Parameters of Interest

Philipp Eisenhauer

Housekeeping

Since nearly all of you choose to review experimental studies. Please be sure to read ...

- ▶ Deaton, A. (2010). Instruments, randomization, and learning about development. *Journal of Economic Literature*, 48(2):424– 455
- ▶ Imbens, G. W. (2010). Better LATE than nothing: Some comments on Deaton (2009) and Heckman & Urzua (2009). *Journal of Economic Literature*, 48(2):399–423
- Heckman, J. J. and Urzúa, S. (2010). Comparing IV with structural models: What simple IV can and cannot identify. *Journal of Econometrics*, 156(1):27–37

Student Presentation

- 20 minutes of presentation and discussion
- peer-review of presentation

Collaborative Writing

We will simply group individuals together that present on the same day.

Lectures

constantly updated and available at

https://github.com/policyMetrics/course/wiki

Figure: Book Recommendations

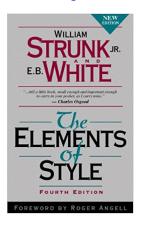
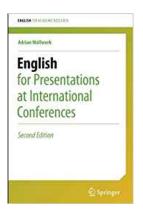
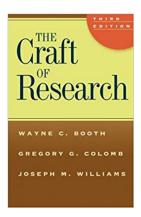
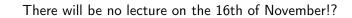




Figure: Book Recommendations







We need to slightly modify the notation, and thus the economics, of our conceptual framework to better align it with the published work on the topic. In the future, we will be more agnostic about what the agents care about and what they know when making their treatment decision.

Earlier Version

$$Y_1 = \mu_1(X) + U_1$$

$$C = \mu_D(Z) + U_C$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcomes

$$Y = DY_1 + (1 - D)Y_0$$
 $S = Y_1 - Y_0 - C$

$$S = Y_1 - Y_0 - C$$

$$D = I[S > 0]$$

The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = \mathrm{I}[\mu_D(X,Z) - V > 0]$$

Parameters of Interest

Philipp Eisenhauer

Heckman (2008) sets out three tasks for us:

- Defining the Set of Hypotheticals or Counterfactuals
 - ⇒ A Scientific Theory
- Identifying Causal Parameters from Real Data
 - ⇒ Mathematical Analysis of Data Point or Set Identification
- Identifying Parameters from Real Data
 - ⇒ Estimation and Testing Theory

Setup

The Generalized Roy Model

Potential Outcomes

$$Y_1 = \mu_1(X) + U_1$$

$$Y_0 = \mu_0(X) + U_0$$

Observed Outcome

$$Y = DY_1 + (1 - D)Y_0$$

Choice

$$D = \mathrm{I}[\mu_D(X,Z) - V > 0]$$

Useful Notation

$$P(X,Z) = \Pr(D = 1 \mid X, Z) = F_V(\mu_D(X, Z))$$

 $U_D = F_V(V)$

Specification

We follow the parameterization in Heckman and Vytlacil (2005):

$$Y_1 = \gamma + \alpha + U_1$$
 $U_1 = \sigma_1 \epsilon$ $\gamma = 0.670$ $\sigma_1 = 0.012$ $Y_0 = \gamma + U_0$ $U_0 = \sigma_0 \epsilon$ $\alpha = 0.200$ $\sigma_0 = -0.050$ $D = I[Z - V > 0]$ $V = \sigma_V \epsilon$ $\epsilon \sim \mathbb{N}(0, 1)$ $\sigma_V = -1.000$

$$Z \sim \mathbb{N}(-0.0026, 0.2700)$$
 $U_D = \Phi\left(\frac{V}{\sigma_V \sigma_\epsilon}\right)$

Individual Heterogeneity

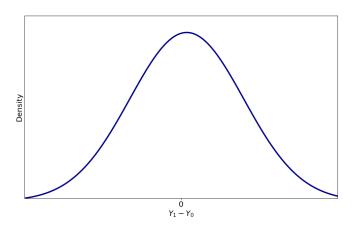
Individual-specific Benefit of Treatment

$$Y_1 - Y_0 = (\mu_1(X) - \mu_0(X)) + (U_1 - U_0)$$

Sources of Heterogeneity

- ► Difference in Observable Characteristics
- Difference in Unobservable Characteristics
 - Uncertainty
 - ▶ Private Information

Figure: Distribution of Benefits



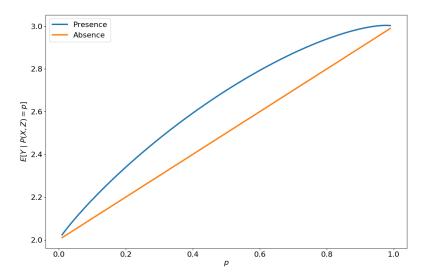
Essential Heterogeneity

Definition: Individuals select their treatment status based on gains unobservable by the econometrician. More formally,

$$Y_1 - Y_0 \not\perp\!\!\!\perp D \mid X = x.$$

⇒ consequences for the choice of the estimation strategy

Figure: Conditional Expectation and Essential Heterogeneity



Conventional Average Treatment Effects

Conventional Average Treatment Effects

$$B^{ATE} = E[Y_1 - Y_0]$$

 $B^{TT} = E[Y_1 - Y_0 \mid D = 1]$
 $B^{TUT} = E[Y_1 - Y_0 \mid D = 0]$

 \Rightarrow correspond to *extreme* policy alternatives

Selection Problem

$$\begin{split} E[Y \mid D = 1] - E[Y \mid D = 0] &= \underbrace{E[Y_1 - Y_0]}_{B^{ATE}} \\ &+ \underbrace{E[Y_1 - Y_0 \mid D = 1] - E[Y_1 - Y_0]}_{\text{Sorting Gain}} \\ &+ \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}} \end{split}$$

$$E[Y \mid D = 1] - E[Y \mid D = 0] = \underbrace{E[Y_1 - Y_0 \mid D = 1]}_{B^{TT}} + \underbrace{E[Y_0 \mid D = 1] - E[Y_0 \mid D = 0]}_{\text{Selection Bias}}$$

⇒ the bias depends on the parameter of interest

Figure: Distribution of Effects with Essential Heterogeneity

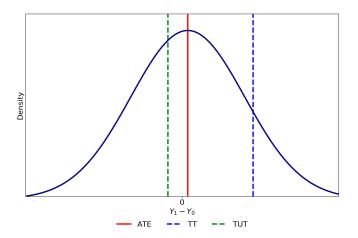


Figure: Distribution of Effects without Essential Heterogeneity

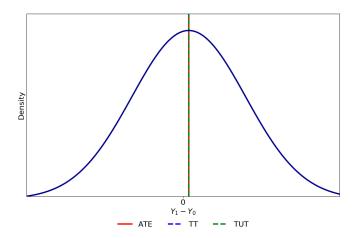
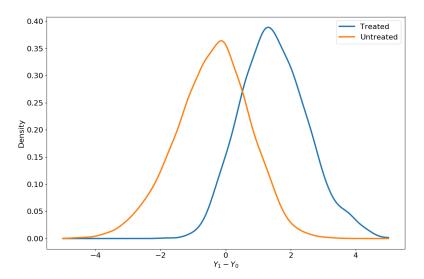


Figure: Distribution of Benefits by Treatment Status



Policy-Relevant Average Treatment Effects

Observed Outcomes

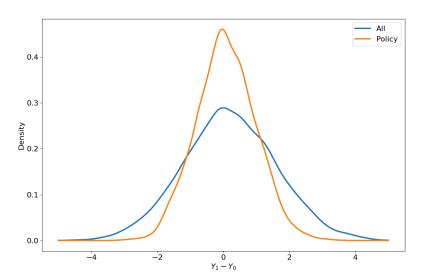
$$Y_B = D_B Y_1 + (1 - D_B) Y_0$$

 $Y_A = D_A Y_1 + (1 - D_A) Y_0$

Effect of Policy

$$B^{PRTE} = \frac{1}{E[D_A] - E[D_B]} (E[Y_A] - E[Y_B])$$

Figure: Distribution of Benefits for Policy



Marginal Effect of Treatment

Marginal Benefit of Treatment

$$B^{MTE}(x, u_S) = E[Y_1 - Y_0 \mid X = x, U_S = u_S]$$

Intuition: Mean gross return to treatment for persons at quantile u_S of the first-stage unobservable V.

Figure: Margin of Indifference

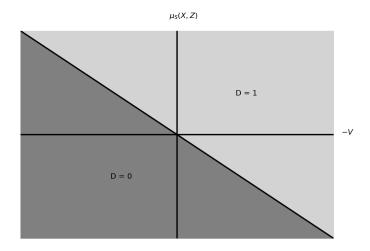
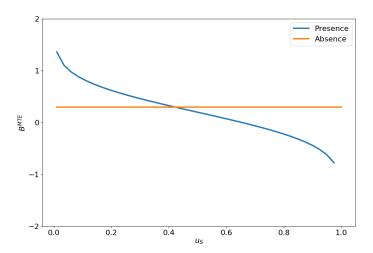


Figure: Marginal Effect of Heterogeneity



Effects of Treatment as Weighted Averages

Parameter Δ_j , can be written as a weighted average of the $B^{MTE}(x, u_S)$.

$$\Delta_j(x) = \int_0^1 B^{MTE}(x, u_S) \omega^j(x, u_S) du_S,$$

where the weights $\omega^{j}(x, u_{S})$ are specific to parameter j and integrate to one.

Effects of Treatment as Weighted Averages

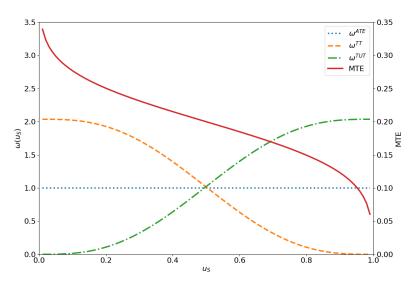
Weights

$$\omega^{ATE}(x, u_S) = 1$$

$$\omega^{TT}(x, u_S) = \frac{1 - F_{P|X=x}(u_S)}{E[P \mid X = x]}$$

$$\omega^{TUT}(x, u_S) = \frac{F_{P|X=x}(u_S)}{E[1 - P \mid X = x]}$$

Figure: Effects of Treatment as Weighted Averages



Local Average Treatment Effect

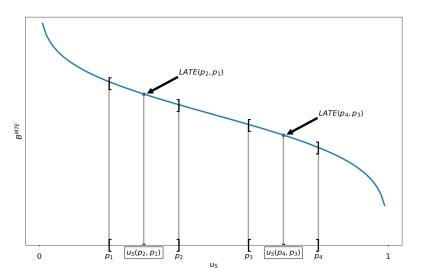
Local Average Treatment Effect

- ► Local Average Treatment Effect: Average effect for those induced to change treatment because of a change in the instrument. ⇒ instrument-dependet parameter
- ▶ Marginal Treatment Effect: Average effect for those individuals with a given unobserved desire to receive treatment.
 - ⇒ deep economic parameter

$$B^{LATE} = \frac{E(Y \mid Z = z) - E[Y \mid Z = z']}{P(z) - P(z')}$$

$$B^{LATE}(x, u_S, u_{S'}) = \frac{1}{u_S - u_{S'}} \int_{u_S}^{u_{S'}} B^{MTE}(x, u) du,$$

Figure: Local Average Treatment Effect



Distributions of Effects

Figure: Distribution of Potential Outcomes

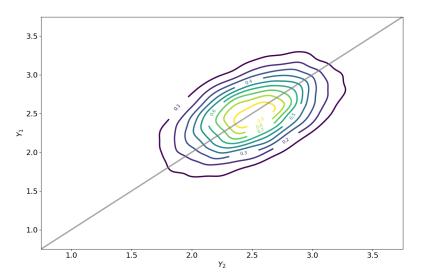
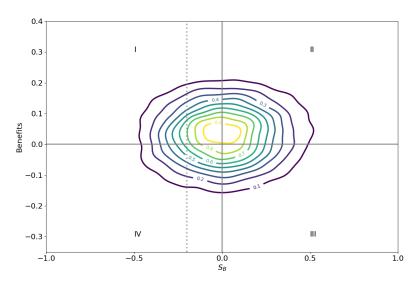


Figure: Distribution of Benefits and Surplus



Appendix

References

- Carneiro, P., Heckman, J. J., and Vytlacil, E. J. (2011). Estimating marginal returns to education. *American Economic Review*, 101(6):2754–2781.
- Deaton, A. (2010). Instruments, randomization, and learning about development. *Journal of Economic Literature*, 48(2):424–455.
- Heckman, J. J. (1990). Selection bias and self-selection. In Eatwell, J., Milgate, M., and Newman, P., editors, *Econometrics*, The New Palgrave Economics Collection, pages 201–224. Palgrave Macmillan, London.
- Heckman, J. J. (2008). Econometric causality. *International Statistical Review*, 76(1):1–27.

- Heckman, J. J. and Taber, C. (2010). Roy model. In Blume, L. E. and Durlauf, S. N., editors, *Microeconometrics*, The New Palgrave Economics Collection, pages 221–228. Palgrave Macmillan, London.
- Heckman, J. J. and Urzúa, S. (2010). Comparing IV with structural models: What simple IV can and cannot identify. *Journal of Econometrics*, 156(1):27–37.
- Heckman, J. J. and Vytlacil, E. J. (2005). Structural equations, treatment effects, and econometric policy evaluation. *Econometrica*, 73(3):669–739.

Imbens, G. W. (2010). Better LATE than nothing: Some comments on Deaton (2009) and Heckman & Urzua (2009). *Journal of Economic Literature*, 48(2):399–423.