Joshua Steier

Charles Pinter: Abstract Algebra,solutions to exercises:Rings

Problem A1:

Prove that the following are commutative rings with unity.

The ring has the addition function: $a + b = a + b - 1$

The ring has the multiplication function: $ab - (a + b) + 2$

Both these functions belong to the integers.

In order for this to be a ring, we should first check that the addition function follows an abelian group.

Let's check the conditions on the addition function to see if it is an abelian group.

First, commutativity $a + b = b + a$

$a + b = a + b - 1$

$b + a = b + a - 1$

$a + b - 1 = b + a - 1$

Next, check associativity

$a + (b + c) = (a + b) + c$

Starting on the left hand side(LHS):

$a + (b + c) = a + b + c - 1 = a + b + c - 2$

Now for the right hand side(RHS):

$(a + b) + c = a + b - 1 + c$

$= a + b - 1 + c - 1$

$= a + b + c - 2$

These two are equal, showing we have associativity.

We now must check for an identity element, and find it if it exists.

$a + e = a$

$e = 1$

Let's attempt to find inverses

$a + a' = e$

$a' = 2 - a$

Now that these conditions are checked, the addition operation follows as an abelian group.

Now, we must check if the multiplication function is associative.

$(a * b) * c = a * (b * c)$

Starting with the LHS:

$a * b = ab - (a + b) + 2$

$(a * b) * c = abc - ac - bc - ab + a + b + c$

Now the RHS:

$a * (b * c)$

$b * c = bc - (b + c) + 2$

$a * (b * c) = abc - ab - ac + a - bc + b + c$

These two are the same, thus this condition is satisfied.

Then, we must check that our functions are distributive over addition.

$a(b + c) = ab + ca$

$a * (b + c) = a * b + c * a$

LHS:

$ab + ac - b - c + 3$

RHS:

$ab + ca$

$ab + ac - b - c + 3$

These results are the same so this must be true.

Now, check if this is a **commutative** ring.

We must simply check if the multiplication operation is commutative.

So, $a * b = b * a$

$a * b = ab - (a + b) + 2$

$b * a = ba - (b + a) + 2$

These results are the same so this is a commutative ring.

Now we check for unity.

First I'll look for an identity element in the multiplication function if it exists.

Exercises: Chapter 18: Ideals and Homomorphisms

A: Prove that each of the following is a subring of the indicated ring

1). $x + y(3)^{\frac{1}{2}}, x, y \in Z$, is a ring in R Let S= $x + y(3)^{\frac{1}{2}}$

Let $a, b \in S$.

$a = x_1 + y_1(3)^{\frac{1}{2}}$ $b = x_2 + y_2(3)^{\frac{1}{2}}$

a). closure under addition:

$a + b = x_1 + y_1(3)^{\frac{1}{2}} + x_2 + y_2(3)^{\frac{1}{2}}$

$(x_1 + x_2) + (y_1 + y_2)(3)^{\frac{1}{2}}$

Takes the original form, so it is closed in addition.

b). closure under multiplication:

$x_1(x_2 + 3^{\frac{1}{2}}y_2) + 3^{\frac{1}{2}}y_1x_2 + 3y_1y_2$

c). closure under negatives:

$-x - y3^{\frac{1}{2}}$

$-1(x + y3^{\frac{1}{2}})$