Algo Tutorial

16 September

6.7. A subsequence is *palindromic* if it is the same whether read left to right or right to left. For instance, the sequence

$$A, C, G, T, G, T, C, A, A, A, A, T, C, G$$

has many palindromic subsequences, including A, C, G, C, A and A, A, A, A (on the other hand, the subsequence A, C, T is *not* palindromic). Devise an algorithm that takes a sequence $x[1 \ldots n]$ and returns the (length of the) longest palindromic subsequence. Its running time should be $O(n^2)$.

```
// Every single character is a palindrome of length 1
L(i, i) = 1 for all indexes i in given sequence

// IF first and last characters are not same
If (X[i] != X[j])  L(i, j) =  max{L(i + 1, j),L(i, j - 1)}

// If there are only 2 characters and both are same
Else if (j == i + 1) L(i, j) = 2

// If there are more than two characters, and first and last
// characters are same
Else L(i, j) =  L(i + 1, j - 1) + 2
```

Overlapping subproblems!

```
             L(0, 5)
           /        \
          /          \
      L(1,5)          L(0,4)
     /    \            /    \
    /      \          /      \
L(2,5)    L(1,4)  L(1,4)  L(0,3)
```

```
for (cl=2; cl<=n; cl++)
{
    for (i=0; i<n-cl+1; i++)
    {
        j = i+cl-1;
        if (str[i] == str[j] && cl == 2)
           L[i][j] = 2;
        else if (str[i] == str[j])
           L[i][j] = L[i+1][j-1] + 2;
        else
           L[i][j] = max(L[i][j-1], L[i+1][j]);
    }
}

return L[0][n-1];
```

15.3-3

Consider a variant of the matrix-chain multiplication problem in which the goal is to parenthesize the sequence of matrices so as to maximize, rather than minimize, the number of scalar multiplications. Does this problem exhibit optimal substructure?

YES!

This modification of the matrix-chain-multiplication problem does still exhibit the optimal substructure property. Suppose we split a maximal multiplication of $A_1, \ldots, A_n$ between A_k and A_{k+1} then, we must have a maximal cost multiplication on either side, otherwise we could substitute in for that side a more expensive multiplication of $A_1, \ldots, A_n$.

Try it later!

15.2-1

Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$.

Answer for reference

An optimal parenthesization of that sequence would be $(A_1A_2)((A_3A_4)(A_5A_6))$ which will require $5*50*6+3*12*5+5*10*3+3*5*6+5*3*6 = 1500+180+150+90+90 = 2010$.