Tutorial 4

Q1-1.

1	2	3
AA	OO	AO

B = Basket number
F = Type of fruit

B	P(B)
1	$\frac{1}{3}$
2	$\frac{1}{3}$
3	$\frac{1}{3}$

F	P(F)
A	1/2 or 0.5
O	1/2 or 0.5

F	B	P(F|B)
A	1	1
O	1	0
A	2	0
O	2	1
A	3	0.5
O	3	0.5

2. $P(B=1, F=a)$

$= P(F=a, B=1)$

$= P(F|B)P(B)$

$= 1 \times \frac{1}{3}$

$= \frac{1}{3}$

3. $P(B=1 \mid F=a)$

Using Baye's

$$P(B=1 \mid F=a) = \frac{P(F=a \mid B=1)P(B=1)}{P(F=a)}$$

$$= \frac{1 \times \frac{1}{3}}{\frac{1}{2}}$$

$$= \frac{1}{3} \div \frac{1}{2}$$

$$= \frac{1}{3} \times \frac{2}{1}$$

$$= \frac{2}{3}$$

Q2. 1.

A	B	$P(A, B)$
a	b	0.01
a	$\neg b$	0.02
$\neg a$	b	0.05
$\neg a$	$\neg b$	0.92

marginalise over C

2. $P(A=a) = 0.03$

$P(B=b) = 0.06$

$P(A=a, B=b) = 0.01$

$\neq P(A=a)P(B=b)$

therefore, not independent

3.

B	$P(B)$
b	0.06 $\Rightarrow 0.01 + 0.01 + 0.04$
$\neg b$	0.94 $\Rightarrow 0.00 + 0.02 + 0.90 + 0.02$

More likely that student does not have bug

4. $P(b|\neg c) = \dfrac{P(b,\neg c)}{P(\neg c)} = \dfrac{0.05}{0.01+0.02+0.04+0.02}$

$= \dfrac{0.05}{0.09} = \dfrac{5}{9}$

B	$P(B\|\neg c)$
b	5/9
¬b	4/9

Similarly,

$P(\neg b|\neg c) = \dfrac{P(\neg b|\neg c)}{P(\neg c)}$

$= \dfrac{0.02+0.02}{0.01+0.02+0.04+0.02}$

$= \dfrac{4}{9}$

More likely that student had the bug. Makes sense because if student did not show up to class, they are likely ill.

5. $P(B|C=\neg c, A=a) = \dfrac{P(b,\neg c, a)}{P(\neg c, a)} = \dfrac{0.01}{0.01+0.02}$

$= \dfrac{1}{3}$

B	$P(B\|C=\neg c, A=a)$
b	$\frac{1}{3}$
¬b	$\frac{2}{3}$

6. A and B are not conditionally independent given C as:-

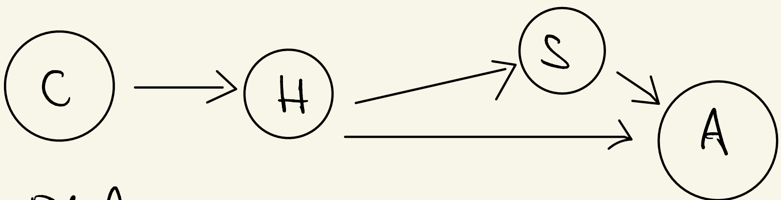
$$P(B=b \mid C=\neg c, A=a) = \frac{1}{3}$$

which is not equal to

$$P(B=b \mid C=c) = \frac{5}{9}$$

Remember, $P(X \mid Y, Z) = P(X \mid Z)$ for conditional independence

Q3- 1.



2a) $P(A, C, S, H)$ is the full joint probability distribution

$$P(C)\,P(H \mid C)\,P(S \mid H)\,P(A \mid S, H)$$

$$= P(c)\ P(f \mid c)\ P(s \mid f)\ P(a \mid f, s)$$

$$= 0.4 \times 0.7 \times 0.9 \times 0.01 = 0.00252$$

2b) $P(A, C, S)$ marginalise over $H = f, v, p$

$0.4 \times 0.7 \times 0.9 \times 0.01$

$+$

$0.4 \times 0.1 \times 0.6 \times 0.40$

$+$

$0.4 \times 0.2 \times 0.3 \times 0.20$

$= 0.001692$

c) $P(C = c, S = s) = \sum_A \sum_H P(C, S, A, H)$

$P(a, c, s) + P(\neg a, c, s)$

from above

0.001692

$0.99 \times 0.9 \times 0.7 \times 0.4$

$+ 0.6 \times 0.6 \times 0.1 \times 0.4$

$+ 0.8 \times 0.3 \times 0.2 \times 0.4$

$= 0.28308$

total: ≈ 0.3

Q4.1 $P(a) = \sum_S P(a|s)P(s) = 0.6 \times 0.3 + 0.7 \times 0.3 = 0.39$

$P(\neg a) = 1 - 0.39$

$= 0.61$

2- $P(\neg g, a|s) = P(\neg g|s)P(a|s) = 0.6 \times 0.3 = 0.18$

$P(\neg g, a|\neg s) = 0.3 \times 0.8 = 0.24$

$P(\neg g) = 0.3 \times 0.3 + 0.8 \times 0.7 = 0.09 + 0.56 = 0.65$

$$P(s|\neg g, a) = \frac{P(\neg g, a|s)P(s)}{P(\neg g, a)} = \frac{0.3 \times 0.18}{0.18 \times 0.3 + 0.24 \times 0.7} = \frac{9}{37}$$

3- $P(\neg g, a, s, c, \neg f)$

$= P(\neg g|s)P(a|s)P(s|c, \neg f)P(c)P(\neg f)$

$= 0.25 \times 0.7 \times 0.7 \times 0.3 \times 0.6$

$= 0.02205$

4- a) No - refer to lecture 13 slides pg 37

b) Yes - same ↑ pg 38

c) Yes - same ↑

d) No - pg 35

e) No - pg 40

f) No - could not derive from $P(C, F|A)$ to $P(C|A)P(F|A)$