

J.A. Thorpe Elementary Topics in Diff Geom ①

1). Level sets and graphs

$$L_c(f) = \{x_1, \dots, x_n \mid f(x_1, \dots, x_n) = c\}$$

Set where f 's takes

$n \geq 2$, contour map a constant value
That constant is called the height $\rightarrow c$.

Exercise - sketch level curves

1.1 $f(x_1, x_2) = x_1$

$$f(x, y) = x$$

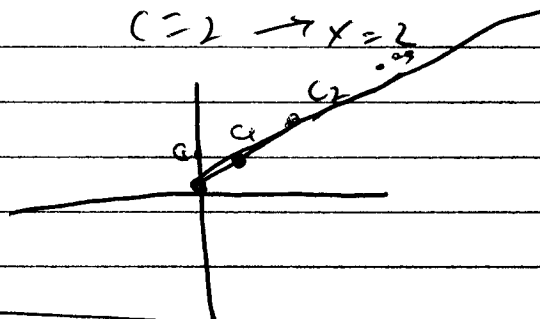
$$c = x$$

$$c = 0, \text{ then } x = 0$$

$$x = 0$$

$$c = 1, \text{ then } x = 1$$

$$c = 2 \rightarrow x = 2$$



1.2

$$f(x, y) = x - y$$

$$c = x - y$$

$$c = 0$$

$$0 = x - y$$

$$+y \quad +y$$

$$y = x \text{ or } x = y \text{ when } c = 0$$

$$1 = x - y$$

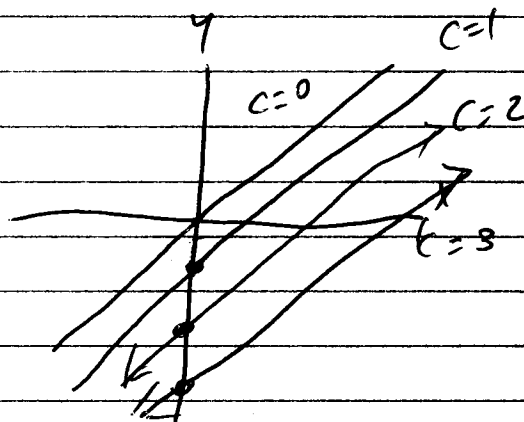
$$+y$$

$$1 + y = x$$

$$1 = x - y$$

$$-x$$

$$1 - x = -y \Rightarrow -1 + x = y$$



1.3) $f(x,y) = x^2 - y^2$

$$C = x^2 - y^2$$

$$0 = x^2 - y^2$$

$$+y^2 \quad +y^2$$

$$y^2 = x^2 \text{ when } C=0$$

$$x^2 - y^2 = 0$$

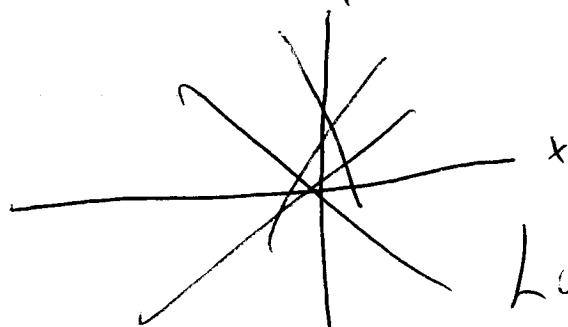
$$+y^2 \quad +y^2$$

$$1 + y^2 = x^2, \text{ or } 1 = x^2 - y^2$$

$$+x^2$$

$$1 - x^2 = -y^2$$

$$y^2 = 1 - x^2 \text{ or } x^2 + y^2 = 1$$



Looks like.

