

# Dummit and Foote: Abstract Algebra Exercise Solutions

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## Chapter 1 Solutions:

**Determine which binary operations are associative**

a).  $a * b = a - b$  for all  $a, b \in \mathbb{Z}$ .

$$a * (b * c) = (a * b) * c$$

$$a * (b - c) = (a - b) * c$$

$$a - (b - c) = (a - b) - c$$

$$a - b + c \neq a - b - c$$

So this operation is not associative.

b).  $a * b = a + b + ab$  for all  $a, b \in \mathbb{R}$

$$a * (b * c) = (a * b) * c$$

$$a * (b + c + bc) = (a + b + ab) * c$$

$$a + b + c + bc + ab + ac + abc = a + b + ab + c + ca + b + abc$$

So this operation is associative.

c).  $a * b = \frac{a+b}{5}$  for all  $a, b \in \mathbb{Q}$

$$a * (b * c) = (a * b) * c$$

$$a * \left(\frac{b+c}{5}\right) = \frac{a+b}{5} * c$$

$$\frac{a + \frac{b+c}{5}}{5} = \frac{\frac{a+b}{5} + c}{5}$$

LHS and RHS  $\neq$

So this operation is not associative.

d).  $(a, b) * (c, d) = (ad + bc, bd)$  on  $\mathbb{Z} \times \mathbb{Z}$

This problem can also be found in Pinter's Dover Book on Abstract Algebra.

$$(a, b) * ((c, d) * (x, y)) = ((a, b) * (c, d)) * (x, y)$$

$$(a, b) * (cy + xd, dy) = (ad + bc, bd) * (x, y)$$

$$(ady + b(cy + xd), bdy) = (y(ad + bc) + xbd, bdy)$$

$$(ady + bcy + bxd, bdy) = (yad + ybc + xbd, bdy)$$

These two are equal so the operation is associative.

e).  $a * b = \frac{a}{b}$  on  $Q - (0)$

$$a * (b * c) = (a * b) * c$$

$$a * \left(\frac{b}{c}\right) = \left(\frac{a}{b}\right) * c$$

$$\frac{\frac{a}{b}}{c} = \frac{\frac{a}{b}}{c}$$

So this operation is associative.

**2). Find which operations are commutative:**

a).  $a * b = a - b$  on  $Z$

$$a * b = b * a$$

$$a - b = b - a$$

This is not true, so this operation is not commutative.

b).  $a * b = a + b + ab$  on  $R$

$$a * b = b * a$$

$$a + b + ab = b + a + ba$$

This is true so the operation is commutative.

c).  $a * b = \frac{a+b}{5}$  on  $Q$

$$a * b = b * a$$

$$\frac{a+b}{5} = \frac{b+a}{5}$$

This is true, so the operation is commutative.

d).  $(a, b) * (c, d) = (ad + bc, bd)$  on  $ZXZ$

$$(a, b) * (c, d) = (c, d) * (a, b)$$

$$(ad + bc, bd) = (cb + da, db)$$

This is true, so the operation is commutative.

e).  $a * b = \frac{a}{b}$  on  $Q$

$$a * b = b * a$$

$$\frac{a}{b} \neq \frac{b}{a}$$

Therefore, this operation is not commutative.