# Washington State University School of Electrical Engineering and Computer Science

**EE 352 Electrical Engineering Laboratory** 

Lab # 1

**Equipment Familiarity and First Order Electrical Circuits** 

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#### Lab Overview

In this lab, four experiments were conducted to validate the first order RC circuits. Experiment one was about getting familiar with the function generator and the oscilloscope. Using LTSPICE the DMM and the oscilloscope were analyzed, as well as the basic behaviors of the RC low pass filter. Finally experiment four was about characterizing the oscilloscope's frequency response and observe its effects on circuit measurements. The circuits were simulated, and the results agreed with the theoretical results with in one-percent error due to tolerances of the simulations.

## **Experiment #1 Getting Familiar with Function Generator and Oscilloscope**

### 1.1 Purpose

The purpose of this experiment was to become familiar with the function generator and the oscilloscope. Understanding them is crucial in conducting electrical engineering experiments. This experiment allowed for visual understanding in using these two pieces of equipment and taking measurements. There was also a gained visual understanding in how these pieces behave with both low and high frequencies.

#### 1.2 Theoretical Background

Two pieces of equipment was shown in the videos. These pieces of equipment were the oscilloscope and the function generator. The function generator as shown in Fig.1 below is used to provide sine waves, rectangular pulse waves, and triangular pulses at a large range of frequencies and amplitudes that serve as inputs to circuits. The internal impedance is 50 Ohms which typically should be factored in when building a circuit that will use the function generator as a signal input. Function generators can also have a DC offset, where not only can the AC signal outputted can fluctuate around 0V but also can oscillate around a certain DC offset.



Figure 1: GW Instek AFG 2000

The other piece of equipment was the oscilloscope, shown in Fig.2 below, is usually connected to the function generator using a Tee BNC splitter which divides the wave or pulse being generated to both the circuit and the oscilloscope. A probe can be connected to the oscilloscope to measure voltages anywhere on the circuit. To adjust the accuracy of the signal, the sec/DIV and the Volt/DIV knobs can be used to zoom in on an area of the signal or zoom out to get a bigger picture of the signal behavior. To save the image of the signal, a thumb drive can be plugged into the oscilloscope with the print button pressed.

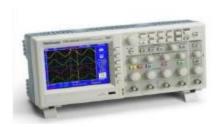


Figure 2: Tektronix TDS 2014B Oscilloscope

The most common signals generated by the function generator is the sine wave, square wave (pulse), and the triangular pulse (ramp). The signals are characterized by their frequencies, amplitudes, and DC offset. The function generator has 2 output channels. The T-splitter is a connector which can be used with the function generator which splits the output terminal into two signals, one which goes to the oscilloscope and the other which goes to the circuit. The oscilloscope is used to measure the signal. The oscilloscope has two knobs which are most important. One is the SEC/DIV knob on the scope which changes the seconds represented by each horizontal division on the scope screen. This knob can be turned to the left to compress the signal or the right to expand the signal on the screen. The other knob is VOL/DIV knob, which is unique to each scope channel, which changes the number of volts per vertical division on the screen for a signal of the assigned channel. It can be turned to the left to shrink the height of the signal or turned to the right to stretch the height of the signal. The x1 probe does not divide the signal input into the oscilloscope, but the x10 probe attenuates the signal by dividing it by 10 and then the scope multiplies the signal by 10 again to display it correctly. AC coupling on the oscilloscope factors out the DC offset of the signal, but DC coupling considers this offset when displaying the signal on the scope.

To get an accurate signal on the oscilloscope, it is best to zoom in as much as possible. The three most used techniques to measure the signals on the oscilloscope are 1) to measure by hand, 2) click 'measure' on the oscilloscope, or 3) using the cursor of the scope. To find the time constant on the oscilloscope; first use the cursor, then go to ch2, select the vertical, then measure peak to peak using cursor 1 and cursor 2, once found calculate V=.63\*Vp-p. Then use the cursor to move to the calculated point V and zoom to find the time change from the lower point to point V. The time between those to points is the rise time for 63% of the peak-to-peak as well as the time constant for the circuit.

# 1.3 Procedure

Not applicable

# 1.4 Results & Analysis

Not Applicable

#### 1.5 Conclusion

Not Applicable

# **Experiment #2: Digital Multimeter (DMM) Verses Oscilloscope**

# 2.1 Purpose

The oscilloscope and the DMM are two vital pieces of equipment when measuring the voltage of a circuit. An understanding of both pieces is important before doing any experiment. When using either piece it is important to understand the pros and cons, a con being that they both have internal impedances that affects the measurement values. Therefore, when measuring the voltage using the DMM and the oscilloscope there is an error introduced to the loading affect of the pieces' internal impedance.

### 2.2 Theoretical Background

The oscilloscope and the Digital multimeter are important pieces of equipment. As explained in Experiment #1, the oscilloscope is used to measure voltages and the digital multimeter is used to measure RMS voltage in AC circuits. The oscilloscope for this experiment carries a R of 10M Ohms and a C of 16 pF and is in parallel with the resistor paired with Vo on the voltage divider circuit shown in Fig.3. Similarly, the DMM carries a loading effect of 10M Ohms for the R and 200 pF for the C. The 10 M Ohms for both pieces is used to represent the x10 probe that would be applied to the circuit. They both have an internal impedance that effects the measured values.

$$Z = R + jZ_C + jZ_L$$
 (1)

The impedance is the resisance added to the reactance from both inductors and capacitors at play in the circuit.

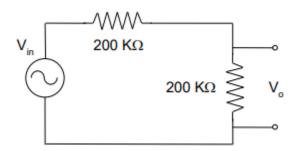


Figure 3: Voltage divider circuit for Part II.

When undergoing both low and high voltages and high and low frequencies, the signal can become less accurate which signifies that there is a limit to accuracy. LTSPICE shows that the oscilloscope and the DMM can be manipulated the change the time and voltage axes by changing the components of the analysis used in the circuit. Ideally the difference in frequency should not affect the voltage divider circuit, due to the error, experimentally there is a bit of an effect.

Also, the reactants of both inductors and capacitors are dependent on frequency. Thusly as frequency changes so does the impedance. When applying the DMM, as the frequency increases, the impedance of it changes with decreases the accuracy of the DMM causing error. In the case that the resistors in the DMM are similar to the voltage divider resistors, the DMM acts like another resistor which drastically effects the circuit. When the resistors in the circuit are very small in comparison to the resistors in the DMM, the resistance in the DMM is big enough to prevent current from going through it so it does not affect the circuit.

#### 2.3 Procedure

- 1. Used LTSPICE to build the circuit shown in Figure 1 with a sine wave source of 80 Hz and 3 volts zero to peak.
- 2. Used transient analysis to show at least 5 sine wave cycles of about 70 ms with a max step size of about 10us to plot Vin and Vout.
- 3. Investigated to loading effect of the oscilloscope with a R of 10Mohms and a C of 16pF across R2. Then plotted it to show Vin and Vout.
- 4. Investigated loading effect of Oscilloscope and the DMM by adding a R of 10Mohms and a C of 200pF also across R2. Then plotted the simulation to show Vin and Vout. Lastly, talked about how 2,3 and 4 compare.
- 5. Increased frequency to 300kHz
- 6. Changed the Sine wave input to 300kHz and 3 volts zero to peak. Also changed transient values to 200us, with max step size of 20ns. Then simulated the circuit to plot Vin and Vout to find the peak value.
- 7. Added loading effect of the Oscilliscope across R2 to plot and find the peak values of Vout and Vin.
- 8. Removed the Oscilloscope then added the DMM across R2 to plot and find peak values of Vout and Vin.
- 9. Changed the sine wave back to 80Hz, and 3volts zero to peak. Also changed the voltage divider resistors to 10Mohms and used the trans to show at least 5 sine wave cycles and set the max step size to be about 10us. Finally added the DMM and plotted Vin and Vout to determine the peak value.

#### 2.4 Results & Analysis

Initially, the circuit of Fig.3 was built in LTSPICE, with the sine wave source set at 80 Hz, and 3 volt zero-to-peak for the input voltage. Then a transient analysis was added to the circuit to show at least five sine wave cycles and with a step size of 10 us as shown in Fig.4.

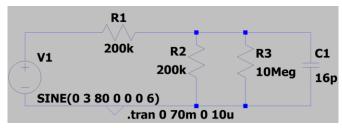


Figure 4: Circuit for Part 3 of the Procedure

Once the circuit shown in Fig.4 was simulated, the plot shown in Fig.5 represents the behavior of both the voltage in and the voltage out of the circuit. The resulting peak value was 1.48V for voltage out which is .02 off or -1.33% from the ideal output voltage of 1.5V.

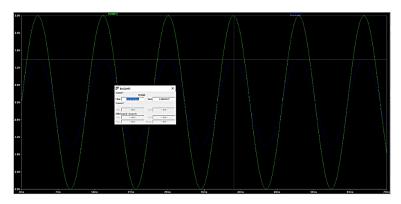


Figure 5: Graph of Part 3 of the Procedure

Next, the DMM was applied to the circuit to analyze the resulting voltages when both the oscilloscope and the DMM is at play as shown in Fig.6. The resulting voltage output when both were applied was 1.47V. This is a .03V difference, or 2%, compared to the ideal value of 1.5V. The reason for this change is because there were more resistors and capacitors at play which increases the impedance, causing a greater change between Vin which gives the idea voltage, and V out.

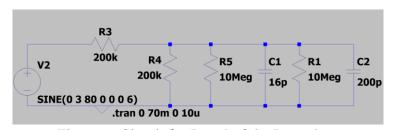


Figure 6: Circuit for Part 4 of the Procedure

Next, the task was to analyze the effect of a voltage output when the frequency is set to 300k Hz as shown in Fig.7. To display the appropriate scale on the plot, the step size was set to 20ns and a stop time of 200us.

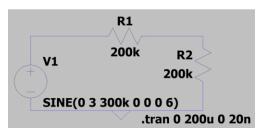


Figure 7: Circuit for Part 6 of the Procedure

The plot shown in Fig.8 displays the resulting Vin and Vout values due to the changes made that are shown in Fig.7. The output voltage was 1.498V which indicates that without the influence of the DMM and the oscilloscope the voltage is more accurate since the difference here is .002V, or .13% compared to the .02V, .03V. The slight difference still existing is due to the small impedance given from the 300k Hz.

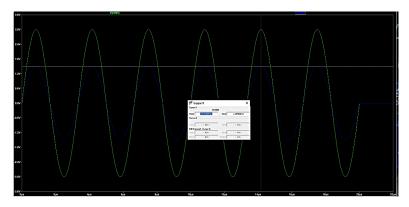


Figure 8: Graph of Part 6 of the Procedure

Fig.9 indicates the circuit shown in Fig.7 with the addition of the oscilloscope.

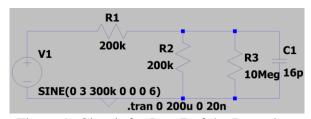


Figure 9: Circuit for Part 7 of the Procedure

The resulting plot shown in Fig. 10 shows the voltage input and output from the circuit in Fig.9 where the output voltage is shown to be 470.6 mV at peak value which is not good because it is less than half the ideal voltage of 1.50V.

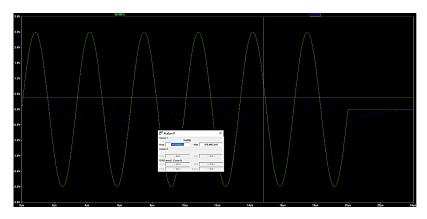


Figure 10: Graph of Part 7 of the Procedure

Then the oscilloscope was replaced with the DMM as shown in Fig.11.

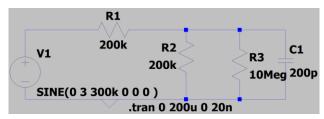


Figure 11: Circuit for Part 8 of the Procedure

The resulting voltage peak output value was 39.989mV as shown in Fig.12 where also the output and input voltages of Fig.11 are graphed. This resulting peak value is even worse and is due to the fact that the DMM had a greater capacitance value than the oscilloscope which increases the impedance effect in the voltage of the circuit.

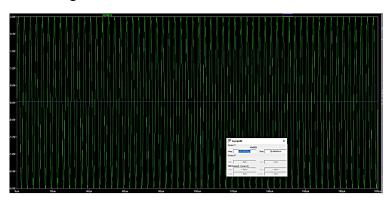


Figure 12: Graph of Part 8 of the Procedure

The last part of this experiment was then to change the frequency back to 80 Hz, then the resistors in the voltage divider were changed to 10M Ohms. For the transient analysis, the stop time was changed back to 70ms and the max step size was changed back to 10us. These changes are all shown in Fig. 13.

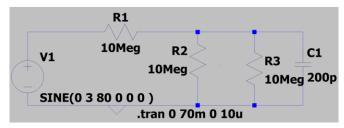


Figure 13: Circuit for Part 9 of the Procedure

Fig.14 below is the resulting plot of the circuit shown in Fig.13. The output peak voltage value was 947.67mV which is better than when the frequency was 300k Hz and the voltage divider resistances were 200k Ohms. Though it is still quite different from the ideal value of 1.5V. This is due to the DMM now acting like another resistor.

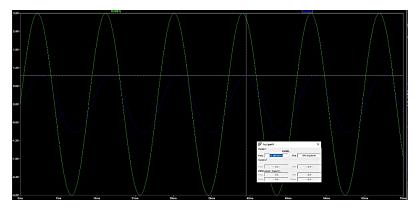


Figure 14: Graph of Part 9 of the Procedure

#### 2.5 Conclusion

The purpose of this experiment was to analyze the effects when comparing the DMM and the oscilloscope under low and high frequencies and under low and high resistances on the voltage divider. Ideally the output voltage should be half the voltage of the voltage source. Under a high frequency of 300kHz, the oscilloscope gave an output voltage of a third of the ideal and the DMM gave an even worse of about an eleventh of the ideal. When the resistances of the voltage divider were equated to the resistance value on the DMM, the output was just under two-thirds the ideal voltage. These scenarios indicate that under high frequencies or high resistances, the devices are not accurate. When the circuit was set to 80hz with the voltage divider resistances set at 200kohms, the DMM and the oscilloscope displayed pretty accurate voltage outputs being only .002V and .03V off from the ideal.

# **Experiment #3: Low Pass RL Circuit**

# 3.1 Purpose

The purpose of this experiment was to figure out how a low-pass filter works in an RL circuit, then to create a bode plot. The bode plot was to be done in two different ways; through an AC analysis decade sweep and by measuring the gain in V/V with frequencies of 0.01fo, 0.1fo, 0.5fo, fo, 2fo, 5fo and 10fo. Then constructing the bode plot in Excel on logscale with the gain being converted into units of dB. The last part of this experiment was to then calculate the time constant of the same RL low pass circuit using the plots simulated in LTSPICE.

### 3.2 Theoretical Background

Various components and devices can be used in circuit. One of these components being resistors that can pass voltage through and have a voltage across them. Another is inductors that are used to store energy, use reactive power and are measure in units of Henrys. A third component is capacitors which are measured in Farads, also use reactive power, and store energy.

Circuits can be expressed in the time domain as well as in the phasor domain. In the phasor domain, the frequency is assumed constant, the real parts measured is impedance and the complex part is reactance. The impedance for the resistor is the resistance R, the impedance for an inductor is dependent on the frequency and the capacitive and inductive elements in the circuit. As shown in equations 2 and 3  $j = \sqrt{-1}$ ,  $\omega = 2\pi f =$  angular frequency, C=capacitance and L = inductance.

$$Z_C = \frac{1}{iwC} (2)$$

$$Z_L = jwL$$
 (3)

When every element of a circuit is converted, the amplitude response can be found by doing the magnitude of Vout/Vin. When the frequency is low the gain is low, and when the frequency is high the gain is high. A circuit is a low pass filter when the gain is 1 at low frequencies and 0 at high frequencies. In this occurrence the cutoff frequency is -3dB. Decibels (dB) are a measuring scale for responses in the logarithmic scale, and the conversion between V/V to dB is:

$$\left(\frac{|V_o|}{|V_i|}\right)_{dB} = 20 \log_{10}\left(\frac{|V_o|}{|V_i|}\right)$$
 (4)

The basic RL low pass circuit used is shown below in Fig. 15 where Vi is the input voltage and Vo is the output voltage. For the circuit shown in Fig.15, the response is dependent on the inductance, resistance, and angular frequency ( $\omega$ ).

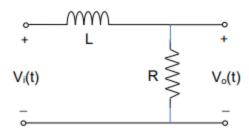


Figure 15: Low-Pass RL circuit

Related to the cutoff frequency is the time constant,  $\tau$ , which is the time it takes to reach 63.4% of the steady state value. The equations used to determine the time constant and the step response are:

$$I(t) = [I(0^+) - I(\infty)]e^{-\frac{t}{\tau}} + I(\infty); \quad \tau = \frac{L}{R}$$
 (5)

The values specified for this circuit were  $R = 300~\Omega$  and L = 1 mH and a Vin of 4V zero-to-peak. Using the angular frequency equation to determine fo, it was determined to be 47.746Hz. Experimentally, the cutoff frequency can be determined by finding when the gain is 63.4% of the DC gain which is then equal to fo. Calculations to find the above values are shown in appendix 1 and 2.

#### 3.3 Procedure

- 1. Using LTSPICE, the circuit of Figure 2 was built with R=300ohms, Vin having an amplitude of 4V zero to peak and L=1mH.
- 2. Used AC analysis with sweep type decade, 1000 points, start frequency of 10Hz and stop frequency of 1MHz. Then plotted Vout/Vin to find 3dB frequency.
- 3. Measured the output voltage by changing the frequency to be .01fo, .1fo, 5fo, fo, 2fo, 5fo, and 10fo. Ran the simulation for each frequency value and each time adjusted to an appropriate stop time and max step size. Also, for each simulation, found the peak value of Vin and Vout then used excel to plot frequency vs amplitude in the Logscale. Example Calculation for stop time and max step size:

Figure 16: Sample calculation for max step size and stop time.

- 4. Compared results to theoretical values.
- 5. Used LTSPICE to modify the input voltage of figure 2 to be a pulse signal with a variance of 0 to 4 volts, 0s delay, time rise 1ns, time fall of 1ns, pulse width of 30us and stop time of 60us. Then used a trans analysis with stop time of 120 us and max step size of .5ns.

6. Plotted Vin and Vout of the circuit to estimate the time constant and steady state response of the system. Lastly, commented on the agreement between the measured step response and the theoretical step response from the pre-lab.

# 3.4 Results & Analysis

The theoretical value for the cutoff frequency (fo) stated in the theoretical background was 47.746kHz. The graphed value is found using the plot in Fig. 18 as 47.64 kHz. Tis indicates only a .22% error from the theoretical value which is likely due to human error in using the cursor to find the value.

The circuit to which this is based is shown in Fig. 17 showing that the simulation is a result of and AC analysis with sweep type of decade, 1000 points, start frequency of 10 Hz and a stop frequency of 1M Hz.

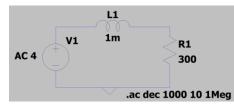


Figure 17: Circuit for Part 1

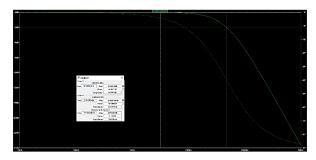


Figure 18: Bode plot graph with the gain in dB vs. the frequency in log scale for 1b.

Below is the circuit for when the frequency varied with a sine wave source. The frequency shown in Fig. 19 is .1fo, with the adjusted stop time of 1.5ms and the step size of 1.5us. The rest of the frequency values used, the corresponding voltage values, step size, stop time, and logscale values are all shown in table 1.

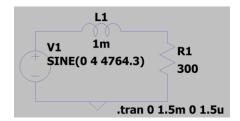


Figure 19: Sample circuit for Part 1c

Table 1: input frequency, transient analysis stop time and timestep, plotted peaks for Vi and Vo, Logscale

frequency	frequency	stop	max	Vi	Vo	Logscale
$(f_0)$	(Hz)	time	timestep	peak	peak	(dB)
$0.01  f_0$	476.4	15ms	15us	(V) 3.997	(V) 3.997	0
$0.1 f_0$	4764	1.5ms	1.5us	3.991	3.98	035
$0.5 f_0$	23820	300us	300ns	3.993	3.577	956
$f_0$	47640	150us	150ns	3.994	2.824	-3.01
2 f <sub>0</sub>	95280	75us	75ns	3.995	1.791	-6.97
5 f <sub>0</sub>	238200	30us	30ns	3.993	0.795	-14.02
10 f <sub>0</sub>	476400	15us	15ns	3.991	0.405	-19.87

Since the error was so small, the theoretical bode plot in Fig.18 is very similar to the experimental bode plot of the frequency vs. logscale values in Fig. 20. This indicates that the measured values in Fig.20 are accurate enough to show the behavior of the circuit.

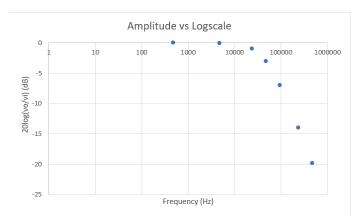


Figure 20: 20log<sub>10</sub>(Vo/Vi) [dB] vs. frequency in log scale [Hz] using data from Table 1.

The circuit below in Fig. 21 is the circuit shown in Fig. 15 with a Vi as a square wave input. The pulse varies from 0 to 4 volts with 0s delay, time rise of 1ns, time fall of 1ns, the pulse width is  $30 \,\mu s$  (>  $5 \,\tau$ ) and period of  $60 \,\mu s$ . The transient analysis has a stop time of  $120 \,\mu s$  and a max step size of  $0.5 \,ns$ .

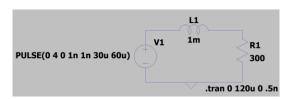


Figure 21: Circuit for Part 2a

Fig. 22 shows the simulation result of Fig. 21 of the behavior of Vin and Vout. The chart on the figure shows the estimated values to then predict the time constant and the steady-state response. Using the cursor, the voltage rise peak-to-peak of Vo was measured to be 3.999V. 3.999V\*0.63 = 2.52V which is the desired voltage value of the time constant. The cursor was moved to 2.52V on the plot of Vo, and the time rise to this voltage was measured to be about 3.334us. This is the time constant,  $\tau$ =3.335 $\mu$ s. Using the time constant estimate and equation (5) from the theoretical background, the steady state response of the circuit was estimated to be:

$$V_c(t) = 3.999e^{-\frac{t}{3.334*10^{-6}}}V \quad (6).$$

Figure 22: Bode plot graph with the gain in dB vs. the frequency in log scale for part 2b.

The theoretical value for  $\tau$  from the theoretical background was 3.333us, and our estimated  $\tau$ =3.334µs. These are only within 1ns of each other, which is a 0.03% difference. This is most likely due to human errors from using the cursor in LTSPICE to measure  $\tau$  from a plot. Thus, the estimate for  $\tau$  can be considered accurate, as well as the steady state response mentioned in equation (6) because it is based off this value for  $\tau$  and the theoretical equation for a low pass RL circuit steady state response, given in equation (5). The reciprocal of tau is calculated as which is used to find the cutoff frequency as shown in appendix 1:

$$\omega_0 = \frac{1}{\tau} = \frac{1}{3.334us} = 300 krad/s \quad (7)$$

#### 3.5 Conclusion

To conclude this experiment, the measurements done in the experiment reflect the equations shown in the theoretical background. The purpose of the experiment was to experimentally find the 3dB frequency, plot varying frequency with the logscale of the gain in dB, and find tau and the step response equation. The 3dB frequency was found by taking the data and finding the gain in dB for each frequency the finding the frequency that revealed a gain of 3 dB. Tau was found by using equation (5). The gain in dB graph was shown to be very similar to the bode plot graphed in LTSPICE. This experiment validated the theories behind low pass filter circuits.

# Experiment #4: Oscilloscope Characterization 4.1 Purpose

The purpose of experiment#4 was to characterize the oscilloscope's frequency response to see how it effects the circuit measurements, determine the cutoff frequency of both the x1 and x10 loading equivalent circuits and the time constant of both the x1 and x10 loading equivalent circuits.

#### 4.2 Theoretical Background

As stated in the theoretical background in experiment #1, the oscilloscope is a useful tool used to measure voltages throughout circuits when using probes facilitate point of measurement on the circuit. Ideally probes have infinite resistance so that no current flows through them to not affect the circuit. Unfortunately, there is a limit of resistance and there is a factor of capacitance. This results in the probes behaving like a low pass RC filter when used to measure the output voltage in the voltage divider circuit as shown in Fig. 23.

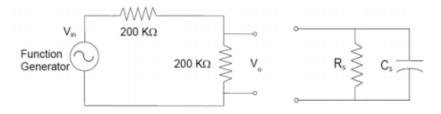


Figure 23: Voltage Divider with Oscilloscope Probe Load

The x1 probe is used for the oscilloscope to directly measure a signal with no attenuation, while the x10 probe attenuates the signal by dividing it by 10 as it is carried to the scope. The x1 probe has an equivalent circuit of a  $1M\Omega$  resistor for the Rs value and a 95pF capacitor for the Cs value, shown in Fig. 23. The x10 probe has an equivalent circuit of a  $10M\Omega$  resistor for the Rs value and a 16pF capacitor for the Cs, shown in Fig. 23.

The DC gain for the x1 probe was 0.445V/V, the cutoff frequency was 18.478k Hz, and the time constant was 8.63us. For the x10 probe, the cutoff frequency is 100k Hz, and the time constant is 1.584us. Calculations for the DC gain is shown in appendix 3 and calculations for the cutoff frequency and the time constant of both probes are shown in appendix 4.

#### 4.3 Procedure

Conducted the following steps for x1 and for x10 probe. For x1 the R in parallel with R2 is 1Mohm and for x10 is 10Mohm. The C value for x1 is 95pF and 10pF for x10.

Used LTSPICE to build the circuit in figure 3 with a 1V for the ac, then added the loading effect with corresponding R and C values, then ran an AC analysis from 10Hz to 1MHz. Used these parameters to plot Vo with about 100 points and determined the DC gain in V/V and in dB by locating the peak value of Vout. Also found the cutoff frequency from the calculated DC gain and compared values with the theoretical values.

- 2. Replaced the input voltage course with a sine wave with 1V zero to peak, and .01fo of the frequency. Also used transient analysis with proper parameters to show 5 to 10 cycles to again determine the DC gain in dB and V/V.
- 3. Repeated step 3 using fo for the frequency and appropriate parameters for the trans analysis.
- 4. Determined the time constant of the circuit using a pulse signal with rise time 1ns, fall time 1ns, pulse width 7\*the theoretical time and stop time be 2\* the pulse width. Then ran the simulation.

# 4.4 Results & Analysis

Figure 24 and Figure 27 shows the circuit of Fig. 23 with the assigned values for 1 and 2 (a) and the 1 and 2 (d) for the x1 probe on the left and the x10 probe on the right. The plots shown in Fig. 25 and Fig.26 indicate the bode plots of the x1 probe and the x10 probe.

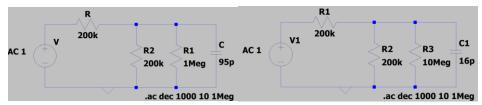


Figure 24: Circuits for Part 1a, x1 (left), and Part 2a, x10 (right)

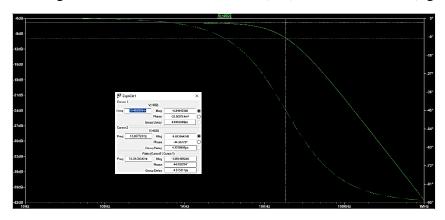


Figure 25: Bode plot of circuit with the x1 probe

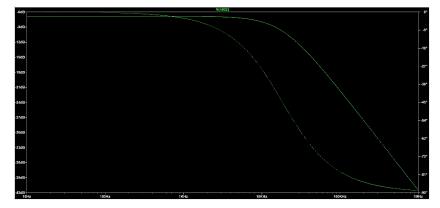


Figure 26: Bode plot of voltage divider circuit with x10 probe

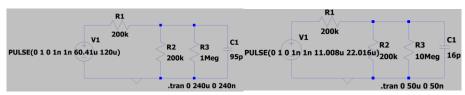


Figure 27: Circuits for Part 1d, x1 (left), and Part 2d, x10 (right)

Table 2 below summarizes the findings of the experiment.

Table 2: Summary of findings from Oscilloscope Characterization Experiment

	x1 probe			x10 probe		
	theoretical	measured	error	theoretical	measured	error (%)
			(%)			
DC gain (V/V)	0.455	0.455	0	0.495	0.495	0
DC gain (dB)	-6.84	-6.848	0.12	-6.108	-6.107	0.02
3dB cutoff	18.478	18.396	0.44	100	100.217	0.22
frequency (kHz)						
Gain at 1% of	-6.84	-6.859	0.28	-6.108	-6.143	0.57
cutoff frequency						
(dB)						
Gain at cutoff	-9.84	-9.871	0.32	-9.108	-9.13	0.24
frequency (dB)						
time constant	8.63	8.675	0.52	1.584	1.596	0.76
(us)						

The DC gain of the x1 probe circuit was measured to be 0.455 V/V. This is the same DC gain as the 0.455 V/V theoretical value for the x1 probe circuit calculated in Appendix 3. Therefore, there is a 0% error between theoretical and measured values. The outcome for the x10 probe is the same with an error of 0% between theoretical and measured DC gains in V/V. The fact that this measurement was done in an LTSPICE simulation could be why there is no measurement error. Simulations remove many sources of error that would be present in physical experiments and allow for a high accuracy between the measured and theoretical values.

The measured cutoff frequency for the x1 probe was found to be 18.396kHz, while the theoretical cutoff frequency was calculated to be 18.478 kHz. This is about a 0.44% error between measured and theoretical cutoff frequencies. The x10 probe had an error of 0.22% between the theoretical and measured values for cutoff frequency. Since the error is less than 1 percent, results from the experiment can be considered accurate and useful. The small error could be due to human error when using the cursor function on the plot when measuring changes in frequency since it is not perfect precision. This same reason can also be used to explain the less than 1% error when measuring the low point of the pulses, the high point of the pulses, and for time constant for both the x1 and x10 probe.

#### 4.5 Conclusion

To conclude this experiment, the measured cutoff frequency for the x1 probe was 18.396 kHz, for the x10 probe it was measured to be 100.217kHz. This indicating that the cutoff frequency for the x10 probe is more than 5 times the cutoff frequency of the x1 probe. The DC gain, 3 dB gain, and tau were calculated and estimated for both the x1 probe and the x10 probe. Also looking at Fig. 25 and Fig.26 the x10 probe gives almost 10 times higher of a gain out than the x1 probe. This concludes that the x10 probe would be much more accurate in measuring a high frequency signal than the x1. The error between the calculated and the estimated was within 1% showing that the LTSPICE can be considered accurate in the values found.

# **Appendix**

Appendix 1: Calculations for phasor gain in experiment #3

$$G(j\omega) = \frac{Vo}{Vi} \rightarrow Vo = G(j\omega)Vi; \quad \text{where Vo and Vi are phasor voltages}$$

$$G(j\omega) = \frac{R}{R + Z_L} = \frac{R}{R + j\omega L} = \frac{1}{1 + \frac{j\omega L}{R}} = \frac{1}{1 + j\omega \tau}$$

$$Let \ \omega_0 = \frac{R}{L} = \frac{1}{\tau}$$

$$G(jw) = \frac{1}{1 + j\frac{\omega}{\omega_0}}$$

$$\omega = 0 \ (DC \ gain): G(j\omega) = \frac{1}{1 + 0} = 1\frac{V}{V} = 0 dB$$

$$\omega = \infty \ (High \ frequency \ gain): G(j\omega) = \frac{1}{1 + \omega} = 0\frac{V}{V} = -\infty \ dB$$

$$\omega = \omega_0 = G(j\omega) = \frac{1}{1 + j} = \frac{1}{\sqrt{2}} \frac{V}{V} = -3 dB$$

Appendix 2: Calculations for the step response of a low pass RL circuit

$$Vi = Vl + VR = L\frac{di}{dt} + iR$$

$$\frac{di}{dt} + \frac{R}{L}i(t) = \frac{Vi}{L}$$

$$i(t) = In(t) + If(t)$$

$$If(t) = I_{\infty}; \quad In(t) = Ae^{\alpha t}; \quad \frac{dIn(t)}{dt} = A\alpha e^{\alpha t}$$

$$A\alpha e^{\alpha t} + \frac{R}{L}Ae^{\alpha t} = 0 \rightarrow \alpha = -\frac{R}{L} = -\frac{1}{\tau}$$

$$I(t) = Ae^{-\frac{Rt}{L}} + I_{\infty}$$

$$I(t) = [I(o^{+}) - I(\infty)]e^{-\frac{Rt}{L}} + I_{\infty}$$

$$\tau = \frac{1}{\omega_{0}} = \frac{L}{R} \rightarrow I(t) = [I(o^{+}) - I(\infty)]e^{-\frac{t}{\tau}} + I_{\infty}$$

Appendix 3: Calculations for DC gain of the x1 and x10 probe for experiment#4

$$G(j\omega) = \frac{Vo}{Vi}(j\omega) = \frac{k}{1 + \frac{j\omega}{\omega_0}}$$

$$DC \ gain \ is \ G(j\omega) \ at \ \omega = 0: \qquad \frac{k}{1 + 0} = k\frac{V}{V}$$

$$x1 \ Probe: \quad R1 = R2 = 200k\Omega. \ Let \ R2' = \frac{R2 * 1M\Omega}{R2 + 1M\Omega} = 166.667k\Omega$$

$$k = \frac{R2'}{R1 + R2'} = \frac{166.667k\Omega}{366.667k\Omega} = 0.455$$

$$x1 \ Probe \ DC \ gain: 0.455 \frac{V}{V}$$

$$x10 \ Probe: \quad R1 = R2 = 200k\Omega. \ Let \ R2' = \frac{R2 * 10M\Omega}{R2 + 10M\Omega} = 196.078k\Omega$$

$$k = \frac{R2'}{R1 + R2'} = \frac{196.078k\Omega}{396.078k\Omega} = 0.495$$

$$x10 \ Probe \ DC \ gain: 0.495 \frac{V}{V}$$

Appendix 4: Calculations for the time constant and cutoff frequencies of the x1 and x10 probe for experiment#4.

$$\tau = R_{eq} * C \qquad R_{eq} = \frac{R2' * R1}{R2' + R1}$$

$$x1 \, Probe: \qquad R2' = 166.667 k\Omega, R1 = 200 k\Omega, C = 95 pF$$

$$R_{eq} = \frac{166.667 k * 200 k}{166.667 k + 200 k} = 90.9 k\Omega$$

$$(90.9 k\Omega)(95 pF) = \tau = 8.63 \ \mu s$$

$$\omega_0 = \frac{1}{\tau} = 115.788 \frac{rad}{s}; \qquad f_0 = \frac{\omega_0}{2\pi} = 18.478 kHz$$

$$x10 \, Probe: \qquad R2' = 196.078 k\Omega, R1 = 200 k\Omega, C = 16 pF$$

$$R_{eq} = \frac{196.078 k * 200 k}{196.078 k + 200 k} = 99.0 k\Omega$$

$$(99.0 k\Omega)(16 pF) = \tau = 1.584 \ \mu s$$

$$\omega_0 = \frac{1}{\tau} = 631.3 k \frac{rad}{s}; \qquad f_0 = \frac{\omega_0}{2\pi} = 100 kHz$$

# Appendix 5: Marked checklist

