

# Parameters of Interest

Philipp Eisenhauer

# *Housekeeping*

Since nearly all of you choose to review experimental studies. Please be sure to read ...

- ▶ Deaton, A. (2010). Instruments, randomization, and learning about development. *Journal of Economic Literature*, 48(2):424–455
- ▶ Imbens, G. W. (2010). Better LATE than nothing: Some comments on Deaton (2009) and Heckman & Urzua (2009). *Journal of Economic Literature*, 48(2):399–423
- ▶ Heckman, J. J. and Urzúa, S. (2010). Comparing IV with structural models: What simple IV can and cannot identify. *Journal of Econometrics*, 156(1):27–37

## Student Presentation

- ▶ 20 minutes of presentation and discussion
- ▶ peer-review of presentation

## Lectures

- ▶ constantly updated and available at

<https://github.com/policyMetrics/course/wiki>

We need to slightly modify the notation, and thus the economics, of our conceptual framework to better align it with the published work on the topic. In the future, we will be more agnostic about what the agents care about and what they know when making their treatment decision.

## Earlier Version

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Cost

$$C = \mu_D(Z) + U_C$$

Observed Outcomes

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$S = Y_1 - Y_0 - C$$

$$D = I[S > 0]$$

## The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

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Heckman (2008) sets out three tasks for us:

- ▶ Defining the Set of Hypotheticals or Counterfactuals  
⇒ A Scientific Theory
- ▶ Identifying Causal Parameters from Real Data  
⇒ Mathematical Analysis of Data Point or Set Identification
- ▶ Identifying Parameters from Real Data  
⇒ Estimation and Testing Theory

# *Setup*

## The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = I[\mu_D(X, Z) - V > 0]$$

## Useful Notation

$$P(X, Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

$$U_D = F_V(V)$$

## Specification

We follow the parameterization in Heckman and Vytlacil (2005):

$$Y_1 = \gamma + \alpha + U_1 \quad U_1 = \sigma_1 \epsilon \quad \gamma = 0.670 \quad \sigma_1 = 0.012$$

$$Y_0 = \gamma + U_0 \quad U_0 = \sigma_0 \epsilon \quad \alpha = 0.200 \quad \sigma_0 = -0.050$$

$$D = I[Z - V > 0] \quad V = \sigma_V \epsilon \quad \epsilon \sim \mathbb{N}(0, 1) \quad \sigma_V = -1.000$$

$$Z \sim \mathbb{N}(-0.0026, 0.2700) \quad U_D = \Phi\left(\frac{V}{\sigma_V \sigma_\epsilon}\right)$$

## *Individual Heterogeneity*

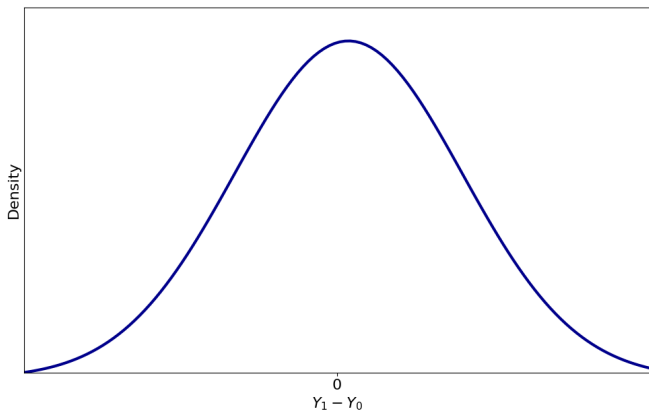
## Individual-specific Benefit of Treatment

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

### Sources of Heterogeneity

- ▶ Difference in Observable Characteristics
- ▶ Difference in Unobservable Characteristics
  - ▶ Uncertainty
  - ▶ Private Information

Figure: Distribution of Benefits





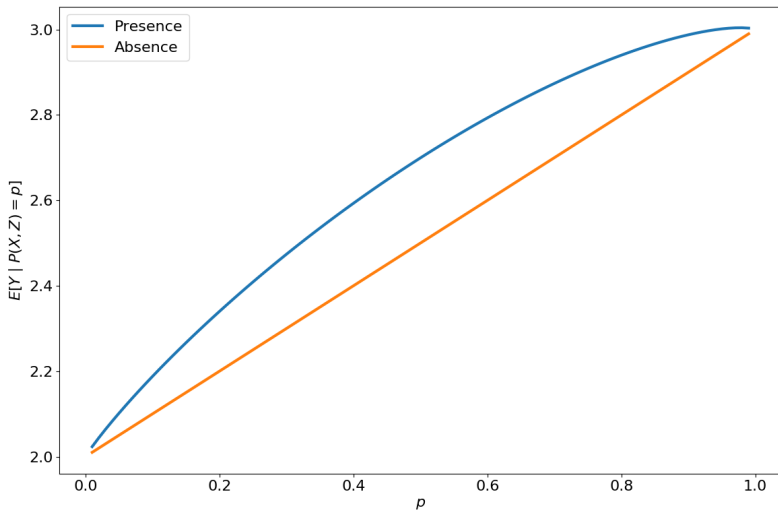
## Essential Heterogeneity

**Definition:** Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp D \mid X = x.$$

⇒ consequences for the choice of the estimation strategy

Figure: Conditional Expectation and Essential Heterogeneity



## *Conventional Average Treatment Effects*

## Conventional Average Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$

$$B^{TT} = E[Y_1 - Y_0 \mid D = 1]$$

$$B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$$

$\Rightarrow$  correspond to *extreme* policy alternatives

## Selection Problem

$$\begin{aligned} E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0]}_{BATE} \\ &+ \underbrace{E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}} \end{aligned}$$

$$\begin{aligned}
 E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}} \\
 &\quad + \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}
 \end{aligned}$$

$\Rightarrow$  the bias depends on the parameter of interest

Figure: Distribution of Effects with Essential Heterogeneity

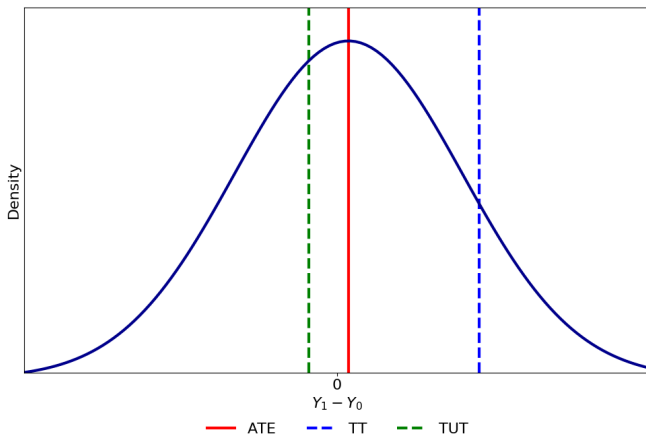


Figure: Distribution of Effects without Essential Heterogeneity

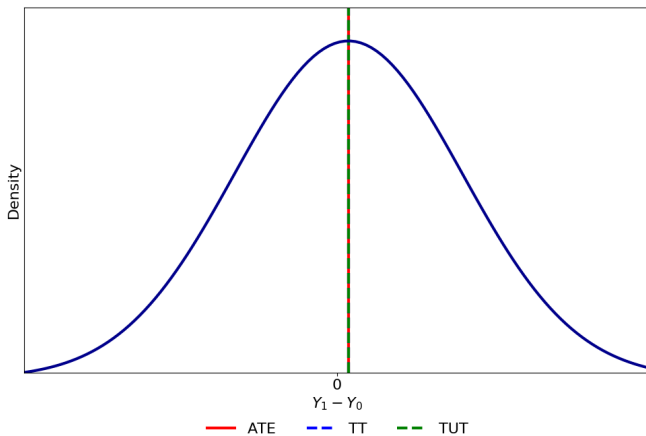
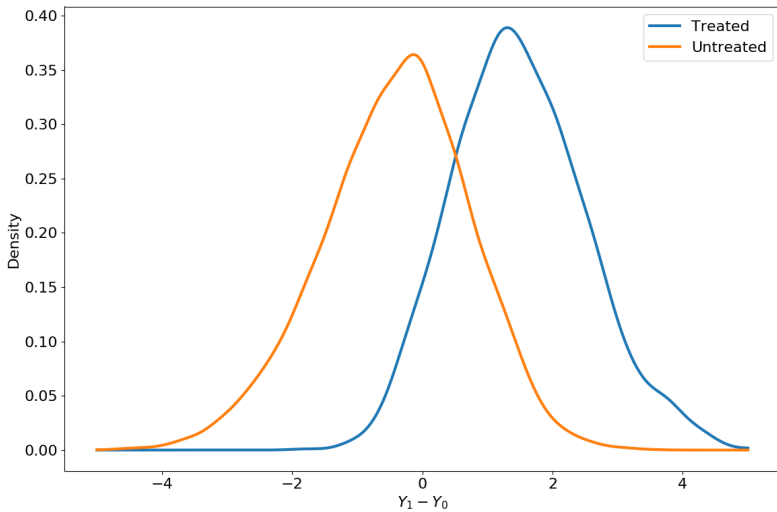




Figure: Distribution of Benefits by Treatment Status



## *Policy-Relevant Average Treatment Effects*

## Observed Outcomes

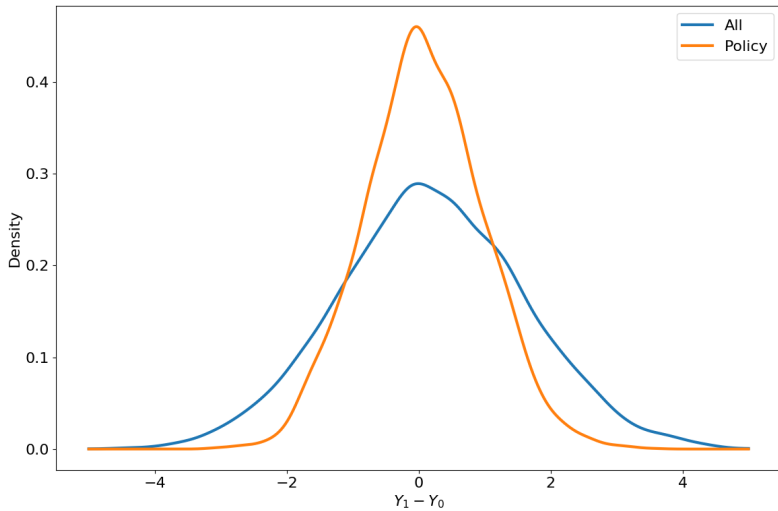
$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$

$$Y_A = D_A Y_1 + (1 - D_A) Y_0$$

## Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Figure: Distribution of Benefits for Policy



## *Marginal Effect of Treatment*

## Marginal Benefit of Treatment

$$B^{MTE}(x, u_S) = E[Y_1 - Y_0 \mid X = x, U_S = u_S]$$

**Intuition:** Mean gross return to treatment for persons at quantile  $u_S$  of the first-stage unobservable  $V$ .

Figure: Margin of Indifference

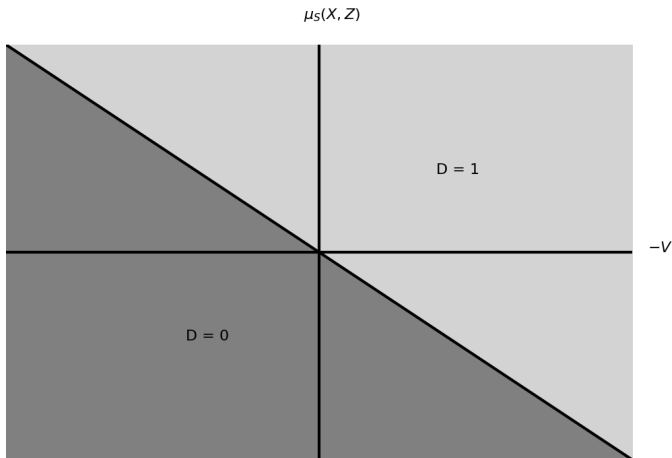
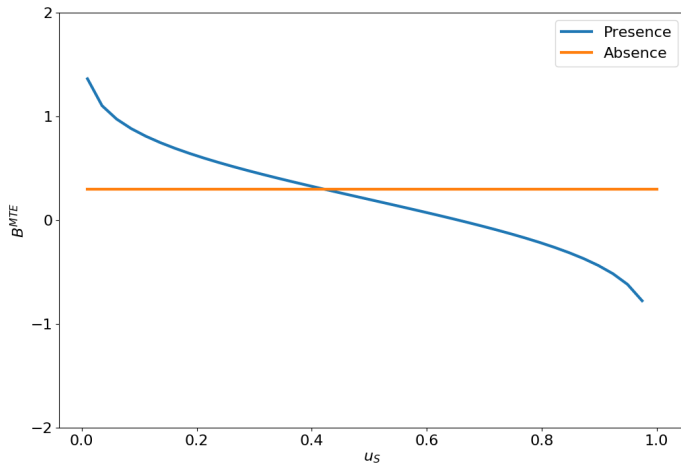


Figure: Marginal Effect of Heterogeneity





## Effects of Treatment as Weighted Averages

Parameter  $\Delta_j$ , can be written as a weighted average of the  $B^{MTE}(x, u_S)$ .

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_S) \omega^j(x, u_S) du_S,$$

where the weights  $\omega^j(x, u_S)$  are specific to parameter  $j$  and integrate to one.

# Effects of Treatment as Weighted Averages

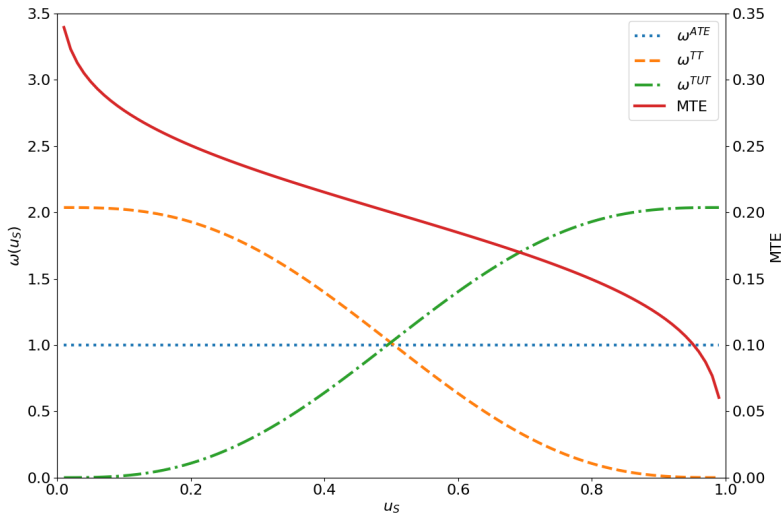
## Weights

$$\omega^{ATE}(x, u_S) = 1$$

$$\omega^{TT}(x, u_S) = \frac{1 - F_{P|X=x}(u_S)}{E[P | X = x]}$$

$$\omega^{TUT}(x, u_S) = \frac{F_{P|X=x}(u_S)}{E[1 - P | X = x]}$$

Figure: Effects of Treatment as Weighted Averages



## *Local Average Treatment Effect*

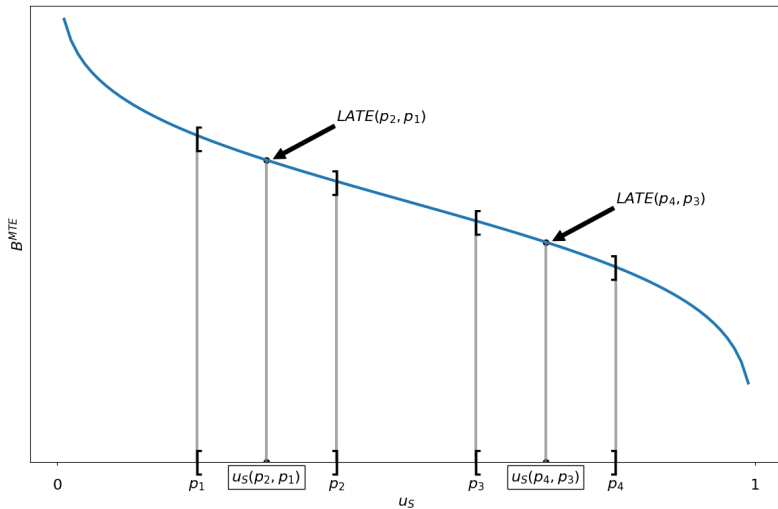
## Local Average Treatment Effect

- ▶ **Local Average Treatment Effect:** Average effect for those induced to change treatment because of a change in the instrument.  $\Rightarrow$  instrument-dependent parameter
- ▶ **Marginal Treatment Effect:** Average effect for those individuals with a given unobserved desire to receive treatment.  $\Rightarrow$  deep economic parameter

$$B^{LATE} = \frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_S, u_{S'}) = \frac{1}{u_S - u_{S'}} \int_{u_S}^{u_{S'}} B^{MTE}(x, u) du,$$

Figure: Local Average Treatment Effect



## *Distributions of Effects*



Figure: Distribution of Potential Outcomes

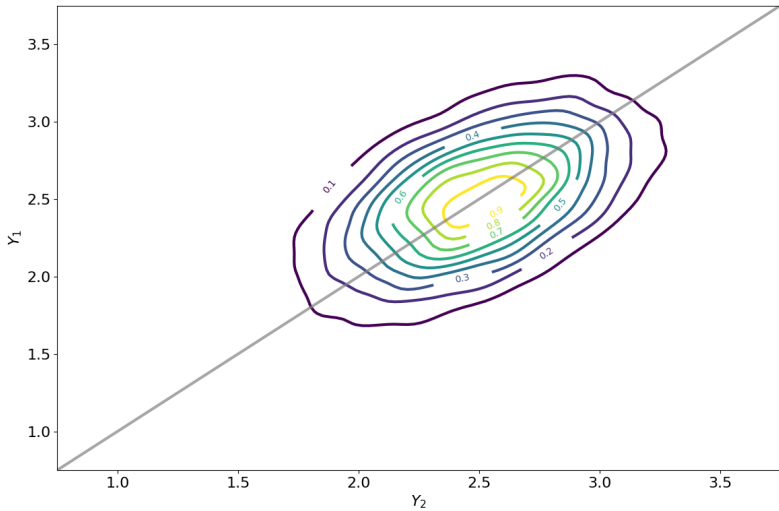
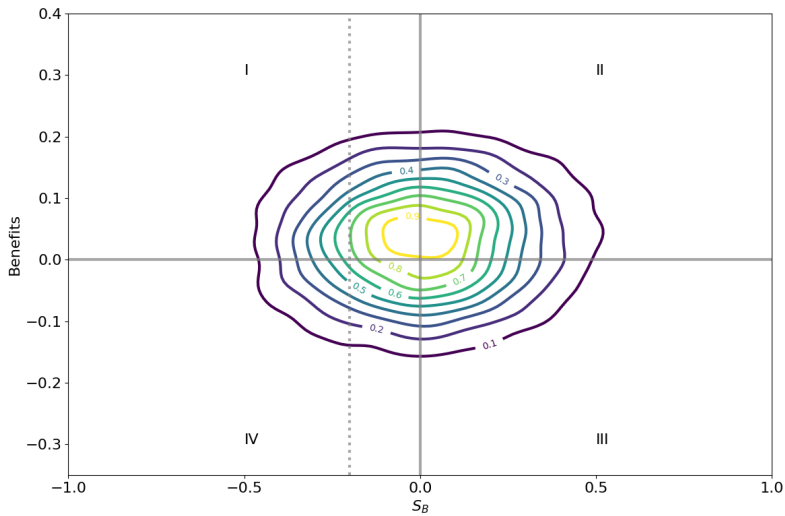


Figure: Distribution of Benefits and Surplus



# Appendix

## *References*

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Imbens, G. W. (2010). Better LATE than nothing: Some comments on Deaton (2009) and Heckman & Urzua (2009). *Journal of Economic Literature*, 48(2):399–423.