

Washington State University
School of Electrical Engineering and Computer
Science
EE 352 Electrical Engineering Laboratory
Lab # 6
Transfer Function Analysis

Name: Sarah Rock
Partner: Isobel Beatz, Zach Nett
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Lab Overview

The focus of this lab was to employ transfer functions to determine the frequency response and step response of second order passive and active circuits, these including a second order RLC high pass filter, a second order Butterworth low pass filter, and a wide-band pass filter. At the end of each experiment, the objective was to then compare the laboratory measurements of the amplitude, phase and step responses of the three circuits with the theoretical responses predicted by the transfer functions in Matlab.

Experiment #1 Second Order RLC High Pass Filter

1.1 Purpose

The purpose of this lab was to obtain a transfer function of a second order RLC high pass filter, then find its amplitude and phase response as a function of frequency as well as the filter's step response. Then the circuit in Fig. 1 is built in LTSPICE to verify that the experimental responses match the predicted responses of the transfer function.

1.2 Theoretical Background

The primary focus for this experiment is to obtain a second order high pass RLC high pass filter's amplitude and phase responses as a function of frequency as well as the step response. The purpose of this kind of circuit is to scoop out unwanted low frequencies; like an audio source. The amplitude and phase is graphed using a bode plot or frequency response curve. The response curve typically extends down from infinity to the cutoff frequency, where the output voltage amplitude (ζ) is .707 of the input signal value ($-3\text{dB}(20\log(v_{\text{out}}/v_{\text{in}}))$) of the input signal. The frequency response curve implies that the filter can pass all signals out to infinity, though in practice that is not the case due to electrical characteristics of the components used.

For the prelab, based on the frequency-dependent behavior of circuit components, the output voltage would be 0 when $\omega = 0$. When $\omega = \infty$, the output voltage would be 1 since equation (1) below would become $\frac{\infty}{\infty} = 1$. Equation (1) is a high pass filter used to determine the behavior of the gain, it is based on the second order RLC circuit shown in Fig. 1.

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{CLs^2}{CLs^2 + RCs + 1} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \quad (1)$$

Here, $s = j\omega$, which can be used interchangeably. In general for a second order system, the characteristic equation is displayed as $D(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$, where ω_n is the natural frequency and ζ is the damping factor. when $\zeta > 1$, the circuit is overdamped. When $\zeta = 1$, the circuit is critically damped, and when $\zeta < 1$ the circuit is underdamped. So when given $\zeta = .2$, $L=10 \text{ mH}$, and $f_n = 2 \text{ kHz}$, it is expected that the step response has an overshoot and the frequency response has a place of peaking. Also given these parameters, $R=50.3 \Omega$ (found by relating the s terms of the characteristic and transfer function), and C is 633 nF (found by relating the singular terms of the characteristic equation and the transfer function). To find the frequency response and the amplitude response, Matlab was used with the code in Appendix A to generate the bode plots shown in Fig. 2 (a) and (b). Here, at $\omega = 0$ the phase is about 180 degrees and the

magnitude is $-\infty$. When $\omega = \infty$, the magnitude is about 0 dB and the phase is about 0 degrees. The step response shows that the circuit is underdamped as it begins by oscillating at 1 AC then converges toward 0.

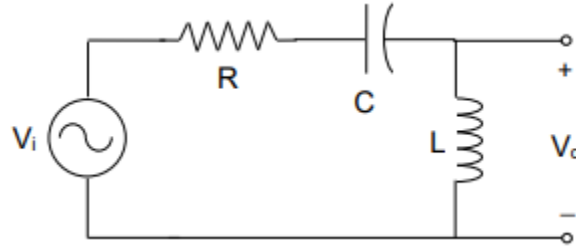


Figure 1: High-pass RLC circuit.

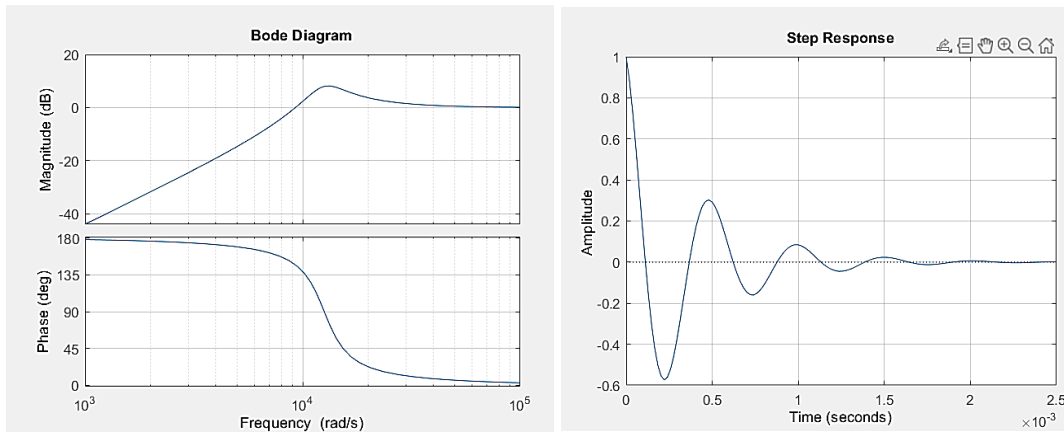


Figure 2 (a) and (b): Matlab Graphs generated from Appendix A code.

To successfully conduct this experiment, the equations used to calculate gain as various frequencies and the phase of the transfer function is shown in equations (2) and (3) below respectively. The rest of the variables were found by using cursors on the LTSPICE graphs generated.

$$G(j\omega) = 20\log\left(\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right) \quad (2)$$

$$\angle H(j\omega) = \pm 360 * \frac{\Delta t}{T} = \pm 360 * \Delta t f \quad (3)$$

Equation (2) was used to find the gain of the circuit in dB and equation (3) was used to calculate the phase angle in degrees. T is the period of the input sinewave which is equal to $1/f$ and Δt is the phase measure between the peak in the input sinusoid and the output sinusoid. It is important to also note that equation (3) is used on its own when the peak output leads the input peak, otherwise when it lags, the equation is multiplied by -1.

1.3 Procedure

1. Fig. 1 was constructed in LTSPICE using the component values chosen in the pre-lab.

2. A 1V AC input voltage was applied and carried with an AC analysis from 20 Hz to 200 kHz with 1000 points per decade to then plot the gain (V_o). This was used to then measure and record the DC gain, the high frequency gain, the slope in dB/decade in the linear region and peaking frequency.
3. The input voltage of the circuit was then changed to a sinewave voltage with 5 volts zero-to-peak to measure the amplitude and phase responses for at least 10-15 different frequencies.
- b. A Transient analysis was applied with around 10 cycles at each frequency and maximum step size to be $T/1000$.
- c. At each frequency the gain in dB and phase response was measured and recorded and the angle was calculated using equation (3).
- d. Using Microsoft Excel, the results were plotted.
4. Finally the step response was measured by changing the input voltage from a sinewave to a rectangular pulse. The parameters were from 0 to 1V peak, delay time of zero, time rise and time fall of 10ns, T_{on} of 5ms and period 10ms. The transient analysis had at least two complete cycles with maximum step size of $T_n/1000$. To then plot the output and input voltage to then compare to the Matlab simulation's step response.

1.4 Results & Analysis

Fig. 3 below shows the Fig. 1 circuit with the calculated R, L and C values found in the pre-lab. Fig. 4 below shows the resulting phase and gain of the circuit to which was used to decide the 10 to 15 frequencies needed to design the phase and gain plots in Excel. For Fig. 4 had a DC gain of $-\infty$, a slope of 39.969 dB/dec between 30 Hz and 300 Hz in the linear region, high frequency of 2.29 mdB (which is about 0 dB), and peak frequency of 2.09 kHz at 8.201 dB. This was all found by using the cursors provided in LTSPICE. The slope only had an error of .0775% from the ideal slope of 40 dB/decade, the high frequency had a .229% error from the ideal 0 dB, and peak frequency had a 4.5% error from the ideal 2 Hz. The errors are due to measurement error and the step size being too big, causing potential skipping in the frequency behavior.

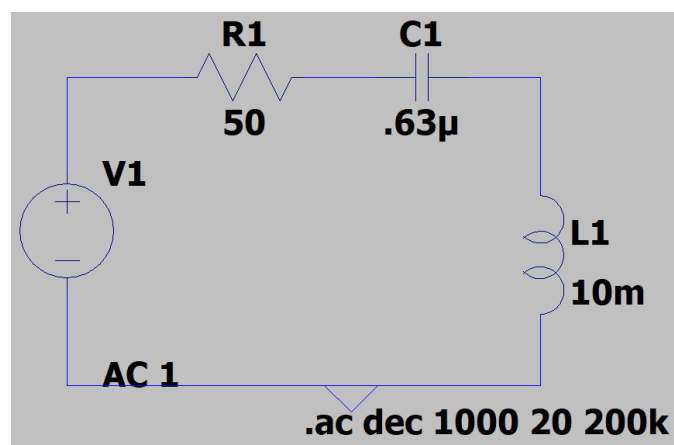


Figure 3: Fig. 1 Circuit with AC Input Voltage

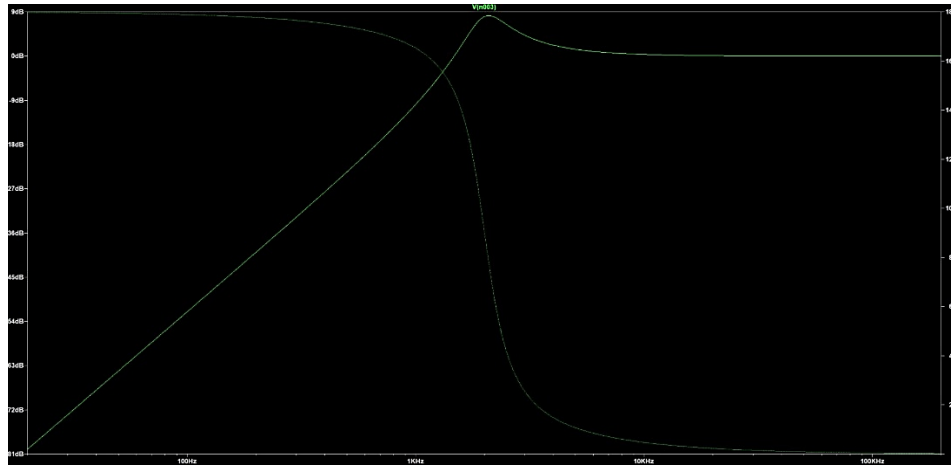


Figure 4: Phase and Gain(dB) vs Frequency of Fig. 2 Circuit

Fig. 5 shows the circuit of Fig. 1 with an input sinewave voltage. This circuit was used for each frequency chosen, which as the frequency changed, the step size and max time changed to correspond with the frequency using $f=1/T$. Table 1 below then shows the resulting step size, max step, frequency, period, calculated gain, and phase. For the gain, V_{in} was 5 V and the V_{out} was the most right peak output voltage of the sinewave graph. To find the gain and phase, equation (2) and (3) were used. Fig. 6 below shows the resulting graphs of gain versus frequency (a) and phase versus frequency (b).

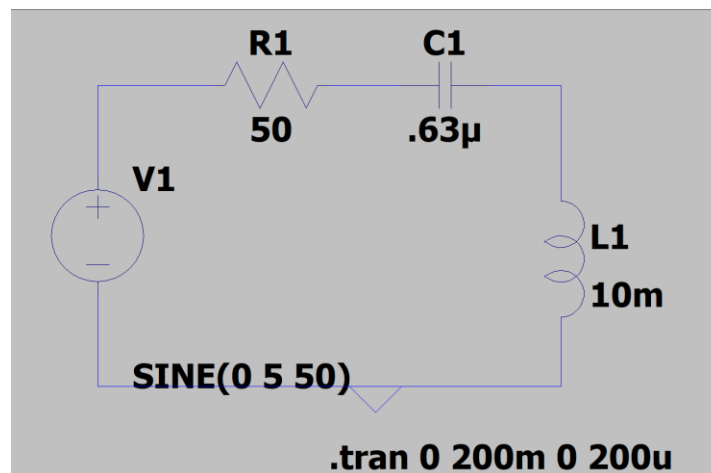


Figure 5: Fig. 1 Circuit with Sinewave Input Voltage with 50 Hz.

Table 1: Showing Period, Max Time, Timestep, chosen Frequencies, Gain, and Phase ($\angle H(j\omega)$).

Period (sec)	Max Time (sec)	Timestep (sec)	Frequency (Hz)	Gain (Vo/Vin)	20log(Gain) (dB)	$\angle H(j\omega)$ (deg)*-1
0.02	0.2	0.0002	50	0.000625	-64.08	178.35
0.01	0.1	0.0001	100	0.00248	-52.11	174.24
0.002	0.02	0.00002	500	0.066	-23.61	172.8
0.001	0.01	0.00001	1000	0.32	-9.90	165.6
0.000667	0.006667	6.67E-06	1500	1.052	0.44	146.75
0.000571	0.005714	5.71E-06	1750	1.808	5.14	122.18
0.0005	0.005	0.000005	2000	2.508	7.99	93.17
0.000476	0.004762	4.76E-06	2100	2.567	8.19	72.72
0.0004	0.004	0.000004	2500	2.086	6.39	39.78
0.0002	0.002	0.000002	5000	1.1692	1.36	11.34
0.0001	0.001	0.000001	10000	1.05	0.42	5.2
0.00002	0.0002	2E-07	50000	0.988	-0.105	-0.54

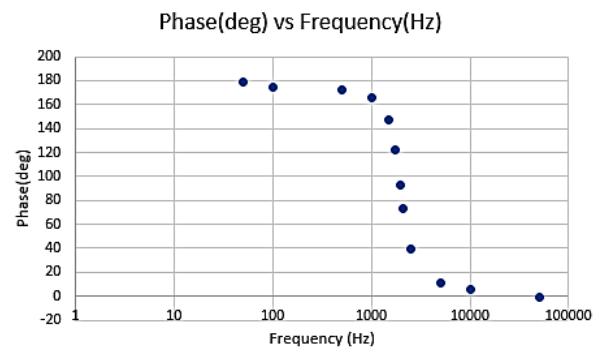
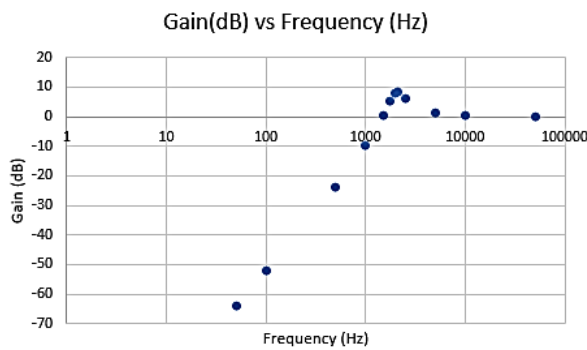


Figure 6: Excel Graphs from Frequency, Gain and Phase Calculations from Table 1.

The plot generated in Fig. 2 and Fig. 4 compared to the plots in Fig. 6 basically show the same results. Then to find the step response of the circuit, Fig. 7 was made in LTSPICE with a input voltage rectangular pulse with 20 ms max time and 500 ns step size. Fig. 8 is the resulting step response of Fig. 7, and when comparing to the step response in Fig. 2(b), the graphs also look the same.

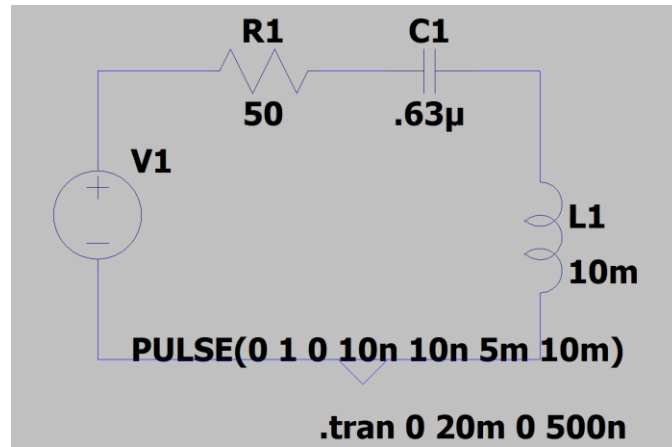


Figure 7: Fig. 1 Circuit with Input Pulse Voltage to determine Step Response.

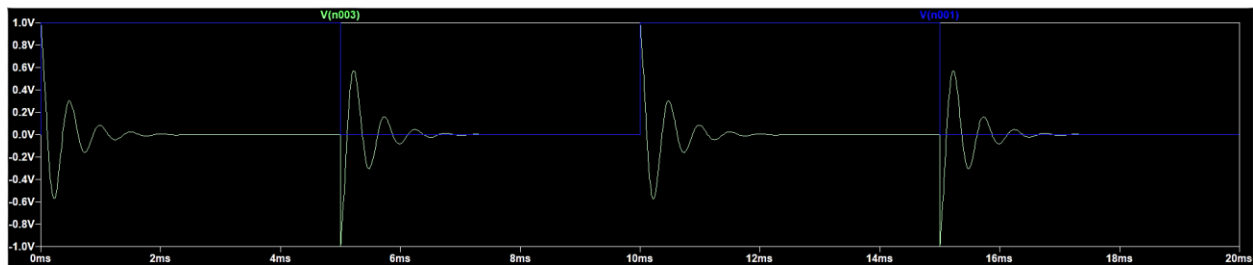


Figure 8: Step response of Fig. 7 Circuit.

1.5 Conclusion

To conclude this experiment, when comparing the phase, magnitude, and step response graphs generated in Matlab, LTSPICE and Excel, they all appear to be the same. The frequency and gain values of the LTSPICE graph showing phase and gain have a less than 5% error in relation to the ideal values due to measurement and general step error.

Experiment #2 Second Order Active Butterworth Low Pass Filter

2.1 Purpose

The purpose of this experiment was to design and build a second order active Butterworth low pass filter based on the 2nd order op-amp low pass filter circuit in Fig. #. Using this designed circuit, the objective is to then obtain the amplitude and phase responses as a function of frequency as well as the filter's step response. Then to verify the experimental responses match with the predicted responses from the transfer function.

2.2 Theoretical Background

For a Butterworth filter, the higher the number of cascaded stages there are within the filter design, and the closer the filter becomes to the ideal response. The frequency response of the Butterworth Filter approximation function is also often referred to as “maximally flat” (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible from 0Hz (DC) until the cut-off frequency at -3dB with no ripples. Higher frequencies beyond the cut-off point rolls-off down to zero in the stop band at 20dB/decade or 6dB/octave. This is because it has a quality factor (ζ) of just 0.707. However, one main disadvantage of the Butterworth filter is that it achieves this pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band. It also has poor phase characteristics as well. In practice however, Butterworth's ideal frequency response is unattainable as it produces excessive passband ripple. To determine the n^{th} order of a Butterworth circuit, equation (4) below was used.

$$N = \left\lceil \frac{\ln\left(\frac{1}{\delta_2^2}\right) - 1}{2 * \ln\left(\frac{\omega_s}{\omega_c}\right)} \right\rceil \quad (4)$$

Here, cutoff frequency is ω_c , stopband frequency is ω_s , and stopband attenuation is δ_2 . Then the transfer function for both the general form of the bandpass filter and the transfer function of the specific circuit shown in Fig. 9 which is the lowpass Butterworth filter used for this experiment, is shown in equation (5).

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\frac{1}{R^2 C_1 C_2}}{s^2 + \frac{2}{RC_1}s + \frac{1}{R^2 C_1 C_2}} \quad (5)$$

Here ω_n is the natural frequency and ζ is the damping factor.

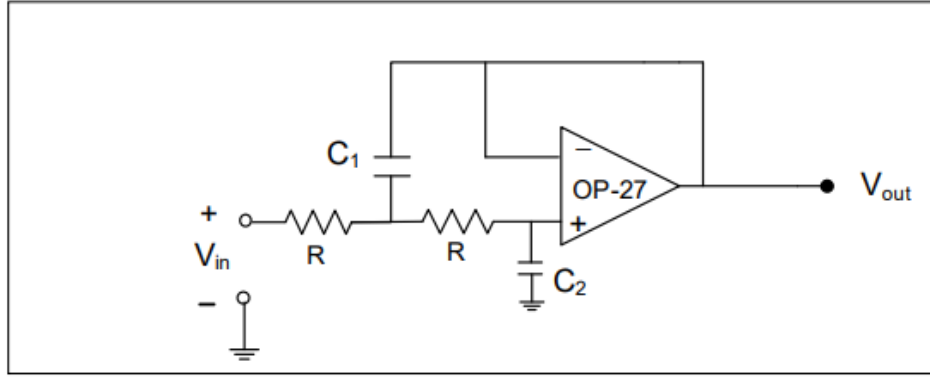


Figure 9: 2nd order op-amp low pass filter.

For the pre-lab, when $f_c = 80 \text{ Hz}$, $f_s = 300 \text{ Hz}$, and $\delta_2 = -20 \text{ dB} = .1 \text{ V/V}$, using equation (4) to determine the n^{th} order, N is calculated to be 1.36 which rounds to 2. Then using these same values and the characteristic equation of the general form in equation (5), the poles are found to be $s = 355430.6351 \pm 355430.6351i$. Then using components of the transfer function for the circuit as well as the same values, C_1 is chosen to be 20 nF to then have C_2 be 10 nF and R be 140.67Ω . Then using the code shown in Appendix B from Matlab, Fig. 10 below shows the amplitude, phase, and step response of the Butterworth lowpass filter.

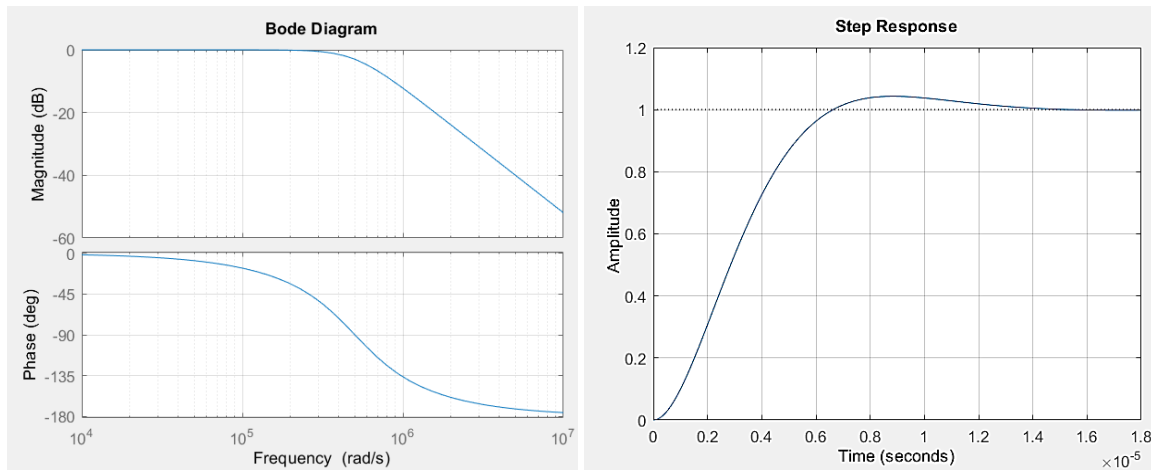


Figure 10: (a) and (b): Matlab Graphs generated from Appendix B code.

2.3 Procedure

1. LTSPICE was used to build Fig. 9 with OP27 using the component values calculated in the pre-lab. The OP27 was set to be connected to $V_{CC} = +12\text{V}$ and $V_{EE} = -12\text{V}$ power supplies.
2. The input voltage was 1V AC with 1000 points per decade and sweep of frequency from 1 kHz to 1Meg Hz.
3. The gain (V_o) was then plotted to find the gain and phase at DC, high frequency, the cutoff frequency, and the slope (dB/Dec) in the linear region.

4. The input voltage of the circuit was then changed to a sinewave voltage with 5 volts zero-to-peak to measure the amplitude and phase responses for at least 10-15 different frequencies.
- b. A Transient analysis was applied with around 10 cycles at each frequency and maximum step size to be $T/1000$.
- c. At each frequency the gain in dB and phase response was measured and recorded and the angle was calculated using equation (3).
- d. Using Microsoft Excel, the results were plotted.
5. Finally the step response was measured by changing the input voltage from a sinewave to a rectangular pulse. The parameters were from 0 to 1V peak, delay time of zero, time rise and time fall of 1 ns, Ton of 50 μ s and period 100 μ s. The transient analysis had at least two complete cycles with maximum step size of $T_n/1000$. To then plot the output and input voltage to then compare to the Matlab simulation's step response.

2.4 Results & Analysis

Fig. 11 below shows the Fig. 9 circuit with the calculated R , C_1 and C_2 values found in the pre-lab. Fig. 12 below shows the resulting phase and gain of the circuit to which was used to decide the 10 to 15 frequencies needed to design the phase and gain plots in Excel. For Fig. 12 had a DC gain of 14.42 μ dB with phase -1.02° , a slope of -31.5 dB/dec between 60 kHz and 600 kHz in the linear region, high frequency of -33.11 dB with phase -222.84° , and cutoff frequency of -3.006 dB with phase -91.3° . This was all found by using the cursors provided in LTSPICE. The slope had an error of 21.25% from the ideal slope of -40 dB/decade, the high frequency did not go to $-\infty$ because of gain bandwidth product which causes it to slope up, and the high frequency angle had a 23.8% error from the ideal -180° . The errors are due to measurement error and the step size being too big, causing potential skipping in the frequency behavior as well as the gain bandwidth product.

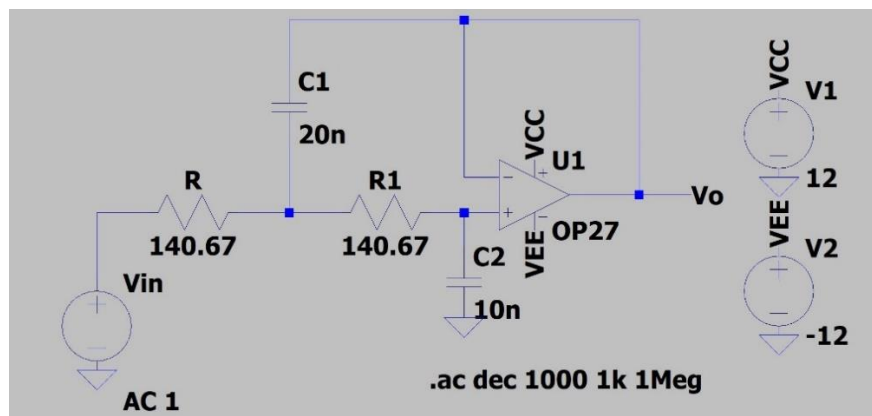


Figure 11: Fig. 9 Circuit with AC Input Voltage

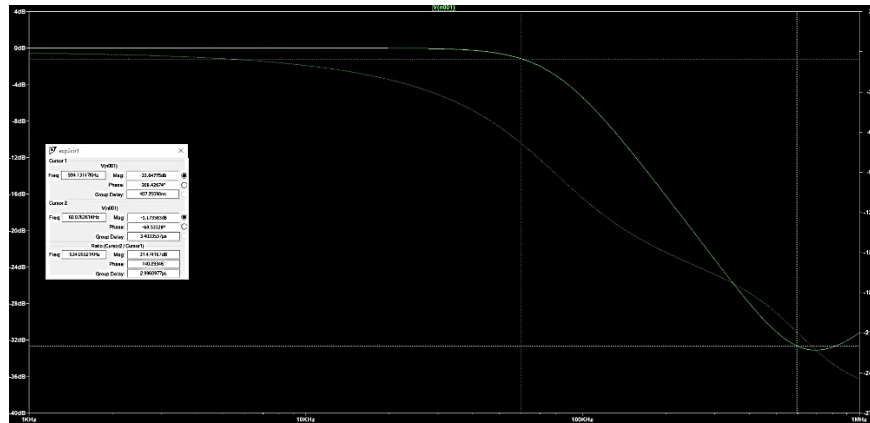


Figure 12: Phase and Gain(dB) vs Frequency of Fig. 11 Circuit

Fig. 13 shows the circuit of Fig. 9 with an input sinewave voltage. This circuit was used for each frequency chosen, which as the frequency changed, the step size and max time changed to correspond with the frequency using $f=1/T$. Table 2 below then shows the resulting period, max time, timestep, chosen frequencies, magnitude of V_{out} , peak time of V_{in} , peak time of V_{out} , t-period, gain, and phase. For the gain, V_{in} was 5 V and the V_{out} was the most right peak output voltage of the sinewave graph. To find the gain and phase, equation (2) and (3) were used. The phase angle equation was multiplied by -1 because the output was lagging the input. Fig. 14 below shows the resulting graphs of gain versus frequency (a) and phase versus frequency (b).

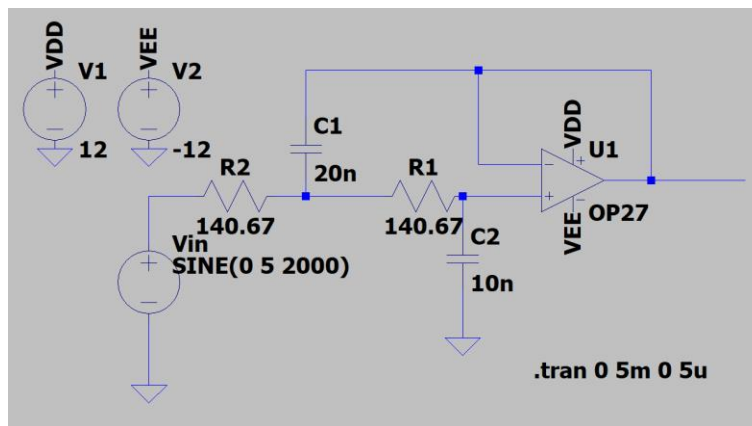


Figure 13: Sinewave Input Voltage with 2000 Hz of Fig. 9 Circuit.

Table 2: Showing Period, Max Time, Timestep, chosen Frequencies, Magnitude of Vout, Peak Time of Vin, Peak Time of Vout, T-period, Gain, and Phase ($\angle H(j\omega)$).

Period (sec)	Max Time (sec)	Timestep (sec)	Frequency (Hz)	Mag (Vout) (V)	Time (Vout) (sec)	Time (Vin) (sec)	T-period (sec)	20log(Vo/Vin) (dB)	$\angle H(j\omega) * -1$ (deg)
0.0005	0.005	5E-6	2000	4.997	0.004628	0.00462	7.46E-06	-0.00521	-5.3699
0.0002	0.002	2E-6	5000	5.0002	0.001853	0.00185	2.57E-06	0.000347	-4.61934
0.0001	0.001	1E-6	10000	5.0004	9.28E-04	9.25E-04	2.81E-06	0.000625	-10.1104
0.00004	0.0004	4E-07	25000	4.982	3.28E-04	3.70E-04	2.98E-06	-0.03063	-26.82
3.33E-05	0.000333	3.33E-07	30000	4.960	3.11E-04	3.08E-04	2.94E-06	-0.07027	-31.7047
0.000025	0.00025	2.5E-07	40000	4.857	2.22E-04	2.31E-04	3.05E-06	-0.25251	-43.92
2.22E-05	0.000222	2.22E-07	45000	4.782	2.09E-04	2.06E-04	3.12E-06	-0.38775	-50.4773
1.82E-05	0.000182	1.82E-07	55000	4.536	1.71E-04	1.68E-04	3.15E-06	-0.84619	-62.4017
1.54E-05	0.000154	1.54E-07	65000	4.182	1.46E-04	1.42E-04	3.17E-06	-1.55109	-74.178
1.33E-05	0.000133	1.33E-07	75000	3.7425	1.26E-04	1.23E-04	3.16E-06	-2.51619	-85.334
0.00001	0.0001	1E-07	100000	2.609	9.55E-05	9.25E-05	3.03E-06	-5.65068	-109.193
7.14E-06	7.14E-05	7.14E-08	140000	1.4006	6.87E-05	6.61E-05	2.59E-06	-11.0529	-130.796
4.55E-06	4.55E-05	4.55E-08	220000	.495	4.40E-05	4.20E-05	1.91E-06	-20.0884	-151.195
3.33E-06	3.33E-05	3.33E-08	300000	.202	3.24E-05	3.08E-05	1.52E-06	-27.8552	-163.975

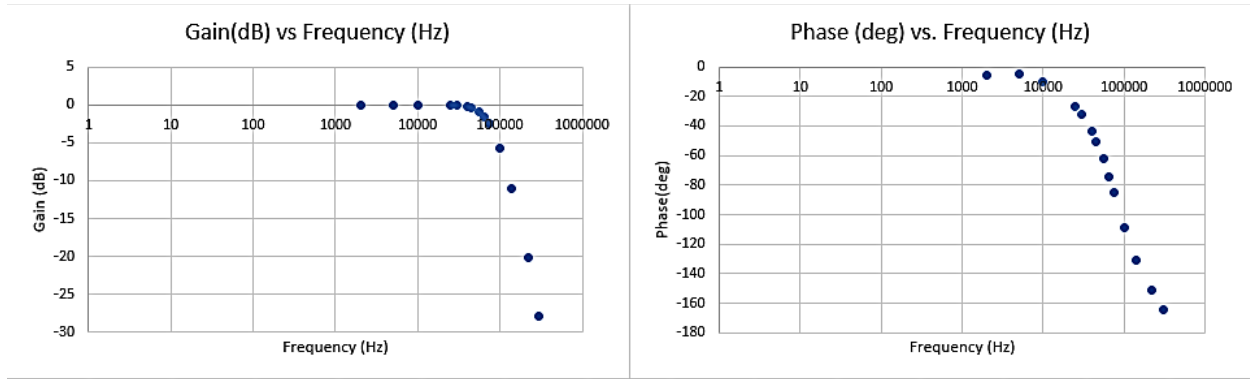


Figure 14: Excel Graphs from Frequency, Gain and Phase Calculations from Table 2.

The plots generated in Fig. 10 and Fig. 12 compared to the plots in Fig. 14 basically show the same results. Then to find the step response of the circuit, Fig. 15 was made in LTSPICE with a input voltage rectangular pulse with $200 \mu s$ max time and step size $.1 \mu s$. Fig. 16 is the resulting step response of Fig. 15, and when comparing to the step response in Fig. 10(b), the graphs also look the same. The behavior of the graph did show an overshoot as expected because $\zeta < 1$.

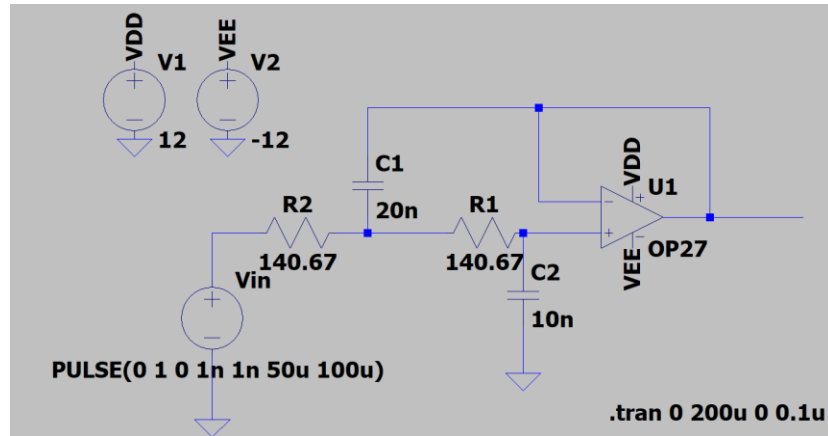


Figure 15: Fig. 9 Circuit with Input Pulse Voltage to determine Step Response.

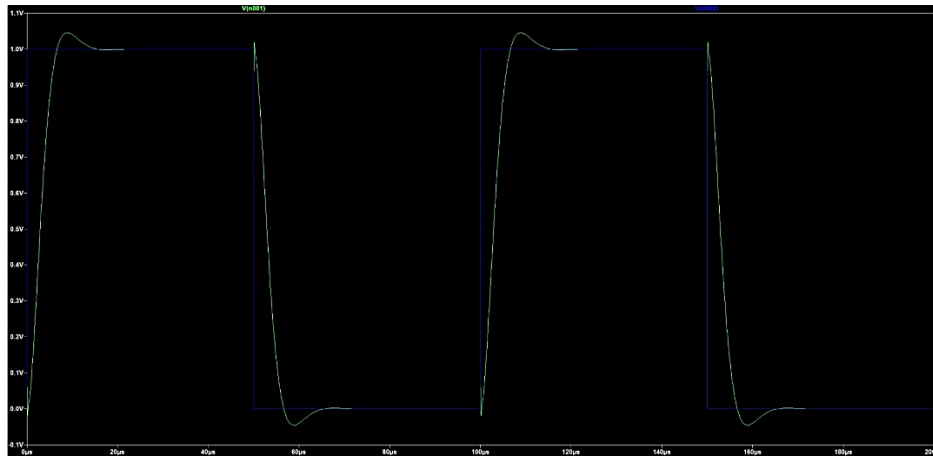


Figure 16: Step response of Fig. 15 Circuit.

2.5 Conclusion

To conclude this experiment, the significant error between the slope and the ideal slope is due to measurement error and the step size being too big, causing potential skipping in the frequency behavior as well as the gain bandwidth product. Also, the linear region did not last at least a decade so the chosen region overlapped on some of the curved region of the graph. Gain bandwidth product causes the behavior of the graph to curve up at the end, preventing the high frequency from approaching negative infinity and the angle from approaching -180° .

Experiment #3 Wide-band Band Pass Filter

3.1 Purpose

The purpose of this lab is to serially connect the high pass filter from experiment 1 with the low pass filter from experiment 2, in order to construct a wide-band bandpass filter. Using this designed circuit, the objective is to then obtain the amplitude and phase responses as a function of frequency as well as the filter's step response. Then to verify the experimental responses match with the predicted responses from the transfer function.

3.2 Theoretical Background

In a Band Pass Filter circuit, the overall width of the actual pass band between the upper and lower -3dB corner points of the filter determines the Quality Factor or Q-point of the circuit. This Q Factor is a measure of how selective or un-selective the band pass filter is towards a given spread of frequencies. The lower the value of the Q factor the wider is the bandwidth of the filter and consequently the higher the Q factor the narrower and more “selective” is the filter. As the quality factor of an active band pass filter relates to the sharpness of the filter's response around its center resonant frequency (f_r) it can also be thought of as the damping factor or damping coefficient because the more damping the filter has the flatter is its response and likewise, the less damping the filter has the sharper is its response. When analyzing active filters, generally a normalized circuit is considered which produces an ideal frequency response having a rectangular shape, and a transition between the passband and the stopband that has an abrupt or very steep roll-off slope. However, these ideal responses are not possible in the real world, so approximations are used to give the best frequency response possible for the type of filter being designed. This experiment focuses on the wide-band pass filter which is the experiment 1 and 2 circuits being serially connected with a voltage follower, as shown in Fig. 17 which is the most common wide-band bandpass filter.

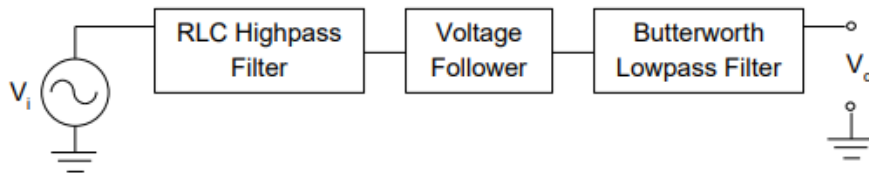


Figure 17: Wide-band bandpass filter.

For the pre-lab, the transfer function of the wide-band bandpass filter was calculated using the code found in appendix c. Also using Matlab, the amplitude, phase and step response were plotted based on appendix c code with the resulting graphs shown in Fig. 18. The continuous-time transfer function found is shown in equation (6) below.

$$H_b(s) = \frac{2.527E11s^2}{s^4 + 7.159E5s^3 + 2.564E11s^2 + 1.37E15s + 3.992E19} \quad (6)$$

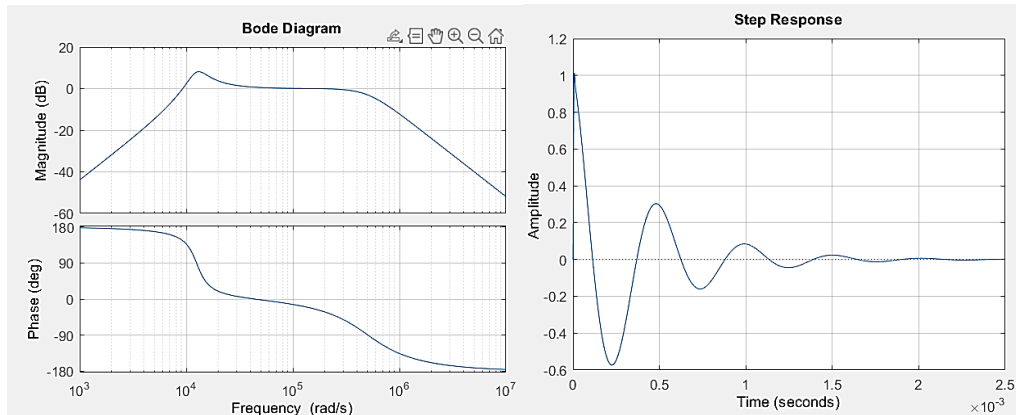


Figure 18: (a) and (b): Matlab Graphs generated from Appendix C code.

3.3 Procedure

1. LTSPICE was used to build Fig. 17 use the designed RLC high-pass filter from experiment 1, the voltage follower and your designed Butterworth lowpass filter from experiment 2.
2. The input voltage was 1V AC with 1000 points per decade and sweep of frequency from 100 Hz to 1Meg Hz.
3. The gain (V_o) was then plotted to find the gain and phase at DC, high frequency, the cutoff frequency, and the slope (dB/Dec) in the linear region at both ends.
4. The input voltage of the circuit was then changed to a sinewave voltage with 1 volt zero-to-peak to measure the amplitude and phase responses for at least 10-15 different frequencies.
- b. A Transient analysis was applied with around 10 cycles at each frequency and maximum step size to be $T/1000$.
- c. At each frequency the gain in dB and phase response was measured and recorded and the angle was calculated using equation (3).
- d. Using Microsoft Excel, the results were plotted.
5. Finally the step response was measured by changing the input voltage from a sinewave to a rectangular pulse. The parameters were from 0 to 1V peak, delay time of zero, time rise and time fall of 1 ns, T_{on} of 5 ms and period 10 ms. The transient analysis had at least two complete cycles with maximum step size of $T_n/1000$. To then plot the output and input voltage to then compare to the Matlab simulation's step response.

3.4 Results & Analysis

Fig. 19 below shows the Fig. 17 circuit. Fig. 20 below shows the resulting phase and gain of the circuit to which was used to decide the 10 to 15 frequencies needed to design the phase and gain plots in Excel. For Fig. 20, the right side had a DC gain of $-\infty$ dB, a slope of -31.2 dB/dec between 60 kHz and 600 kHz in the linear region, high frequency of -32 dB. The left side had a DC gain of $-\infty$ dB, a slope of 42.2 dB/dec between 100 Hz and 1 kHz in the linear region, high

frequency of -52 dB. For both sides, the cutoff frequency was 8.1816 dB at 2.08 kHz with phase 76.33°, and the phase for both sides high frequency was -227.37°. This was all found by using the cursors provided in LTSPICE. The slope had an error of 22% from the ideal slope of -40 dB/decade for the right side and 30% for the left side, the high frequency did not go to $-\infty$ because of the op-amp limiting response, and the high frequency angle had a 26.3% error from the ideal -180°. The cutoff frequency had a 4.16% error from the ideal 2 kHz because of the peak gain effect with $\zeta < .707$. The errors are due to measurement error and the step size being too big, causing potential skipping in the frequency behavior as well as the op-amp limiting response.

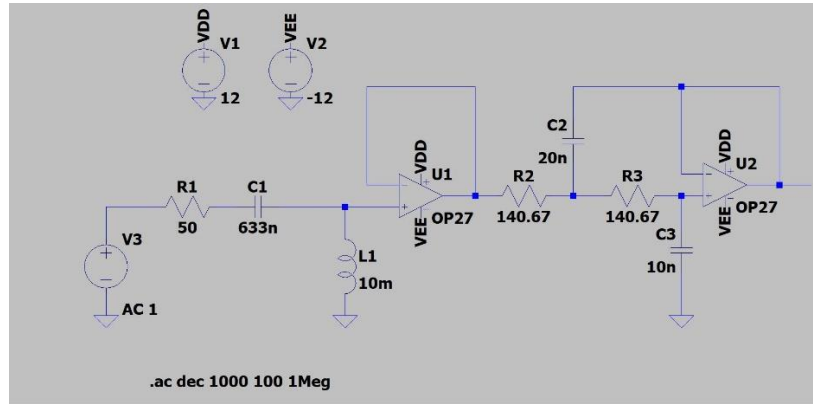


Figure 19: Fig. 17 Circuit with AC Input Voltage

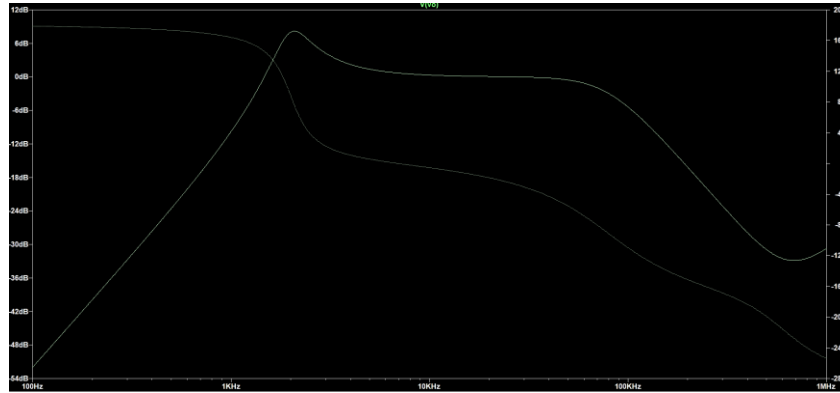


Figure 20: Phase and Gain(dB) vs Frequency of Fig. 17 Circuit

Fig. 21 shows the circuit of Fig. 19 with an input sinewave voltage. This circuit was used for each frequency chosen, which as the frequency changed, the step size and max time changed to correspond with the frequency using $f=1/T$. Table 3 below then shows the resulting period, max time, timestep, chosen frequencies, magnitude of V_{out} , peak time of V_{in} , peak time of V_{out} , t-period, gain, and phase. For the gain, V_{in} was 1 V and the V_{out} was the most right peak output voltage of the sinewave graph. To find the gain and phase, equation (2) and (3) were used. The phase angle equation was multiplied by -1 because the output was lagging the input after 4000 Hz, from 200 to 4000 Hz the equation was not multiplied by -1 because the output was leading

the input. Fig. 22 below shows the resulting graphs of gain versus frequency (a) and phase versus frequency (b).

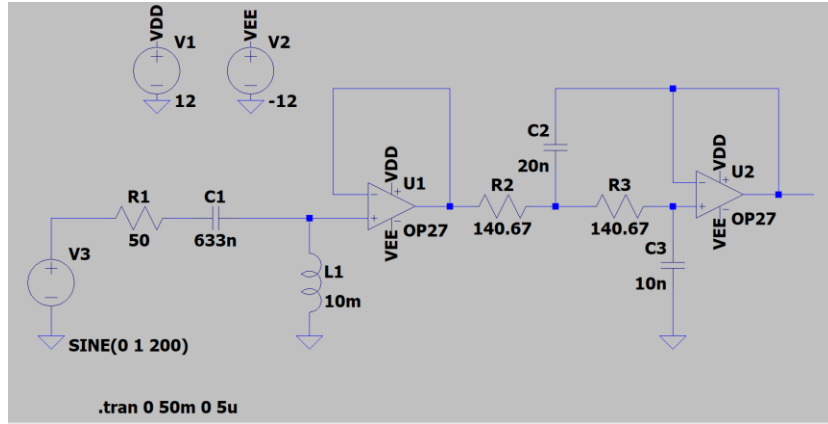


Figure 21: Sinewave Input Voltage with 200 Hz of Fig. 19 Circuit.

Table 3: Showing Period, Max Time, Timestep, chosen Frequencies, Magnitude of Vout, Peak Time of Vin, Peak Time of Vout, T-period, Gain, and Phase ($\angle H(j\omega)$).

Period (sec)	Max Time (sec)	Timestep (sec)	Frequency (Hz)	Mag (Vout) (V)	Time (Vout) (sec)	Time (Vin) (sec)	T-period (sec)	20log(Vo/Vin) (dB)	$\angle H(j\omega)$ (deg)
0.005	0.05	0.00005	200	1.01E-2	4.38E-02	4.63E-02	2.46E-03	-3.99E+01	1.77E+02
0.002	0.02	0.00002	500	6.63E-2	1.76E-02	1.85E-02	9.40E-04	-2.36E+01	1.69E+02
0.001	0.01	0.00001	1000	.322	8.79E-03	9.25E-03	4.56E-04	-9.84E+00	1.64E+02
0.000667	0.006667	6.67E-06	1500	1.06	5.90E-03	6.17E-03	2.70E-04	5.18E-01	1.46E+02
0.0005	0.005	0.000005	2000	2.51	4.50E-03	4.62E-03	1.22E-04	8.00E+00	8.81E+01
0.000476	0.004762	4.76E-06	2100	2.56	4.31E-03	4.40E-03	9.77E-05	8.18E+00	7.39E+01
0.000444	0.004444	4.44E-06	2250	2.43	4.04E-03	4.11E-03	-7.23E-05	7.72E+00	5.85E+01
0.000385	0.003846	3.85E-06	2600	1.96	3.52E-03	3.56E-03	3.93E-05	5.85E+00	3.68E+01
0.000333	0.003333	3.33E-06	3000	1.62	3.06E-03	3.08E-03	2.16E-05	4.21E+00	2.34E+01
0.00025	0.0025	2.5E-06	4000	1.29	2.31E-03	2.31E-03	7.37E-06	2.21E+00	1.06E+01
0.0001	0.001	0.000001	10000	1.05	9.26E-04	9.25E-04	1.43E-06	4.49E-01	-5.14E+00
0.000025	0.00025	2.5E-07	40000	.978	2.34E-04	2.31E-04	2.97E-06	-1.90E-01	-4.27E+01
0.00002	0.0002	2E-07	50000	.925	1.88E-04	1.85E-04	3.11E-06	-6.78E-01	-5.60E+01
0.00001	0.0001	1E-07	100000	.522	9.56E-05	9.25E-05	3.06E-06	-5.65E+00	-1.10E+02
0.000005	0.00005	5E-08	200000	.148	4.83E-05	4.62E-05	2.10E-06	-1.66E+01	-1.52E+02
3.33E-06	3.33E-05	3.33E-08	300000	6.48E-2	3.25E-05	3.08E-05	1.62E-06	-2.38E+01	-1.75E+02

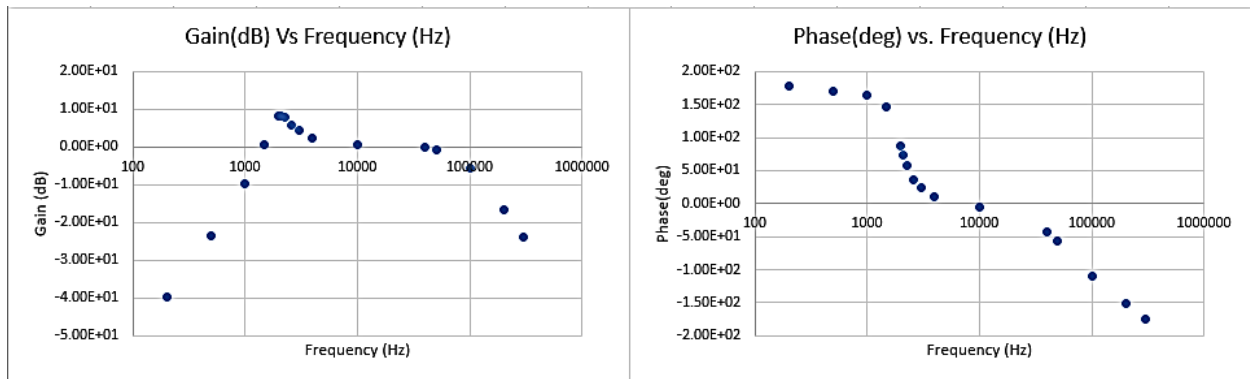


Figure 22 (a) and (b): Excel Graphs from Frequency, Gain and Phase Calculations from Table 3.

The plots generated in Fig. 18 and Fig. 20 compared to the plots in Fig. 22 basically show the same results. Then to find the step response of the circuit, Fig. 15 was made in LTSPICE with a input voltage rectangular pulse with 20 *ms* max time and step size .2 μ s. Fig. 24 is the resulting step response of Fig. 23, and when comparing to the step response in Fig. 18(b), the graphs also look the same. The behavior of the graph did show that it was underdamped in relation with the time domain because there is a lot of oscillation.

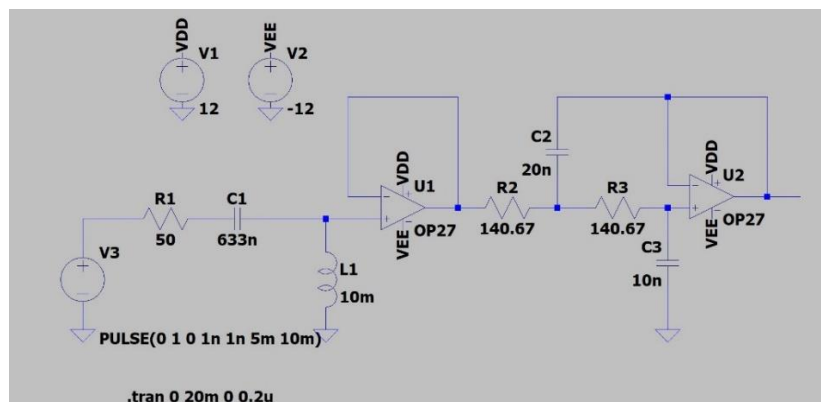


Figure 23: Fig. 19 Circuit with Input Pulse Voltage to determine Step Response.



Figure 24: Step response of Fig. 23 Circuit.

3.5 Conclusion

To conclude this experiment, the output voltage switched from leading the input voltage to lagging the input voltage, requiring the phase angle equation to be multiplied by -1. The significant error between the gain, frequency and gain values are due to multiple reasons. These being the peak gain effect, the op-amp limiting response, measurement errors, and the step size being too big.

Appendix

Appendix A

Matlab Simulation for Prelab Experiment 1.

```
%EE352_6_1 Simulating RLC circuit
%Step #1 enter the values
R=50;
L=0.01;
fn=2000;
wn=2*pi*fn;
C=1/(L*wn*wn)
zeta=R/(2*wn*L)
%Step #2 enter the coefficients of the numerator and
%the denominator of the
%Transfer function.
N=[1 0 0];
D=[1 R/L 1/(L*C)];
%Step #3 use the tf() command to define the transfer
function of the system
Sys=tf(N,D);
% Step #4 obtain the step response and bode plot of the
system
step(Sys)
grid on;
figure
bode(Sys)
grid on
```

Appendix B

Matlab Simulation for Prelab Experiment 2.

```
clear all
close all
clc
%EE352_6_1 Simulating RLC circuit
%Step #1 enter the values
R=140.67;
fn=80000;
wn=2*pi*fn;
C1=20e-9;
C2=10e-9;
zeta=1/sqrt(2);
%Step #2 enter the coefficients of the numerator and
%the denominator of the
%Transfer function.
N=[1/(R^2*C1*C2)];
D=[1 2/(R*C1) 1/(R^2*C1*C2)];
%Step #3 use the tf() command to define the transfer
function of the system
Sys=tf(N,D);
% Step #4 obtain the step response and bode plot of the
system
step(Sys)
grid on;
figure
bode(Sys)
grid on
```

Appendix C

Matlab Simulation for Prelab Experiment 3.

```
clc
close all
clear all
% Define System 1
N1=[1 0 0];
R1=50; L1=0.01; Cx=633E-9;
D1=[1 R1/L1 1/(L1*Cx)];
sys1=tf(N1,D1)
%Define System 2
C1=20E-9;
R=140.67
C2=10E-9
wn=1/(R*R*C1*C2);
N2=wn;
D2=[1 2/(R*C1) wn]
sys2=tf(N2,D2);
% Cascade the two systems
sys=series(sys1,sys2)
step(sys);
grid on;
figure
bode(sys)
grid on;
```

Appendix D

Lab Assignment 6: Transfer Function Analysis

Lab 6 Checklist:

Name: Sarah Raxk

☒ Lab title and introduction

- Lab title, your name, date, and lab partner.
- Brief introduction (two or three sentences) explaining the purpose of this lab.

☒ i. RLC Highpass Filter (40 pts total)

- ☒ 1. Diagram of circuit with values of R, L and C to obtain desired damping ratio and natural frequency.
- ☒ 2. Circuit transfer function as a ratio of polynomials in s with numerical coefficients.
- ☒ 3. Transfer function behavior at high and low frequencies.
- ☒ 4. Measured R, R_L , L, and C for circuit implementation.
- ☒ 5. Measured amplitude and phase responses (in graphical form).
- ☒ 6. Measured step response (presented in graphical form).
- ☒ 7. Comparison between measured frequency of oscillations in step response and natural frequency (include % difference, assessment of linearity of circuit)
- ☒ 8. Simulated frequency and step responses (MATLAB).

☒ ii. Second Order Active Butterworth Low Pass Filter (40 pts total)

- ☒ 1. Butterworth filter circuit diagram.
- ☒ 2. Designed circuit transfer function with numerical coefficients.
- ☒ 3. Theoretical resistance, capacitance values for your design.
- ☒ 4. Measured resistance, capacitance values
- ☒ 5. Measured amplitude and phase responses in graphical form.
- ☒ 6. Measured step response in graphical form.
- ☒ 7. Simulated frequency and step responses (MATLAB).
- ☒ 8. Comparison of theoretical and measured responses
 - Cutoff frequency
 - Passband (low frequency) gain
 - Slope of magnitude response above cutoff frequency
 - Step response

☒ iii. Wide-band Bandpass Filter (20 pts total)

- ☒ 1. Block diagram of circuit (similar to Fig. 3).
- ☒ 2. Measured amplitude response in graphical form.
- ☒ 3. Measured step response in graphical form.
- ☒ 4. Simulated (MATLAB) amplitude and step responses.
- ☒ 5. Compare measured and simulated responses:
 - Cutoff frequencies (low and high)
 - Step response oscillation frequency compared to both filters' natural frequencies.