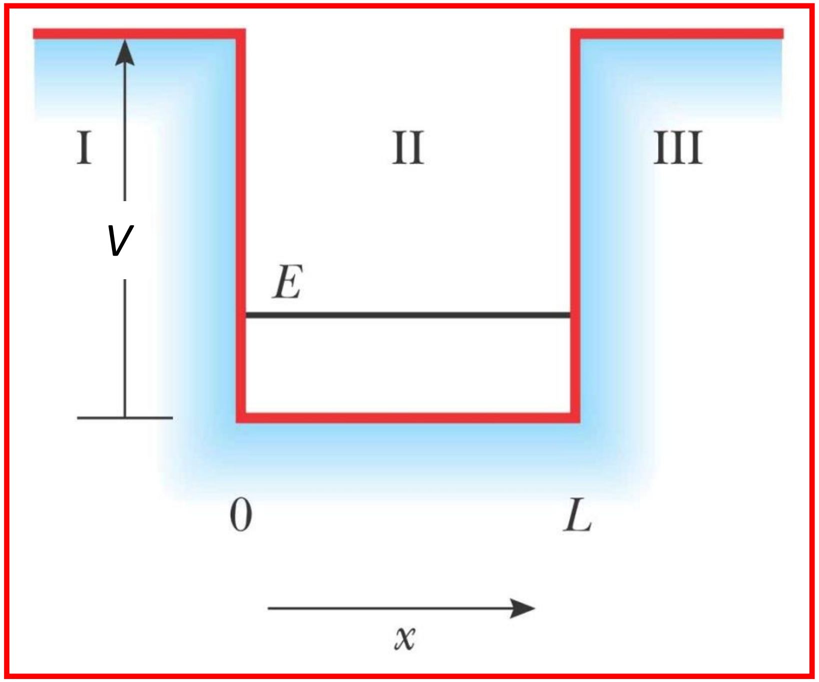
Modern Physics

Lecture 18

Finite Potential Well

Time Independent Schrödinger Equation

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial^2 x}+V(x)\psi(x)=E\psi(x)}$$



Finite potential well diagram

Defining Well

Region I

For $x < 0$; $V(x) = V$

Region II

For $0 < x < L$; $V(x) = 0$

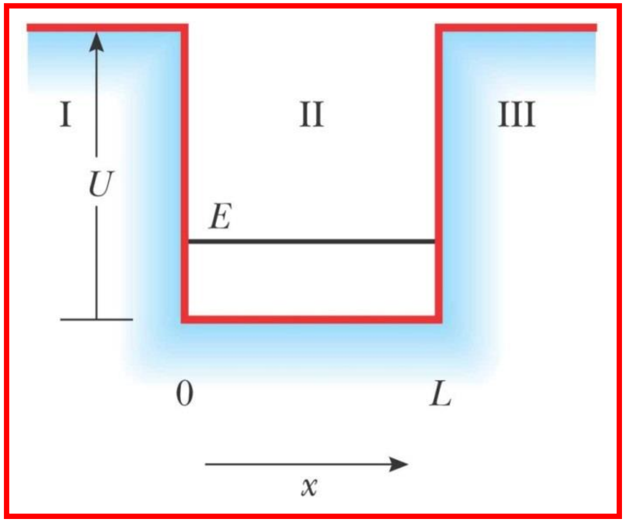
Region III

For $x > L$; $V(x) = V$

Time Independent Schrödinger Equation will take the following form

In region I

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2}+V\psi_I=E\psi_I$$



In region II

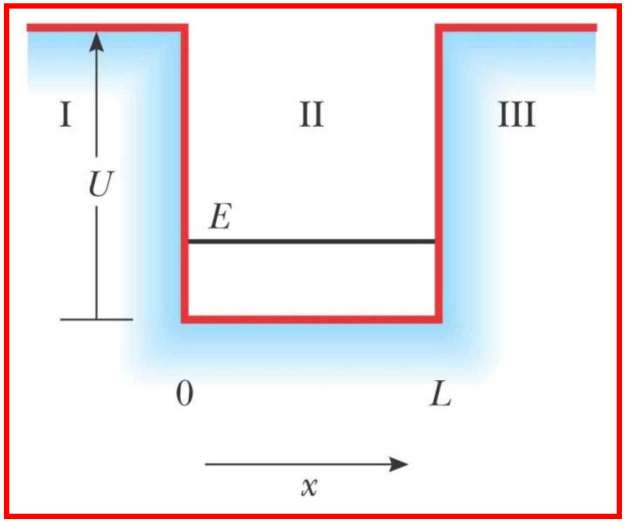
$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2}=E\psi_{II}$$

In region III

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{III}}{dx^2}+V\psi_{III}=E\psi_{III}$$

Region II

- $V(x) = 0$
  - This is the same situation as previously for infinite potential well



$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} = E\psi_{II}$$

- The general solution is

  $\psi_{II}(x) = G \sin kx + H \cos kx$
  - where G and H are constants

- The boundary conditions , however, no longer require that $\psi(x)$ be zero at the ends of the well

Regions I and III

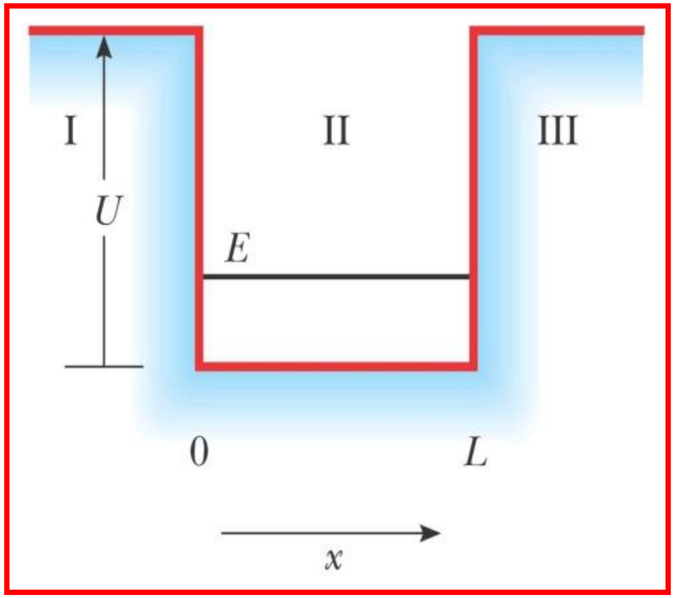
- The Schrödinger equation for these regions is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V\psi=E\psi$$

- It can be re-written as

$$\frac{d^2\psi}{dx^2}=\frac{2m(V-E)}{\hbar^2}\psi=C^2\psi,$$

$$\text{where}\quad C^2\equiv\frac{2m(V-E)}{\hbar^2}$$



The general solution of this equation will be

$$\psi(x) = Ae^{Cx} + Be^{-Cx}$$

For region I

Where A and B are constants

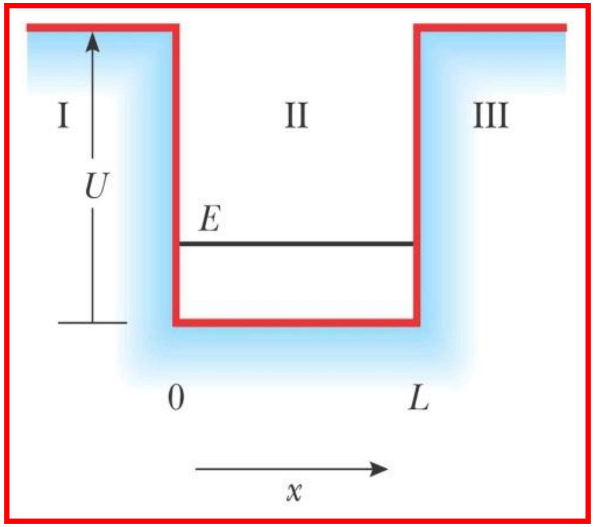
$$\psi(x) = De^{Cx} + Fe^{-Cx}$$

For region III

- Requiring that wave function must be finite at $x \rightarrow \infty$ and $x \rightarrow -\infty$, this means
- In region I, general solution is

$$\psi(x) = Ae^{Cx} + Be^{-Cx}$$

- $B = 0$, and $\psi_I(x) = Ae^{Cx}$
  – This is necessary to avoid an infinite value for $\psi(x)$ for large negative values of x

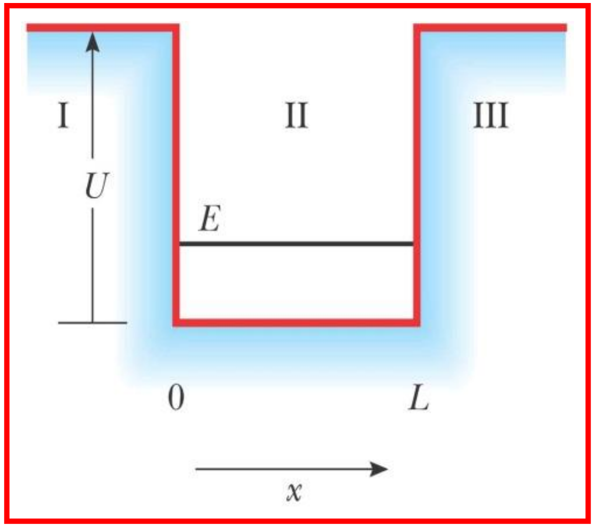


Similarly in region III,

$$\psi(x) = De^{Cx} + Fe^{-Cx}$$

$D = 0$, and $\psi_{III}(x) = \mathrm{F}e^{-Cx}$

This is necessary to avoid an infinite value for $\psi(x)$ for large positive values of x



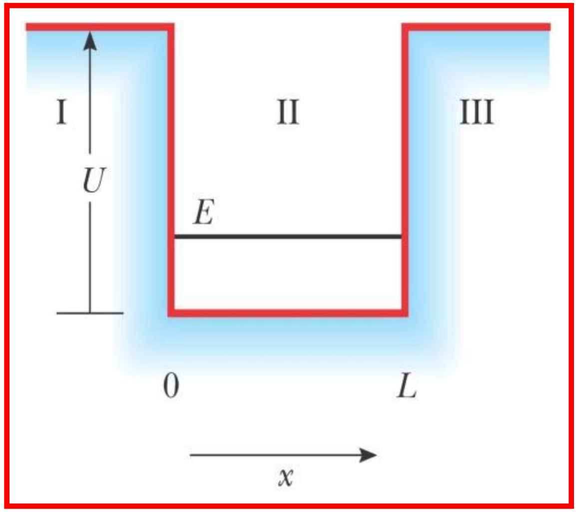
Therefore we have

Region I

$$\boxed{\psi_I(x) = Ae^{Cx}}$$

Region II

$$\boxed{\psi_{II}(x) = G\sin(kx) + H\cos(kx)}$$



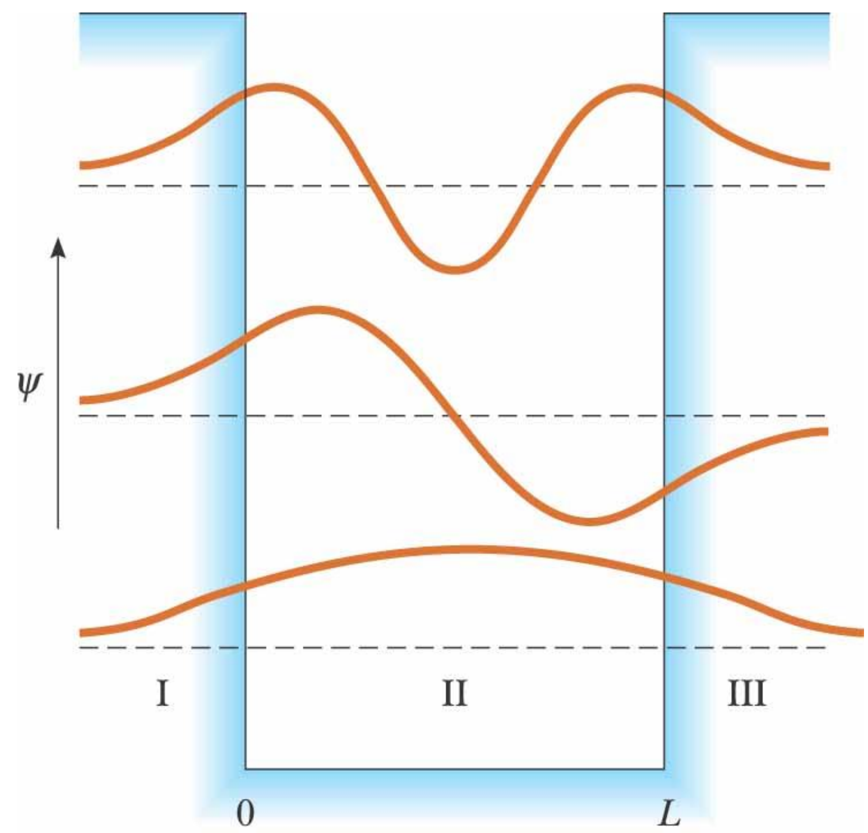
Region III

$$\boxed{\psi_{III}(x) = Fe^{-Cx}}$$

Finite Potential Well

Graphical Results for $\psi\,(x)$

- Outside the potential well, classical physics forbids the presence of the particle
- Quantum mechanics shows the wave function decays exponentially to approach zero



Finite Potential Well
Graphical Results for Probability Density, $|\psi(x)|^2$

- The probability densities for the lowest three states are shown
- The functions are smooth at the boundaries
- Outside the box, the probability to find the particle decreases exponentially, but it is not zero!